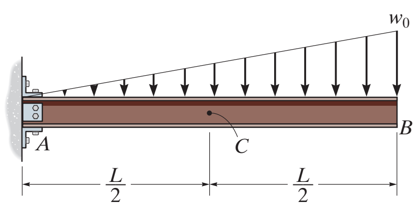
7–1. Determine the internal normal force, shear force, and moment at point C in the cantilever beam.



SOLUTION

The intensity of the triangular distributed loading at C can be computed using the similar triangles shown in Fig. a,

$$\frac{w_C}{L/2} = \frac{w_0}{L} \text{ or } w_C = w_0/2$$

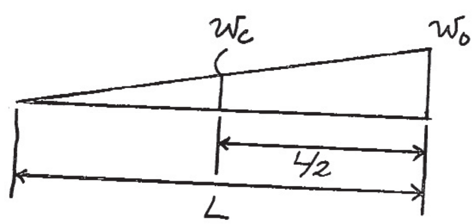
With reference to Fig. b,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_C = 0$ **Ans.**

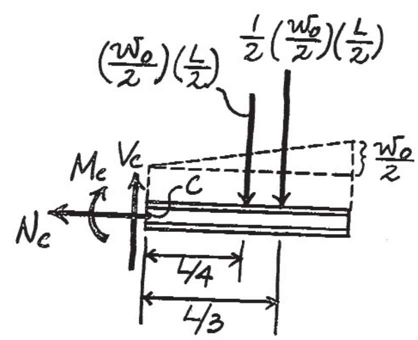
$+\uparrow \Sigma F_y = 0; \quad V_C - \left(\frac{w_0}{2}\right)\left(\frac{L}{2}\right) - \frac{1}{2}\left(\frac{w_0}{2}\right)\left(\frac{L}{2}\right) = 0 \qquad V_C = \frac{3w_0L}{8}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \quad -M_C - \left(\frac{w_0}{2}\right)\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) - \frac{1}{2}\left(\frac{w_0}{2}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) = 0 \quad M_C = -\frac{5}{48}w_0L^2$ **Ans.**

The negative sign indicates that $\mathbf{M}_C$ acts in the opposite sense to that shown on the free-body diagram.



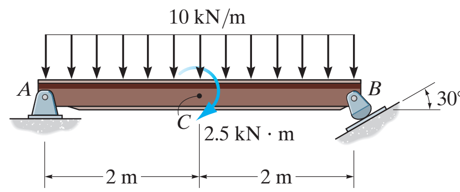
(a)



(b)

Ans:
$N_C = 0$
$V_C = \frac{3w_0L}{8}$
$M_C = -\frac{5}{48}w_0L^2$

7–2. Determine the internal normal force, shear force, and moment at point C in the simply supported beam. Point C is located just to the right of the 2.5-kN · m couple moment.



SOLUTION

Writing the moment equation of equilibrium about point A with reference to Fig. a,

$\circlearrowleft + \Sigma M_A = 0; \quad F_B \cos 30° (4) - 10(4)(2) - 2.5 = 0 \quad F_B = 23.816 \text{ kN}$

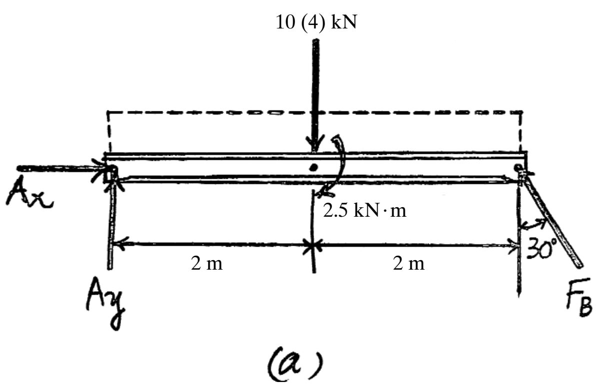
Using the result of $\mathbf{F}_B$ and referring to Fig. b,

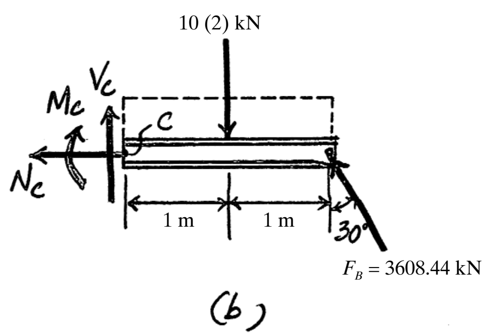
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -N_C - 23.816 \sin 30° = 0 \quad N_C = -11.908 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_C + 23.816 \cos 30° - 10(2) = 0 \quad V_C = -0.625 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \quad 23.816 \cos 30° (2) - 10(2)(1) - M_C = 0 \quad M_C = 21.25 \text{ kN}\cdot\text{m}$ **Ans.**

The negative sign indicates that $\mathbf{N}_C$ and $\mathbf{V}_C$ act in the opposite sense to that shown on the free – body diagram.

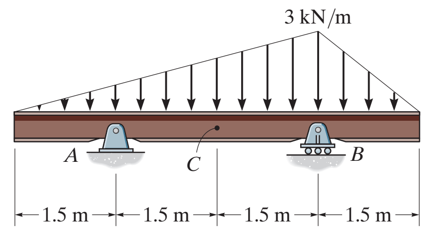




Ans:

$N_C = -11.908$ kN
$V_C = -0.625$ kN
$M_C = 21.25$ kN · m

7–3. Determine the internal normal force, shear force, and moment at point C in the double-overhang beam.



SOLUTION

The intensity of the triangular distributed loading at C can be computed using the similar triangles shown in Fig. b,

$$\frac{w_C}{3} = \frac{3}{4.5} \text{ or } w_C = 2 \text{ kN/m}$$

With reference to Fig. a,

$$\circlearrowleft +\Sigma M_B = 0; \quad \frac{1}{2}(3)(4.5)(1.5) - \frac{1}{2}(3)(1.5)(0.5) - A_y(3) = 0 \quad A_y = 3 \text{ kN}$$

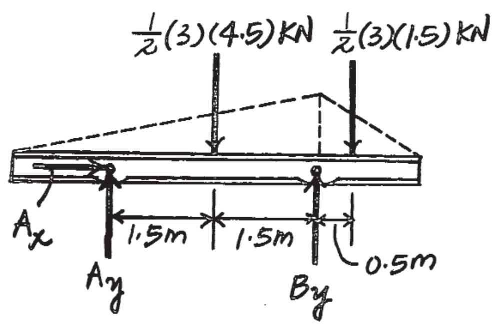
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x = 0$$

Using the results of $\mathbf{A}_x$ and $\mathbf{A}_y$ and referring to Fig. c,

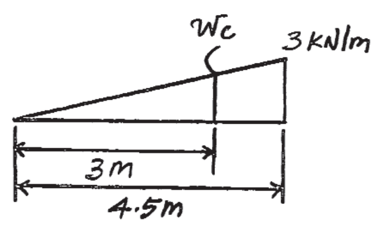
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_C = 0$$ **Ans.**

$$+\uparrow \Sigma F_y = 0; \quad 3 - \frac{1}{2}(2)(3) - V_C = 0 \quad V_C = 0$$ **Ans.**

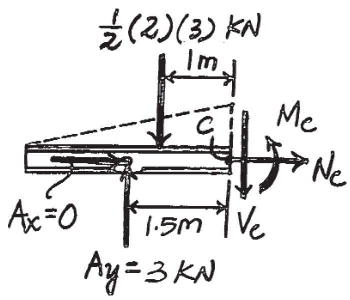
$$\circlearrowleft +\Sigma M_C = 0; \quad M_C + \frac{1}{2}(2)(3)(1) - 3(1.5) = 0 \quad M_C = 1.5 \text{ kN} \cdot \text{m}$$ **Ans.**



(a)



(b)

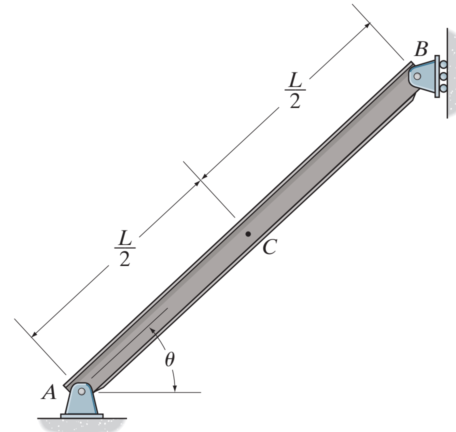


(c)

Ans:

$N_C = 0$
$V_C = 0$
$M_C = 1.5 \text{ kN} \cdot \text{m}$

***7–4.** The beam has a weight w per unit length. Determine the internal normal force, shear force, and moment at point C due to its weight.



SOLUTION

With reference to Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \quad B_x(L \sin\theta) - wL\cos\theta\left(\frac{L}{2}\right) = 0 \quad B_x = \frac{wL\cos\theta}{2\sin\theta}$$

Using this result and referring to Fig. *b*,

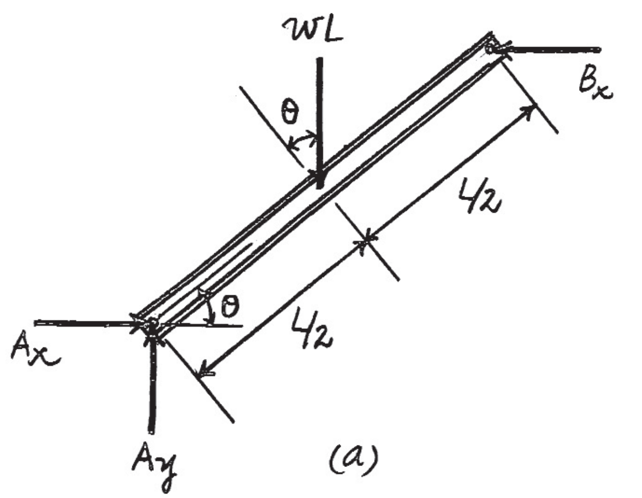
$$\xrightarrow{+} \Sigma F_{x'} = 0; \quad -N_C - \frac{wL\cos\theta}{2\sin\theta}(\cos\theta) - w\left(\frac{L}{2}\right)\sin\theta = 0 \quad N_C = -\frac{wL}{2}\csc\theta \qquad \textbf{Ans.}$$

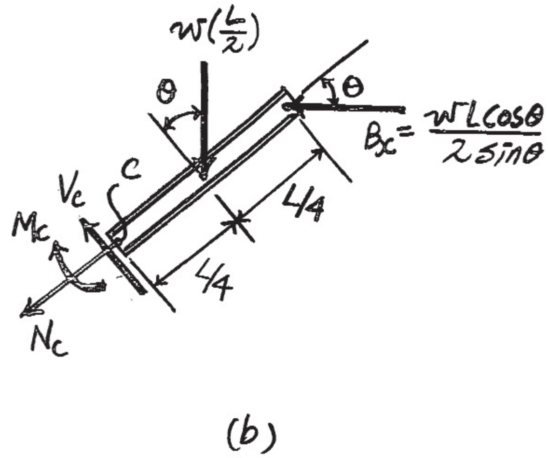
$$+\uparrow \Sigma F_{y'} = 0; \quad V_C - w\left(\frac{L}{2}\right)\cos\theta + \frac{wL\cos\theta}{2\sin\theta}\sin\theta = 0 \quad V_C = 0 \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_C = 0; \quad \frac{wL\cos\theta}{2\sin\theta}\left(\frac{L}{2}\sin\theta\right) - w\left(\frac{L}{2}\right)\cos\theta\left(\frac{L}{4}\right) - M_C = 0$$

$$M_C = \frac{wL^2}{8}\cos\theta \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{N}_C$ acts in the opposite sense to that shown on the free-body diagram

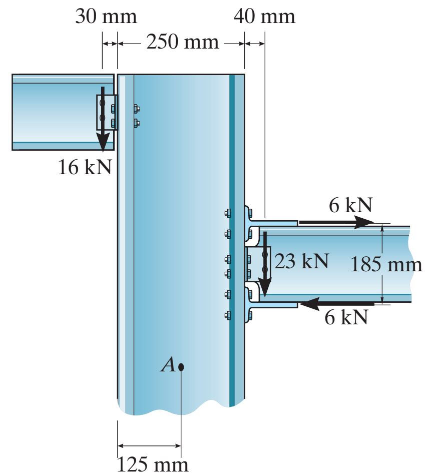




Ans:
$N_C = -\frac{wL}{2}\csc\theta$
$V_C = 0$
$M_C = \frac{wL^2}{8}\cos\theta$

7–5.

Two beams are attached to the column such that structural connections transmit the loads shown. Determine the internal normal force, shear force, and moment acting in the column at a section passing horizontally through point A.



SOLUTION

$\xrightarrow{+} \Sigma F_x = 0; \qquad 6 - 6 - V_A = 0$

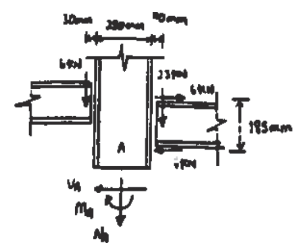
$V_A = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad -N_A - 16 - 23 = 0$

$N_A = -39 \text{ kN}$ **Ans.**

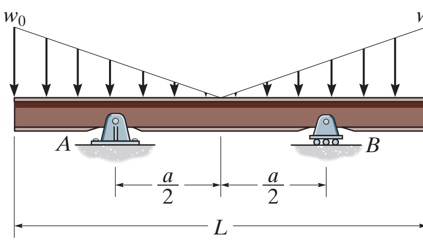
$\circlearrowleft + \Sigma M_A = 0; \qquad -M_A + 16(0.155) - 23(0.165) - 6(0.185) = 0$

$M_A = -2.42 \text{ kN} \cdot \text{m}$ **Ans.**



Ans:
$V_A = 0,$
$N_A = -39 \text{ kN}$
$M_A = -2.425 \text{ kN} \cdot \text{m}$

7–6. Determine the distance a in terms of the beam's length L between the symmetrically placed supports A and B so that the internal moment at the center of the beam is zero.



SOLUTION

In this problem, it is required that the internal moment at point C be equal to zero. With reference to Fig. a,

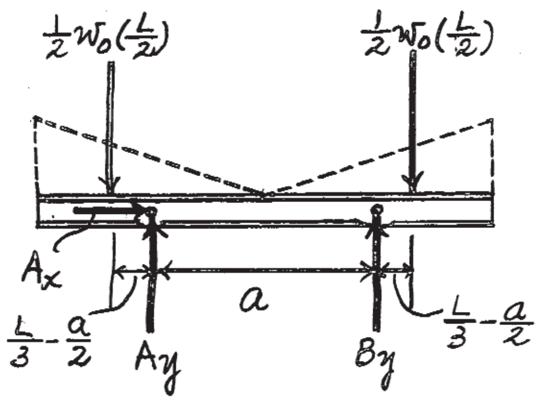
$$\circlearrowleft +\Sigma M_A = 0; \qquad B_y(a) - \frac{1}{2}w_0\left(\frac{L}{2}\right)\left[a + \left(\frac{L}{3} - \frac{a}{2}\right)\right] + \frac{1}{2}w_0\left(\frac{L}{2}\right)\left(\frac{L}{3} - \frac{a}{2}\right) = 0$$

$$B_y = \frac{1}{4}w_0 L$$

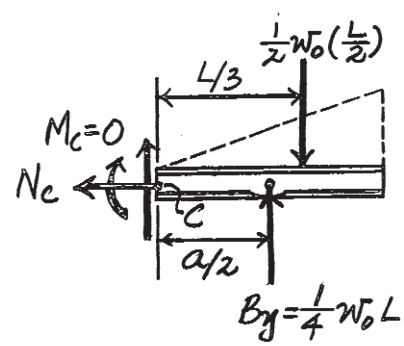
Using this result and referring to Fig. b,

$$\circlearrowleft +\Sigma M_C = 0; \qquad \frac{1}{4}w_0 L\left(\frac{a}{2}\right) - \frac{1}{2}w_0\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) = 0$$

$$a = \frac{2}{3}L$$ **Ans.**



(a)

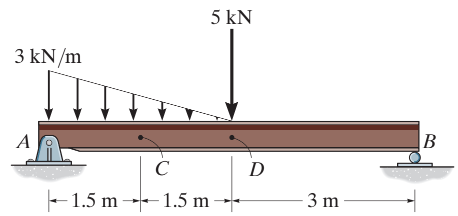


(b)

Ans:

$a = \frac{L}{3}$

7–7. Determine the internal normal force, shear force, and moment at points C and D in the simply supported beam. Point D is located just to the left of the 5-kN force.



SOLUTION

The intensity of the triangular distributed loading at C can be computed using the similar triangles shown in Fig. b,

$$\frac{w_C}{1.5} = \frac{3}{3} \text{ or } w_C = 1.5 \text{ kN/m}$$

With reference to Fig. a,

$\circlearrowleft + \Sigma M_A = 0; \quad B_y(6) - 5(3) - \frac{1}{2}(3)(3)(1) = 0 \qquad B_y = 3.25 \text{ kN}$

Using this result and referring to Fig. c,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_C + 3.25 - \frac{1}{2}(1.5)(1.5) - 5 = 0 \quad V_C = 2.875 \text{ kN}$ **Ans.**

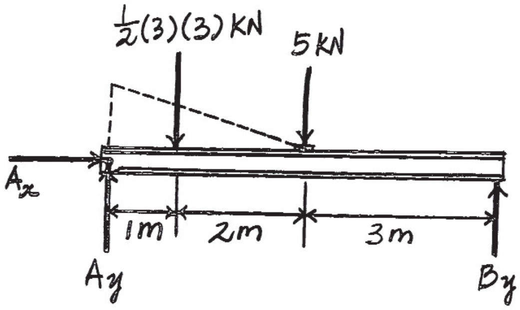
$\circlearrowleft + \Sigma M_C = 0; \quad 3.25(4.5) - \frac{1}{2}(1.5)(1.5)(0.5) - 5(1.5) - M_C = 0 \quad M_C = 6.56 \text{ kN} \cdot \text{m}$ **Ans.**

Also, referring to Fig. d,

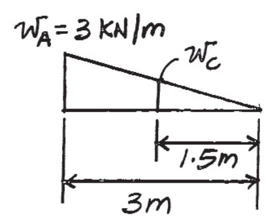
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_D + 3.25 - 5 = 0 \quad V_D = 1.75 \text{ kN}$ **Ans.**

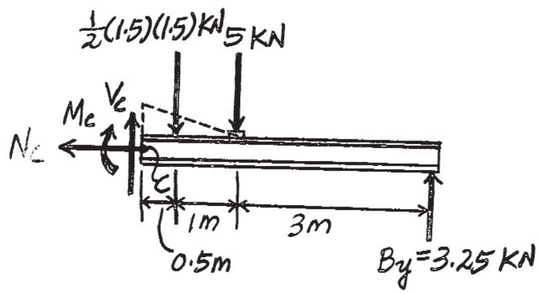
$\circlearrowleft + \Sigma M_D = 0; \quad 3.25(3) - M_D \quad M_D = 9.75 \text{ kN} \cdot \text{m}$ **Ans.**



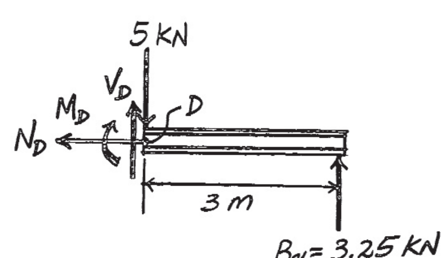
(a)



(b)



(c)



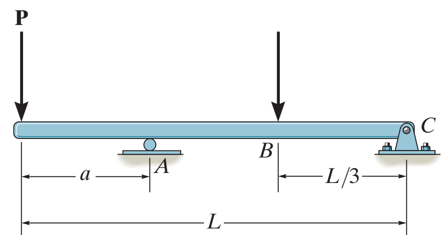
(d)

Ans:

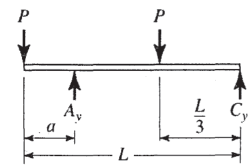
$N_C = 0$
$V_C = 2.875 \text{ kN}$
$M_C = 6.56 \text{ kN} \cdot \text{m}$
$N_D = 0$
$V_D = 1.75 \text{ kN}$
$M_D = 9.75 \text{ kN} \cdot \text{m}$

***7–8.**

Determine the distance a as a fraction of the beam's length L for locating the roller support so that the moment in the beam at B is zero.



SOLUTION



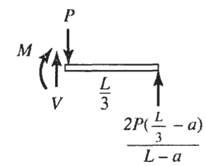
$\circlearrowleft + \Sigma M_A = 0; \quad -P\left(\frac{2L}{3} - a\right) + C_y(L - a) + Pa = 0$

$$C_y = \frac{2P\left(\frac{L}{3} - a\right)}{L - a}$$

$\circlearrowleft + \Sigma M = 0; \qquad M = \frac{2P\left(\frac{L}{3} - a\right)}{L - a}\left(\frac{L}{3}\right) = 0$

$$2PL\left(\frac{L}{3} - a\right) = 0$$

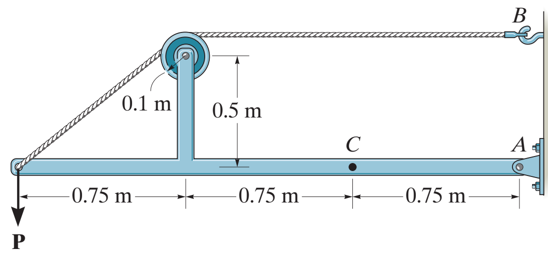
$$a = \frac{L}{3}$$ **Ans.**



Ans:
$a = \frac{L}{3}$

7–9.

Determine the normal force, shear force, and moment at a section passing through point C. Take $P = 8$ kN.



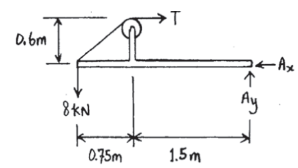
SOLUTION

$\zeta + \Sigma M_A = 0; \qquad -T(0.6) + 8(2.25) = 0$

$T = 30$ kN

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 30$ kN

$+\uparrow \Sigma F_y = 0; \qquad A_y = 8$ kN

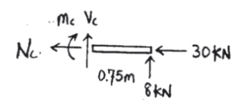


$\xrightarrow{+} \Sigma F_x = 0; \qquad -N_C - 30 = 0$

$N_C = -30$ kN **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_C + 8 = 0$

$V_C = -8$ kN **Ans.**



$\zeta + \Sigma M_C = 0; \qquad -M_C + 8(0.75) = 0$

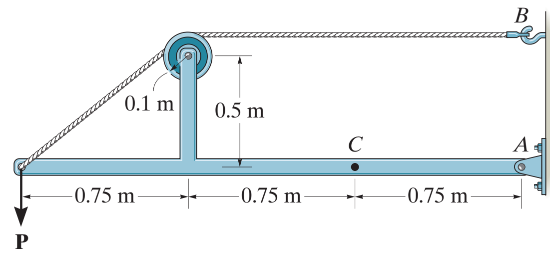
$M_C = 6\ \text{kN}\cdot\text{m}$ **Ans.**

Ans:

$N_C = -30$ kN
$V_C = -8$ kN
$M_C = 6\ \text{kN}\cdot\text{m}$

7–10.

The cable will fail when subjected to a tension of 2 kN. Determine the largest vertical load P the frame will support and calculate the internal normal force, shear force, and moment at a section passing through point C for this loading.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0;\qquad -2(0.6) + P(2.25) = 0$

$P = 0.533 \text{ kN}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;\qquad A_x = 2 \text{ kN}$

$+\uparrow \Sigma F_y = 0;\qquad A_y = 0.533 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0;\qquad -N_C - 2 = 0$

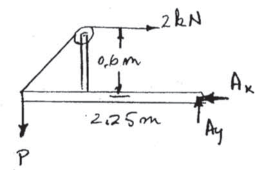
$N_C = -2 \text{ kN}$ **Ans.**

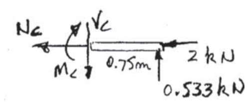
$+\uparrow \Sigma F_y = 0;\qquad V_C - 0.533 = 0$

$V_C = -0.533 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;\qquad -M_C + 0.533(0.75) = 0$

$M_C = 0.400 \text{ kN} \cdot \text{m}$ **Ans.**



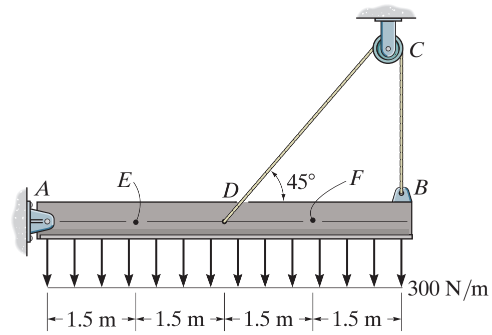


Ans:

$P = 0.533 \text{ kN}$
$N_C = -2 \text{ kN}$
$V_C = -0.533 \text{ kN}$
$M_C = 0.400 \text{ kN} \cdot \text{m}$

7–11.

Determine the internal normal force, shear force, and moment at points E and F in the beam.



SOLUTION

With reference to Fig. *a*,

$\zeta + \Sigma M_A = 0;\quad T(6) + T\sin 45°(3) - 300(6)(3) = 0 \qquad T = 664.92\text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 664.92\cos 45° - A_x = 0 \qquad A_x = 470.17\text{ N}$

$+\uparrow \Sigma F_y = 0;\quad A_y + 664.92\sin 45° + 664.92 - 300(6) = 0 \qquad A_y = 664.92\text{ N}$

Use these result and referring to Fig. *b*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad N_E - 470.17 = 0$

$N_E = 470\text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad 664.92 - 300(1.5) - V_E = 0$

$V_E = 215\text{ N}$ **Ans.**

$\zeta + \Sigma M_E = 0;\quad M_E + 300(1.5)(0.75) - 664.92(1.5) = 0$

$M_E = 660\text{ N}\cdot\text{m}$ **Ans.**

Also, by referring to Fig. *c*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad N_F = 0$ **Ans.**

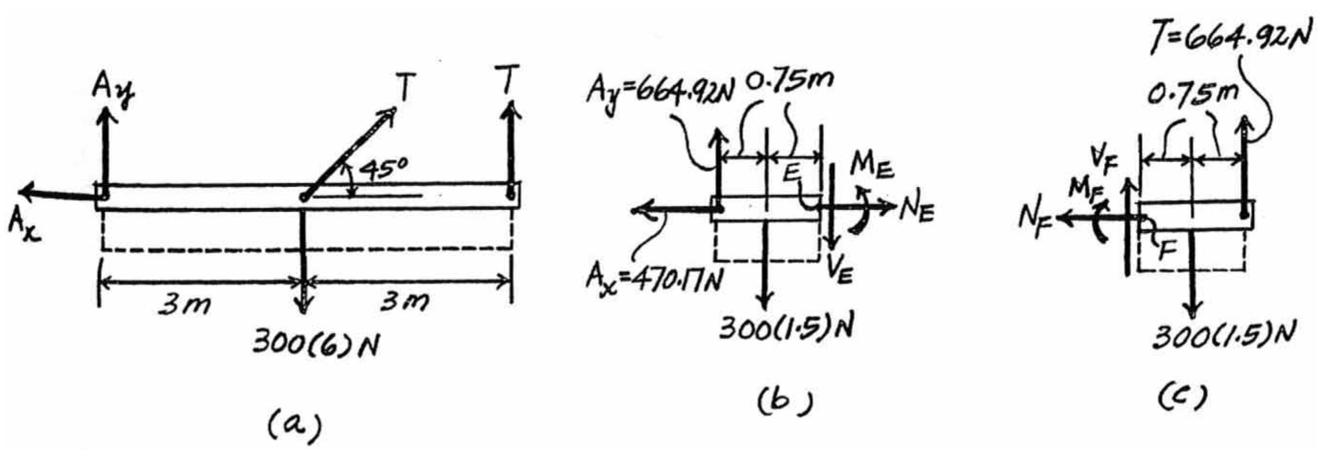
$+\uparrow \Sigma F_y = 0;\quad V_F + 664.92 - 300 = 0$

$V_F = -215\text{ N}$ **Ans.**

$\zeta + \Sigma M_F = 0;\quad 664.92(1.5) - 300(1.5)(0.75) - M_F = 0$

$M_F = 660\text{ N}\cdot\text{m}$ **Ans.**

The negative sign indicates that $\mathbf{V}_F$ acts in the opposite sense to that shown on the free-body diagram.

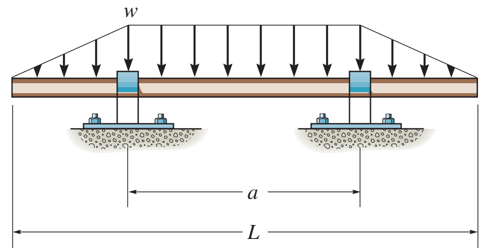


Ans:

$N_E = 470\text{ N}, V_E = 215\text{ N}$
$M_E = 660\text{ N}\cdot\text{m}, N_F = 0$
$V_F = -215\text{ N}, M_F = 660\text{ N}\cdot\text{m}$

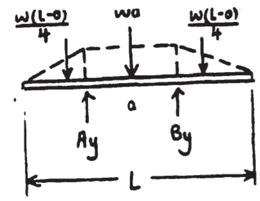
***7–12.**

Determine the distance a between the bearings in terms of the shaft's length L so that the moment in the *symmetric* shaft is zero at its center.



SOLUTION

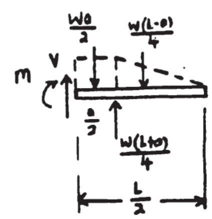
Due to symmetry, $A_y = B_y$



$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y - \frac{w(L-a)}{4} - wa - \frac{w(L-a)}{4} = 0$$

$$A_y = B_y = \frac{w}{4}(L + a)$$

$$\circlearrowleft + \Sigma M = 0; \qquad -M - \frac{wa}{2}\left(\frac{a}{4}\right) - \frac{w(La)}{4}\left(\frac{a}{2} + \frac{L}{6} - \frac{a}{6}\right) + \frac{w}{4}(L + a)\left(\frac{a}{2}\right) = 0$$



Since $M = 0$;

$$3a^2 + (L - a)(L + 2a) - 3a\,(L + a) = 0$$

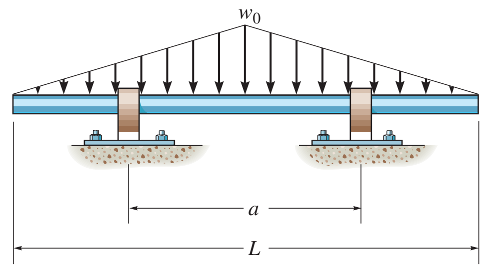
$$2a^2 + 2a\,L - L^2 = 0$$

$$a = 0.366\,L \qquad \textbf{Ans.}$$

Ans:
$a = 0.366\,L$

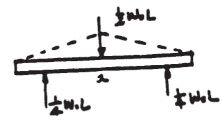
7–13.

Determine the distance a between the supports in terms of the shaft's length L so that the bending moment in the *symmetric* shaft is zero at the shaft's center. The intensity of the distributed load at the center of the shaft is w_0. The supports are journal bearings.



SOLUTION

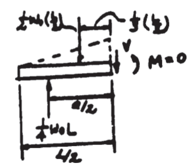
Support reactions: FBD(a)



Moments Function:

$$\zeta+\Sigma M = 0; \qquad 0 + \frac{1}{2}(w_0)\left(\frac{L}{2}\right)\left(\frac{1}{3}\right)\left(\frac{L}{2}\right) - \frac{1}{4}w_0 L\left(\frac{a}{2}\right) = 0$$

$$a = \frac{L}{3}$$ **Ans.**

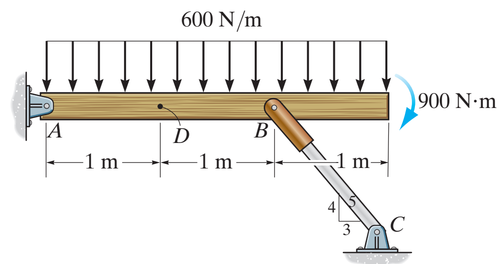


Ans:

$a = \dfrac{L}{3}$

7–14.

Determine the internal normal force, shear force, and moment at point D in the beam.



SOLUTION

Writing the equations of equilibrium with reference to Fig. *a,* we have

$\circlearrowleft + \Sigma M_A = 0; \quad F_{BC}\left(\frac{4}{5}\right)(2) - 600(3)(1.5) - 900 = 0 \quad F_{BC} = 2250\text{ N}$

$\circlearrowleft + \Sigma M_B = 0; \quad 600(3)(0.5) - 900 - A_y(2) = 0 \quad A_y = 0$

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 2250\left(\frac{2}{5}\right) = 0 \quad A_x = 1350\text{ N}$

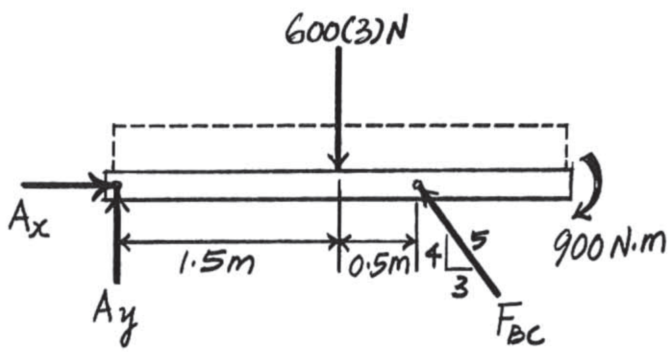
Using these results and referring to Fig. b, we have

$\xrightarrow{+} \Sigma F_x = 0; \quad N_D + 1350 = 0 \quad N_D = -1350\text{ N} = -1.35\text{ kN}$ **Ans.**

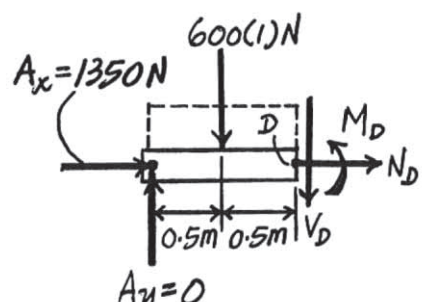
$+\uparrow \Sigma F_y = 0; \quad -V_D - 600(1) = 0 \quad V_D = -600\text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0; \quad M_D + 600(1)(0.5) = 0 \quad M_D = -300\text{ N}\cdot\text{m}$ **Ans.**

The negative sign indicates that $\mathbf{N}_D$, $\mathbf{V}_D$, and $\mathbf{M}_D$ act in the opposite sense to that shown on the free-body diagram.



(a)



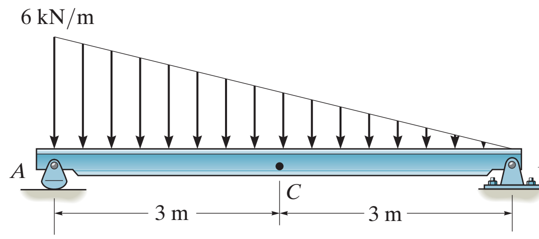
(b)

Ans:

$N_D = -1350\text{ N} = -1.35\text{ kN}$
$V_D = -600\text{ N}$
$M_D = -300\text{ N}\cdot\text{m}$

7–15.

Determine the internal normal force, shear force, and moment at point C.



SOLUTION

Support Reactions. Referring to the *FBD* of the entire beam shown in Fig. *a*,

$$\zeta+\Sigma M_A = 0; \qquad B_y(6) - \frac{1}{2}(6)(6)(2) = 0 \qquad B_y = 6.00\text{ kN}$$

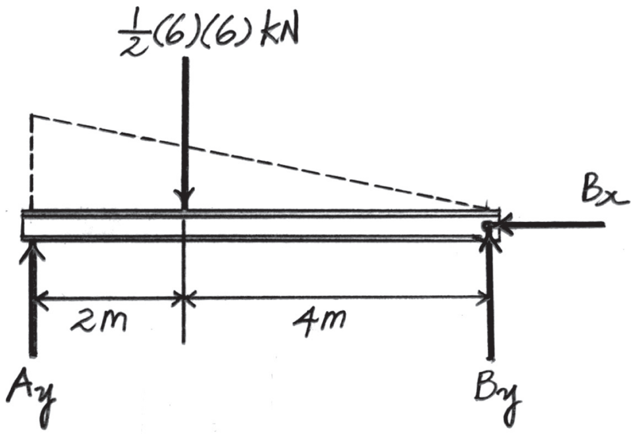
$$\xrightarrow{+}\Sigma F_x = 0; \qquad B_x = 0$$

Internal Loadings. Referring to the *FBD* of right segment of the beam sectioned through C, Fig. *b*

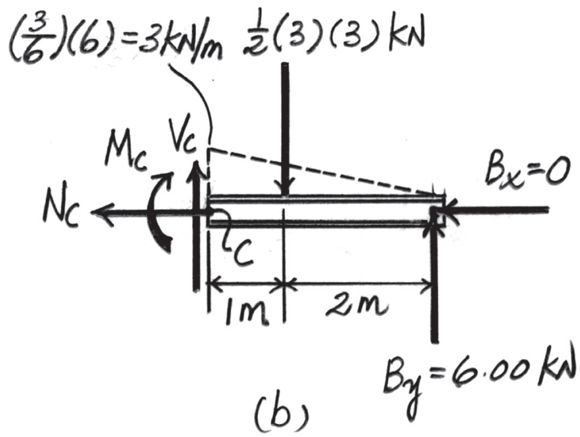
$$\xrightarrow{+}\Sigma F_x = 0; \qquad N_C = 0 \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \qquad V_C + 6.00 - \frac{1}{2}(3)(3) = 0 \qquad V_C = -1.50\text{ kN} \qquad \textbf{Ans.}$$

$$\zeta+\Sigma M_C = 0; \quad 6.00(3) - \frac{1}{2}(3)(3)(1) - M_C = 0 \quad M_C = 13.5\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

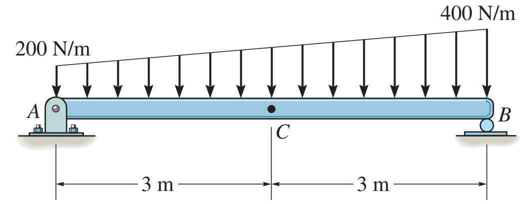


(b)

Ans:
$N_C = 0$
$V_C = -1.50\text{ kN}$
$M_C = 13.5\text{ kN}\cdot\text{m}$

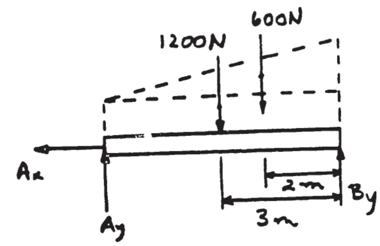
***7–16.**

Determine the internal normal force, shear force, and moment at point C of the beam.



SOLUTION

Beam:

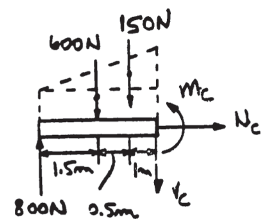


$\circlearrowleft + \Sigma M_B = 0;\qquad 600\,(2) + 1200\,(3) - A_y\,(6) = 0$

$A_y = 800\text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad A_x = 0$

Segment AC:



$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad 800 - 600 - 150 - V_C = 0$

$V_C = 50\text{ N}$ **Ans.**

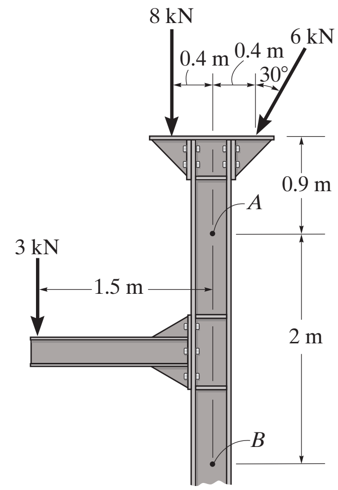
$\circlearrowleft + \Sigma M_C = 0;\qquad -800\,(3) + 600\,(1.5) + 150\,(1) + M_C = 0$

$M_C = 1350\text{ N}\cdot\text{m} = 1.35\text{ kN}\cdot\text{m}$ **Ans.**

Ans:
$N_C = 0$
$V = 50\text{ N}$
$M_C = 1.35\text{ kN}\cdot\text{m}$

7–17.

Determine the internal normal force, shear force, and moment at points A and B in the column.



SOLUTION

Applying the equation of equilibrium to Fig. *a* gives

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad V_A - 6 \sin 30° = 0 \quad V_A = 3 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad N_A - 6 \cos 30° - 8 = 0 \quad N_A = 13.2 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 8(0.4) + 6 \sin 30°(0.9) - 6 \cos 30°(0.4) - M_A = 0$

$M_A = 3.82 \text{ kN} \cdot \text{m}$ **Ans.**

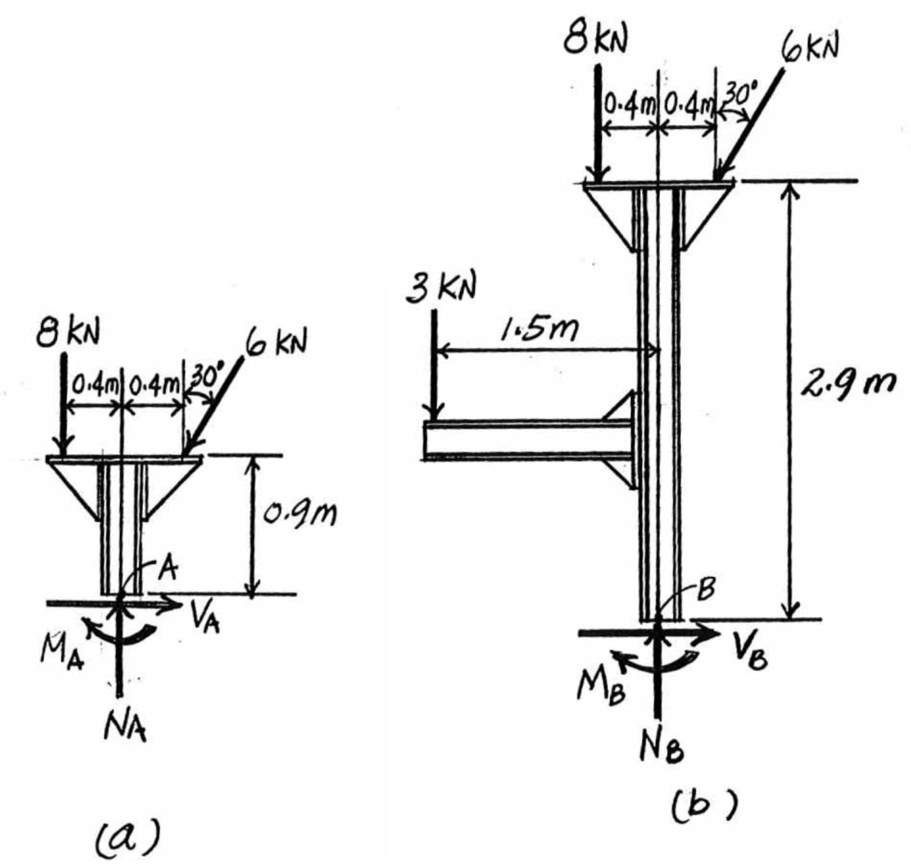
and to Fig. *b*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad V_B - 6 \sin 30° = 0 \quad V_B = 3 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad N_B - 3 - 8 - 6 \cos 30° = 0 \quad N_B = 16.2 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_B = 0;$ $\quad 3(1.5) + 8(0.4) + 6 \sin 30°(2.9) - 6 \cos 30°(0.4) - M_B = 0$

$M_B = 14.3 \text{ kN} \cdot \text{m}$ **Ans.**

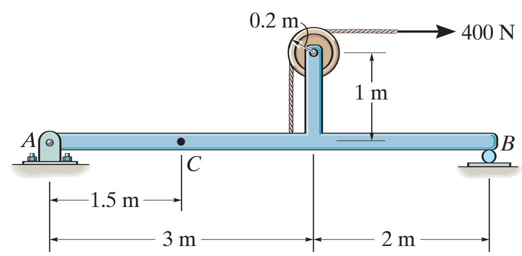


Ans:

$V_A = 3 \text{ kN}, N_A = 13.2 \text{ kN}, M_A = 3.82 \text{ kN} \cdot \text{m}$
$V_B = 3 \text{ kN}, N_B = 16.2 \text{ kN}, M_B = 14.3 \text{ kN} \cdot \text{m}$

7–18.

Determine the internal normal force, shear force, and moment at point C.



SOLUTION

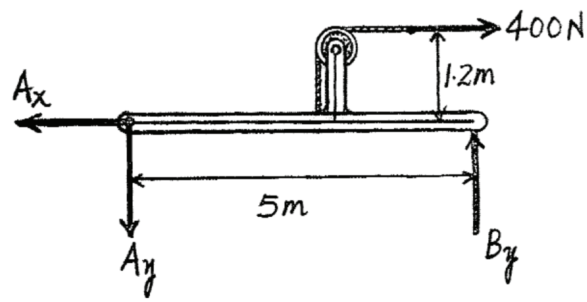
Beam:

$\xrightarrow{+} \Sigma F_x = 0; \qquad -A_x + 400 = 0$

$A_x = 400 \text{ N}$

$\zeta + \Sigma M_B = 0; \qquad A_y(5) - 400(1.2) = 0$

$A_y = 96 \text{ N}$



Segment *AC*:

$\xrightarrow{+} \Sigma F_x = 0; \qquad N_C - 400 = 0$

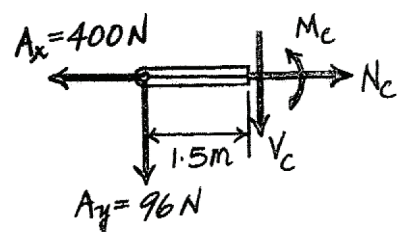
$N_C = 400 \text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad -96 - V_C = 0$

$V_C = -96 \text{ N}$ **Ans.**

$\zeta + \Sigma M_C = 0; \qquad M_C + 96(1.5) = 0$

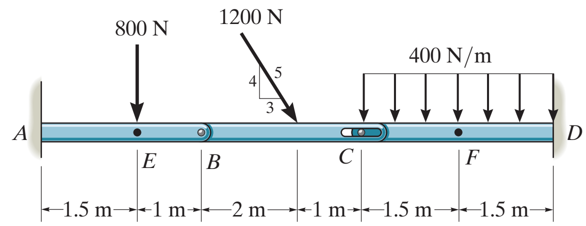
$M_C = -144 \text{ N} \cdot \text{m}$ **Ans.**



Ans:
$N_C = 400 \text{ N}$
$V_C = -96 \text{ N}$
$M_C = -144 \text{ N} \cdot \text{m}$

7–19.

Determine the internal normal force, shear force, and moment at points E and F of the compound beam. Point E is located just to the left of 800 N force.



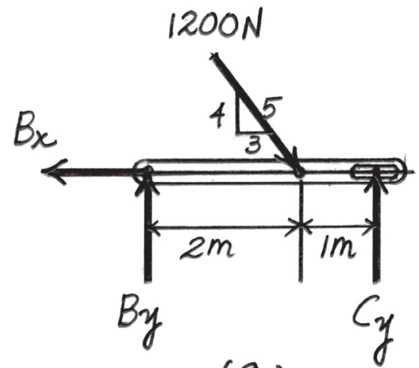
SOLUTION

Support Reactions. Referring to the *FBD* of member *BC* shown in Fig. *a*,

$$\zeta+\Sigma M_B = 0; \quad C_y(3) - 1200\left(\frac{4}{5}\right)(2) = 0 \quad C_y = 640 \text{ N}$$

$$\zeta+\Sigma M_C = 0; \quad 1200\left(\frac{4}{5}\right)(1) - B_y(3) = 0 \quad B_y = 320 \text{ N}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 1200\left(\frac{3}{5}\right) - B_x = 0 \quad B_x = 720 \text{ N}$$



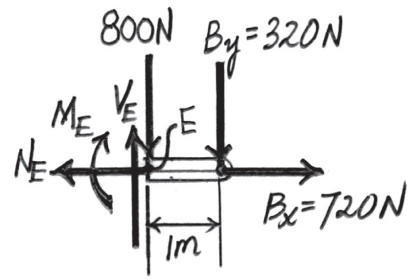
(a)

Internal Loadings. Referring to the right segment of member *AB* sectioned through *E*, Fig. *b*

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 720 - N_E = 0 \quad N_E = 720 \text{ N} \quad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \quad V_E - 800 - 320 = 0 \quad V_E = 1120 \text{ N} = 1.12 \text{ kN} \quad \textbf{Ans.}$$

$$\zeta+\Sigma M_E = 0; \quad -M_E - 320(1) = 0 \quad M_E = -320 \text{ N}\cdot\text{m} \quad \textbf{Ans.}$$



(b)

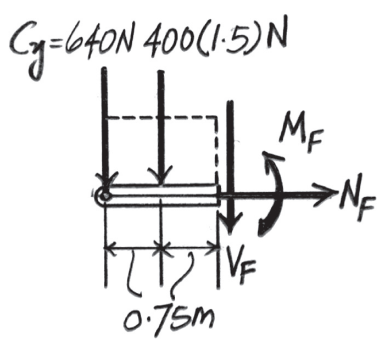
Referring to the left segment of member *CD* sectioned through *F*, Fig. *c*,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad N_F = 0 \quad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \quad -V_F - 640 - 400(1.5) = 0 \quad V_F = -1240 \text{ N} = -1.24 \text{ kN} \quad \textbf{Ans.}$$

$$\zeta+\Sigma M_F = 0; \quad M_F + 400(1.5)(0.75) + 640(1.5) = 0$$

$$M_F = -1410 \text{ N}\cdot\text{m} = -1.41 \text{ kN}\cdot\text{m} \quad \textbf{Ans.}$$

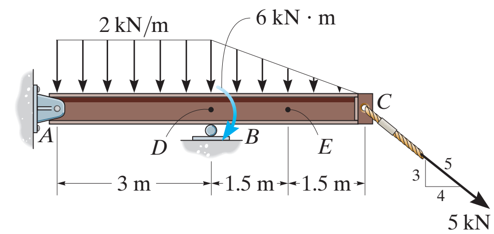


(c)

Ans:
$N_E = 720$ N
$V_E = 1.12$ kN
$M_E = -320$ N·m
$N_F = 0$
$V_F = -1.24$ kN
$M_F = -1.41$ kN·m

***7–20.**

Determine the internal normal force, shear force, and moment at points D and E in the overhang beam. Point D is located just to the left of the roller support at B, where the couple moment acts.



SOLUTION

The intensity of the triangular distributed load at E can be found using the similar triangles in Fig. b. With reference to Fig. a,

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(3) - 2(3)(1.5) - 6 - \frac{1}{2}(2)(3)(4) - 5\left(\frac{3}{5}\right)(6) = 0$$

$$B_y = 15 \text{ kN}$$

Using this result and referring to Fig. c,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 5\left(\frac{4}{5}\right) - N_D = 0 \qquad N_D = 4 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad V_D + 15 - \frac{1}{2}(2)(3) - 5\left(\frac{3}{5}\right) = 0 \qquad V_D = -9 \text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_D = 0; \qquad -M_D - 6 - \frac{1}{2}(2)(3)(1) - 5\left(\frac{3}{5}\right)(3) = 0 \qquad M_D = -18 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

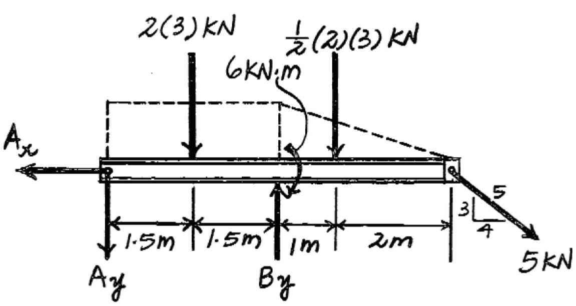
Also, by referring to Fig. d, we can write

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 5\left(\frac{4}{5}\right) - N_E = 0 \qquad N_E = 4 \text{ kN} \qquad \textbf{Ans.}$$

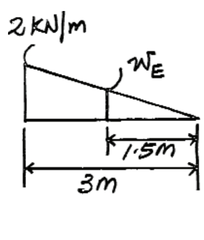
$$+\uparrow \Sigma F_y = 0; \qquad V_E - \frac{1}{2}(1)(1.5) - 5\left(\frac{3}{5}\right) = 0 \qquad V_E = 3.75 \text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_E = 0; \qquad -M_E - \frac{1}{2}(1)(1.5)(0.5) - 5\left(\frac{3}{5}\right)(1.5) = 0 \qquad M_E = -4.875 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

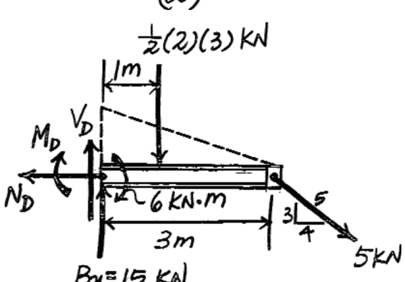
The negative sign indicates that $\mathbf{V}_D$, $\mathbf{M}_D$, and $\mathbf{M}_E$ act in the opposite sense to that shown on the free-body diagram.



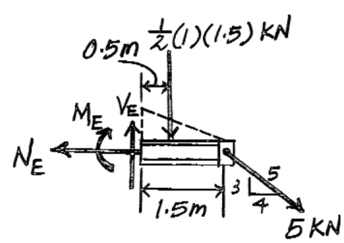
(a)



(b)



(c)

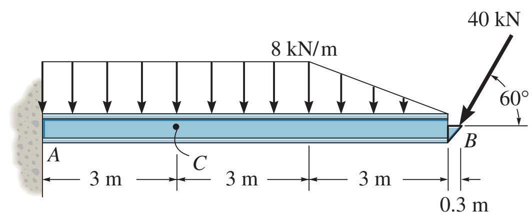


(d)

Ans:
$N_D = 4$ kN
$V_D = -9$ kN
$M_D = -18$ kN · m
$N_E = 4$ kN
$V_E = 3.75$ kN
$M_E = -4.875$ kN · m

7–21.

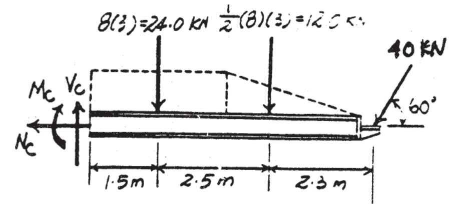
Determine the internal normal force, shear force, and bending moment at point C.



SOLUTION

Free body Diagram: The support reactions at A need not be computed.

Internal Forces: Applying equations of equilibrium to segment BC, we have



$\xrightarrow{+} \Sigma F_x = 0; \qquad -40\cos 60° - N_C = 0 \qquad N_C = -20.0\text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_C - 24.0 - 12.0 - 40\sin 60° = 0$

$V_C = 70.6\text{ kN}$ **Ans.**

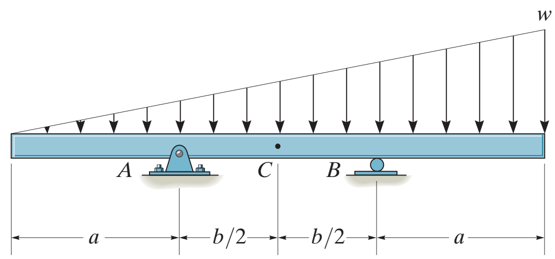
$\circlearrowleft + \Sigma M_C = 0; \qquad -24.0(1.5) - 12.0(4) - 40\sin 60°(6.3) - M_C = 0$

$M_C = -302\text{ kN}\cdot\text{m}$ **Ans.**

Ans:
$N_C = -20.0\text{ kN}$
$V_C = 70.6\text{ kN}$
$M_C = -302\text{ kN}\cdot\text{m}$

7–22.

Determine the ratio of a/b for which the shear force will be zero at the midpoint C of the beam.



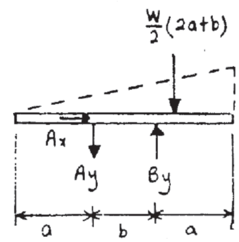
SOLUTION

$\zeta + \Sigma M_B = 0;$ $$-\frac{w}{2}(2a+b)\left[\frac{2}{3}(2a+b)-(a+b)\right]+A_y(b)=0$$

$$A_y = \frac{w}{6b}(2a+b)(a-b)$$

$\xrightarrow{+} \Sigma F_x = 0;$ $$A_x = 0$$

$+\uparrow \Sigma F_y = 0;$ $$-\frac{w}{6b}(2a+b)(a-b)-\frac{w}{4}\left(a+\frac{b}{2}\right)-V_C=0$$



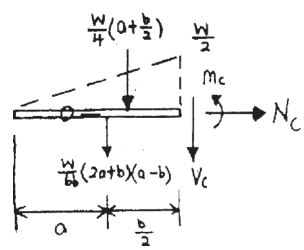
Since $V_C = 0$,

$$-\frac{1}{6b}(2a+b)(a-b)=\frac{1}{4}(2a+b)\left(\frac{1}{2}\right)$$

$$-\frac{1}{6b}(a-b)=\frac{1}{8}$$

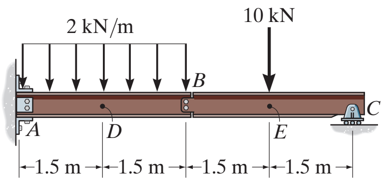
$$-a+b=\frac{3}{4}b$$

$$\frac{a}{b}=\frac{1}{4}$$ **Ans.**



Ans:
$\frac{a}{b}=\frac{1}{4}$

7–23. Determine the internal normal force, shear force, and moment at points D and E in the compound beam. Point E is located just to the left of the 10-kN concentrated load. Assume the support at A is fixed and the connection at B is a pin.



SOLUTION

With reference to Fig. b,

$\xrightarrow{+} \Sigma F_x = 0; \quad B_x = 0$

$\circlearrowleft + \Sigma M_B = 0; \quad C_y(3) - 10(1.5) = 0 \quad C_y = 5 \text{ kN}$

$\circlearrowleft + \Sigma M_C = 0; \quad 10(1.5) - B_y(3) = 0 \quad B_y = 5 \text{ kN}$

Using these results and referring to Fig. c,

$\xrightarrow{+} \Sigma F_x = 0; \quad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_D - 2(1.5) - 5 = 0 \quad V_D = 8 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0; \quad -M_D - 2(1.5)(0.75) - 5(1.5) = 0 \quad M_D = -9.75 \text{ kN}$ **Ans.**

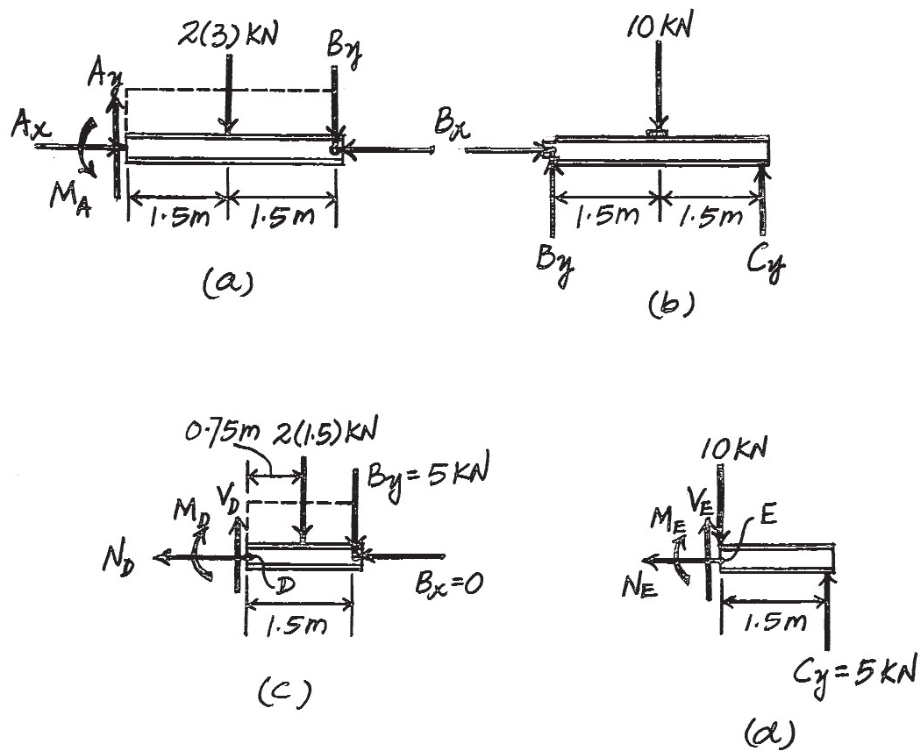
Also, by referring to Fig. d,

$\xrightarrow{+} \Sigma F_x = 0; \quad N_E = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_E - 10 + 5 = 0 \quad V_E = 5 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0; \quad 5(1.5) - M_E = 0 \quad M_E = 7.5 \text{ kN} \cdot \text{m}$ **Ans.**

The negative sign indicates that $\mathbf{M}_P$ acts in the opposite sense to that shown in the free-body diagram.

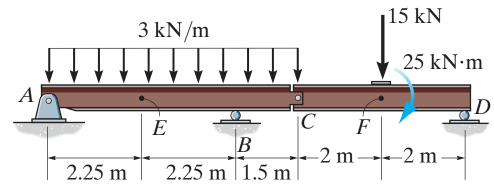


Ans:

$N_D = 0$, $V_D = 8$ kN, $M_D = -9.75$ kN,
$N_E = 0$, $V_E = 5$ kN, $M_E = 7.5$ kN $\cdot$ m

***7–24.**

Determine the internal normal force, shear force, and moment at points E and F in the compound beam. Point F is located just to the left of the 15-kN force and 25-kN·m couple moment.



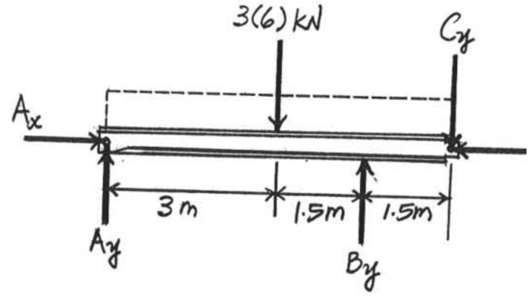
SOLUTION

With reference to Fig. b, we have

$\xrightarrow{+} \Sigma F_x = 0;\quad C_x = 0$

$\circlearrowleft + \Sigma M_C = 0;\quad D_y(4) - 15(2) - 25 = 0 \qquad D_y = 13.75\text{ kN}$

$\circlearrowleft + \Sigma M_D = 0;\quad 15(2) - 25 - C_y(4) = 0 \qquad C_y = 1.25\text{ kN}$



(a)

Using these results and referring to Fig. a, we have

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x = 0$

$\circlearrowleft + \Sigma M_B = 0;\quad 3(6)(1.5) - 1.25(1.5) - A_y(4.5) = 0 \qquad A_y = 5.583\text{ kN}$

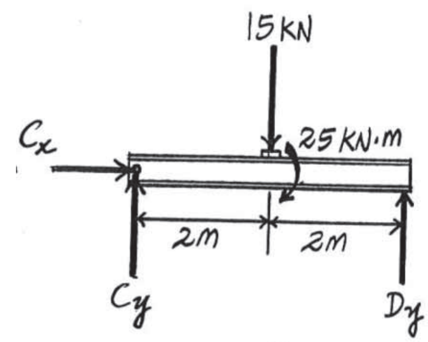
With these results and referring to Fig. c,

$\xrightarrow{+} \Sigma F_x = 0;\quad N_E = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad 5.583 - 3(2.25) - V_E = 0 \qquad V_E = -1.17\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0;\quad M_E + 3(2.25) - 3(2.25)(8.125) = 0$

$M_E = 4.97\text{ kN}\cdot\text{m}$ **Ans.**



(b)

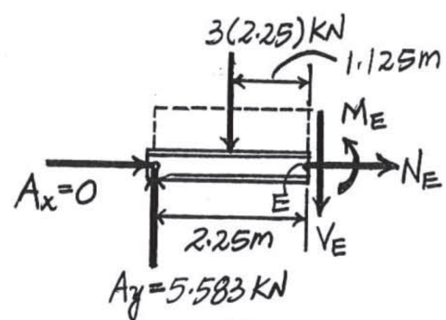
Also, using the result of D_y referring to Fig. d, we have

$\xrightarrow{+} \Sigma F_x = 0;\quad N_F = 0$ **Ans.**

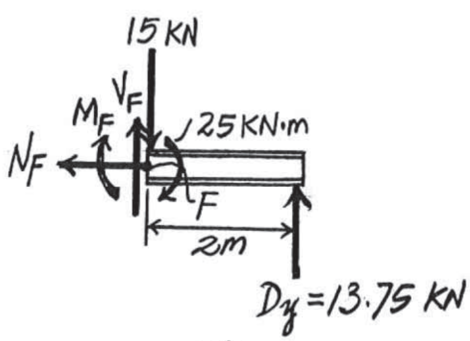
$+\uparrow \Sigma F_y = 0;\quad V_F - 15 + 13.75 = 0 \qquad V_F = 1.25\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_F = 0;\quad 13.75(2) - 25 - M_F = 0 \qquad M_F = 2.5\text{ kN}\cdot\text{m}$ **Ans.**

The negative sign indicates that $\mathbf{V}_E$ acts in the opposite sense to that shown in the free-body diagram.



(C)



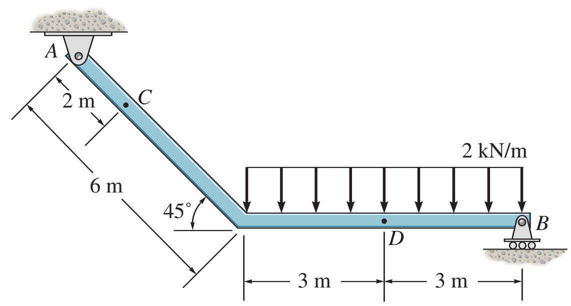
(d)

Ans:

$N_E = 0$
$V_E = -1.17\text{ kN}$
$M_E = 4.97\text{ kN}\cdot\text{m}$
$N_F = 0$
$V_F = 1.25\text{ kN}$
$M_F = 2.5\text{ kN}\cdot\text{m}$

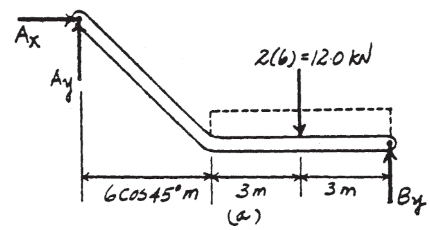
7–25.

Determine the internal normal force, shear force, and the moment at points C and D.



SOLUTION

Support Reactions: FBD (a).



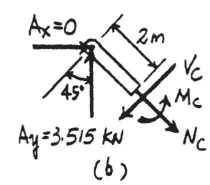
$\circlearrowleft + \Sigma M_A = 0;\qquad B_y(6 + 6\cos 45°) - 12.0(3 + 6\cos 45°) = 0$

$$B_y = 8.485\text{ kN}$$

$+\uparrow \Sigma F_y = 0;\qquad A_y + 8.485 - 12.0 = 0\qquad A_y = 3.515\text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0\qquad A_x = 0$

Internal Forces: Applying the equations of equilibrium to segment AC [FBD (b)], we have



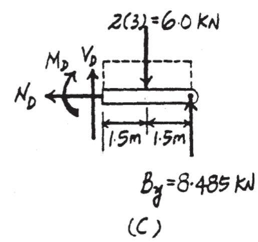
$\nearrow + \Sigma F_{x'} = 0;\qquad 3.515\cos 45° - V_C = 0\qquad V_C = 2.49\text{ kN}$ **Ans.**

$\nwarrow + \Sigma F_{y'} = 0;\qquad 3.515\sin 45° - N_C = 0\qquad N_C = 2.49\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;\qquad M_C - 3.515\cos 45°(2) = 0$

$M_C = 4.97\text{ kN}\cdot\text{m}$ **Ans.**

Applying the equations of equilibrium to segment BD [FBD (c)], we have



$\xrightarrow{+} \Sigma F_x = 0;\qquad N_D = 0$ **Ans.**

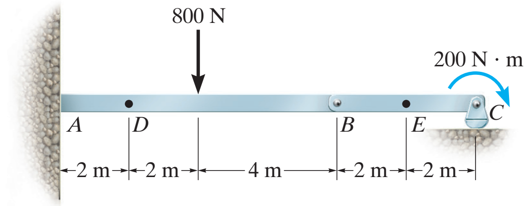
$+\uparrow \Sigma F_y = 0;\qquad V_D + 8.485 - 6.00 = 0\qquad V_D = -2.49\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0;\qquad 8.485(3) - 6(1.5) - M_D = 0$

$M_D = 16.5\text{ kN}\cdot\text{m}$ **Ans.**

Ans:
$V_C = 2.49\text{ kN}$
$N_C = 2.49\text{ kN}$
$M_C = 4.97\text{ kN}\cdot\text{m}$
$N_D = 0$
$V_D = -2.49\text{ kN}$
$M_D = 16.5\text{ kN}\cdot\text{m}$

7–26. Determine the internal normal force, shear force, and moment at points E and D of the compound beam.



SOLUTION

Given:

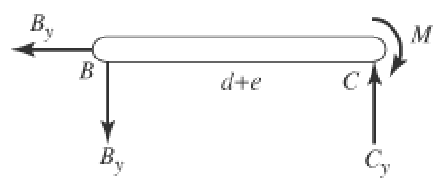
$M = 200 \text{ N·m}$ $\quad c = 4 \text{ m}$

$F = 800 \text{ N}$ $\quad d = 2 \text{ m}$

$a = 2 \text{ m}$ $\quad e = 2 \text{ m}$

$b = 2 \text{ m}$

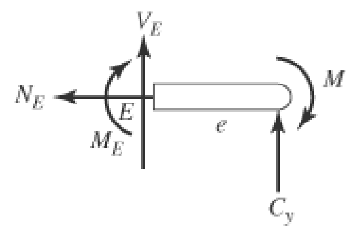
Segment BC :



$-M + C_y(d + e) = 0 \qquad C_y = \dfrac{M}{d + e}$

$-B_y + C_y = 0 \qquad B_y = C_y$

Segment EC :

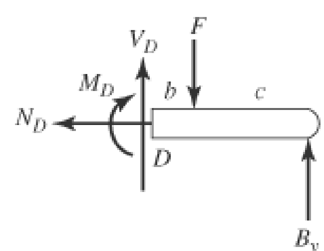


$-N_E = 0 \qquad N_E = 0\text{N} \qquad N_E = 0.00$ **Ans.**

$V_E + C_y = 0 \qquad V_E = -C_y \qquad V_E = -50.00 \text{ N}$ **Ans.**

$-M_E - M + C_y e = 0 \qquad M_E = C_y e - M \qquad M_E = -100.00 \text{N·m}$ **Ans.**

Segment DB :



$-N_D = 0 \qquad N_D = 0\text{N} \qquad N_D = 0.00$ **Ans.**

$V_D - F + B_y = 0 \qquad V_D = F - B_y \qquad V_D = 750.00 \text{ N}$ **Ans.**

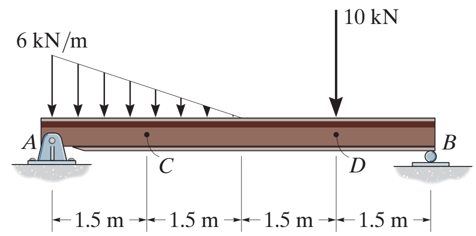
$-M_D - F b + B_y(b + c) = 0$

$M_D = -F b + B_y(b + c) \qquad M_D = -1300 \text{N·m}$ **Ans.**

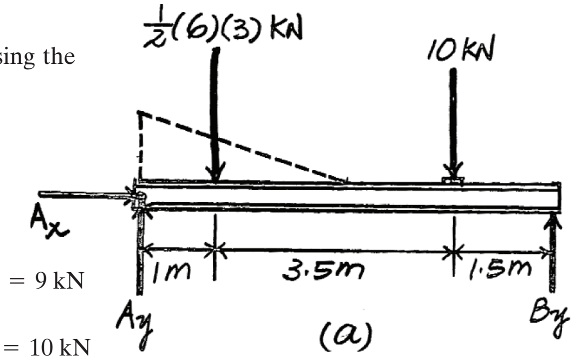
Ans:
$N_E = 0, V_E = -50 \text{ N}, M_E = -100 \text{ N·m}$
$N_D = 0, V_D = 750 \text{ N}, M_D = -1300 \text{ N·m}$

7–27.

Determine the internal normal force, shear force, and moment at points C and D in the simply supported beam. Point D is located just to the left of the 10-kN concentrated load.



SOLUTION



The intensity of the triangular distributed loading at C can be computed using the similar triangles shown in Fig. b,

$$\frac{w_C}{1.5} = \frac{6}{3} \text{ or } w_C = 3 \text{ kN/m}$$

With reference to Fig. a,

$\zeta + \Sigma M_A = 0; \quad B_y(6) - 10(4.5) - \frac{1}{2}(6)(3)(1) = 0 \quad B_y = 9 \text{ kN}$

$\zeta + \Sigma M_B = 0; \quad \frac{1}{2}(6)(3)(5) + 10(1.5) - A_y(6) = 0 \quad A_y = 10 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad A_x = 0$

Using these results and referring to Fig. c,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad 10 - \frac{1}{2}(3)(1.5) - 3(1.5) - V_C = 0 \quad V_C = 3.25 \text{ kN}$ **Ans.**

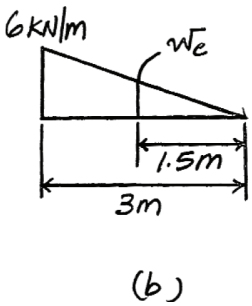
$\zeta + \Sigma M_C = 0; \quad M_C + 3(1.5)(0.75) + \frac{1}{2}(3)(1.5)(1) - 10(1.5) = 0 \quad M_C = 9.375 \text{ kN} \cdot \text{m}$ **Ans.**

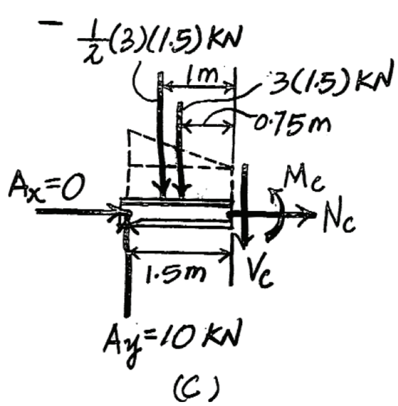
Also, by referring to Fig. d,

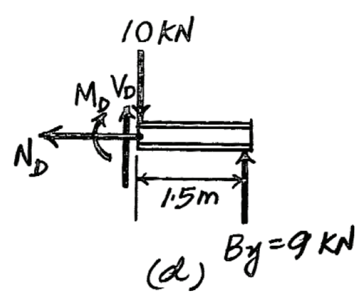
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_D + 9 - 10 = 0 \quad V_D = 1 \text{ kN}$ **Ans.**

$\zeta + \Sigma M_D = 0; \quad 9(1.5) - M_D = 0 \quad M_D = 13.5 \text{ kN} \cdot \text{m}$ **Ans.**



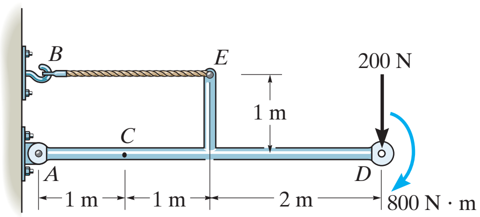




Ans:
$N_C = 0$
$V_C = 3.25 \text{ kN}$
$M_C = 9.375 \text{ kN} \cdot \text{m}$
$N_D = 0$
$V_D = 1 \text{ kN}$
$M_D = 13.5 \text{ kN} \cdot \text{m}$

***7–28.**

Determine the internal normal force, shear force, and moment at point C.



SOLUTION

Support Reactions. Referring to the *FBD* of the entire assembly shown in Fig. *a*,

$$\circlearrowleft +\Sigma M_A = 0; \quad F_{BE}(1) - 200(4) - 800 = 0 \quad F_{BE} = 1600\text{ N}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad A_x - 1600 = 0 \quad A_x = 1600\text{ N}$$

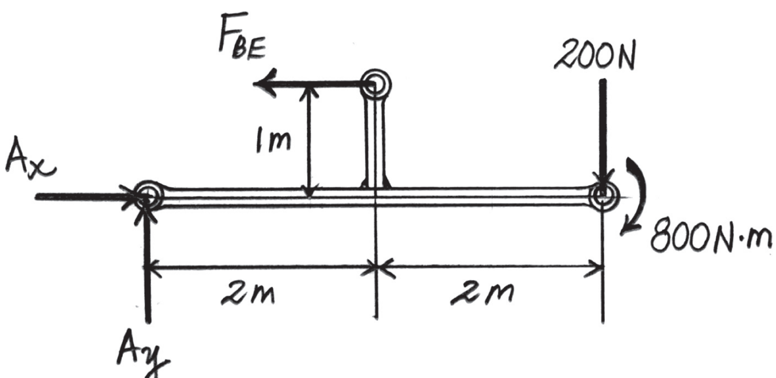
$$+\uparrow\Sigma F_y = 0; \quad A_y - 200 = 0 \quad A_y = 200\text{ N}$$

Internal Loading. Referring to the *FBD* of the left segment of the assembly sectioned through C, Fig. *b*,

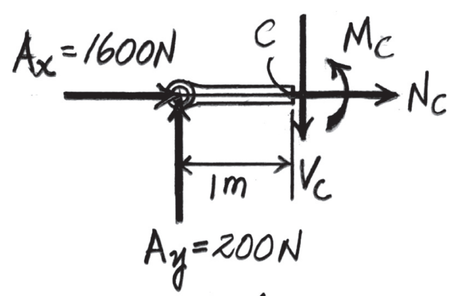
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 1600 + N_C = 0 \quad N_C = -1600\text{ N} = -1.60\text{ kN} \quad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \quad 200 - V_C = 0 \quad V_C = 200\text{ N} \quad \textbf{Ans.}$$

$$\circlearrowleft +\Sigma M_A = 0; \quad M_C - 200(1) = 0 \quad M_C = 200\text{ N}\cdot\text{m} \quad \textbf{Ans.}$$



(a)

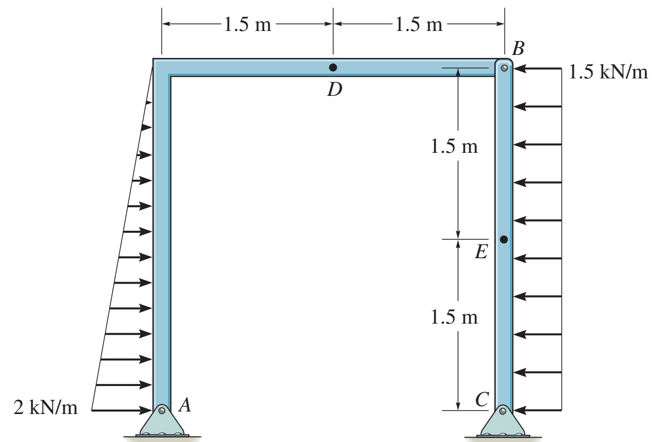


(b)

Ans:
$N_C = -1.60$ kN
$V_C = 200$ N
$M_C = 200$ N·m

7–29.

Determine the internal normal force, shear force, and moment at point D of the two-member frame.



SOLUTION

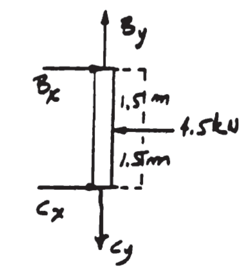
Member BC:

$\zeta + \Sigma M_C = 0;$ $\quad 4.5\,(1.5) - B_x\,(3) = 0$

$B_x = 2.25\text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 2.25 + C_x - 4.5 = 0$

$C_x = 2.25\text{ kN}$



Member AB:

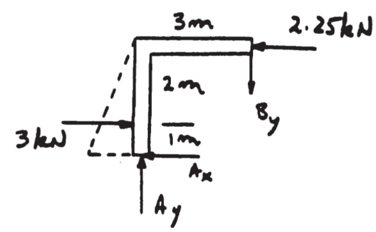
$\zeta + \Sigma M_A = 0;$ $\quad 2.25\,(3) - 3\,(1) - B_y\,(3) = 0$

$B_y = 1.25\text{ kN}$

Segment DB:

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad -N_D - 2.25 = 0$

$N_D = -2.25\text{ kN}$ **Ans.**

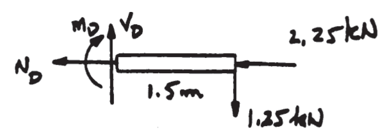


$+\uparrow \Sigma F_y = 0;$ $\quad V_D - 1.25 = 0$

$V_D = 1.25\text{ kN}$ **Ans.**

$\zeta + \Sigma M_D = 0;$ $\quad -M_D - 1.25\,(1.5) = 0$

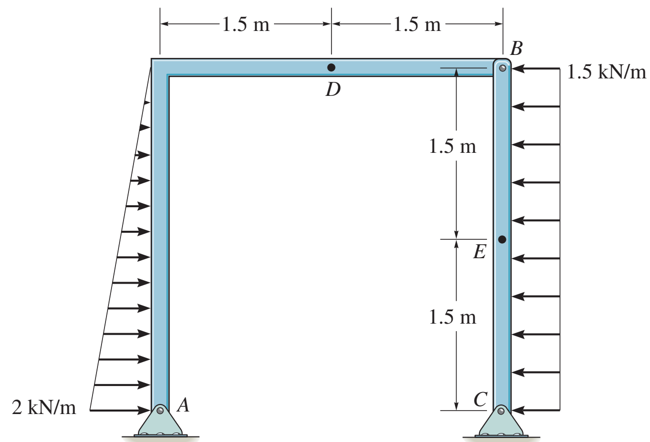
$M_D = -1.88\text{ kN}\cdot\text{m}$ **Ans.**



Ans:
$N_D = -2.25\text{ kN}$
$V_D = 1.25\text{ kN}$
$-1.88\text{ kN}\cdot\text{m}$

7–30.

Determine the internal normal force, shear force, and moment at point E.



SOLUTION

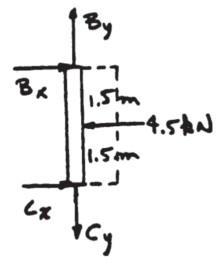
Member BC:

$\zeta + \Sigma M_C = 0;\quad 4.5\,(1.5) - B_x\,(3) = 0$

$B_x = 2.25\text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0;\quad 2.25 + C_x - 4.5 = 0$

$C_x = 2.25\text{ kN}$



Member AB:

$\zeta + \Sigma M_A = 0;\quad 2.25\,(3) - 3\,(1) - B_y\,(3) = 0$

$B_y = 1.25\text{ kN}$

Segment BE:

$+\uparrow \Sigma F_y = 0;\quad 1.25 - N_E = 0$

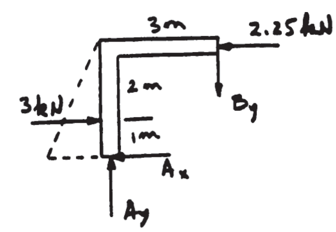
$N_E = 1.25\text{ kN}$ **Ans.**

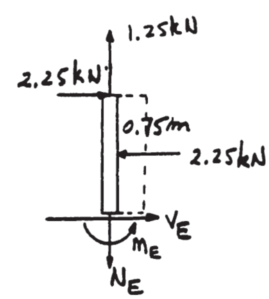
$\xrightarrow{+} \Sigma F_x = 0;\quad V_E + 2.25 - 2.25 = 0$

$V_E = 0$ **Ans.**

$+\Sigma M_g = 0;\quad M_g - 2.25\,(0.75) = 0$

$M_g = 1.6875\text{ kN}\cdot\text{m} = 1.69\text{ kN}\cdot\text{m}$ **Ans.**

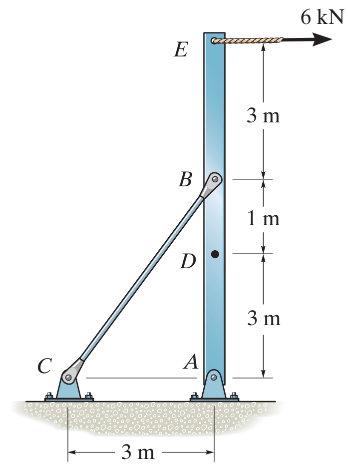




Ans:
$N_E = 1.25\text{ kN}$
$V_E = 0$
$M_B = 1.69\text{ kN}\cdot\text{m}$

7–31.

Determine the internal normal force, shear force, and moment at point D.



SOLUTION

Support Reactions. Notice that member BC is a two force member. Referring to the FBD of member ABE shown in Fig. a,

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{3}{5}\right)(4) - 6(7) = 0 \qquad F_{BC} = 17.5 \text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - 17.5\left(\frac{3}{5}\right) + 6 = 0 \qquad A_x = 4.50 \text{ kN}$$

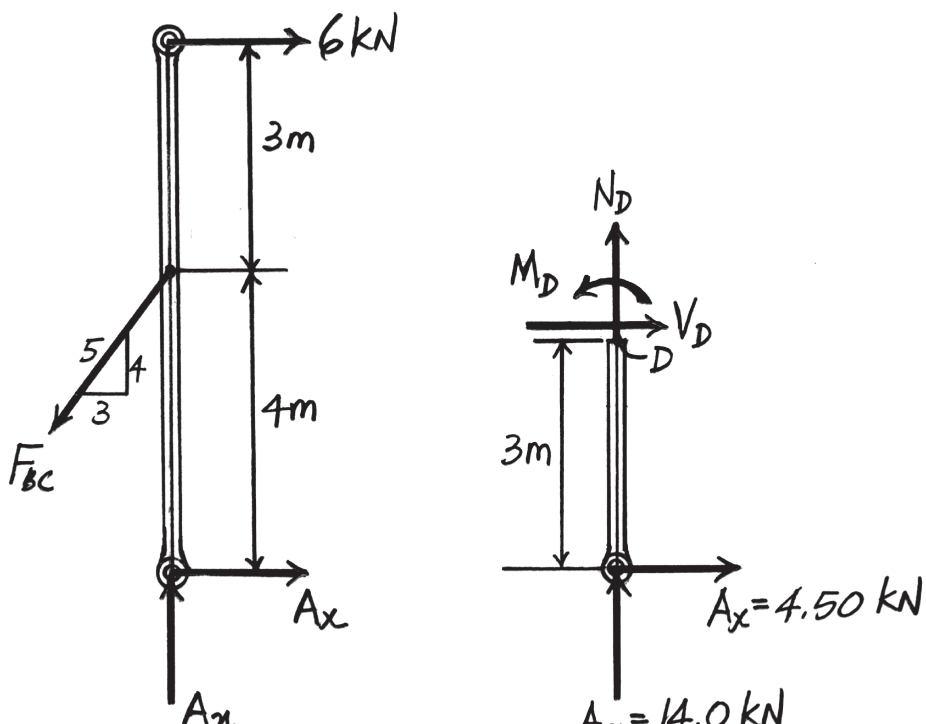
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 17.5\left(\frac{4}{5}\right) = 0 \qquad A_y = 14.0 \text{ kN}$$

Internal Loadings. Referring to the FBD of the lower segment of member ABE sectioned through D, Fig. b,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 4.50 + V_D = 0 \qquad V_D = -4.50 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_D + 14.0 = 0 \qquad N_D = -14.0 \text{ kN} \qquad \textbf{Ans.}$$

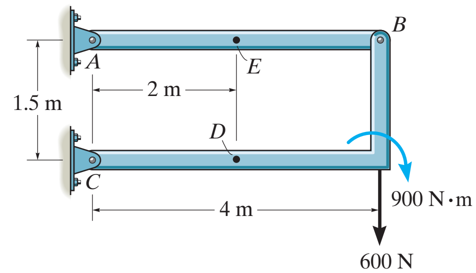
$$\circlearrowleft + \Sigma M_D = 0; \qquad M_D + 4.50(3) = 0 \qquad M_D = -13.5 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$



Ans:
$V_D = -4.50$ kN
$N_D = -14.0$ kN
$M_D = -13.5$ kN·m

***7–32.**

Determine the internal normal force, shear force, and moment acting at points D and E of the frame.



SOLUTION

Support Reactions. Notice that member AB is a two force member. Referring to the FBD of member BC,

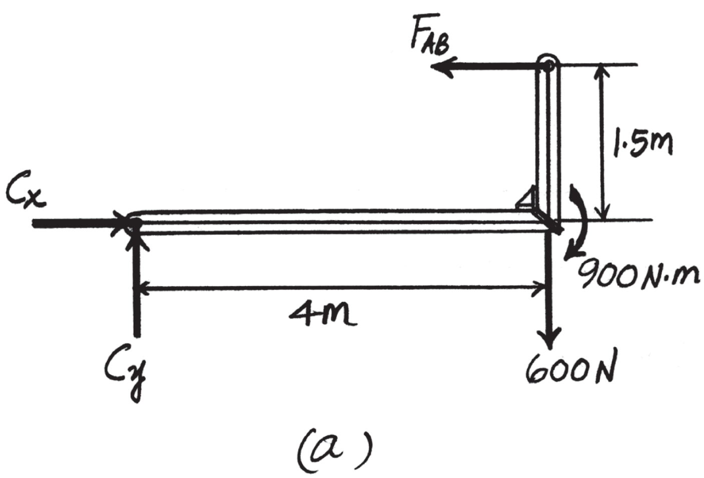
$\circlearrowleft+\Sigma M_C = 0;$	$F_{AB}(1.5) - 900 - 600(4) = 0$	$F_{AB} = 2200\text{ N}$
$\xrightarrow{+}\Sigma F_x = 0;$	$C_x - 2200 = 0$	$C_x = 2200\text{ N}$
$+\uparrow\Sigma F_y = 0;$	$C_y - 600 = 0$	$C_y = 600\text{ N}$

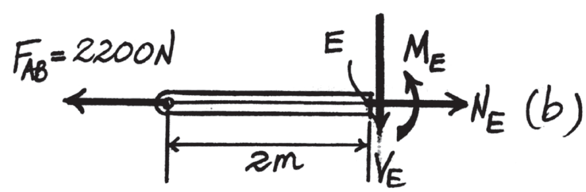
Internal Loadings. Referring to the left segment of member AB sectioned through E, Fig. b,

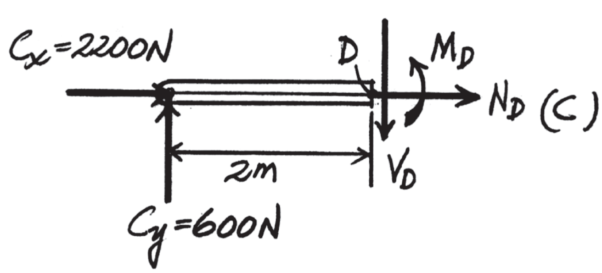
$\xrightarrow{+}\Sigma F_x = 0;$	$N_E - 2200 = 0$	$N_E = 2200\text{ N} = 2.20\text{ kN}$	**Ans.**
$+\uparrow\Sigma F_y = 0;$	$V_E = 0$		**Ans.**
$\circlearrowleft+\Sigma M_E = 0;$	$M_E = 0$		**Ans.**

Referring to the left segment of member BC sectioned through D, Fig. c

$\xrightarrow{+}\Sigma F_x = 0;$	$N_D + 2200 = 0$	$N_D = -2200\text{ N} = -2.20\text{ kN}$	**Ans.**
$+\uparrow\Sigma F_y = 0;$	$600 - V_D = 0$	$V_D = 600\text{ N}$	**Ans.**
$\circlearrowleft+\Sigma M_D = 0;$	$M_D - 600(2) = 0$	$M_D = 1200\text{ N}\cdot\text{m} = 1.20\text{ kN}\cdot\text{m}$	**Ans.**



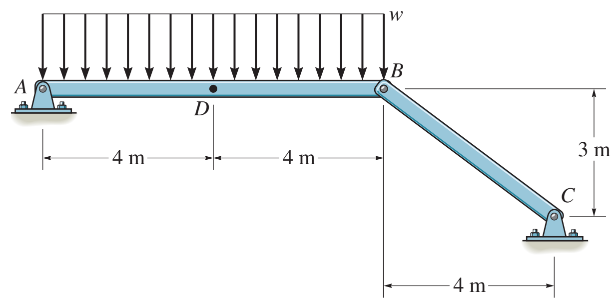




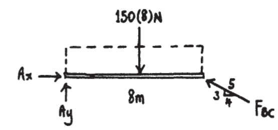
Ans:
$N_E = 2.20\text{ kN}$
$V_E = 0$
$M_E = 0$
$N_D = -2.20\text{ kN}$
$V_D = 600\text{ N}$
$M_D = 1.20\text{ kN}\cdot\text{m}$

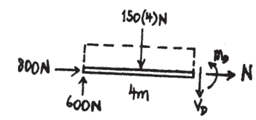
7–33.

Determine the normal force, shear force, and moment at a section passing through point D. Take $w = 150$ N/m.



SOLUTION





$\circlearrowleft + \Sigma M_A = 0; \qquad -150(8)(4) + \frac{3}{5}F_{BC}(8) = 0$

$F_{BC} = 1000 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - \frac{4}{5}(1000) = 0$

$A_x = 800 \text{ N}$

$+\uparrow \Sigma F_y = 0; \qquad A_y - 150(8) + \frac{3}{5}(1000) = 0$

$A_y = 600 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad N_D = -800 \text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad 600 - 150(4) - V_D = 0$

$V_D = 0$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0; \qquad -600(4) + 150(4)(2) + M_D = 0$

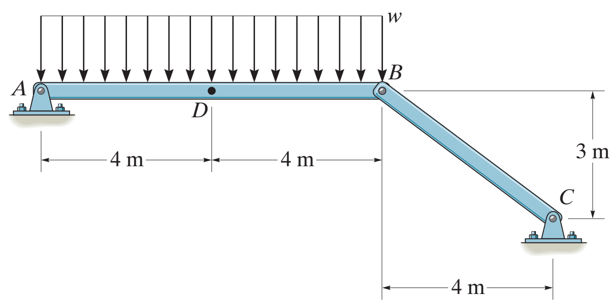
$M_D = 1200 \text{ N}\cdot\text{m} = 1.20 \text{ kN}\cdot\text{m}$ **Ans.**

Ans:

$N_D = -800$ N
$V_D = 0$
$M_D = 1.20$ kN · m

7–34.

The beam AB will fail if the maximum internal moment at D reaches 800 N·m or the normal force in member BC becomes 1500 N. Determine the largest load w it can support.



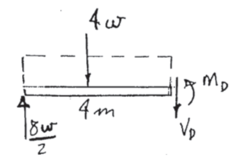
SOLUTION

Assume maximum moment occurs at D;

$\circlearrowleft + \Sigma M_D = 0;$ $\quad M_D - 4w(2) = 0$

$$800 = 4w(2)$$

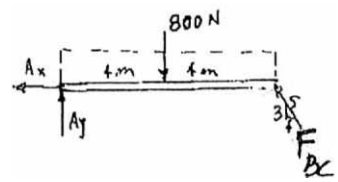
$$w = 100 \text{ N/m}$$



$\circlearrowleft + \Sigma M_A = 0;$ $\quad -800(4) + F_{BC}(0.6)(8) = 0$

$$F_{BC} = 666.7 \text{ N} < 1500 \text{ N} \qquad \textbf{(O.K.!)}$$

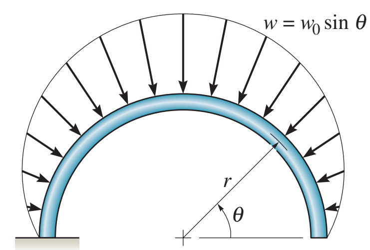
$$w = 100 \text{ N/m} \qquad \textbf{Ans.}$$



Ans:
$w = 100$ N/m

7–35.

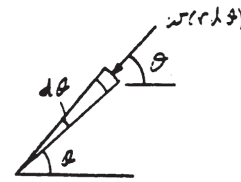
The distributed loading $w = w_0 \sin \theta$, measured per unit length, acts on the curved rod. Determine the internal normal force, shear force, and moment in the rod at $\theta = 45°$.



SOLUTION

$w = w_0 \sin \theta$

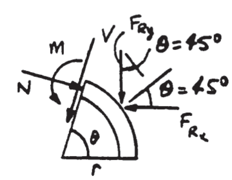
Resultants of distributed loading:



$$F_{Rx} = \int_0^\theta w_0 \sin \theta (r\, d\theta) \cos \theta = r w_0 \int_0^\theta \sin \theta \cos \theta \, d\theta = \frac{1}{2} r\, w_0 \sin^2 \theta$$

$$F_{Ry} = \int_0^\theta w_0 \sin \theta (r\, d\theta) \sin \theta = r w_0 \int_0^\theta \sin^2 \theta \, d\theta = r w_0 \left[\frac{1}{2}\theta - \frac{1}{4} \sin 2\theta \right]$$

$\nearrow + \Sigma F_x = 0; \qquad -V + F_{Rx} \cos 45° + F_{Ry} \sin 45° = 0$



$$V = \left(\frac{1}{2} r\, w_0 \sin^2 45° \right) \cos 45° + w_0 \left(\frac{1}{2}\frac{\pi}{4} - \frac{1}{4} \sin 90° \right) \sin 45°$$

$V = 0.278\, w_0 r$ **Ans.**

$+\nwarrow \Sigma F_y = 0; \qquad -N - F_{Ry} \cos 45° + F_{Rx} \sin 45° = 0$

$$N = -r\, w_0 \left[\frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{4} \sin 90° \right] \cos 45° + \left(\frac{1}{2} r\, w_0 \sin^2 45° \right) \sin 45°$$

$N = 0.0759\, w_0 r$ **Ans.**

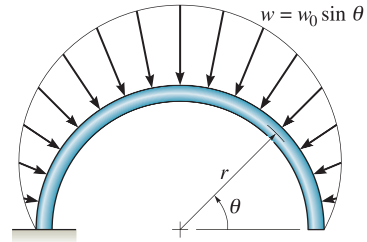
$\circlearrowleft + \Sigma M_O = 0; \qquad M - (0.0759\, r\, w_0)(r) = 0$

$M = 0.0759 \quad w_0 r^2$ **Ans.**

Ans:
$V = 0.278\, w_0 r$
$N = 0.0759\, w_0 r$
$M = 0.0759\, w_0 r^2$

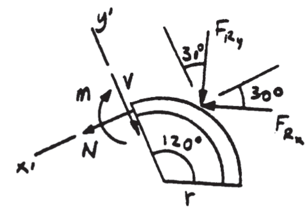
***7–36.**

Solve Prob. 7–35 for $\theta = 120°$.



SOLUTION

Resultants of distributed load:



$$F_{Rx} = \int_0^\theta w_0 \sin\theta(r\,d\theta)\cos\theta = rw_0 \int_0^\theta \sin\theta\cos\theta = \frac{1}{2}rw_0\sin^2\theta$$

$$F_{Ry} = \int_0^\theta w_0 \sin\theta(r\,d\theta)\sin\theta = rw_0 \int_0^\theta \sin^2\theta\,d\theta = rw_0\left[\frac{1}{2}\theta\frac{1}{4}\sin 2\theta\right]rw_0(\sin\theta)\Big|_0^\theta = r\,w_0(\sin\theta)$$

$$F_{Rx} = \tfrac{1}{2}r\,w_0\sin^2 120° = 0.375\,r\,w_0$$

$$F_{Ry} = r\,w_0\left[\frac{1}{2}(\pi)\left(\frac{120°}{180°}\right) - \frac{1}{4}\sin 240°\right] = 1.2637\,r\,w_0$$

$\overset{+}{\swarrow}\Sigma F_{x'} = 0;$ $\quad N + 0.375\,rw_0\cos 30° + 1.2637\,r\,w_0\sin 30° = 0$

$N = -0.957\,r\,w_0$ **Ans.**

$_{+}\nwarrow\Sigma F_{y'} = 0;$ $\quad -V + 0.375\,rw_0\sin 30° - 1.2637\,r\,w_0\cos 30° = 0$

$V = -0.907\,rw_0$ **Ans.**

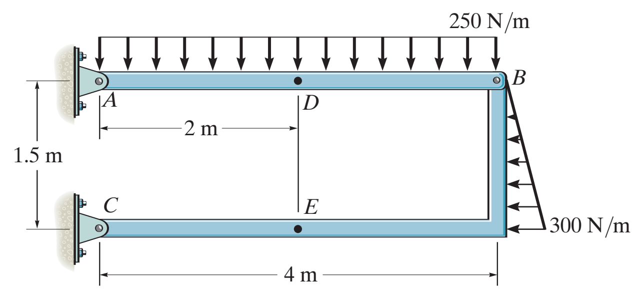
$\circlearrowleft + \Sigma M_O = 0;$ $\quad -M - 0.957\,r\,w_0(r) = 0$

$M = -0.957\,r^2w_0$ **Ans.**

Ans:
$N = -0.957\,r\,w_0$
$V = -0.907\,rw_0$
$M = -0.957\,r^2w_0$

7–37.

Determine the internal normal force, shear force, and moment at point D of the two-member frame.

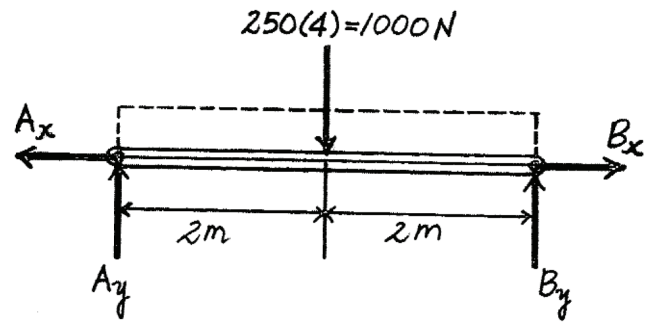


SOLUTION

Member *AB*:

$\circlearrowleft +\Sigma M_A = 0; \qquad B_y\,(4) - 1000\,(2) = 0$

$B_y = 500\text{ N}$



Member *BC*:

$\circlearrowleft +\Sigma M_C = 0; \qquad -500\,(4) + 225\,(0.5) + B_x\,(1.5) = 0$

$B_x = 1258.33\text{ N}$

Segment *DB*:

$\xrightarrow{+} \Sigma F_x = 0; \qquad -N_D + 1258.33 = 0$

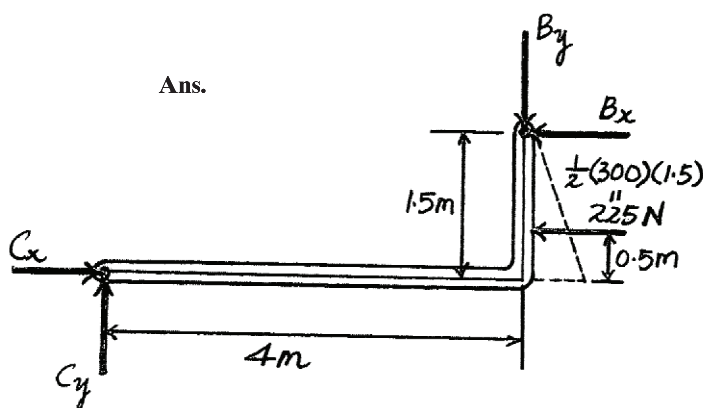
$N_D = 1.26\text{ kN}$ **Ans.**

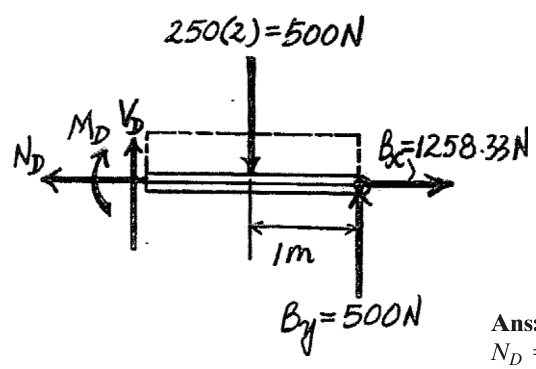
$+\uparrow \Sigma F_y = 0; \qquad V_D - 500 + 500 = 0$

$V_D = 0$ **Ans.**

$\circlearrowleft +\Sigma M_D = 0; \qquad -M_D + 500\,(1) = 0$

$M_D = 500\text{ N}\cdot\text{m}$ **Ans.**

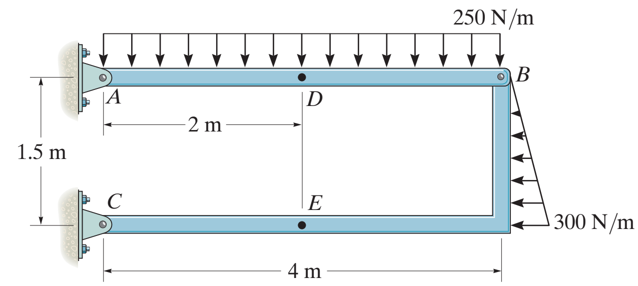




Ans:
$N_D = 1.26\text{ kN}$
$V_D = 0$
$M_D = 500\text{ N}\cdot\text{m}$

7–38.

Determine the internal normal force, shear force, and moment at point E of the two-member frame.

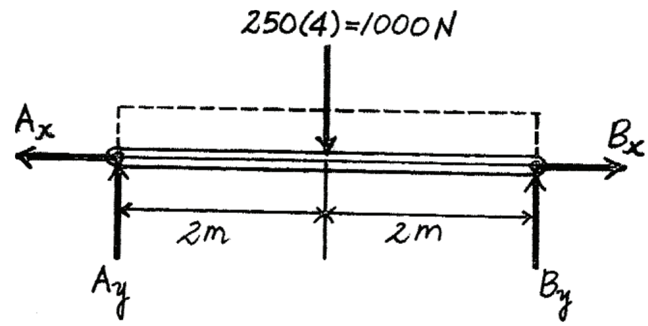


SOLUTION

Member *AB*:

$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(4) - 1000(2) = 0$

$B_y = 500 \text{ N}$



Member *BC*:

$\circlearrowleft + \Sigma M_C = 0; \qquad -500(4) + 225(0.5) + B_x(1.5) = 0$

$B_x = 1258.33 \text{ N}$

Segment *EB*:

$\xrightarrow{+} \Sigma F_x = 0; \qquad -N_E - 1258.33 - 225 = 0$

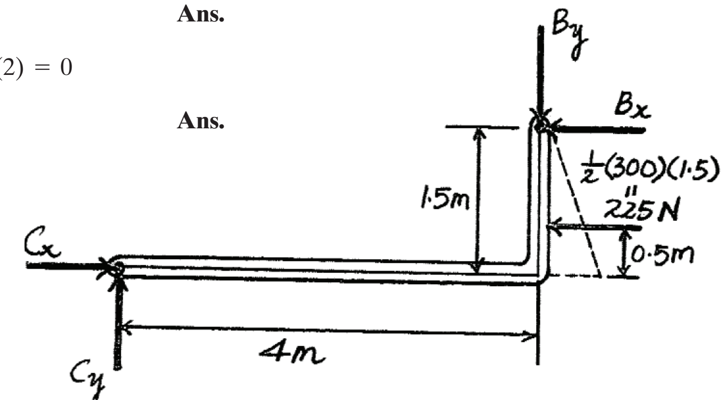
$N_E = -1.48 \text{ kN}$ **Ans.**

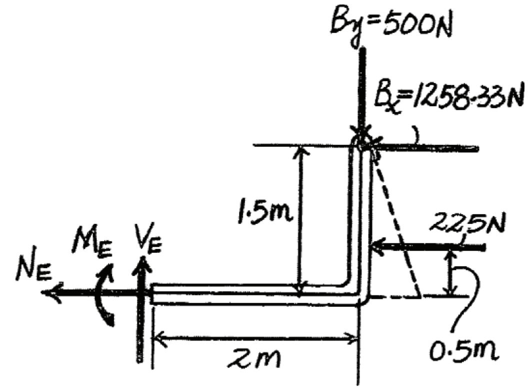
$+\uparrow \Sigma F_y = 0; \qquad V_E - 500 = 0$

$V_E = 500 \text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0; \qquad -M_E + 225(0.5) + 1258.33(1.5) - 500(2) = 0$

$M_E = 1000 \text{ N} \cdot \text{m}$ **Ans.**

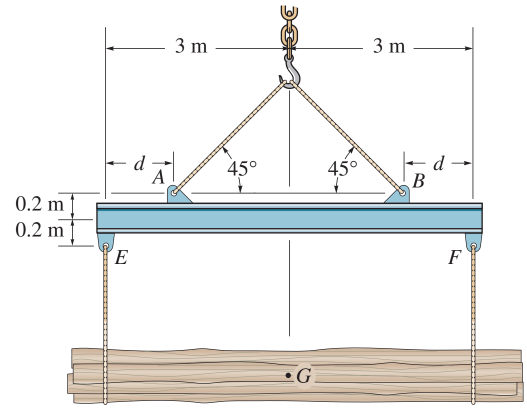




Ans:
$N_E = -1.48 \text{ kN}$
$V_E = 500 \text{ N}$
$M_E = 1000 \text{ N} \cdot \text{m}$

7–39.

The strongback or lifting beam is used for materials handling. If the suspended load has a weight of 2 kN and a center of gravity of G, determine the placement d of the padeyes on the top of the beam so that there is no moment developed within the length AB of the beam. The lifting bridle has two legs that are positioned at 45°, as shown.

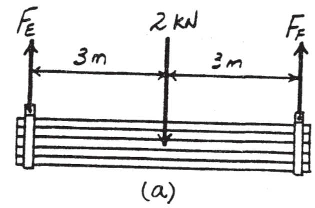


SOLUTION

Support Reactions: From FBD (a),

$\circlearrowleft + \Sigma M_E = 0; \quad F_F(6) - 2(3) = 0 \quad F_E = 1.00 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad F_F + 1.00 - 2 = 0 \quad F_F = 1.00 \text{ kN}$

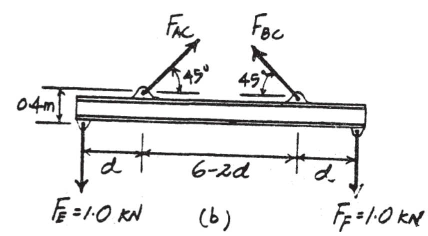


From FBD (b),

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AC} \cos 45° - F_{BC} \cos 45° = 0 \quad F_{AC} = F_{BC} = F$

$+\uparrow \Sigma F_y = 0; \quad 2F \sin 45° - 1.00 - 1.00 = 0$

$F_{AC} = F_{BC} = F = 1.414 \text{ kN}$

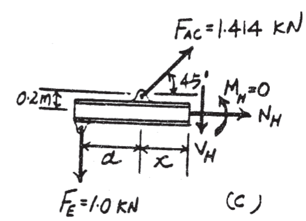


Internal Forces: This problem requires $M_H = 0$. Summing moments about point H of segment EH[FBD (c)], we have

$\circlearrowleft + \Sigma M_H = 0; \quad 1.00(d + x) - 1.414 \sin 45°(x)$

$- 1.414 \cos 45°(0.2) = 0$

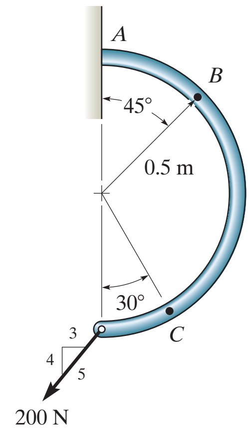
$d = 0.200 \text{ m}$ **Ans.**



Ans:
$d = 0.200 \text{ m}$

***7–40.**

Determine the internal normal force, shear force, and moment acting at points B and C on the curved rod.



SOLUTION

Support Reactions. Not required

Internal Loadings. Referring to the *FBD* of bottom segment of the curved rod sectioned through C, Fig. *a*

$+\nearrow \Sigma F_x = 0;\ N_C - 200 \sin(36.87° + 30°) = 0 \quad N_C = 183.92\text{ N} = 184\text{ N}$ **Ans.**

$+\nwarrow \Sigma F_y = 0;\ -V_C - 200 \cos(36.87° + 30°) \quad V_C = -78.56\text{ N} = -78.6\text{ N}$ **Ans.**

$$\circlearrowleft + \Sigma M_C = 0; \quad 200\left(\frac{4}{5}\right)(0.5 \sin 30°) - 200\left(\frac{3}{5}\right)[0.5(1 - \cos 30°)] + M_C = 0$$

$M_C = -31.96\text{ N}\cdot\text{m} = -32.0\text{ N}\cdot\text{m}$ **Ans.**

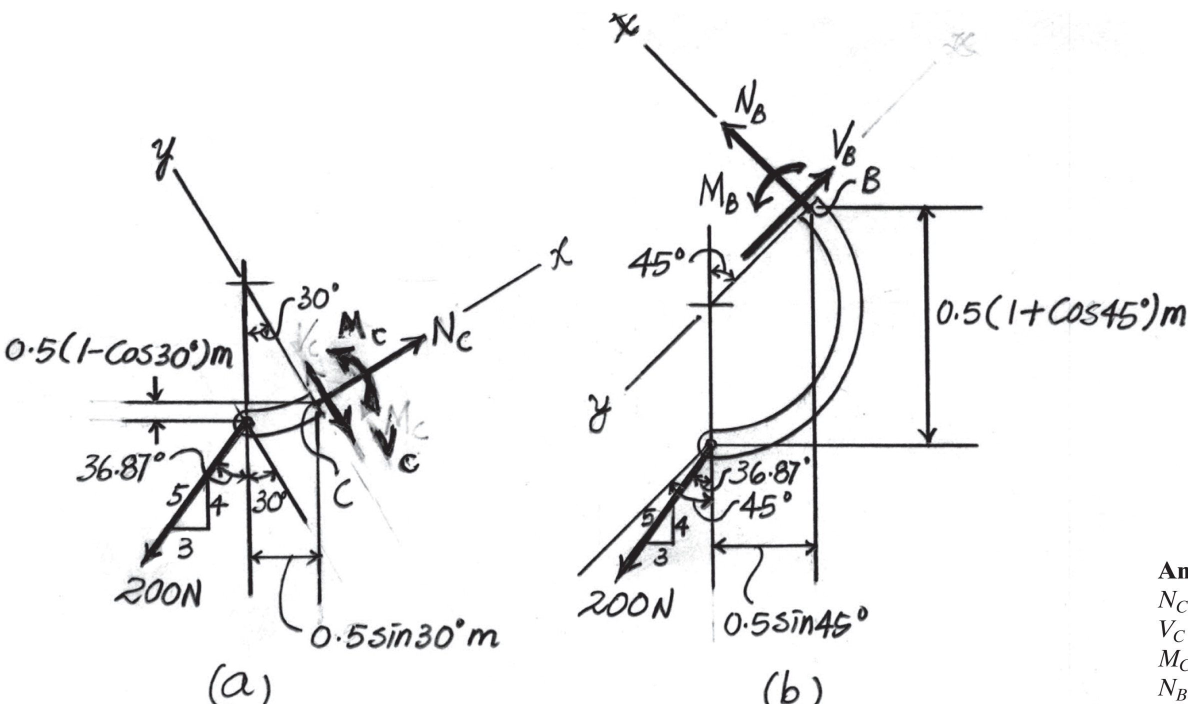
Referring to the *FBD* of bottom segment of the curved rod sectioned through B, Fig. *b*

$\nwarrow + \Sigma F_x = 0; \quad N_B - 200 \sin(45° - 36.87°) = 0 \quad N_B = 28.28\text{ N} = 28.3\text{ N}$ **Ans.**

$+\swarrow \Sigma F_y = 0; \quad -V_B + 200 \cos(45° - 36.87°) = 0 \quad V_B = 197.99\text{ N} = 198\text{ N}$ **Ans.**

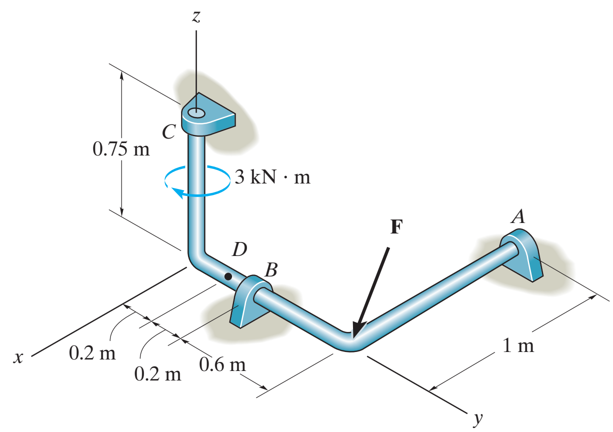
$$\circlearrowleft + \Sigma M_B = 0; \quad M_B + 200\left(\frac{4}{5}\right)(0.5 \sin 45°) - 200\left(\frac{3}{5}\right)[0.5(1 + \cos 45°] = 0$$

$M_B = 45.86\text{ N}\cdot\text{m} = 45.9\text{ N}\cdot\text{m}$ **Ans.**



Ans:
$N_C = 184\text{ N}$
$V_C = -78.6\text{ N}$
$M_C = -32.0\text{ N}\cdot\text{m}$
$N_B = 28.3\text{ N}$
$V_B = 198\text{ N}$
$M_B = 45.9\text{ N}\cdot\text{m}$

7–41. Determine the x, y, z components of internal loading in the rod at point D. $\mathbf{F} = \{7\mathbf{i} - 12\mathbf{j} - 5\mathbf{k}\}$ kN.



SOLUTION

Given:

$$M = 3 \text{ kN}\cdot\text{m}$$

$$\mathbf{F} = \begin{pmatrix} 7 \\ -12 \\ -5 \end{pmatrix} \text{kN}$$

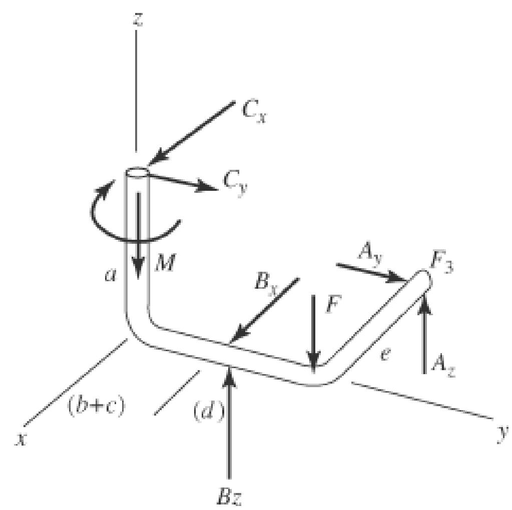
$$a = 0.75 \text{ m}$$

$$b = 0.2 \text{ m}$$

$$c = 0.2 \text{ m}$$

$$d = 0.6 \text{ m}$$

$$e = 1 \text{ m}$$



Guesses

$C_x = 1$ N $\quad C_y = 1$ N $\quad B_x = 1$ N

$B_z = 1$ N $\quad A_y = 1$ N $\quad A_z = 1$ N

Given

$$\begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} + \begin{pmatrix} C_x \\ C_y \\ 0 \end{pmatrix} + \mathbf{F} = \mathbf{0}$$

$$\begin{pmatrix} -e \\ b+c+d \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} 0 \\ b+c \\ 0 \end{pmatrix} \times \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix} \times \begin{pmatrix} C_x \\ C_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ b+c+d \\ 0 \end{pmatrix} \times \mathbf{F} + \begin{pmatrix} 0 \\ 0 \\ -M \end{pmatrix} = \mathbf{0}$$

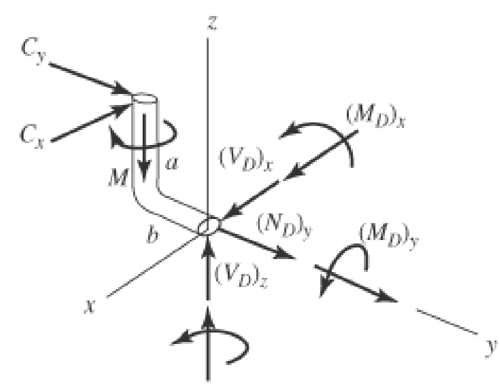
$$\begin{pmatrix} A_y \\ A_z \\ B_x \\ B_z \\ C_x \\ C_y \end{pmatrix} = \mathrm{Find}(A_y, A_z, B_x, B_z, C_x, C_y) \qquad \begin{pmatrix} A_y \\ A_z \\ B_x \\ B_z \\ C_x \\ C_y \end{pmatrix} = \begin{pmatrix} -53.60 \\ 87.00 \\ 109.00 \\ -82.00 \\ -116.00 \\ 65.60 \end{pmatrix} \mathrm{kN}$$

Guesses

$V_{Dx} = 1\ \mathrm{N}$ $\quad N_{Dy} = 1\ \mathrm{N}$

$V_{Dz} = 1\mathrm{N}$ $\quad M_{Dx} = 1\ \mathrm{N \cdot m}$

$M_{Dy} = 1\ \mathrm{N \cdot m}$ $\quad M_{Dz} = 1\ \mathrm{N \cdot m}$



Given

$$\begin{pmatrix} C_x \\ C_y \\ 0 \end{pmatrix} + \begin{pmatrix} V_{Dx} \\ N_{Dy} \\ V_{Dz} \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} 0 \\ -b \\ a \end{pmatrix} \times \begin{pmatrix} C_x \\ C_y \\ 0 \end{pmatrix} + \begin{pmatrix} M_{Dx} \\ M_{Dy} \\ M_{Dz} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -M \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} V_{Dx} \\ N_{Dy} \\ V_{Dz} \\ M_{Dx} \\ M_{Dy} \\ M_{Dz} \end{pmatrix} = \mathrm{Find}(V_{Dx}, N_{Dy}, V_{Dz}, M_{Dx}, M_{Dy}, M_{Dz})$$

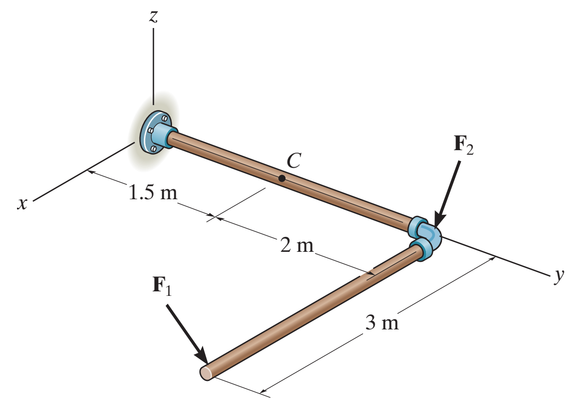
$$\begin{pmatrix} V_{Dx} \\ N_{Dy} \\ V_{Dz} \end{pmatrix} = \begin{pmatrix} 116.00 \\ -65.60 \\ 0.00 \end{pmatrix} \mathrm{kN}$$ **Ans.**

$$\begin{pmatrix} M_{Dx} \\ M_{Dy} \\ M_{Dz} \end{pmatrix} = \begin{pmatrix} 49.20 \\ 87.00 \\ 26.20 \end{pmatrix} \mathrm{kN \cdot m}$$ **Ans.**

Ans:

$V_{Dx} = 116.00$ kN, $N_{Dy} = -65.60$ kN, $V_{Dz} = 0.00$, $M_{Dx} = 49.20$, $M_{Dy} = 87.00$, $M_{Dz} = 26.20$

7–42. Determine the x, y, z components of internal loading at a section passing through point C in the pipe assembly. Neglect the weight of the pipe. Take $\mathbf{F}_1 = \{-80\mathbf{i} + 200\mathbf{j} - 300\mathbf{k}\}$ kN and $\mathbf{F}_2 = \{250\mathbf{i} - 150\mathbf{j} - 200\mathbf{k}\}$ kN.



SOLUTION

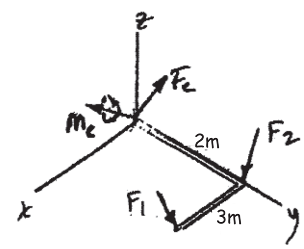
$\Sigma \mathbf{F}_R = 0;\quad \mathbf{F}_C + \mathbf{F}_1 + \mathbf{F}_2 = 0$

$\mathbf{F}_C = \{-170\mathbf{i} - 50\mathbf{j} + 500\mathbf{k}\}$ kN

$C_x = -170$ kN **Ans.**

$C_y = -50$ kN **Ans.**

$C_z = 500$ kN **Ans.**



$\Sigma \mathbf{M}_R = 0;\quad \mathbf{M}_C + \mathbf{r}_{C1} \times \mathbf{F}_1 + \mathbf{r}_{C2} \times \mathbf{F}_2 = 0$

$$\mathbf{M}_C + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 0 \\ -80 & 200 & -300 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 0 \\ 250 & -150 & -200 \end{vmatrix} = 0$$

$\mathbf{M}_C = \{1000\mathbf{i} - 900\mathbf{j} - 260\mathbf{k}\}\ \mathbf{kN \cdot m}$

$M_{Cx} = 1\ \text{MN} \cdot \text{m}$ **Ans.**

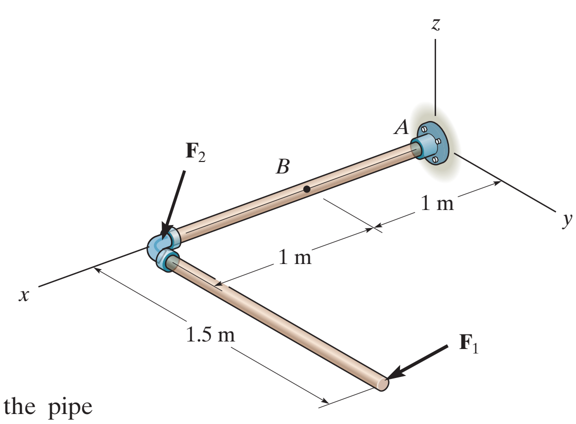
$M_{Cy} = 900\ \text{kN} \cdot \text{m}$ **Ans.**

$M_{Cz} = -260\ \text{kN} \cdot \text{m}$ **Ans.**

Ans:
$C_x = -170$ kN
$C_y = -50$ kN
$C_z = 500$ kN
$M_{Cx} = 1\ \text{MN} \cdot \text{m}$
$M_{Cy} = 900\ \text{kN} \cdot \text{m}$
$M_{Cz} = -260\ \text{kN} \cdot \text{m}$

7–43.

Determine the x, y, z components of internal loading at a section passing through point B in the pipe assembly. Neglect the weight of the pipe. Take $\mathbf{F}_1 = \{200\mathbf{i} - 100\mathbf{j} - 400\mathbf{k}\}$ N and $\mathbf{F}_2 = \{300\mathbf{i} - 500\mathbf{k}\}$ N.

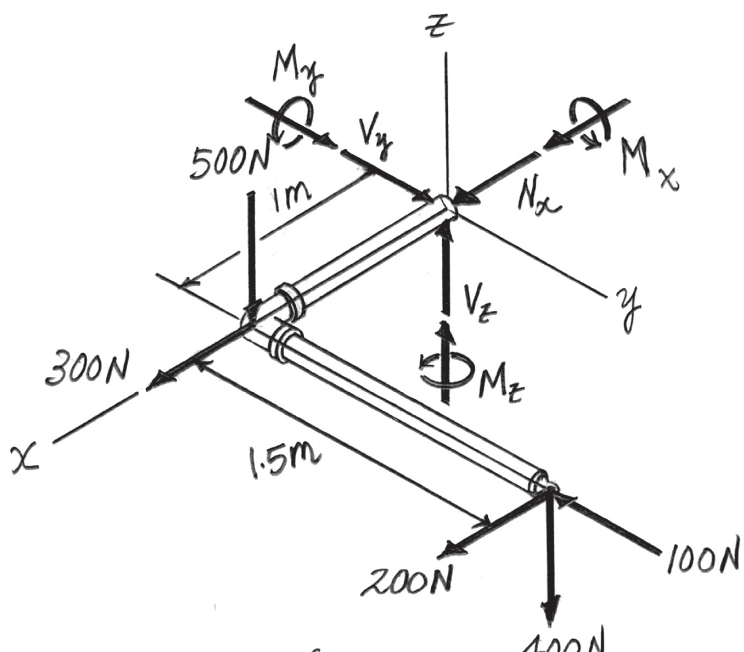


SOLUTION

Internal Loadings. Referring to the *FBD* of the free end segment of the pipe assembly sectioned through B, Fig. a,

$\Sigma F_x = 0;$	$N_x + 300 + 200 = 0$	$N_x = -500\text{ N}$	**Ans.**
$\Sigma F_y = 0;$	$V_y - 100 = 0$	$V_y = 100\text{ N}$	**Ans.**
$\Sigma F_z = 0;$	$V_z - 500 - 400 = 0$	$V_z = 900\text{ N}$	**Ans.**
$\Sigma M_x = 0;$	$M_x - 400(1.5) = 0$	$M_x = 600\text{ N}\cdot\text{m}$	**Ans.**
$\Sigma M_y = 0;$	$M_y + 500(1) + 400(1) = 0$	$M_y = -900\text{ N}\cdot\text{m}$	**Ans.**
$\Sigma M_z = 0$	$M_z - 200(1.5) - 100(1) = 0$	$M_z = 400\text{ N}\cdot\text{m}$	**Ans.**

The negative signs indicate that $\mathbf{N}_x$ and $\mathbf{M}_y$ act in the opposite sense to those shown in *FBD*.

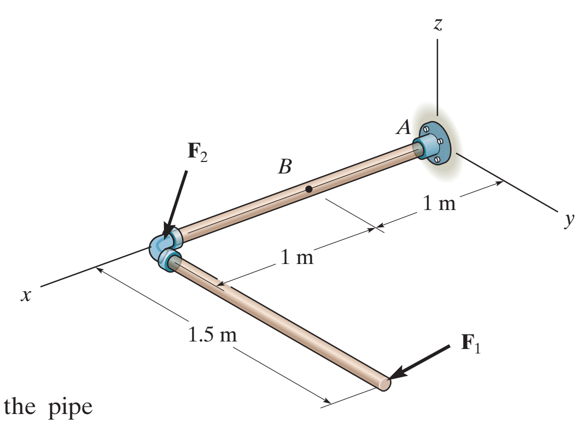


(a)

Ans:
$N_x = -500\text{ N}$
$V_y = 100\text{ N}$
$V_z = 900\text{ N}$
$M_x = 600\text{ N}\cdot\text{m}$
$M_y = -900\text{ N}\cdot\text{m}$
$M_z = 400\text{ N}\cdot\text{m}$

***7–44.**

Determine the x, y, z components of internal loading at a section passing through point B in the pipe assembly. Neglect the weight of the pipe. Take $\mathbf{F}_1 = \{100\mathbf{i} - 200\mathbf{j} - 300\mathbf{k}\}$ N and $\mathbf{F}_2 = \{100\mathbf{i} + 500\mathbf{j}\}$ N.



SOLUTION

Internal Loadings. Referring to the *FBD* of the free end segment of the pipe assembly sectioned through B, Fig. *a*

$\Sigma F_x = 0;\quad N_x + 100 + 100 = 0\qquad N_x = -200\text{ N}$ **Ans.**

$\Sigma F_y = 0;\quad V_y + 500 - 200 = 0\qquad V_y = -300\text{ N}$ **Ans.**

$\Sigma F_z = 0;\quad V_z - 300 = 0\qquad V_z = 300\text{ N}$ **Ans.**

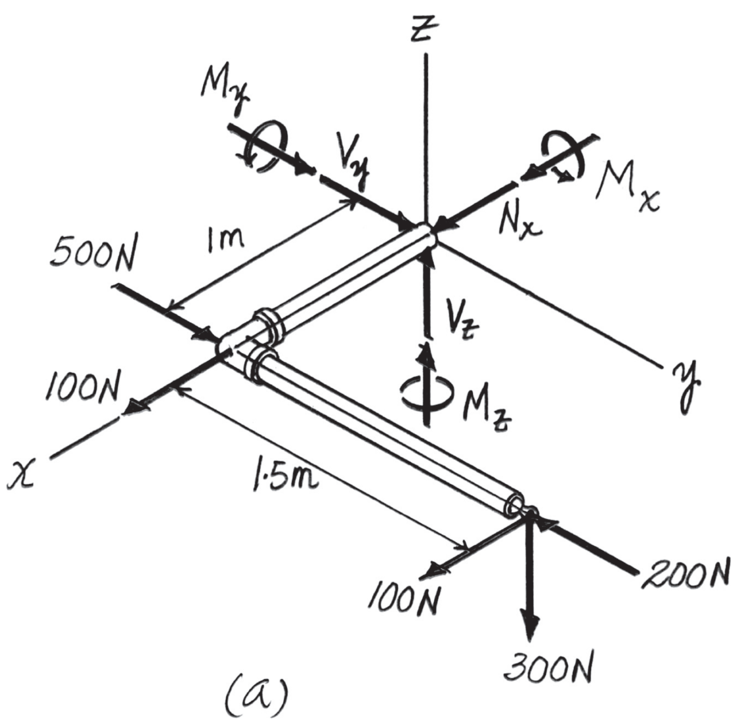
$\Sigma M_x = 0;\quad M_x - 300(1.5) = 0\qquad M_x = 450\text{ N}\cdot\text{m}$ **Ans.**

$\Sigma M_y = 0;\quad M_y + 300(1) = 0\qquad M_y = -300\text{ N}\cdot\text{m}$ **Ans.**

$\Sigma M_z = 0;\quad M_z + 500(1) - 100(1.5) - 200(1) = 0$

$M_z = -150\text{ N}\cdot\text{m}$ **Ans.**

The negative signs indicates that $\mathbf{N}_x$, $\mathbf{V}_y$, $\mathbf{M}_y$ and $\mathbf{M}_z$ act in the senses opposite to those shown in *FBD*.

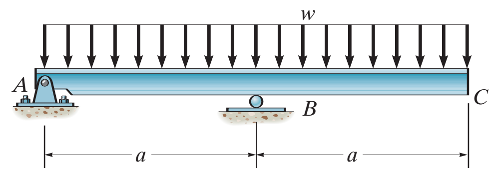


(a)

Ans:
$N_x = -200$ N
$V_y = -300$ N
$V_z = 300$ N
$M_x = 450$ N·m
$M_y = -300$ N·m
$M_z = -150$ N·m

7–45.

Draw the shear and moment diagrams for the beam.



SOLUTION

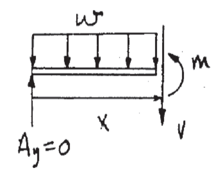
$0 \le x < a$

$+\uparrow \Sigma F_y = 0;\qquad -V - wx = 0$

$V = -wx$ **Ans.**

$\zeta + \Sigma M = 0;\qquad M + wx\left(\dfrac{x}{2}\right) = 0$

$M = -\dfrac{w}{2}x^2$ **Ans.**



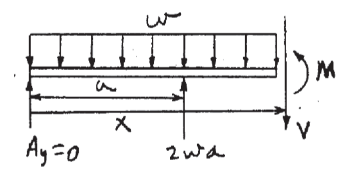
$a < x \le 2a$

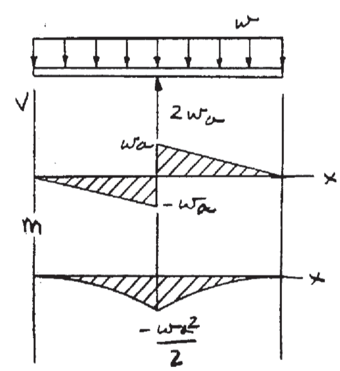
$+\uparrow \Sigma F_y = 0;\qquad -V + 2\,wa - wx = 0$

$V = w\,(2a - x)$ **Ans.**

$\zeta + \Sigma M = 0;\qquad M + wx\left(\dfrac{x}{2}\right) - 2\,wa\,(x - a) = 0$

$M = 2\,wa\,x - 2\,wa^2 - \dfrac{w}{2}x^2$ **Ans.**





Ans:

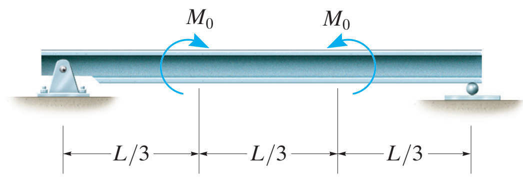
$0 \le x < a$: $V = -wx$, $M = -\dfrac{w}{2}x^2$

$a < x \le 2a$: $V = w(2a - x)$

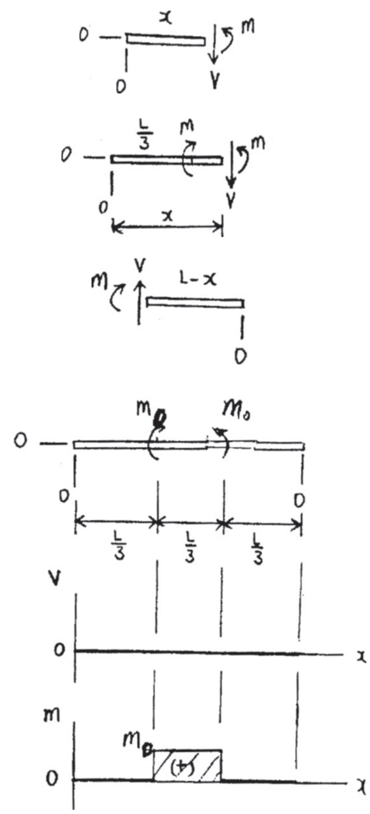
$M = 2wax - 2wa^2 - \dfrac{w}{2}x^2$

7–46.

Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set $M_0 = 500\ \text{N} \cdot \text{m}$, $L = 8\ \text{m}$.



SOLUTION



(a)

For $0 \le x \le \dfrac{L}{3}$

$+\uparrow \Sigma F_y = 0; \qquad V = 0$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad M = 0$ **Ans.**

For $\dfrac{L}{3} < x < \dfrac{2L}{3}$

$+\uparrow \Sigma F_y = 0; \qquad V = 0$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad M = M_0$ **Ans.**

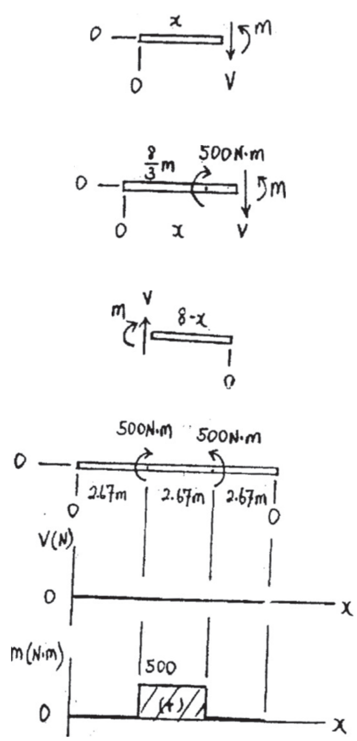
For $\dfrac{2L}{3} < x \le L$

$+\uparrow \Sigma F_y = 0; \qquad V = 0$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad M = 0$ **Ans.**

(b)

Set $M_0 = 500\ \text{N} \cdot \text{m}, L = 8\ \text{m}$



For $0 \le x < \dfrac{8}{3}\ \text{m}$

$+\uparrow \Sigma F_y = 0; \qquad V = 0$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad M = 0$ **Ans.**

For $\dfrac{8}{3}\ \text{m} < x < \dfrac{16}{3}\ \text{m}$

$+\uparrow \Sigma F_y = 0; \qquad V = 0$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad M = 500\ \text{N} \cdot \text{m}$ **Ans.**

For $\dfrac{16}{3}\ \text{m} < x \le 8\ \text{m}$

$+\uparrow \Sigma F_y = 0; \qquad V = 0$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad M = 0$ **Ans.**

Ans:

$0 \le x < \dfrac{L}{3}$: $V = 0, M = 0$

$\dfrac{L}{3} < x < \dfrac{2L}{3}$: $V = 0, M = M_0$

$\dfrac{2L}{3} < x \le L$: $V = 0, M = 0$

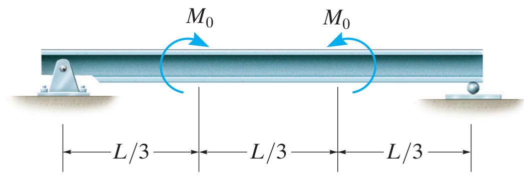
$0 \le x < \dfrac{8}{3}$ m: $V = 0, M = 0$

$\dfrac{8}{3}$ m $< x < \dfrac{16}{3}$ m: $V = 0, M = 500\ \text{N} \cdot \text{m}$

$\dfrac{16}{3}$ m $< x \le 8$ m: $V = 0, M = 0$

7–47.

If $L = 9$ m, the beam will fail when the maximum shear force is $V_{max} = 5$ kN or the maximum bending moment is $M_{max} = 2$ kN·m. Determine the magnitude M_0 of the largest couple moments it will support.



SOLUTION

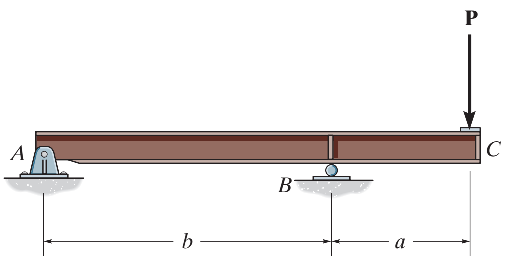
See solution to Prob. 7–48 a.

$M_{max} = M_0 = 2$ kN·m **Ans.**

Ans:
$M_{max} = 2$ kN·m

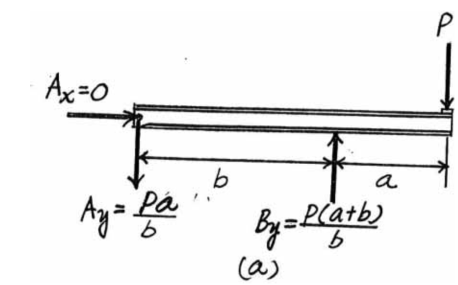
***7–48.**

Draw the shear and moment diagrams for the overhang beam.



SOLUTION

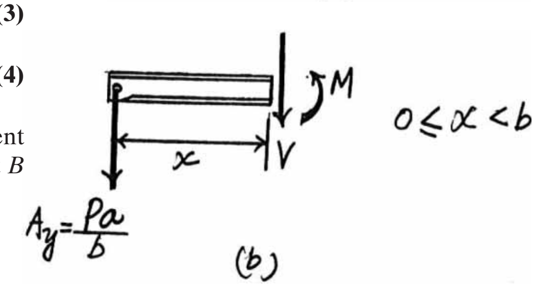
Since the loading discontinues at B, the shear stress and moment equation must be written for regions $0 \le x < b$ and $b < x \le a + b$ of the beam. The free-body diagram of the beam's segment sectioned through an arbitrary point in these two regions are shown in Figs. *b* and *c*.



Region $0 \le x < b$, Fig. *b*

$$+\uparrow \Sigma F_y = 0; \qquad -\frac{Pa}{b} - V = 0 \qquad V = -\frac{Pa}{b} \qquad \textbf{(1)}$$

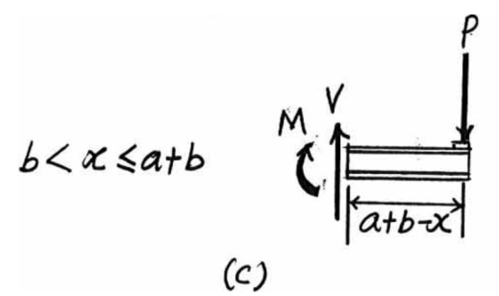
$$\circlearrowleft + \Sigma M = 0; \qquad M + \frac{Pa}{b}x = 0 \qquad M = -\frac{Pa}{b}x \qquad \textbf{(2)}$$



Region $b < x \le a + b$, Fig. *c*

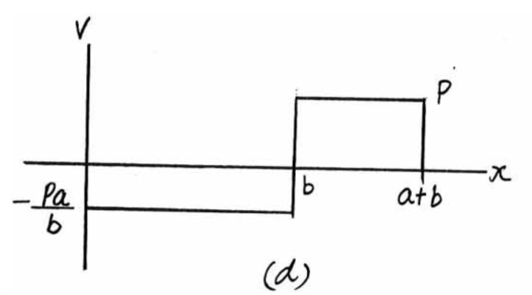
$$\Sigma F_y = 0; \qquad V - P = 0 \qquad V = P \qquad \textbf{(3)}$$

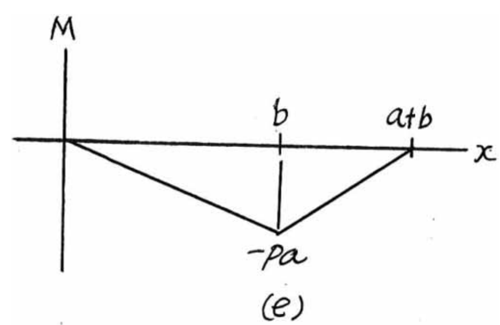
$$\circlearrowleft + \Sigma M = 0; \qquad -M - P(a + b - x) = 0 \qquad M = -P(a + b - x) \qquad \textbf{(4)}$$



The shear diagram in Fig. *d* is plotted using Eqs. (1) and (3), while the moment diagram shown in Fig. *e* is plotted using Eqs. (2) and (4). The value of moment at B is evaluated using either Eqs. (2) or (4) by substituting $x = b$; i.e.,

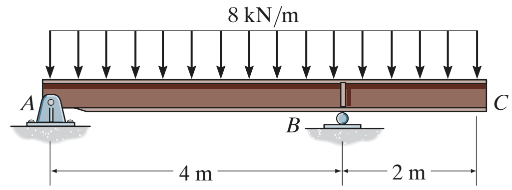
$$M|_{x=b} = -\frac{Pa}{b}(b) = -Pa \text{ or } M|_{x=b} = -P(a + b - b) = -Pa$$





7–49.

Draw the shear and moment diagrams for the overhang beam.



SOLUTION

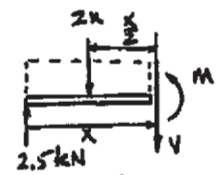
$0 \le x < 5$ m:

$+\uparrow \Sigma F_y = 0; \qquad 2.5 - 2x - V = 0$

$V = 2.5 - 2x$

$\zeta + \Sigma M = 0; \qquad M + 2x\left(\frac{1}{2}x\right) - 2.5x = 0$

$M = 2.5x - x^2$



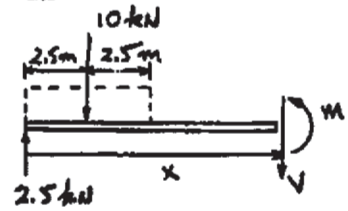
$5 \le x < 10$ m:

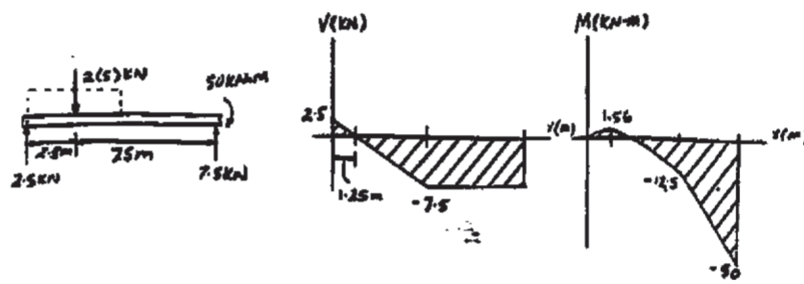
$+\uparrow \Sigma F_y = 0; \qquad 2.5 - 10 - V = 0$

$V = -7.5$

$\zeta + \Sigma M = 0; \qquad M + 10(x - 2.5) - 2.5x = 0$

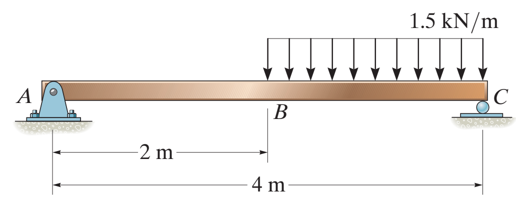
$M = -7.5x - 25$





7–50.

Draw the shear and moment diagrams for the beam.

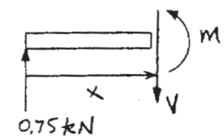


SOLUTION

$0 \leq x \leq 2$ m:

$+\uparrow \Sigma F_y = 0; \qquad 0.75 - V = 0$

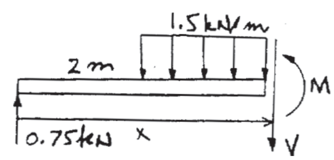
$V = 0.75 \text{ kN}$ **Ans.**



$\zeta + \Sigma M = 0; \qquad M - 0.75x = 0$

$M = 0.75\,x \text{ kN}\cdot\text{m}$ **Ans.**

$2 \text{ m} < x < 4$ m:

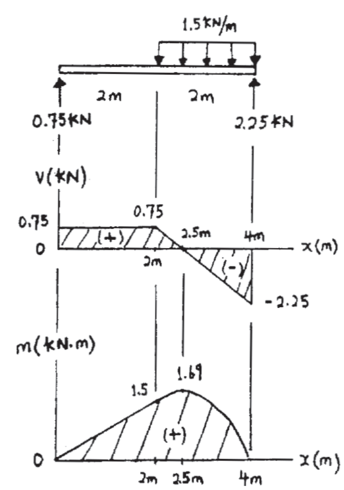


$+\uparrow \Sigma F_y = 0; \qquad 0.75 - 1.5\,(x - 2) - V = 0$

$V = 3.75 - 1.5\,x \text{ kN}$ **Ans.**

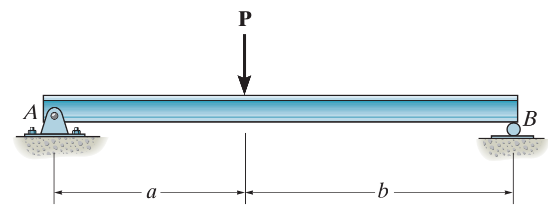
$\zeta + \Sigma M = 0; \qquad M + \dfrac{1.5}{2}(x - 2)^2 - 0.75\,x = 0$

$M = -0.75\,x^2 + 3.75\,x - 3 \text{ kN}\cdot\text{m}$ **Ans.**



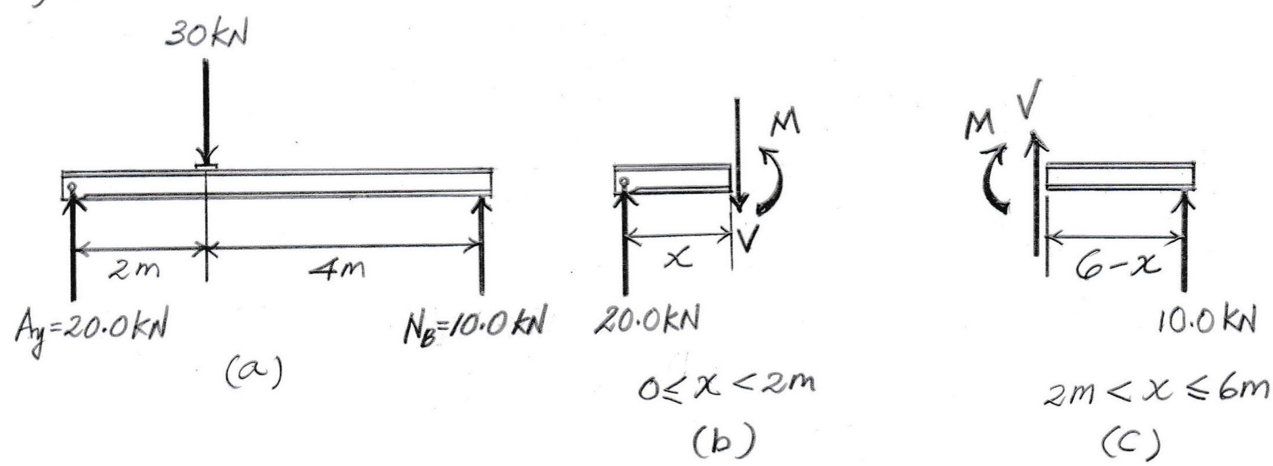
Ans:
$V = 0.75$ kN
$M = 0.75\,x \text{ kN}\cdot\text{m}$
$V = 3.75 - 1.5\,x$ kN
$M = -0.75x^2 + 3.75x - 3 \text{ kN}\cdot\text{m}$

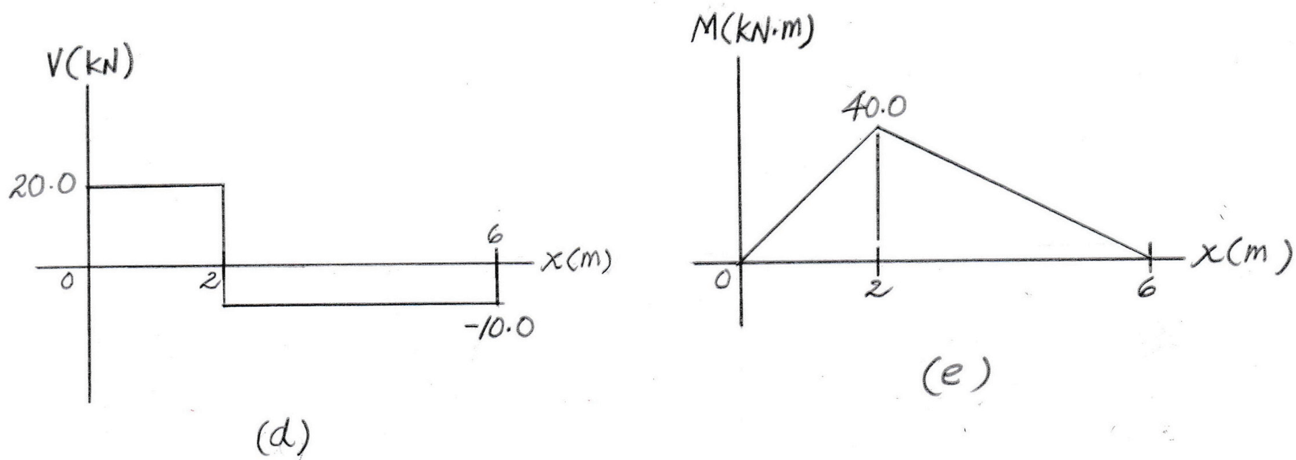
7–51. Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set $P = 30$ kN, $a = 2$ m, $b = 4$ m.



SOLUTION

b)





7–51. Continued

The support reactions are indicated on the *FBD* of the beam, Fig. *a*. Since the loading discontinues, the shear and moment functions must be written for regions $0 \leq x < 2$ m and $2 \text{ m} < x \leq 6$ m of the beam. The *FBD* of the beam's segment sectioned through an arbitrary point in these two regions are shown in Fig. *b* and *c*.

For region $0 \leq x < 2$ m, Fig. *b*,

$+\uparrow \Sigma F_y = 0; \qquad 20.0 - V = 0 \qquad V = 20.0 \text{ kN}$ **(1)**

$\circlearrowleft + \Sigma M = 0; \qquad M - 20.0x = 0 \qquad M = \{20.0x\} \text{ kN} \cdot \text{m}$ **(2)**

For region $2 \text{ m} < x \leq 6$ m, Fig. *c*,

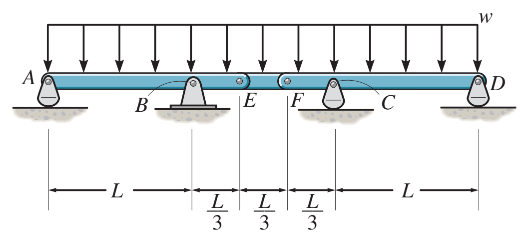
$+\uparrow \Sigma F_y = 0; \qquad V + 10.0 = 0 \qquad V = -10.0 \text{ kN}$ **(3)**

$\circlearrowleft + \Sigma M = 0; \qquad 10.0(6 - x) - M = 0 \qquad M = \{60.0 - 10.0x\} \text{ kN} \cdot \text{m}$ **(4)**

The shear diagram shown in Fig. *d* is plotted using Eqs. (1) and (3), while the moment diagram shown in Fig. *e* is plotted using Eqs. (2) and (4). The value of moment at $x = 2$ m can be evaluated using either Eqs. (2) or (4).

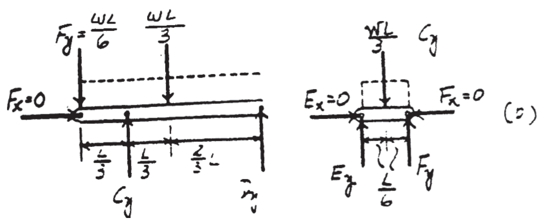
***7–52.**

Draw the shear and moment diagrams for the compound beam. The beam is pin-connected at E and F.



SOLUTION

Support Reactions: From FBD (b),



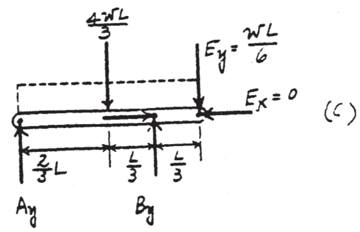
$$\circlearrowleft + \Sigma M_E = 0; \qquad F_y\left(\frac{L}{3}\right) - \frac{wL}{3}\left(\frac{L}{6}\right) = 0 \qquad F_y = \frac{wL}{6}$$

$$+\uparrow \Sigma F_y = 0; \qquad E_y + \frac{wL}{6} - \frac{wL}{3} = 0 \qquad E_y = \frac{wL}{6}$$

From FBD (a),

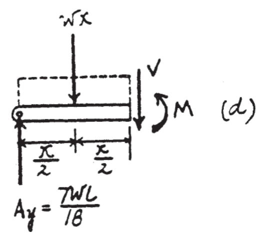
$$\circlearrowleft + \Sigma M_C = 0; \qquad D_y(L) + \frac{wL}{6}\left(\frac{L}{3}\right) - \frac{4wL}{3}\left(\frac{L}{3}\right) = 0 \qquad D_y = \frac{7wL}{18}$$

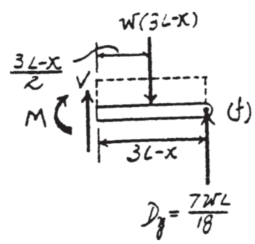
From FBD (c),



$$\circlearrowleft + \Sigma M_B = 0; \qquad \frac{4wL}{3}\left(\frac{L}{3}\right) - \frac{wL}{6}\left(\frac{L}{3}\right) - A_y(L) = 0 \qquad A_y = \frac{7wL}{18}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y + \frac{7wL}{18} - \frac{4wL}{3} - \frac{wL}{6} = 0 \qquad B_y = \frac{10wL}{9}$$





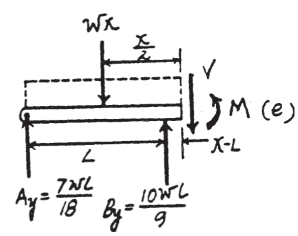
Shear and Moment Functions: For $0 \le x < L$ [FBD (d)],

$$+\uparrow \Sigma F_y = 0; \qquad \frac{7wL}{18} - wx - V = 0$$

$$V = \frac{w}{18}(7L - 18x) \qquad \textbf{Ans.}$$

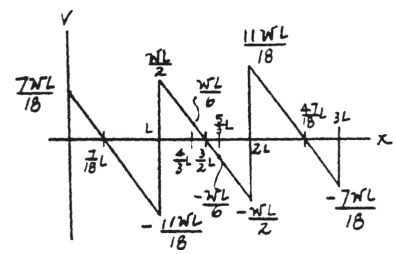
$$\circlearrowleft + \Sigma M = 0; \qquad M + wx\left(\frac{x}{2}\right) - \frac{7wL}{18}x = 0$$

$$M = \frac{w}{18}(7Lx - 9x^2) \qquad \textbf{Ans.}$$



7–52. Continued

For $\boldsymbol{L \le x < 2L}$ [FBD (e)],



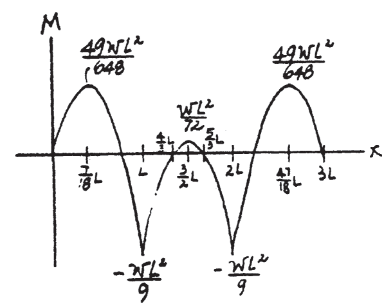
$$+\uparrow \Sigma F_y = 0; \qquad \frac{7wL}{18} + \frac{10wL}{9} - wx - V = 0$$

$$V = \frac{w}{2}(3L - 2x) \qquad \textbf{Ans.}$$

$$\circlearrowleft +\Sigma M = 0; \qquad M + wx\left(\frac{x}{2}\right) - \frac{7wL}{18}x - \frac{10wL}{9}(x - L) = 0$$

$$M = \frac{w}{18}(27Lx - 20L^2 - 9x^2) \qquad \textbf{Ans.}$$

For $\boldsymbol{2L < x \le 3L}$ [FBD (f)],



$$+\uparrow \Sigma F_y = 0; \qquad V + \frac{7wL}{18} - w(3L - x) = 0$$

$$V = \frac{w}{18}(47L - 18x) \qquad \textbf{Ans.}$$

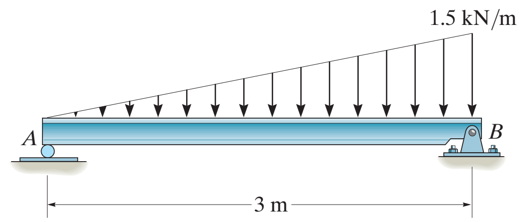
$$\circlearrowleft +\Sigma M = 0; \quad \frac{7wL}{18}(3L - x) - w(3L - x)\left(\frac{3L - x}{2}\right) - M = 0$$

$$M = \frac{w}{18}(47Lx - 9x^2 - 60L^2) \qquad \textbf{Ans.}$$

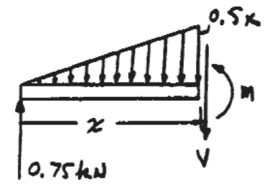
Ans:
For $0 \le x < L$
$$V = \frac{w}{18}(7L - 18x)$$
$$M = \frac{w}{18}\left(7Lx - 9x^2\right)$$
For $L < x < 2L$
$$V = \frac{w}{2}(3L - 2x)$$
$$M = \frac{w}{18}\left(27Lx - 20L^2 - 9x^2\right)$$
For $2L < x \le 3L$
$$V = \frac{w}{18}(47L - 18x)$$
$$M = \frac{w}{18}\left(47Lx - 9x^2 - 60L^2\right)$$

7–53.

Draw the shear and moment diagrams for the beam.



SOLUTION



$+\uparrow \Sigma F_y = 0; \qquad 0.75 - \frac{1}{2}x\,(0.5x) - V = 0$

$$V = 0.75 - 0.25x^2$$

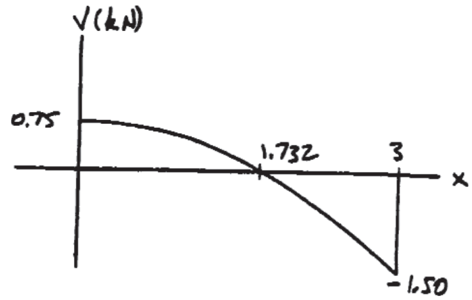
$$V = 0 = 0.75 - 0.25x^2$$

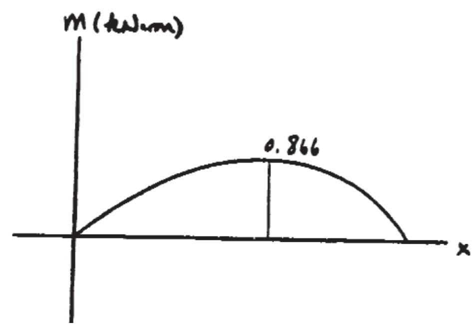
$$x = 1.732 \text{ m}$$

$\zeta +\Sigma M = 0; \qquad M + \left(\frac{1}{2}\right)(0.5\,x)\,(x)\left(\frac{1}{3}x\right) - 0.75\,x = 0$

$$M = 0.75\,x - 0.08333\,x^3$$

$$M_{max} = 0.75(1.732) - 0.08333(1.732)^3 = 0.866$$

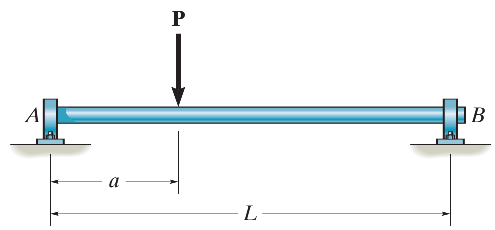




Ans:
$x = 1.732$ m
$M_{max} = 0.75(1.732) - 0.08333(1.732)^3 = 0.866$

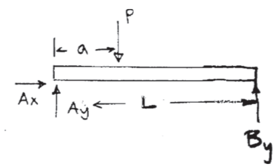
7–54.

Draw the shear and moment diagrams for the shaft (a) in terms of the parameters shown; (b) set $P = 9$ kN, $a = 2$ m, $L = 6$ m. There is a thrust bearing at A and a journal bearing at B.



SOLUTION

(a)



$\circlearrowleft + \Sigma M_B = 0; \quad (A_y)(L) - P(L - a) = 0$

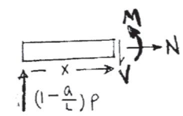
$$A_y = \left(\frac{L - a}{L}\right)P$$

$$A_y = \left(1 - \frac{a}{L}\right)P$$

$+\uparrow \Sigma F_y = 0; \quad A_y + B_y - P = 0$

$$B_y = P - A_y = \left(\frac{a}{L}\right)P$$

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$

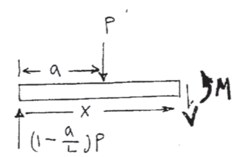


For $0 \le x \le a$

$+\uparrow \Sigma F_y = 0; \quad \left(1 - \frac{a}{L}\right)P - V = 0$

$$V = \left(1 - \frac{a}{L}\right)P \qquad \textbf{Ans.}$$

$\xrightarrow{+} \Sigma F_x = 0; \quad A = 0$



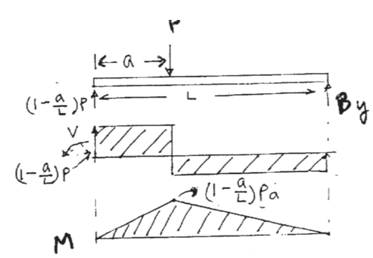
$\circlearrowleft + \Sigma M = 0; \quad \left(1 - \frac{a}{L}\right)Px - M = 0$

$$M = \left(1 - \frac{a}{L}\right)Px \qquad \textbf{Ans.}$$

For $a < x < L$

$+\uparrow \Sigma F_y = 0; \quad \left(1 - \frac{a}{L}\right)P - P - V = 0$

$$V = -\left(\frac{a}{L}\right)P \qquad \textbf{Ans.}$$



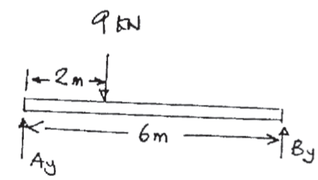
$\circlearrowleft + \Sigma M = 0; \quad \left(1 - \frac{a}{L}\right)Px - P(x - a) - M = 0$

$$M = Px - \left(\frac{a}{L}\right)Px - Px + Pa$$

7–54. Continued

(b)

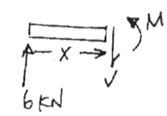
$$M = P\left(a - \frac{a}{L}x\right) \qquad \textbf{Ans.}$$



$\circlearrowleft + \Sigma M_B = 0;\quad A_y(6) - 9(4) = 0$

$A_y = 6\text{ kN}$

$+\uparrow \Sigma F_y = 0;\quad B_y = 3\text{ kN}$

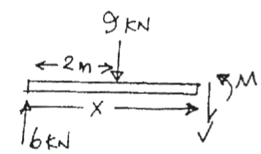


For $0 \leq x \leq 2$ m

$+\uparrow \Sigma F_y = 0;\quad 6 - V = 0$

$V = 6\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M = 0;\quad 6x - M = 0$



$M = 6x\text{ kN}\cdot\text{m}$ **Ans.**

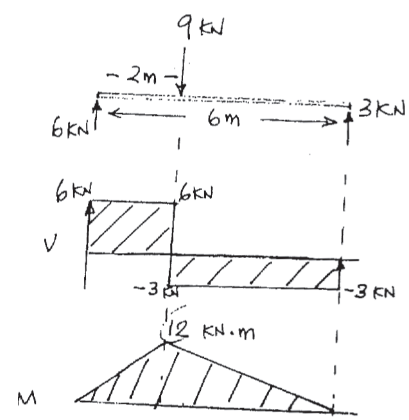
For $2\text{ m} < x \leq 6$ m

$+\uparrow \Sigma F_y = 0;\quad 6 - 9 - V = 0$

$V = -3\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M = 0;\quad 6x - 9(x - 2) - M = 0$

$M = 18 - 3x\text{ kN}\cdot\text{m}$ **Ans.**



Ans:

$0 \leq x < a$: $V = \left(1 - \frac{a}{L}\right)P$

$M = \left(1 - \frac{a}{L}\right)Px$

$a < x \leq L$: $V = -\left(\frac{a}{L}\right)P$

$M = P\left(a - \frac{a}{L}x\right)$

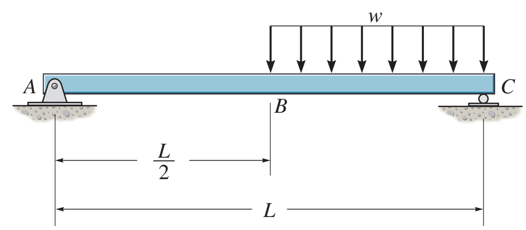
$0 \leq x < 2$ m: $V = 6$ kN, $M = \{6x\}$ kN·m

$2\text{ m} < x \leq 6$ m: $V = -3$ kN

$M = \{18 - 3x\}$ kN·m

7–55.

Draw the shear and moment diagrams for the beam.

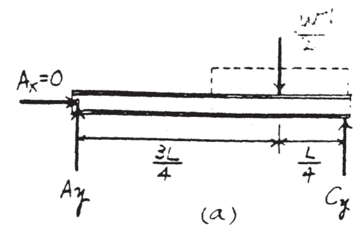


SOLUTION

Support Reactions: From FBD (a),

$$\circlearrowleft + \Sigma M_A = 0; \quad C_y(L) - \frac{wL}{2}\left(\frac{3L}{4}\right) = 0 \qquad C_y = \frac{3wL}{8}$$

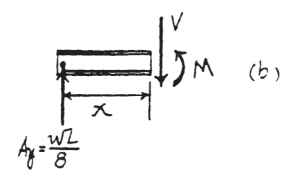
$$+\uparrow \Sigma F_y = 0; \qquad A_y + \frac{3wL}{8} - \frac{wL}{2} = 0 \qquad A_y = \frac{wL}{8}$$



Shear and Moment Functions: For $\mathbf{0 \le x < \frac{L}{2}}$ [FBD (b)],

$$+\uparrow \Sigma F_y = 0; \qquad \frac{wL}{8} - V = 0 \qquad V = \frac{wL}{8}$$ **Ans.**

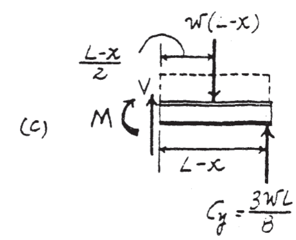
$$\circlearrowleft + \Sigma M = 0; \qquad M - \frac{wL}{8}(x) = 0 \qquad M = \frac{wL}{8}x$$ **Ans.**



For $\mathbf{\frac{L}{2} < x \le L}$ [FBD (c)],

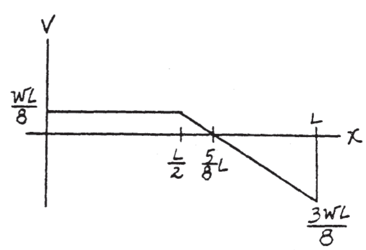
$$+\uparrow \Sigma F_y = 0; \qquad V + \frac{3wL}{8} - w(L - x) = 0$$

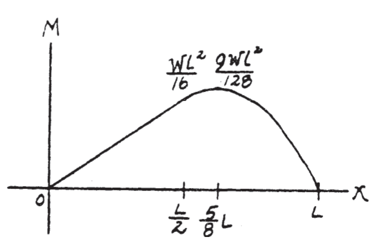
$$V = \frac{w}{8}(5L - 8x)$$ **Ans.**



$$\circlearrowleft + \Sigma M = 0; \qquad \frac{3wL}{8}(L - x) - w(L - x)\left(\frac{L - x}{2}\right) - M = 0$$

$$M = \frac{w}{8}(-L^2 + 5Lx - 4x^2)$$ **Ans.**





Ans:

$V = \frac{wL}{8}$

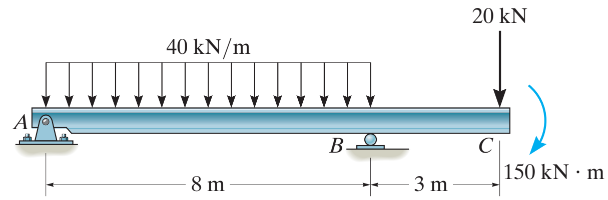
$M = \frac{wL}{8}x$

$V = \frac{w}{8}(5L - 8x)$

$M = \frac{w}{8}(-L^2 + 5Lx - 4x^2)$

***7–56.**

Draw the shear and moment diagrams for the beam.



SOLUTION

$0 \le x < 8$

$+\uparrow \Sigma F_y = 0; \quad 133.75 - 40x - V = 0$

$V = 133.75 - 40x$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \quad M + 40x\left(\frac{x}{2}\right) - 133.75x = 0$

$M = 133.75x - 20x^2$ **Ans.**

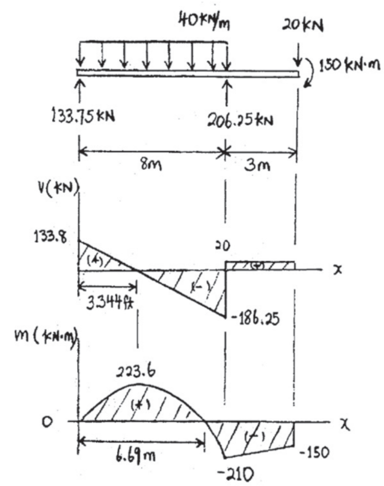
$8 < x \le 11$

$+\uparrow \Sigma F_y = 0; \quad V - 20 = 0$

$V = 20$ **Ans.**

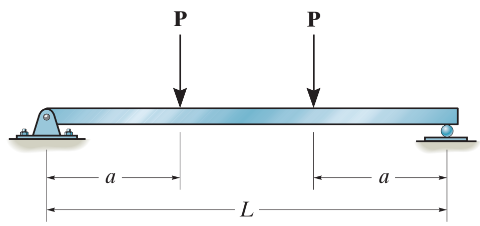
$\circlearrowleft + \Sigma M = 0; \quad M + 20(11 - x) + 150 = 0$

$M = 20x - 370$ **Ans.**

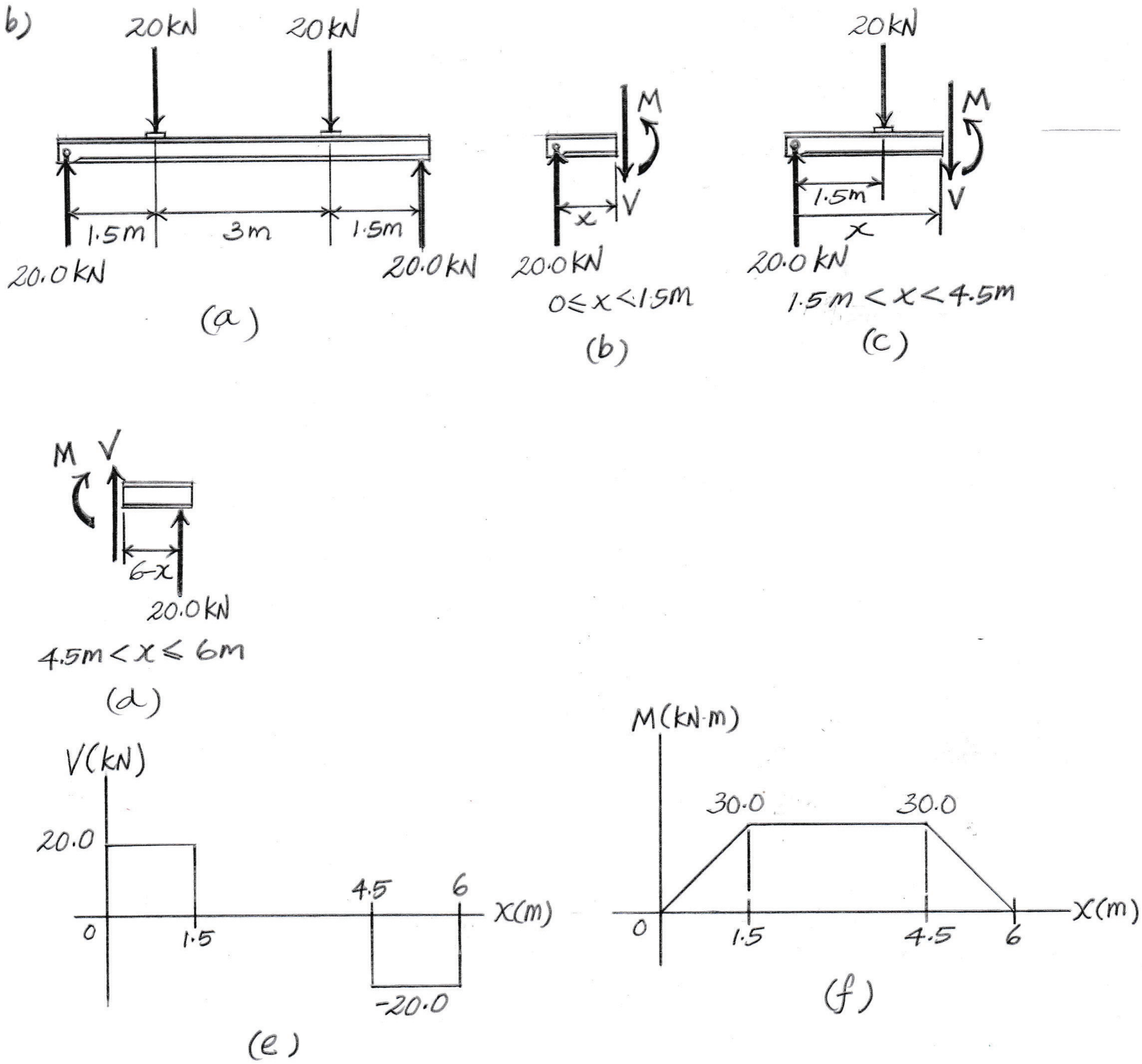


Ans:
For $0 \le x < 8$ m
$V = (133.75 - 40x)$ kN
$M = (133.75x - 20x^2)$ kN·m
For 8 m $< x \le 11$ m
$V = 20$ kN
$M = (20x - 370)$ kN·m

7–57. Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set $P = 20\text{ kN}$, $a = 1.5\text{ m}$, $L = 6\text{ m}$.



SOLUTION



7–57. Continued

The support reactions are indicated on the *FBD* of the beam, Fig. *a*. Since the loading discontinue, the shear and moment functions must be written for regions $0 \leq x < 1.5$ m, $1.5 < x < 4.5$ m and 4.5 m $< x \leq 6$ m of the beam. The *FBD* of the beam's segment sectioned through an arbitrary point in these two regions are shown in Fig. *b*, *c* and *d*.

For region $0 \leq x < 1.5$ m, Fig. *b*

$$+\uparrow \Sigma F_y = 0; \qquad 20.0 - V = 0 \qquad V = \{20.0\}\ \text{kN} \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M - 20.0x = 0 \qquad M = \{20.0x\}\ \text{kN} \cdot \text{m} \qquad \textbf{(2)}$$

For region $1.5 < x < 4.5$ m, Fig. *c*

$$+\uparrow \Sigma F_y = 0; \qquad 20.0 - 20 - V = 0 \qquad V = 0 \qquad \textbf{(3)}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M + 20(x - 1.5) - 20.0x = 0 \qquad M = \{30.0\}\ \text{kN} \cdot \text{m} \qquad \textbf{(4)}$$

For region 4.5 m $< x \leq 6$ m, Fig. *d*

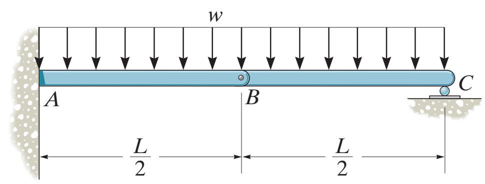
$$+\uparrow \Sigma F_y = 0; \qquad V + 20.0 = 0 \qquad V = \{-20.0\}\ \text{kN} \qquad \textbf{(5)}$$

$$\circlearrowleft + \Sigma M = 0; \qquad 20.0(6 - x) - M = 0 \qquad M = \{120 - 20.0x\}\ \text{kN} \cdot \text{m} \qquad \textbf{(6)}$$

The shear diagram shown in Fig. *e* is plotted using Eqs. (1), (3) and (5), while the moment diagram shown in Fig. *f* is plotted using Eqs. (2), (4) and (6). The values of moment at $x = 1.5$ m and $x = 4.5$ m can be evaluated using Eqs. (2) and (6) respectively.

7–58.

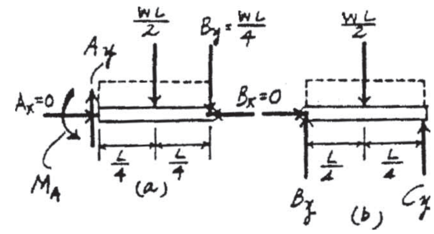
Draw the shear and bending-moment diagrams for beam ABC. Note that there is a pin at B.



SOLUTION

Support Reactions: From FBD (a),

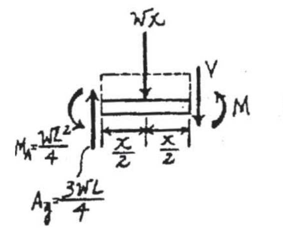
$$\zeta + \Sigma M_C = 0; \qquad \frac{wL}{2}\left(\frac{L}{4}\right) - B_y\left(\frac{L}{2}\right) = 0 \qquad B_y = \frac{wL}{4}$$



From FBD (b),

$$+\uparrow \Sigma F_y = 0; \qquad A_y - \frac{wL}{2} - \frac{wL}{4} = 0 \qquad A_y = \frac{3wL}{4}$$

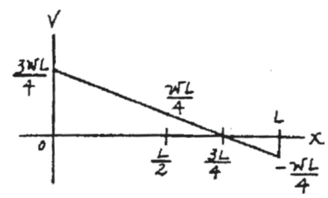
$$\zeta + \Sigma M_A = 0; \qquad M_A - \frac{wL}{2}\left(\frac{L}{4}\right) - \frac{wL}{4}\left(\frac{L}{2}\right) = 0 \qquad M_A = \frac{wL^2}{4}$$



Shear and Moment Functions: For $0 \leq x \leq L$ [FBD (c)],

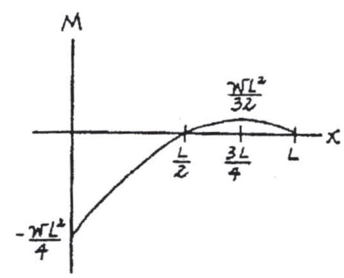
$$+\uparrow \Sigma F_y = 0; \qquad \frac{3wL}{4} - wx - V = 0$$

$$V = \frac{w}{4}(3L - 4x) \qquad \textbf{Ans.}$$



$$\zeta + \Sigma M = 0; \qquad \frac{3wL}{4}(x) - wx\left(\frac{x}{2}\right) - \frac{wL^2}{4} - M = 0$$

$$M = \frac{w}{4}\left(3Lx - 2x^2 - L^2\right) \qquad \textbf{Ans.}$$

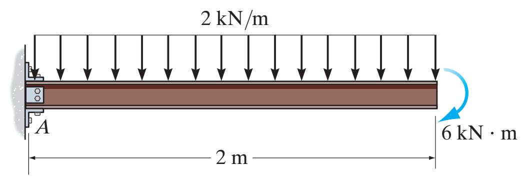


Ans:

$V = \frac{w}{4}(3L - 4x)$

$M = \frac{w}{4}\left(3Lx - 2x^2 - L^2\right)$

7–59. Draw the shear and moment diagrams for the cantilever beam.



SOLUTION

The free - body diagram of the beam's right segment sectioned through an arbitrary point shown in Fig. *a* will be used to write the shear and moment equations of the beam.

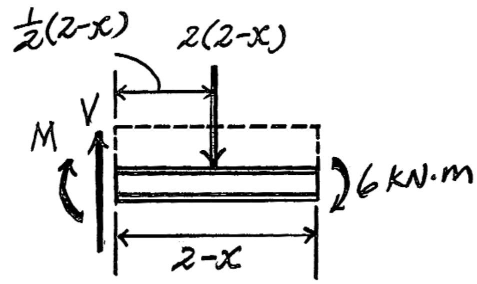
$$+\uparrow \Sigma F_y = 0; \quad V - 2(2-x) = 0 \quad V = \{4 - 2x\}\,\text{kN} \qquad \textbf{(1)}$$

$$+\Sigma M = 0; \quad -M - 2(2-x)\left[\frac{1}{2}(2-x)\right] - 6 = 0 \quad M = \{-x^2 + 4x - 10\}\,\text{kN}\cdot\text{m} \qquad \textbf{(2)}$$

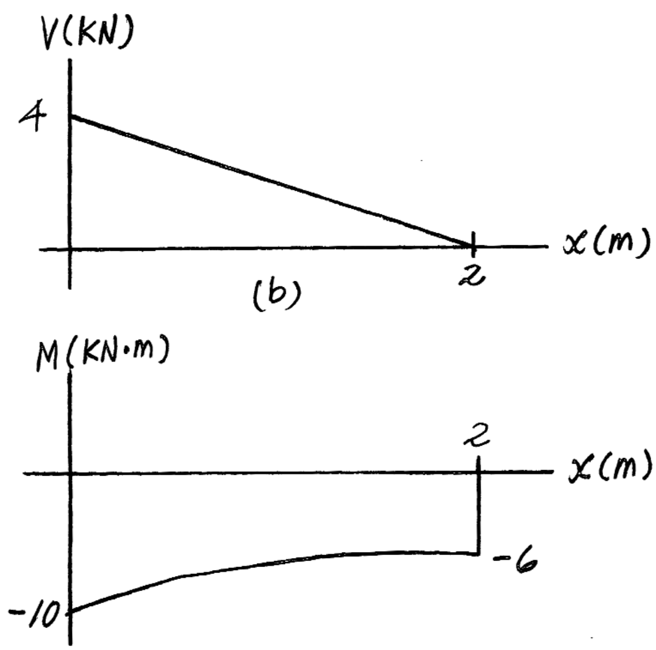
The shear and moment diagrams shown in Figs. *b* and *c* are plotted using Eqs. (1) and (2), respectively. The value of the shear and moment at $x = 0$ is evaluated using Eqs. (1) and (2).

$$V|_{x=0} = 4 - 2(0) = 4\,\text{kN}$$

$$M|_{x=0} = [-0 + 4(0) - 10] = -10\,\text{kN}\cdot\text{m}$$



(a)



(b)

(c)

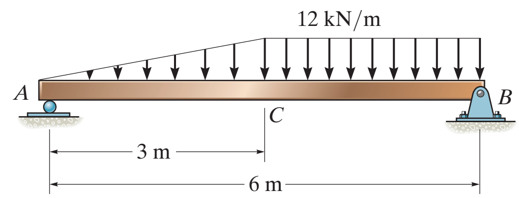
Ans:

$V = (4 - 2x)\,\text{kN}$

$M = (-x^2 + 4x - 10)\,\text{kN}\cdot\text{m}$

***7–60.**

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions. Referring to the *FBD* of the entire beam shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_B = 0; \quad 12(3)(1.5) + \frac{1}{2}(12)(3)(4) - A_y(6) = 0 \qquad A_y = 21.0 \text{ kN}$$

$$\circlearrowleft + \Sigma M_A = 0; \quad B_y(6) - \frac{1}{2}(12)(3)(2) - 12(3)(4.5) = 0 \qquad B_y = 33.0 \text{ kN}$$

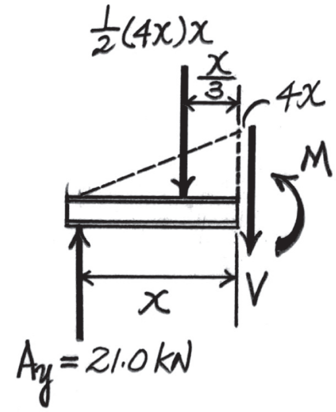
$$\xrightarrow{+} \Sigma F_x = 0; \quad B_x = 0$$

Shear And Moment Functions. The beam will be sectioned at two arbitrary distances x in region AC ($0 \leq x$ 3 m) and region CB (3 m $< x \leq 6$ m).
For region $0 \leq x < 3$ m, Fig. *b*

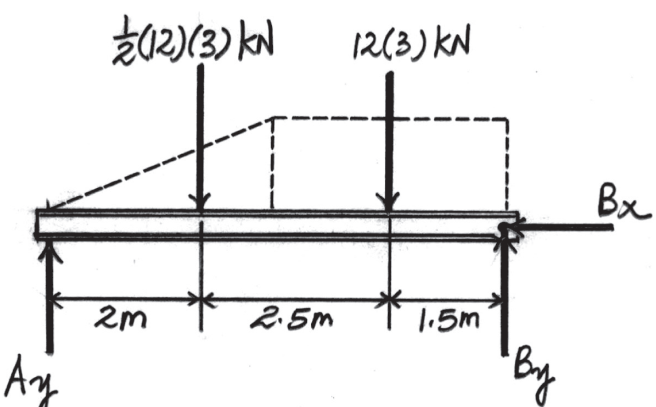
$$+\uparrow \Sigma F_y = 0; \quad 21.0 - \frac{1}{2}(4x)(x) - V = 0 \quad V = \left\{21.0 - 2x^2\right\} \text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_O = 0 \quad M + \left[\frac{1}{2}(4x)(x)\right]\left(\frac{x}{3}\right) - 21.0x = 0$$

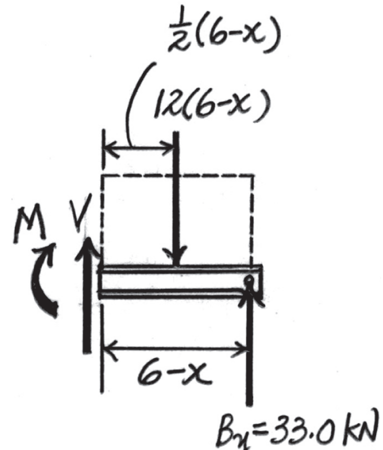
$$M = \left\{21.0x - \frac{2}{3}x^3\right\} \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$



(b)



(a)



(c)

7–60. Continued

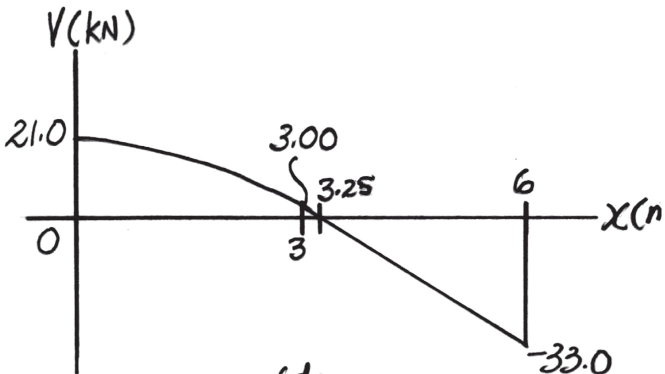
For region $3\text{ m} < x \leq 6\text{ m}$, Fig. *c*

$+\uparrow \Sigma F_y = 0; \qquad V + 33.0 - 12(6 - x) = 0 \qquad V = \{39.0 - 12x\}\text{ kN}$ **Ans.**

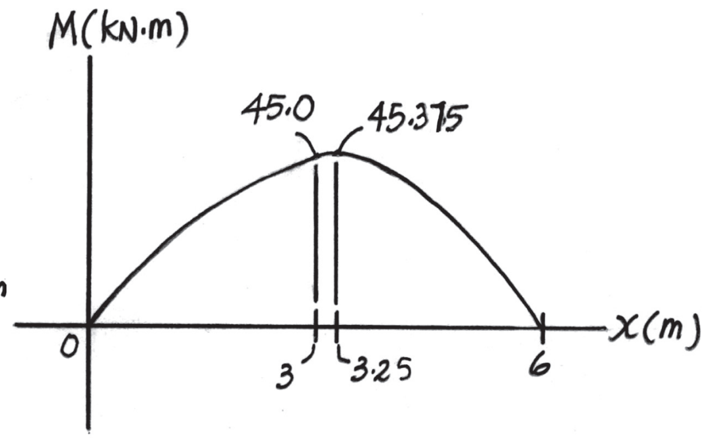
$\circlearrowleft + \Sigma M_O = 0 \qquad 33.0(6 - x) - [12(6 - x)]\left[\frac{1}{2}(6 - x)\right] - M = 0$

$$M = \left\{-6x^2 + 39x - 18\right\}\text{ kN}\cdot\text{m}$$ **Ans.**

Plotting the shear and moment functions obtained, the shear and moment diagram shown in Fig. *d* and *e* resulted.



(d)

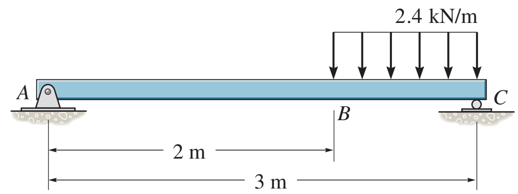


(e)

Ans:
For $0 \leq x < 3\text{ m}$
$V = \left\{21.0 - 2x^2\right\}\text{ kN}$
$M = \left\{21.0x - \frac{2}{3}x^3\right\}\text{ kN}\cdot\text{m}$
For $3\text{ m} < x \leq 6\text{ m}$
$V = \{39.0 - 12x\}\text{ kN}$
$M = \left\{-6x^2 + 39x - 18\right\}\text{ kN}\cdot\text{m}$

7–61.

Draw the shear and moment diagrams for the beam.

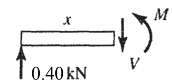


SOLUTION

Support Reactions:

$\zeta + \Sigma M_A = 0; \qquad C_y(3) - 2.4(2.5) = 0 \qquad C_y = 2.00\text{ kN}$

$+\uparrow \Sigma F_y = 0; \qquad A_y - 2.4 + 2.00 = 0 \qquad A_y = 0.40\text{ kN}$

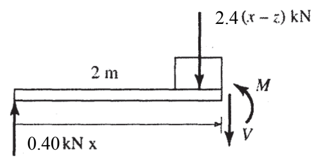


(a)

Shear and Moment Functions: For $\mathbf{0 \le x < 2\text{ m}}$ [FBD (a)],

$+\uparrow \Sigma F_y = 0; \qquad 0.4 - V = 0 \qquad V = 0.4\text{ kN}$ **Ans.**

$\zeta + \Sigma M = 0; \qquad M - 0.4x = 0 \qquad M = (0.4x)\text{ kN}\cdot\text{m}$ **Ans.**



(b)

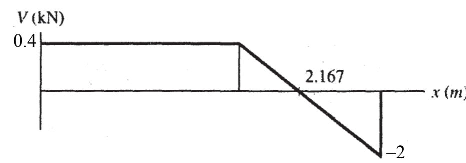
For $\mathbf{2\text{ m} < x \le 3\text{ m}}$ [FBD (b)],

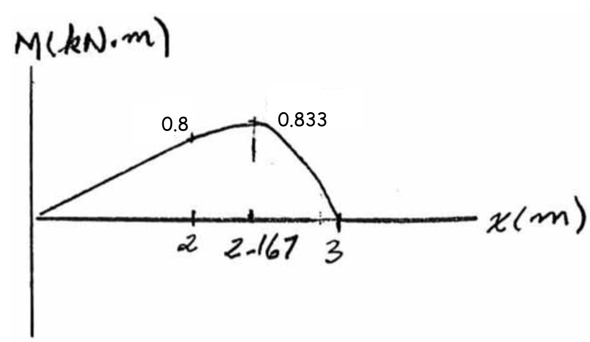
$+\uparrow \Sigma F_y = 0; \qquad 0.4 - 2.4(x - 2) - V = 0$

$V = \{5.20 - 2.40x\}\text{ kN}$ **Ans.**

$\zeta + \Sigma M = 0; \qquad 0.4x - 2.4(x - 2)\left(\dfrac{x - 2}{2}\right) - M = 0$

$M = \{-1.2x^2 + 5.2\,x - 4.8\}\text{ kN}\cdot\text{m}$ **Ans.**



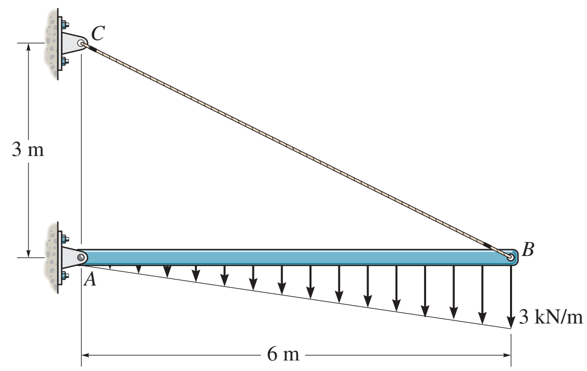


Ans:

$V = 0.4\text{ kN}$
$M = \{0.4x\}\text{ kN}\cdot\text{m}$
$V = \{5.20 - 2.40x\}\text{ kN}$
$M = \{-1.2x^2 + 5.2x - 4.8\}\text{ kN}\cdot\text{m}$

7–62.

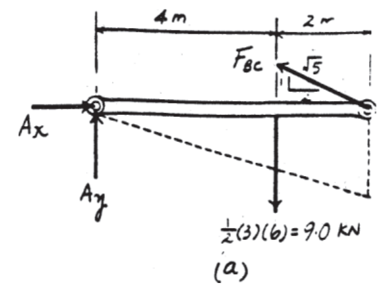
Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions: From FBD (a),

$\zeta + \Sigma M_B = 0; \quad 9.00(2) - A_y(6) = 0 \qquad A_y = 3.00 \text{ kN}$



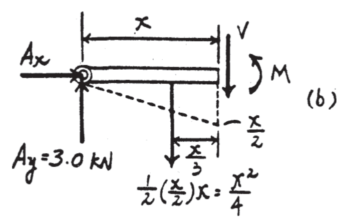
Shear and Moment Functions: For $0 \le x \le 6$ m [FBD (b)],

$$+\uparrow \Sigma F_y = 0; \qquad 3.00 - \frac{x^2}{4} - V = 0$$

$$V = \left\{3.00 - \frac{x^2}{4}\right\} \text{kN} \qquad \textbf{Ans.}$$

The maximum moment occurs when $V = 0$, then

$$0 = 3.00 - \frac{x^2}{4} \qquad x = 3.464 \text{ m}$$

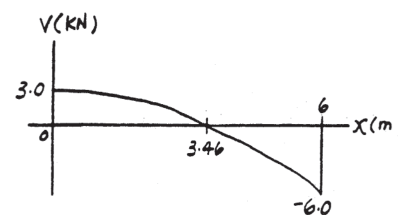


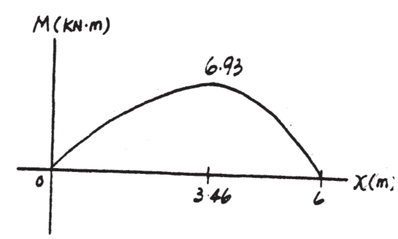
$$\zeta + \Sigma M = 0; \qquad M + \left(\frac{x^2}{4}\right)\left(\frac{x}{3}\right) - 3.00x = 0$$

$$M = \left\{3.00x - \frac{x^3}{12}\right\} \text{kN} \cdot \text{m} \qquad \textbf{Ans.}$$

Thus,

$$M_{\max} = 3.00(3.464) - \frac{3.464^3}{12} = 6.93 \text{ kN} \cdot \text{m}$$





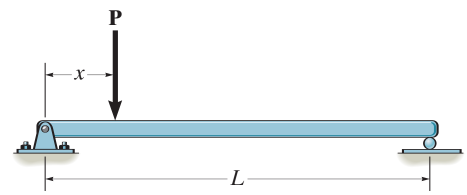
Ans:

$$V = \left\{3.00 - \frac{x^2}{4}\right\} \text{kN}$$

$$M = \left\{3.00x - \frac{x^3}{12}\right\} \text{kN} \cdot \text{m}$$

7–63.

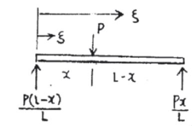
The beam will fail when the maximum internal moment is $M_{\max}$. Determine the position x of the concentrated force **P** and its smallest magnitude that will cause failure.



SOLUTION

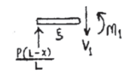
For $\xi < x$,

$$M_1 = \frac{P\xi(L - x)}{L}$$



For $\xi > x$,

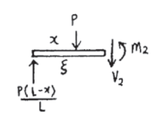
$$M_2 = -\frac{Px}{L}(L - \xi)$$



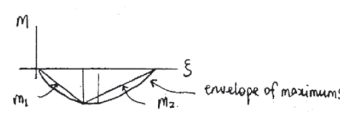
Note that $M_1 = M_2$ when $x = \xi$

$$M_{max} = M_1 = M_2 = \frac{Px}{L}(L - x)$$

$$\frac{dM_{max}}{dx} = \frac{P}{L}(L - 2x) = 0$$



$$x = \frac{L}{2}$$ **Ans.**



Thus, $M_{max} = \frac{P}{L}\left(\frac{L}{2}\right)\left(L - \frac{L}{2}\right) = \frac{P}{2}\left(\frac{L}{2}\right)$

$$P = \frac{4M_{max}}{L}$$ **Ans.**

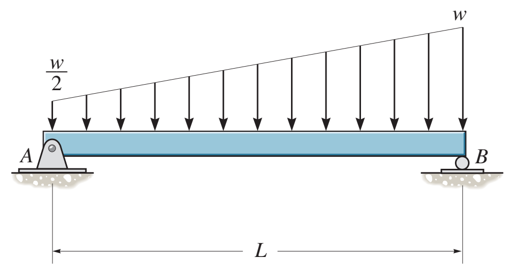
Ans:

$x = \frac{L}{2}$

$P = \frac{4M_{\max}}{L}$

***7–64.**

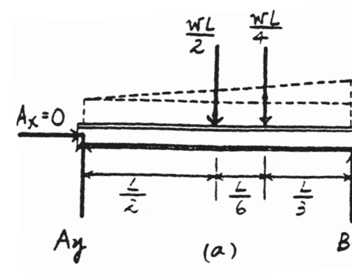
Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions: From FBD (a),

$$\zeta + \Sigma M_B = 0; \quad \frac{wL}{4}\left(\frac{L}{3}\right) + \frac{wL}{2}\left(\frac{L}{2}\right) - A_y(L) = 0 \qquad A_y = \frac{wL}{3}$$



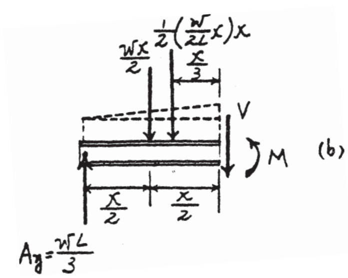
Shear and Moment Functions: For $0 \le x \le L$ [FBD (b)],

$$+\uparrow \Sigma F_y = 0; \qquad \frac{wL}{3} - \frac{w}{2}x - \frac{1}{2}\left(\frac{w}{2L}x\right)x - V = 0$$

$$V = \frac{w}{12L}(4L^2 - 6Lx - 3x^2) \qquad \textbf{Ans.}$$

The maximum moment occurs when $V = 0$, then

$$0 = 4L^2 - 6Lx - 3x^2 \qquad x = 0.5275L$$



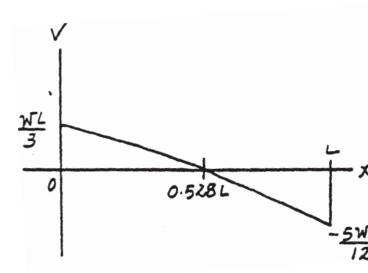
$$\zeta + \Sigma M = 0; \qquad M + \frac{1}{2}\left(\frac{w}{2L}x\right)x\left(\frac{x}{3}\right) + \frac{wx}{2}\left(\frac{x}{2}\right) - \frac{wL}{3}(x) = 0$$

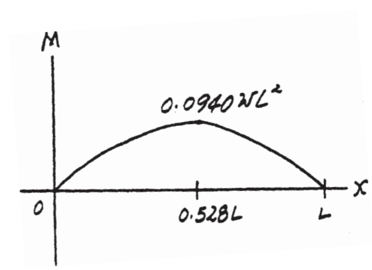
$$M = \frac{w}{12L}(4L^2x - 3Lx^2 - x^3) \qquad \textbf{Ans.}$$

Thus,

$$M_{\max} = \frac{w}{12L}[4L^2(0.5275L) - 3L(0.5275L)^2 - (0.5275L)^3]$$

$$= 0.0940wL^2 \qquad \textbf{Ans.}$$





Ans:

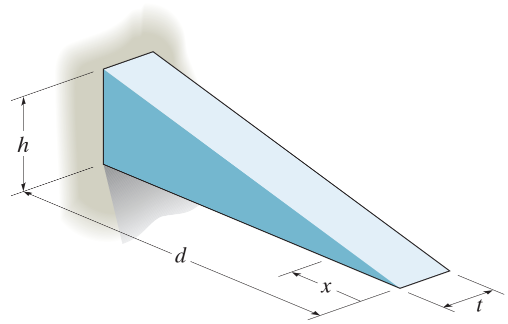
$V = \frac{w}{12L}(4L^2 - 6Lx - 3x^2)$

$M = \frac{w}{12L}(4L^2x - 3Lx^2 - x^3)$

$M_{\max} = 0.0940\,wL^2$

7–65.

The cantilevered beam is made of material having a specific weight γ. Determine the shear and moment in the beam as a function of x.

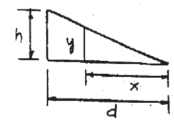


SOLUTION

By similar triangles

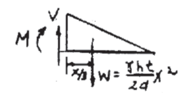
$$\frac{y}{x} = \frac{h}{d} \qquad y = \frac{h}{d}x$$

$$W = \gamma V = \gamma\left(\frac{1}{2}yxt\right) = \gamma\left[\frac{1}{2}\left(\frac{h}{d}x\right)xt\right] = \frac{\gamma ht}{2d}x^2$$



$$+\uparrow\Sigma F_y = 0; \qquad V - \frac{\gamma ht}{2d}x^2 = 0 \qquad V = \frac{\gamma ht}{2d}x^2$$ **Ans.**

$$\circlearrowleft + \Sigma M = 0; \qquad -M - \frac{\gamma ht}{2d}x^2\left(\frac{x}{3}\right) = 0 \qquad M = -\frac{\gamma ht}{6d}x^3$$ **Ans.**

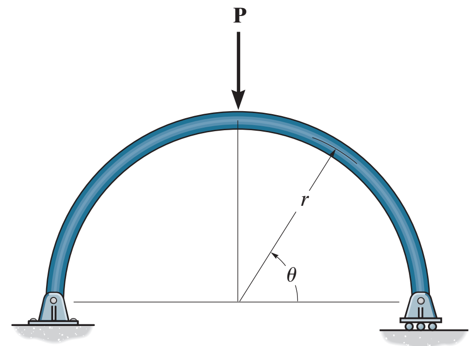


Ans:

$$V = \frac{\gamma ht}{2d}x^2$$

$$M = -\frac{\gamma ht}{6d}x^3$$

7–66.

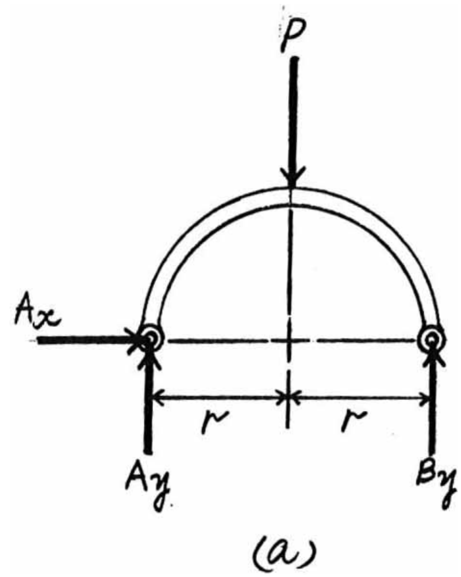


Determine the internal normal force, shear force, and moment in the curved rod as a function of θ, where $0° \leq \theta \leq 90°$.

SOLUTION

With reference to Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(2r) - p(r) = 0 \qquad B_y = p/2$

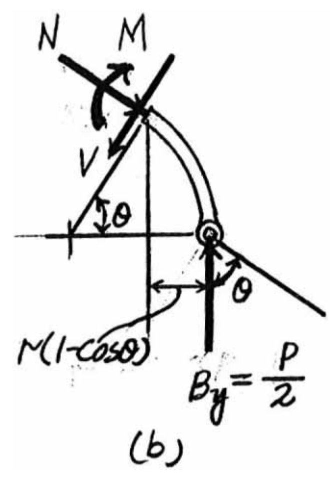


Using this result and referring to Fig. *b*, we have

$\Sigma F_{x'} = 0; \qquad \dfrac{p}{2}\sin\theta - V = 0 \qquad V = \dfrac{p}{2}\sin\theta$ **Ans.**

$\Sigma F_{y'} = 0; \qquad \dfrac{p}{2}\cos\theta - N = 0 \qquad N = \dfrac{p}{2}\cos\theta$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad \dfrac{p}{2}[r(1 - \cos\theta)] - M = 0 \qquad M = \dfrac{pr}{2}(1 - \cos\theta)$ **Ans.**



Ans:

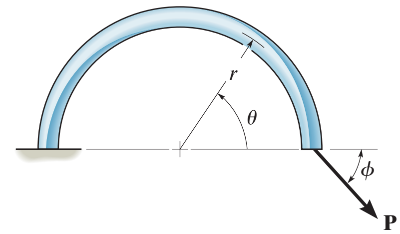
$V = \dfrac{P}{2}\sin\theta$

$N = \dfrac{p}{2}\cos\theta$

$M = \dfrac{pr}{2}(1 - \cos\theta)$

7–67.

Determine the internal normal force, shear force, and moment in the curved rod as a function of θ. The force $\mathbf{P}$ acts at the constant angle ϕ.



SOLUTION

Support Reactions. Not required

Internal Loadings. Referring to the *FBD* of the free and segment of the sectioned curved rod, Fig. *a*,

$+\nwarrow\Sigma F_y = 0;\quad N - P\sin(\theta + \phi) = 0\quad N = P\sin(\theta + \phi)$ **Ans.**

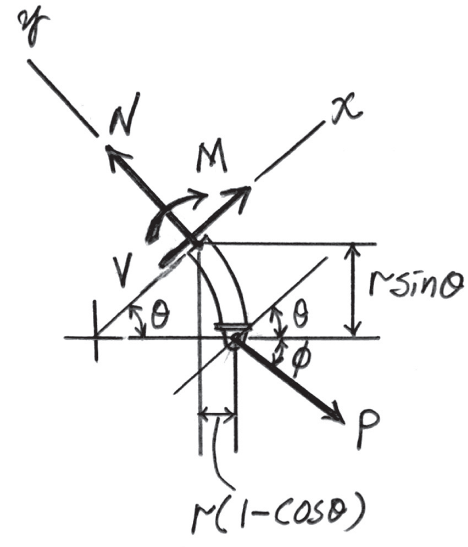
$+\nearrow\Sigma F_x = 0;\quad P\cos(\theta + \phi) + V = 0\quad V = -P\cos(\theta + \phi)$ **Ans.**

$\circlearrowleft+\Sigma M_O = 0;\quad P\cos\phi(r\sin\theta) - P\sin\phi[r(1 - \cos\theta)] - M = 0$

$$M = Pr(\sin\theta\cos\phi + \cos\theta\sin\phi - \sin\phi)$$

Using the identity $\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$,

$$M = Pr[\sin(\theta + \phi) - \sin\phi]$$ **Ans.**

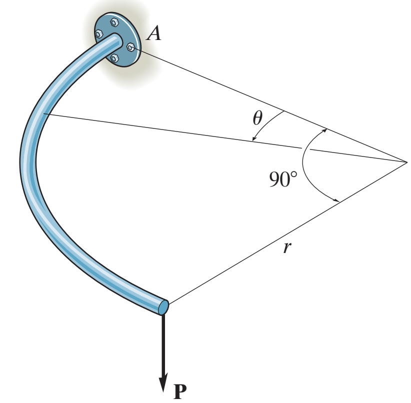


(a)

Ans:
$N = P\sin(\theta + \phi)$
$V = -P\cos(\theta + \phi)$
$M = Pr[\sin(\theta + \phi) - \sin\phi]$

***7–68.**

The quarter circular rod lies in the horizontal plane and supports a vertical force **P** at its end. Determine the magnitudes of the components of the internal shear force, moment, and torque acting in the rod as a function of the angle θ.



SOLUTION

$\Sigma F_z = 0;\qquad V = |P|$ **Ans.**

$\Sigma M_x = 0;\qquad M + P\,(r \sin(90° - \theta)) = 0$

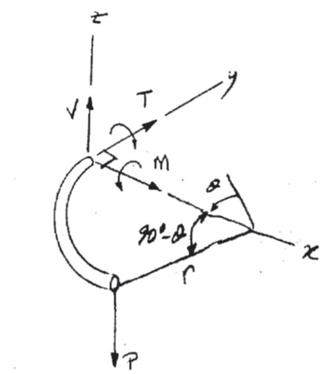
$M = -P\,r \cos\theta$

$M = |P\,r\cos\theta|$ **Ans.**

$\Sigma M_y = 0;\qquad T + P\,r\,(1 - \cos(90° - \theta)) = 0$

$T = -P\,r\,(1 - \sin\theta)$

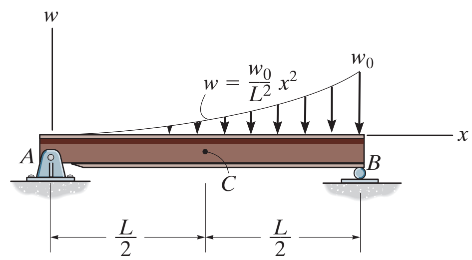
$T = |P\,r\,(1 - \sin\theta)|$ **Ans.**



Ans:
$V = |P|$
$M = |P\,r\cos\theta|$
$T = |P\,r\,(1 - \sin\theta)|$

7–69.

Draw the shear and moment diagrams for the beam.



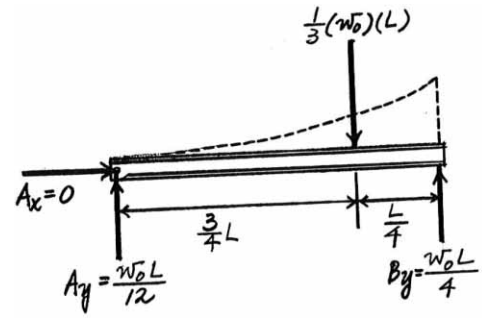
SOLUTION

The free-body diagram of the beam's segment sectioned through an arbitrary point shown in Fig. b will be used to write the shear and moment equations. The magnitude of the resultant force of the parabolic distributed loading and the location of its point of application are given in the inside back cover of the book.

Referring to Fig. b, we have

$$+\uparrow \Sigma F_y = 0; \quad \frac{w_0 L}{12} - \frac{1}{3}\left(\frac{w_0}{L^2}x^2\right)x - V = 0 \qquad V = \frac{w_0}{12L^2}\left(L^3 - 4x^3\right) \quad \textbf{(1)}$$

$$\zeta + \Sigma M = 0; \quad M + \frac{1}{3}\left(\frac{w_0}{L^2}x^2\right)(x)\left(\frac{x}{4}\right) - \frac{w_0 L}{12}x = 0 \quad M = \frac{w_0}{12L^2}\left(L^3 x - x^4\right) \quad \textbf{(2)}$$



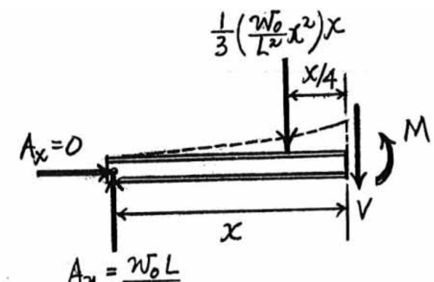
(a)

The shear and moment diagrams shown in Figs. c and d are plotted using Eqs. (1) and (2), respectively. The location at which the shear is equal to zero can be obtained by setting $V = 0$ in Eq. (1).

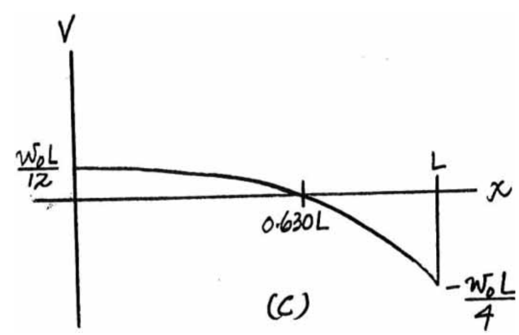
$$0 = \frac{w_0}{12L^2}\left(L^3 - 4x^3\right) \qquad x = 0.630L$$

The value of the moment at $x = 0.630L$ is evaluated using Eq. (2).

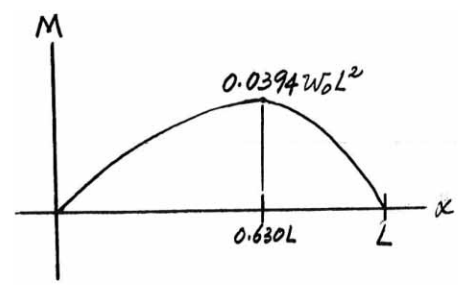
$$M\big|_{x=0.630L} = \frac{w_0}{12L^2}\left[L^3(0.630L) - (0.630L)^4\right] = 0.0394 w_0 L^2$$



(b)



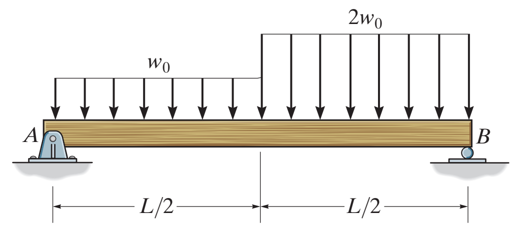
(c)



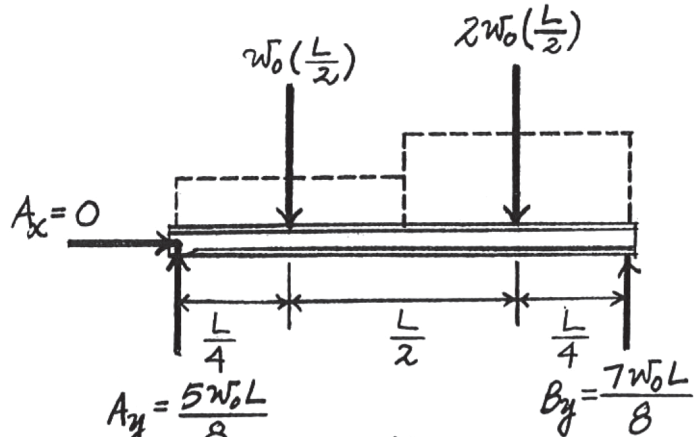
(d)

7–70.

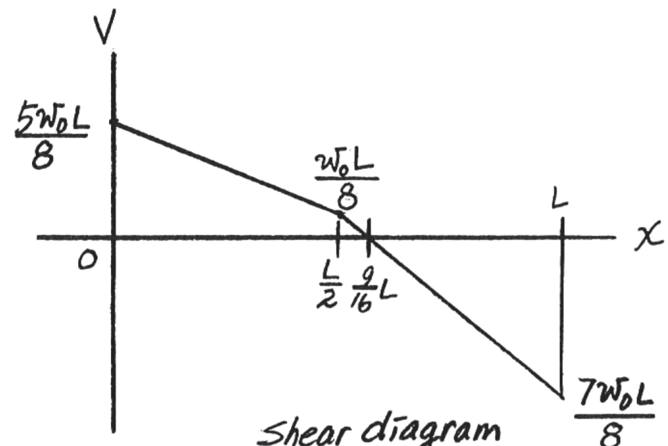
Draw the shear and moment diagrams for the simply-supported beam.



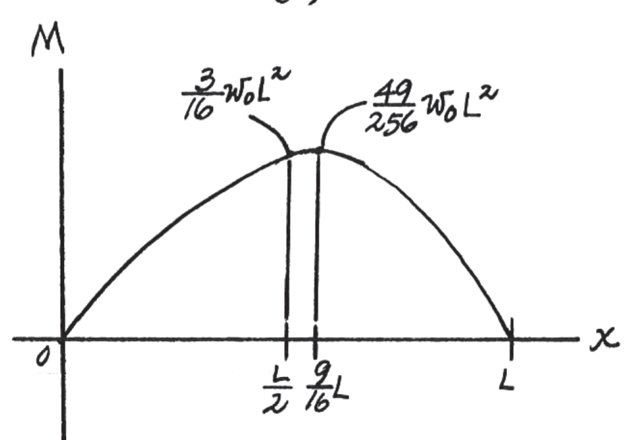
SOLUTION



(a)



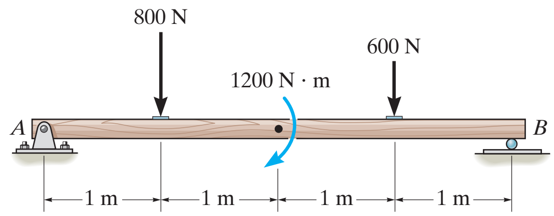
Shear diagram
(b)



Moment diagram
(c)

7–71.

Draw the shear and moment diagrams for the beam.



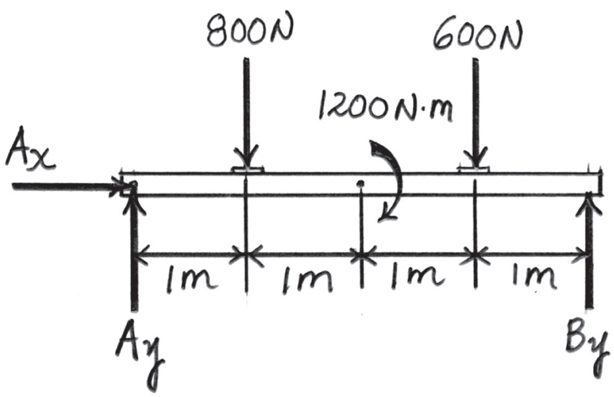
SOLUTION

Support Reactions. Referring to the *FBD* of the simply supported beam shown in Fig. *a*

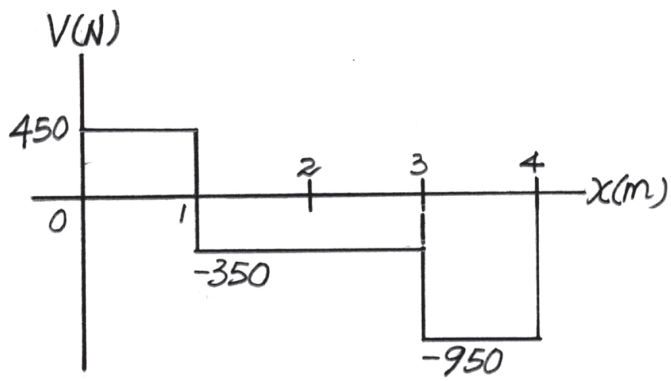
$\circlearrowleft+\Sigma M_A = 0;\quad B_y(4) - 800(1) - 600(3) - 1200 = 0 \qquad B_y = 950\text{ N}$

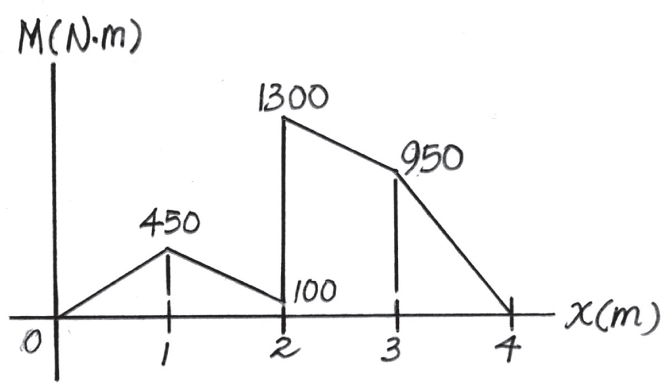
$\circlearrowleft+\Sigma M_B = 0;\quad 600(1) + 800(3) - 1200 - A_y(4) = 0 \qquad A_y = 450\text{ N}$

$\xrightarrow{+}\Sigma F_x = 0;\qquad A_x = 0$



(a)

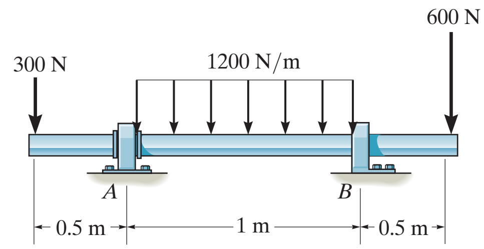




Ans:
$x = 1^-$
$V = 450\text{ N}$
$M = 450\text{ N}\cdot\text{m}$
$x = 3^+$
$V = -950\text{ N}$
$M = 950\text{ N}\cdot\text{m}$

***7–72.**

Draw the shear and moment diagrams for the beam. The supports at A and B are a thrust bearing and journal bearing, respectively.



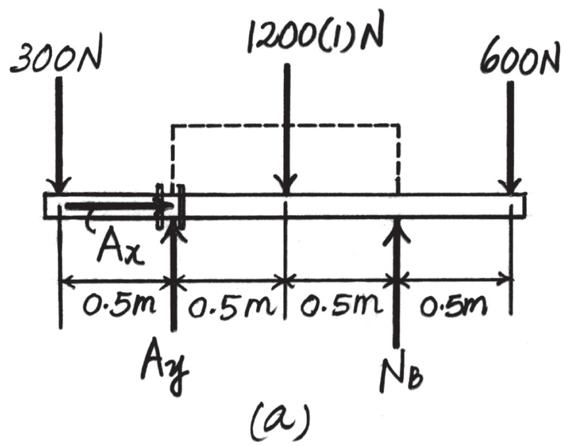
SOLUTION

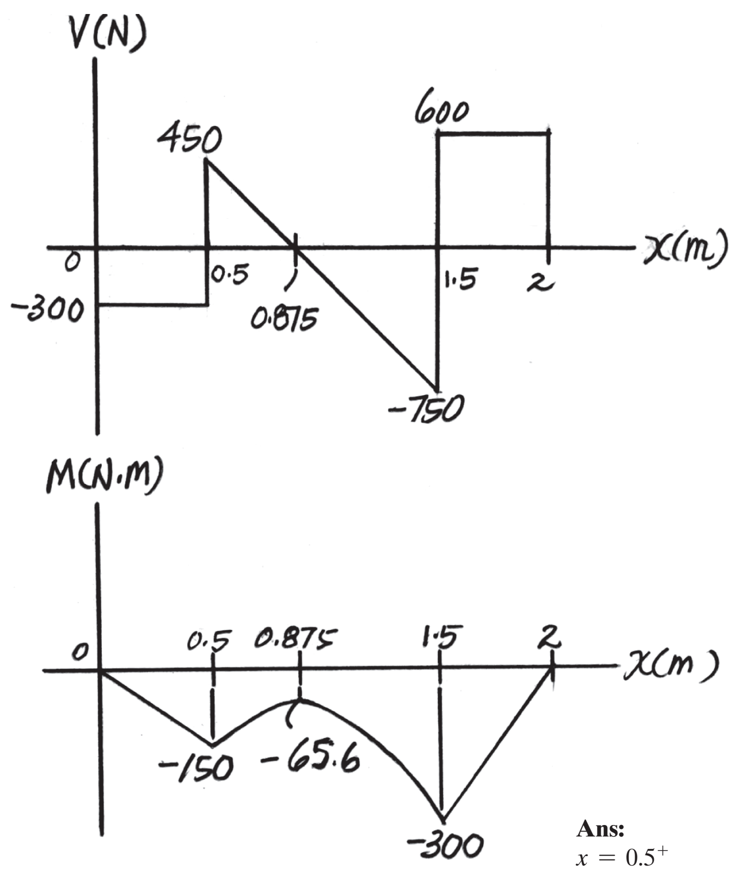
Support Reactions. Referring to the *FBD* of the shaft shown in Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0;\quad N_B(1) + 300(0.5) - 1200(1)(0.5) - 600(1.5) = 0 \quad N_B = 1350\text{ N}$

$\circlearrowleft + \Sigma M_B = 0;\quad 1200(1)(0.5) + 300(1.5) - 600(0.5) - A_y(1) = 0 \quad A_y = 750\text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x = 0$

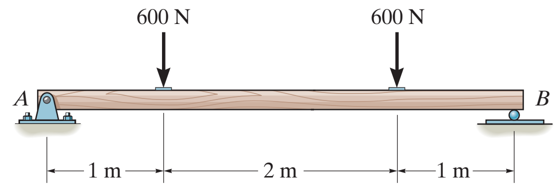




Ans:
$x = 0.5^+$
$V = 450\text{ N}$
$M = -150\text{ N}\cdot\text{m}$
$x = 1.5^-$
$V = -750\text{ N}$
$M = -300\text{ N}\cdot\text{m}$

7–73.

Draw the shear and moment diagrams for the beam.



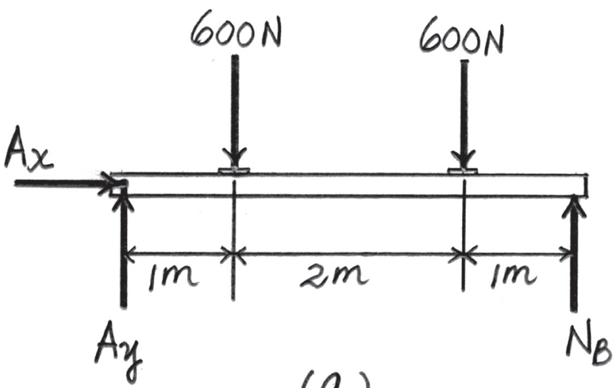
SOLUTION

Support Reactions. Referring to the *FBD* of the beam shown in Fig. *a*

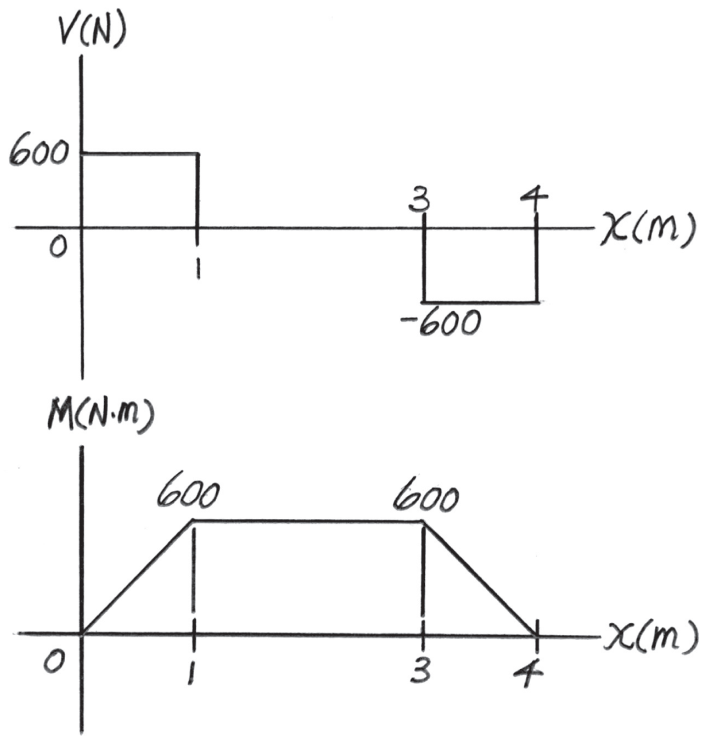
$\circlearrowleft + \Sigma M_A = 0; \quad N_B(4) - 600(3) - 600(1) = 0 \quad N_B = 600 \text{ N}$

$\circlearrowleft + \Sigma M_B = 0; \quad 600(1) + 600(3) - A_y(4) = 0 \quad A_y = 600 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0 \quad A_x = 0$



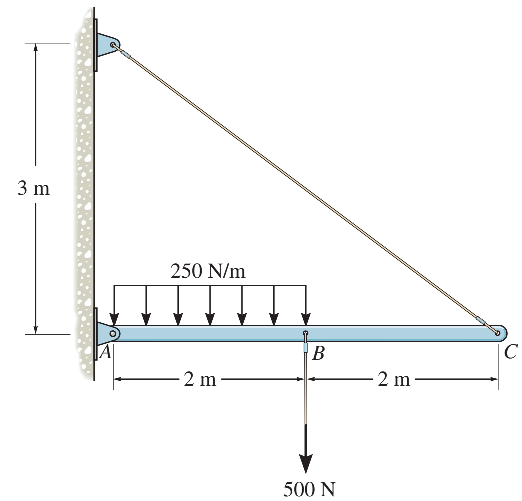
(a)



Ans:
$x = 1^-$
$V = 600 \text{ N}$
$M = 600 \text{ N} \cdot \text{m}$

7–74.

Draw the shear and moment diagrams for the beam.

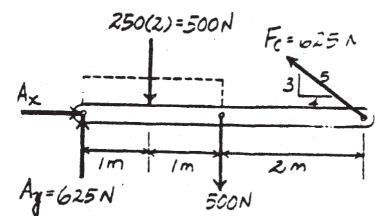


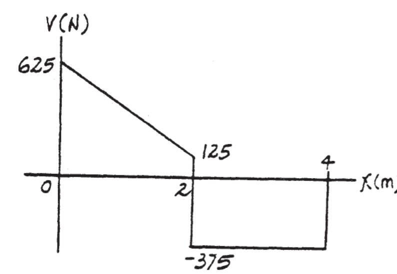
SOLUTION

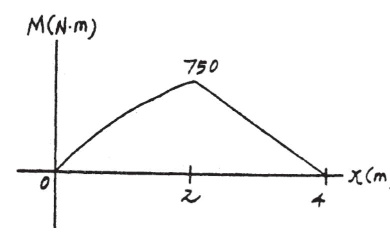
Support Reactions:

$\circlearrowleft + \Sigma M_A = 0; \qquad F_C\left(\frac{3}{5}\right)(4) - 500(2) - 500(1) = 0 \qquad F_C = 625\text{ N}$

$+\uparrow \Sigma F_y = 0; \qquad A_y + 625\left(\frac{3}{5}\right) - 500 - 500 = 0 \qquad A_y = 625\text{ N}$



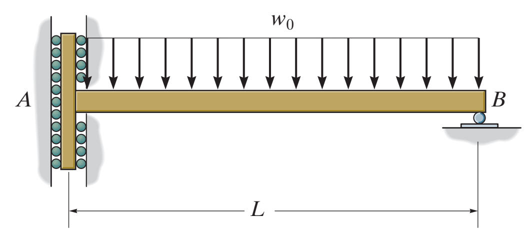




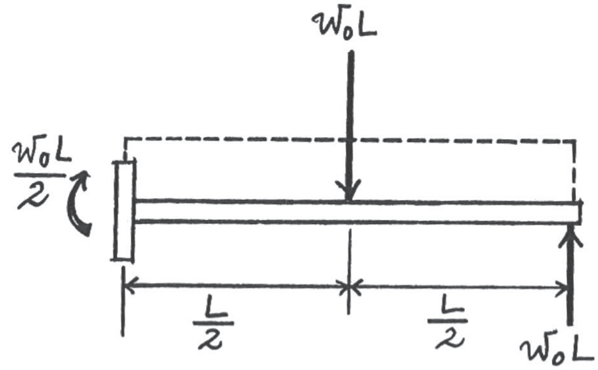
Ans:
$x = 2^+$
$V = -375\text{ N}$
$M = 750\text{ N}\cdot\text{m}$

7–75.

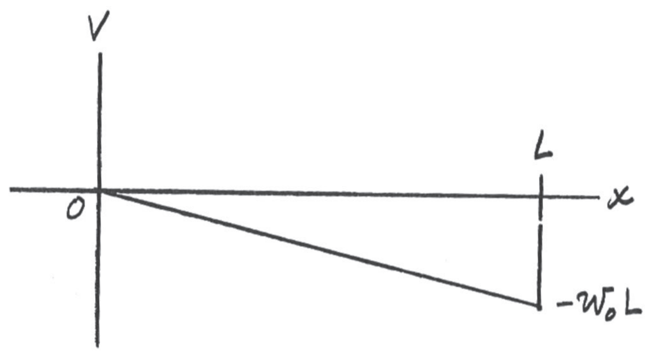
Draw the shear and moment diagrams for the beam. The support at A offers no resistance to vertical load.



SOLUTION

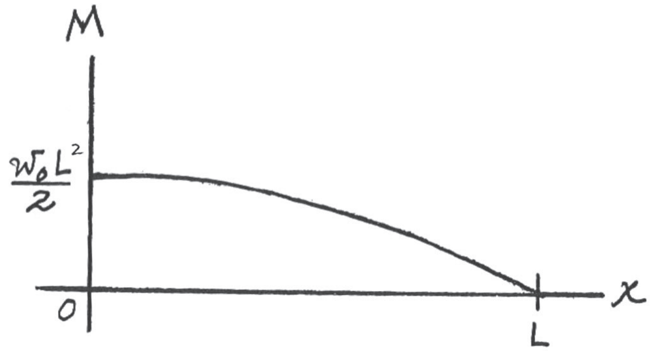


(a)



Shear diagram

(b)

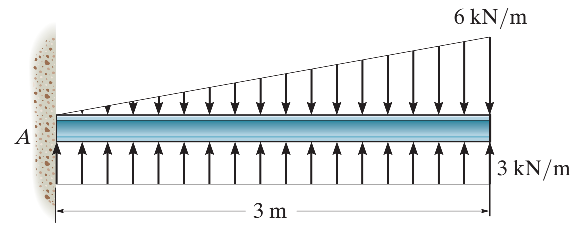


Moment diagram

(c)

***7–76.**

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions. The *FBD* of the beam acted upon the equivalent loading (by superposition) is shown in Fig. *a*. Equilibrium gives

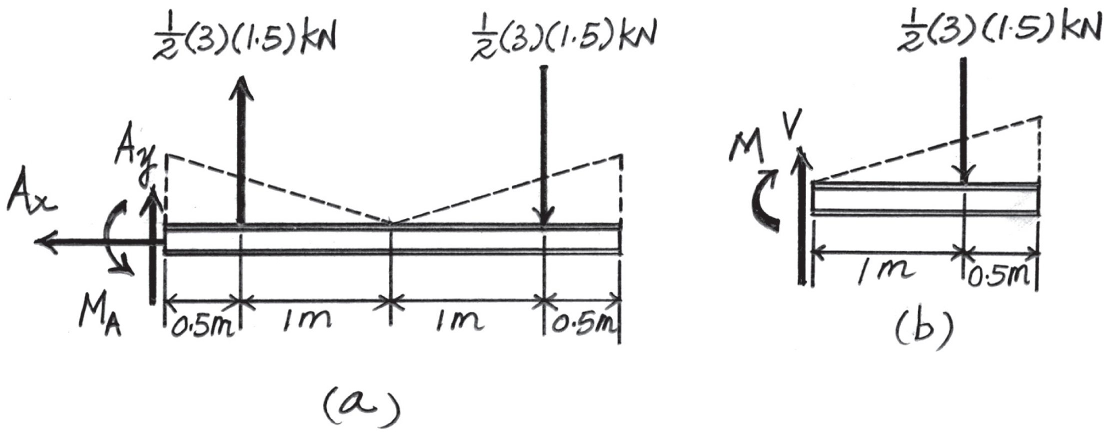
$$\zeta + \Sigma M_A = 0; \quad M_A + \frac{1}{2}(3)(1.5)(0.5) - \frac{1}{2}(3)(1.5)(2.5) = 0 \quad M_A = 4.50 \text{ kN} \cdot \text{m}$$

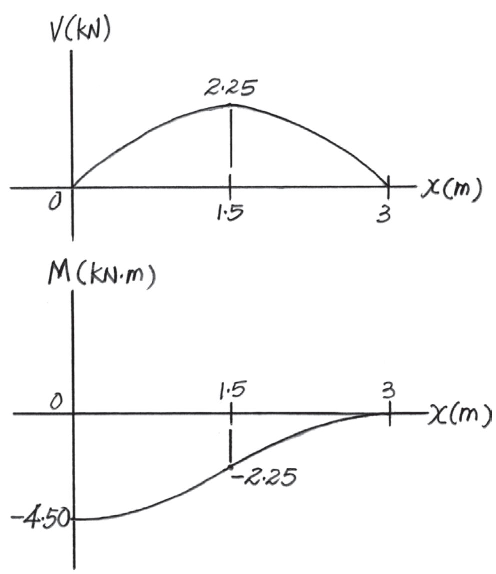
$$+\uparrow \Sigma F_y = 0; \qquad A_y = 0$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$$

Internal Loadings. Referring to the *FBD* of the right segment of the beam sectioned at $x = 1.5$ m, the internal moment at this section is

$$\zeta + \Sigma M_O = 0; \qquad -M - \frac{1}{2}(3)(1.5)(1) = 0 \qquad M = -2.25 \text{ kN} \cdot \text{m}$$

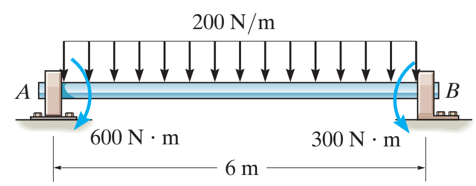




Ans:
$x = 1.5$
$V = 2.25$ kN
$M = -2.25$ kN · m

7–77.

Draw the shear and moment diagrams for the beam. The supports at A and B are a thrust and journal bearing, respectively.



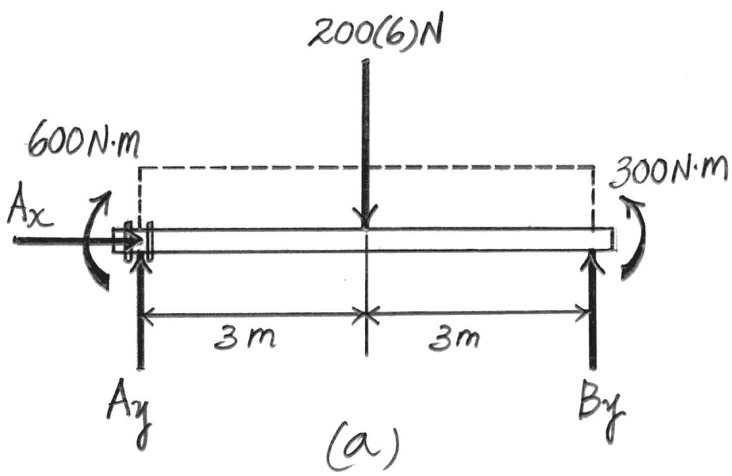
SOLUTION

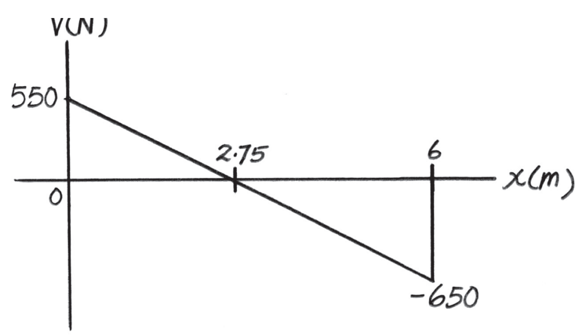
Support Reactions. Referring to the *FBD* of the shaft shown in Fig. *a*,

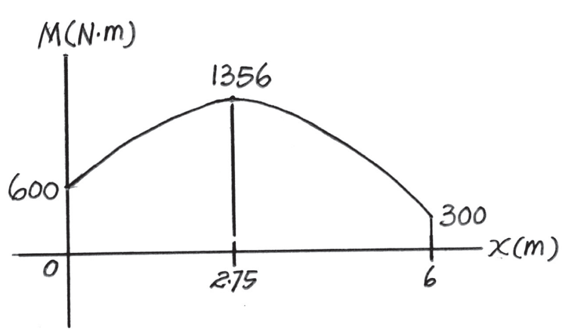
$$\circlearrowleft + \Sigma M_A = 0; \quad B_y(6) + 300 - 200(6)(3) - 600 = 0 \quad B_y = 650 \text{ N}$$

$$\circlearrowleft + \Sigma M_B = 0; \quad 200(6)(3) + 300 - 600 - A_y(6) = 0 \quad A_y = 550 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x = 0$$

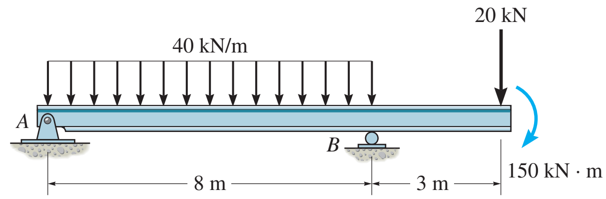






Ans:
$x = 2.75$
$V = 0$
$M = 1356 \text{ N} \cdot \text{m}$

7–78. Draw the shear and moment diagrams for the beam.



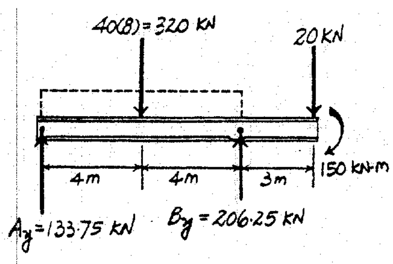
SOLUTION

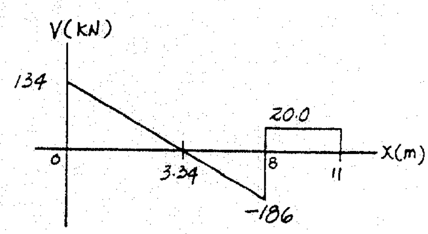
Support Reactions:

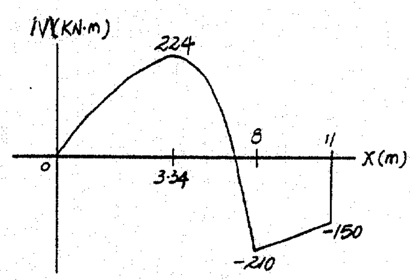
$$\circlearrowleft + \Sigma M_A = 0; \quad B_y(8) - 320(4) - 20(11) - 150 = 0$$

$$B_y = 206.25 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y + 206.25 - 320 - 20 = 0 \quad A_y = 133.75 \text{ kN}$$

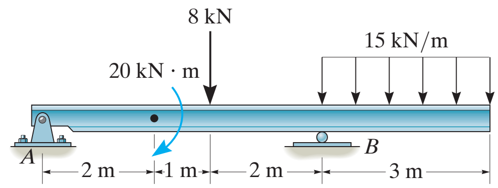




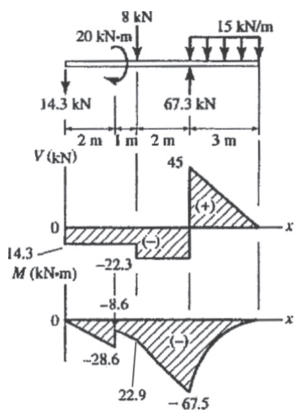


7–79.

Draw the shear and moment diagrams for the beam.



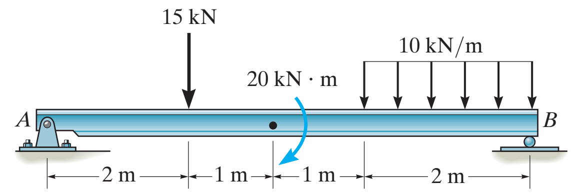
SOLUTION



Ans:
$x = 2^+$
$V = -14.3$
$M = -8.6$

***7–80.**

Draw the shear and moment diagrams for the beam.



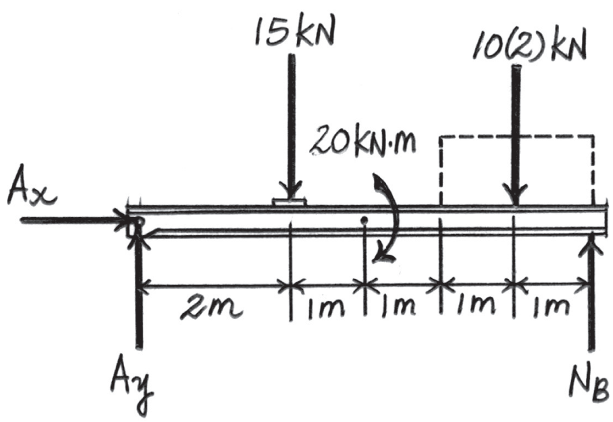
SOLUTION

Support Reaction. Referring to the *FBD* of the beam shown in Fig. *a*,

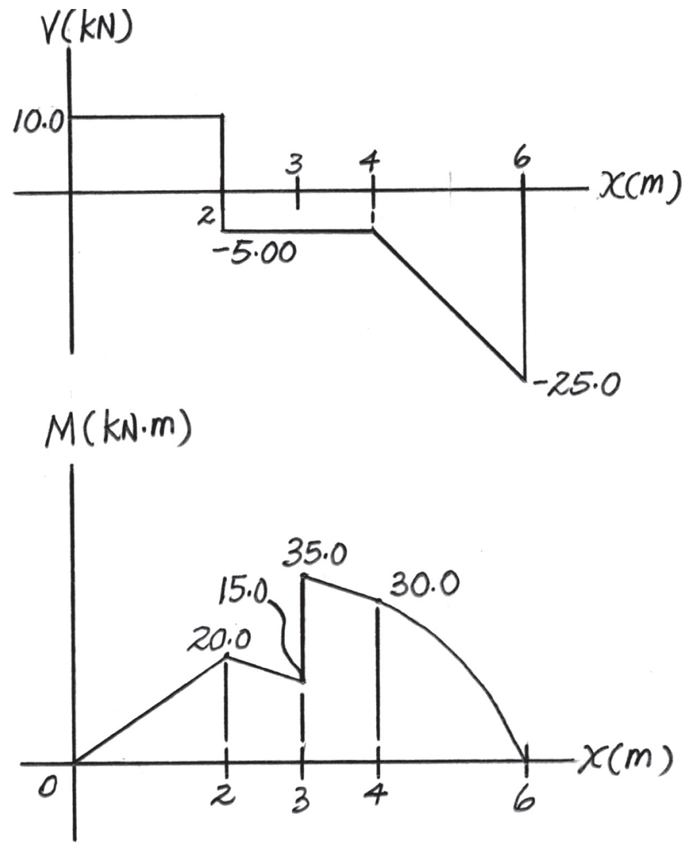
$\zeta + \Sigma M_A = 0; \quad N_B(6) - 15(2) - 20 - 10(2)(5) = 0 \quad N_B = 25.0 \text{ kN}$

$\zeta + \Sigma M_B = 0; \quad 10(2)(1) + 15(4) - 20 - A_y(6) = 0 \quad A_y = 10.0 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$



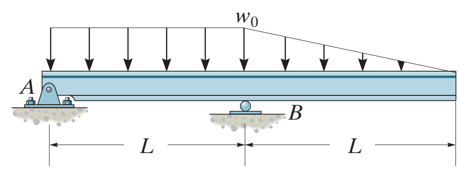
(a)



Ans:
$x = 2^-$
$V = 10.0 \text{ kN}$
$M = 20.0 \text{ kN} \cdot \text{m}$
$x = 3^+$
$V = -5.00 \text{ kN}$
$M = 35.0 \text{ kN} \cdot \text{m}$

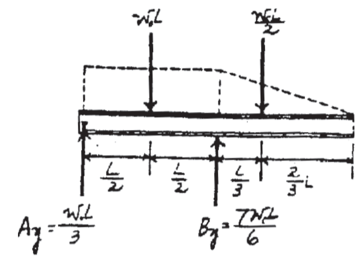
7–81.

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions:

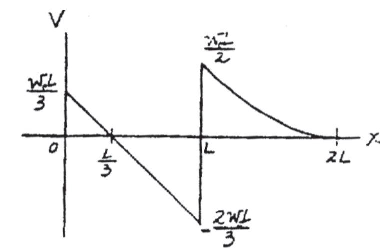


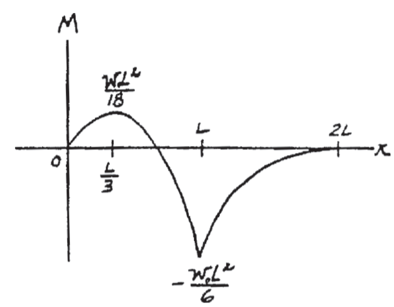
$\zeta + \Sigma M_A = 0; \qquad B_y(L) - w_0L\left(\frac{L}{2}\right) - \frac{w_0L}{2}\left(\frac{4L}{3}\right) = 0$

$$B_y = \frac{7w_0L}{6}$$

$+\uparrow \Sigma F_y = 0; \qquad A_y + \frac{7w_0L}{6} - w_0L - \frac{w_0L}{2} = 0$

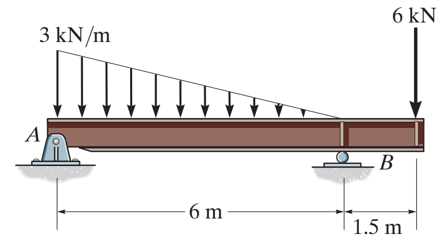
$$A_y = \frac{w_0L}{3}$$





7–82.

Draw the shear and moment diagrams for the overhang beam.



SOLUTION

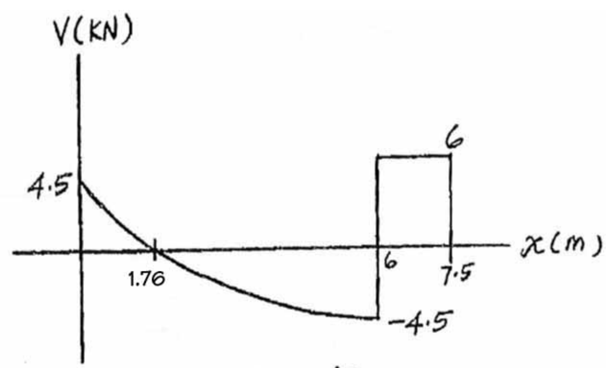
The maximum span moment occurs at the position where shear is equal to zero within the region $0 \leq x < 6\text{m}$ of the beam. This location can be obtained using the method of sections. By setting $V = 0$, Fig. *b*, we have

$+\uparrow \Sigma F_y = 0; \quad 4.5 - \frac{1}{2}\left(\frac{1}{2}x\right)x - \frac{1}{2}(6 - x)(x) = 0 \qquad x = 1.76 \text{ m}$ **Ans.**

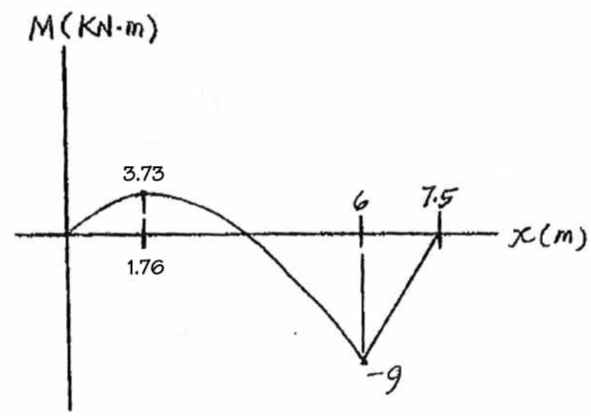
Using this result,

$$+\Sigma M = 0; \quad M|_{x=1.76\text{m}} + \frac{1}{2}(6 - 1.76)(1.76)\left(\frac{1.76}{2}\right) + \frac{1}{4}(1.76)(1.76)\left[\frac{2}{3}(1.76)\right] - 4.5(1.76) = 0$$

$$M|_{x=1.76\text{m}} = 3.73 \text{ kN} \cdot \text{m}$$



Shear diagram
(c)

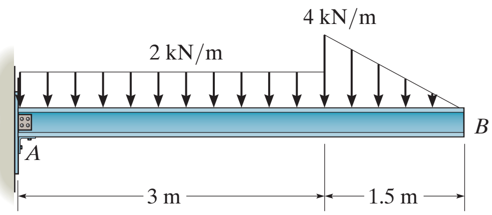


Moment diagram
(d)

Ans:
$x = 1.76$ m

7–83.

Draw the shear and moment diagrams for the beam.

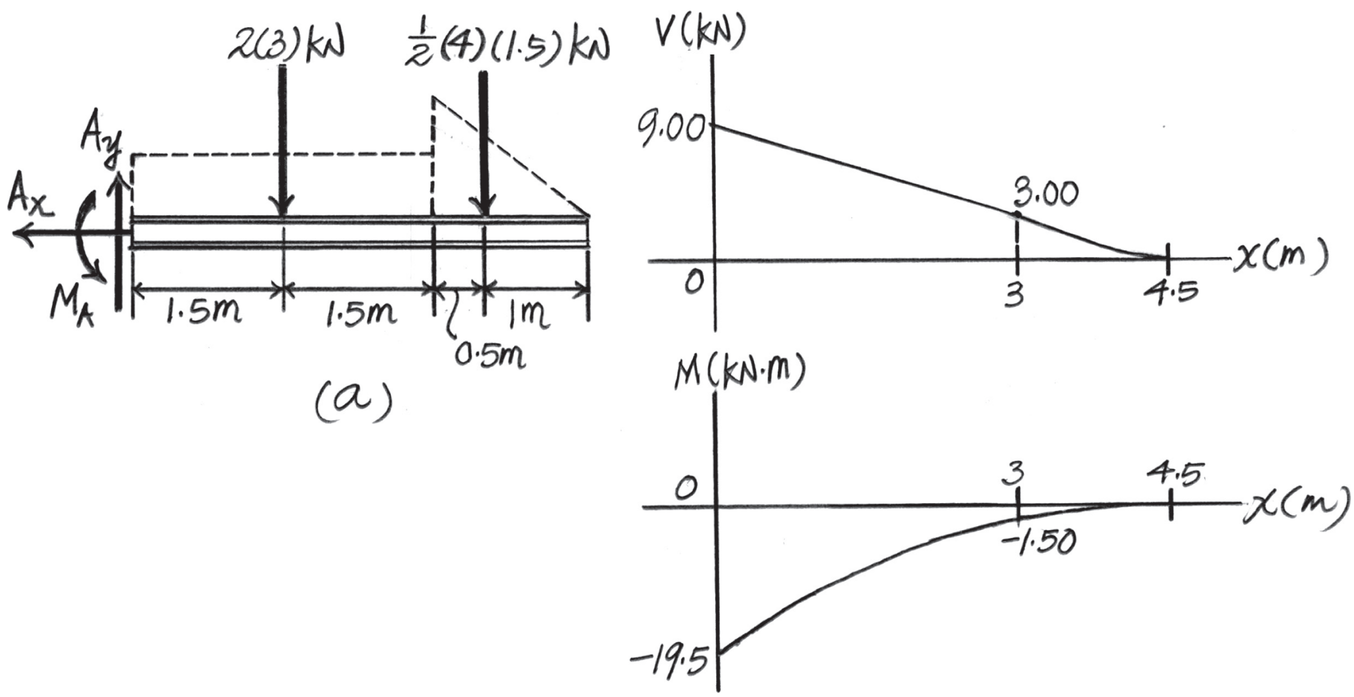


SOLUTION

Support Reactions. Referring to the *FBD* of the cantilevered beam shown in Fig. *a*.

$\zeta+\Sigma M_A = 0; \quad M_A - 2(3)(1.5) - \frac{1}{2}(4)(1.5)(3.5) = 0 \quad M_A = 19.5 \text{ kN} \cdot \text{m}$ **Ans.**

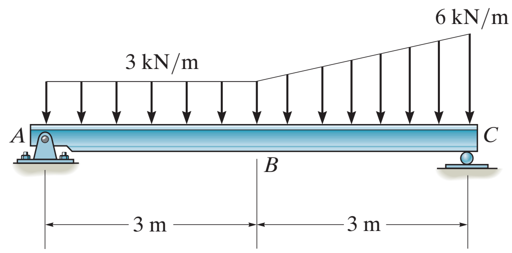
$+\uparrow \Sigma F_y = 0; \quad A_y - 2(3) - \frac{1}{2}(4)(1.5) = 0 \quad A_y = 9.00 \text{ kN}$ **Ans.**



Ans:
$x = 3$
$V = 3.00$ kN
$M = -1.50$ kN·m

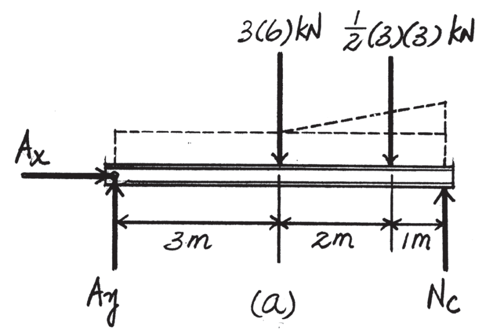
***7–84.**

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions. Referring to the *FBD* of the simply supported beam shown in Fig. *a*,



(a)

$$\zeta+\Sigma M_A = 0; \quad N_C(6) - 3(6)(3) - \frac{1}{2}(3)(3)(5) = 0 \qquad N_C = 12.75 \text{ kN}$$

$$\zeta+\Sigma M_C = 0; \quad \frac{1}{2}(3)(3)(1) + 3(6)(3) - A_y(6) = 0 \qquad A_y = 9.75 \text{ kN}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0$$

Shear And Moment Functions. Referring to the *FBD* of the left segment of the beam sectioned at a distance x within region BC ($3 \text{ m} < x \leq 6 \text{ m}$),

$$+\uparrow\Sigma F_y = 0; \quad 9.75 - 3x - \frac{1}{2}(x-3)(x-3) - V = 0 \quad V = \left\{5.25 = \frac{1}{2}x^2\right\} \text{kN}$$

$$\zeta+\Sigma M_O = 0; \quad M + \frac{1}{2}(x-3)(x-3)\left[\frac{1}{3}(x-3)\right] + 3x\left(\frac{x}{2}\right) - 9.75x = 0$$

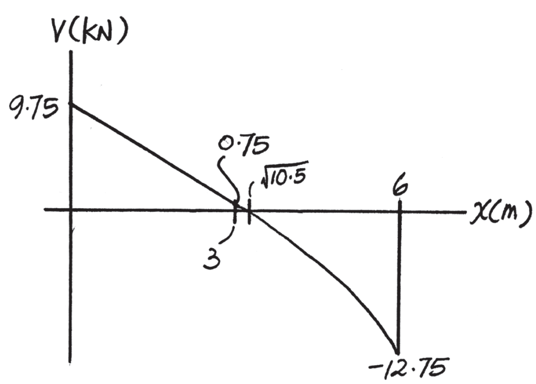
$$M = \left\{-\frac{1}{6}x^3 + 5.25x + 4.50\right\} \text{kN} \cdot \text{m}$$

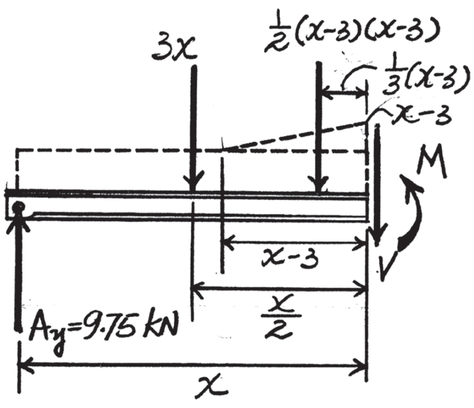
Set $V = 0$, we obtain

$$0 = 5.25 - \frac{1}{2}x^2 \qquad x = \sqrt{10.5} \text{ m} \qquad \textbf{Ans.}$$

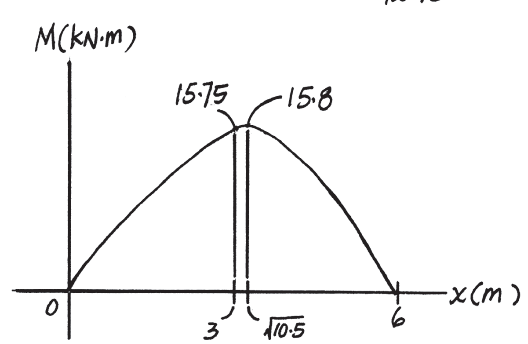
The corresponding moment is

$$M = -\frac{1}{6}\left(\sqrt{10.5}\right)^3 + 5.25\sqrt{10.5} + 4.50 = 15.8 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$





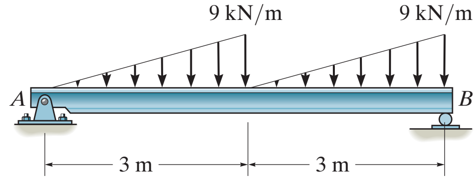
(b)



Ans:
$x = \sqrt{10.5}$ m
$V = 0$
$M = 15.8 \text{ kN} \cdot \text{m}$

7–85.

Draw the shear and moment diagrams for the beam.



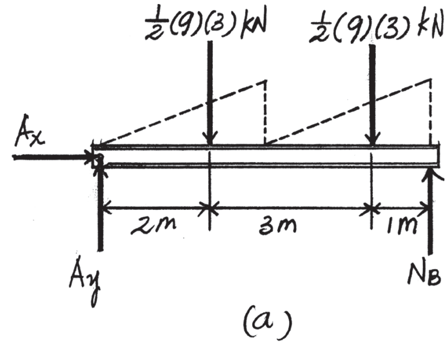
SOLUTION

Support Reactions. Referring to the *FBD* of the simply supported beam shown in Fig. *a*,

$$\zeta+\Sigma M_A = 0; \qquad N_B(6) - \frac{1}{2}(9)(3)(2) - \frac{1}{2}(9)(3)(5) = 0 \qquad N_B = 15.75 \text{ kN}$$

$$\zeta+\Sigma M_B = 0; \qquad \frac{1}{2}(9)(3)(1) + \frac{1}{2}(9)(3)(4) - A_y(6) = 0 \qquad A_y = 11.25 \text{ kN}$$

$$\xrightarrow{+}\Sigma F_x = 0; \qquad A_x = 0$$

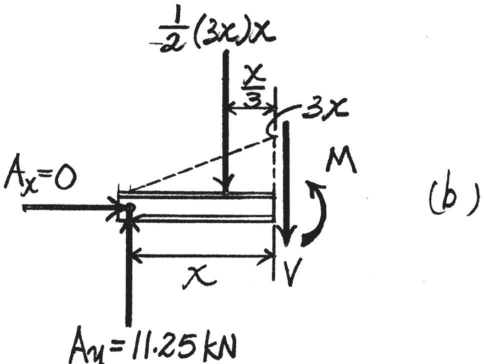


Shear And Moment Functions. Referring to the *FBD* of the left segment of the beam sectioned at a distance x within region $0 < x \leq 3$ m.

$$+\uparrow\Sigma F_y = 0; \qquad 11.25 - \frac{1}{2}(3x)(x) - V = 0 \qquad V = \left\{11.25 - \frac{3}{2}x^2\right\} \text{ kN}$$

$$\zeta+\Sigma M_O = 0: \qquad M + \left[\frac{1}{2}(3x)(x)\right]\left(\frac{x}{3}\right) - 11.25x = 0$$

$$M = \left\{11.25x - \frac{1}{2}x^3\right\} \text{ kN} \cdot \text{m}$$



Set $V = 0$;

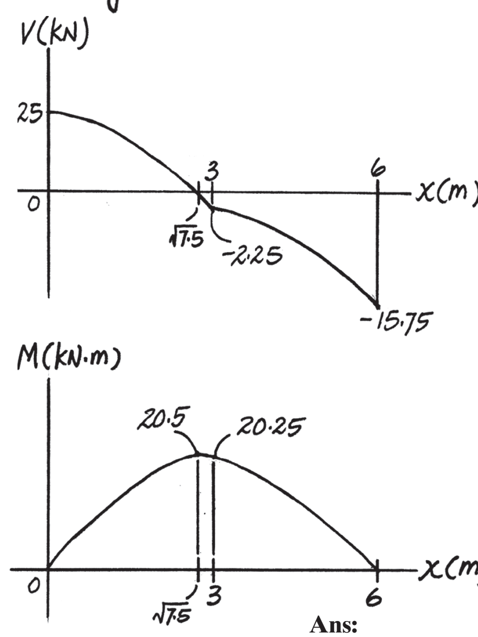
$$0 = 11.25 - \frac{3}{2}x^2 \quad x = \sqrt{7.5} \text{ m}$$

The corresponding moment is

$$M = 11.25\,(\sqrt{7.5}) - \frac{1}{2}(\sqrt{7.5})^3 = 20.5 \text{ kN} \cdot \text{m}$$

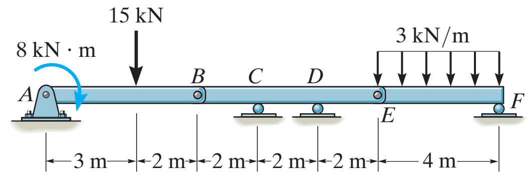
The moment at $x = 3$ m is

$$M = 11.25(3) - \frac{1}{2}(3^3) = 20.25 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$



Ans:
$x = 3$
$V = -2.25$ kN
$M = 20.25$ kN · m

7–86. The beam consists of three segments pin connected at B and E. Draw the shear and moment diagrams for the beam.



SOLUTION

Given: $M_A = 8\ \text{kN}\cdot\text{m}$ $\quad F = 15\ \text{kN}$ $\quad w = 3\ \text{kN/m}$ $\quad a = 3\ \text{m}$ $\quad b = 2\ \text{m}$

$c = 2\ \text{m}$ $\quad d = 2\ \text{m}$ $\quad e = 2\ \text{m}$ $\quad f = 4\ \text{m}$

Guesses $\quad A_y = 1\ \text{N}$ $\quad B_y = 1\ \text{N}$ $\quad C_y = 1\ \text{N}$

$D_y = 1\ \text{N}$ $\quad E_y = 1\ \text{N}$ $\quad F_y = 1\ \text{N}$

Given

$$A_y + C_y + D_y + F_y - F - wf = 0 \qquad F b - M_A - A_y(a+b) = 0$$

$$-wf\left(\frac{f}{2}\right) + F_y f = 0 \qquad M_A + F a + B_y(a+b) = 0$$

$$B_y + C_y + D_y + E_y = 0 \qquad -B_y c + D_y d + E_y(d+e) = 0$$

$$\begin{pmatrix} A_y \\ B_y \\ C_y \\ D_y \\ E_y \\ F_y \end{pmatrix} = \text{Find}(A_y, B_y, C_y, D_y, E_y, F_y) \qquad \begin{pmatrix} A_y \\ B_y \\ C_y \\ D_y \\ E_y \\ F_y \end{pmatrix} = \begin{pmatrix} 4.40 \\ -10.60 \\ 15.20 \\ 1.40 \\ -6.00 \\ 6.00 \end{pmatrix} \text{kN}$$

$x_1 = 0, 0.01a .. a$

$$V_1(x) = A_y \frac{1}{\text{kN}} \qquad M_1(x) = (A_y x + M_A)\frac{1}{\text{kN}\cdot\text{m}}$$

$x_2 = a, 1.01a .. a + b$

$$V_2(x) = (A_y - F)\frac{1}{\text{kN}} \qquad M_2(x) = \left[A_y x + M_A - F(x-a)\right]\frac{1}{\text{kN}\cdot\text{m}}$$

$x_3 = a + b, 1.01(a+b) .. a + b + c$

$$V_3(x) = B_y \frac{1}{\text{kN}} \qquad M_3(x) = B_y(x - a - b)\frac{1}{\text{kN}\cdot\text{m}}$$

$x_4 = a + b + c, 1.01(a+b+c) .. a + b + c + d$

$$V_4(x) = (B_y + C_y)\frac{1}{\text{kN}} \qquad M_4(x) = \left[B_y(x-a-b) + C_y(x-a-b-c)\right]\frac{1}{\text{kN}\cdot\text{m}}$$

7–86. Continued

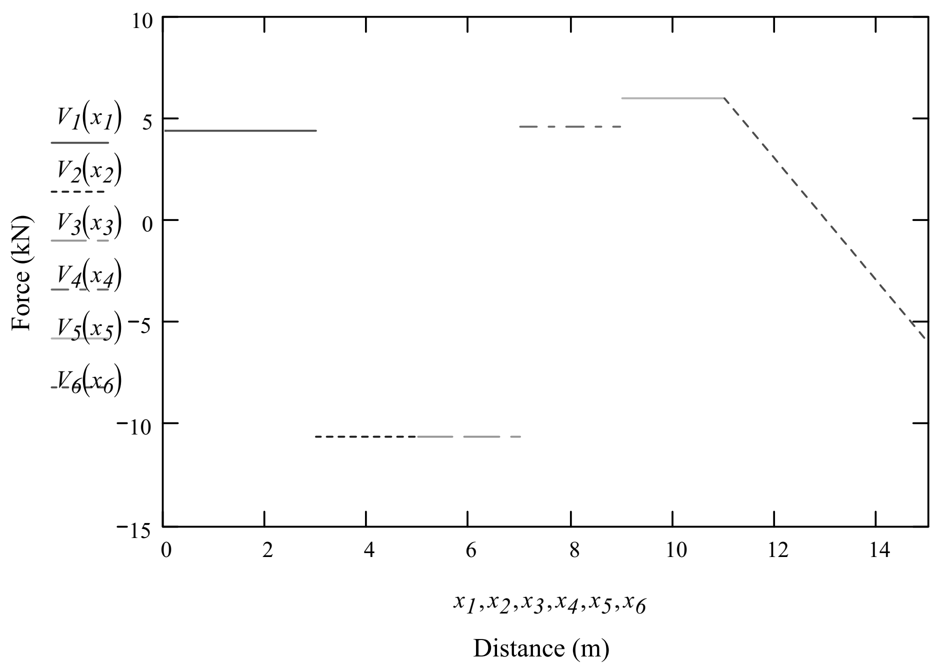
$$x_5 = a + b + c + d, 1.01(a + b + c + d) .. a + b + c + d + e$$

$$V_5(x) = -E_y \frac{1}{\text{kN}} \qquad M_5(x) = E_y(a + b + c + d + e - x)\frac{1}{\text{kN}\cdot\text{m}}$$

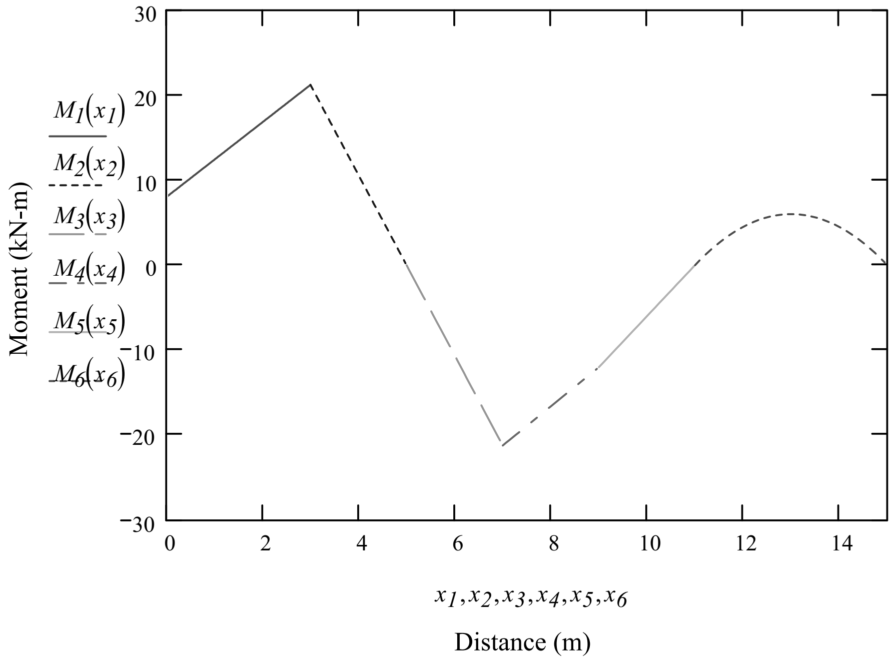
$$x_6 = a + b + c + d + e, 1.01(a + b + c + d + e) .. a + b + c + d + e + f$$

$$V_6(x) = \left[-F_y + w(a + b + c + d + e + f - x)\right]\frac{1}{\text{kN}}$$

$$M_6(x) = \left[F_y(a + b + c + d + e + f - x) - w\frac{(a + b + c + d + e + f - x)^2}{2}\right]\frac{1}{\text{kN}\cdot\text{m}}$$

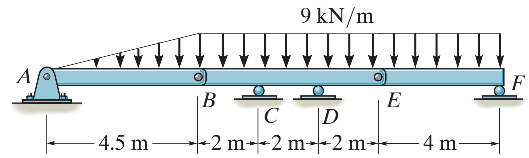


7–86. Continued



7–87.

The beam consists of three segments pin connected at B and E. Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions. Referring to the *FBD* of member *EF*, Fig. *c*

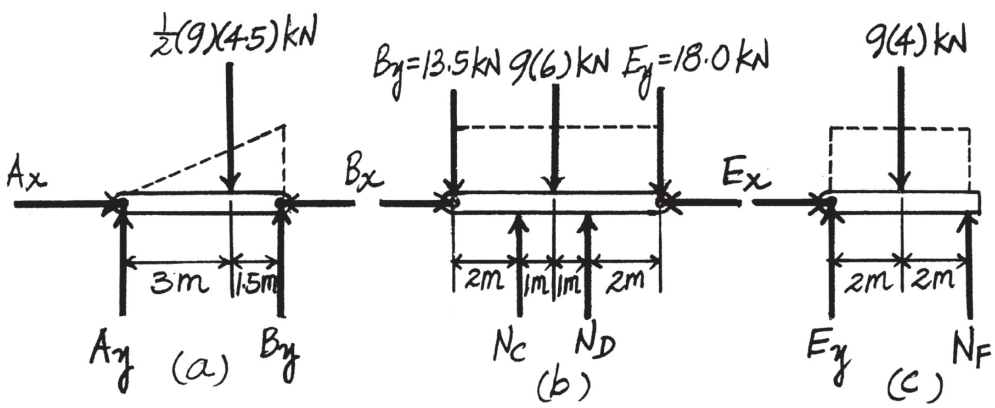
$$\circlearrowleft + \Sigma M_E = 0; \qquad N_F(4) - 9(4)(2) = 0 \qquad N_F = 18.0 \text{ kN}$$

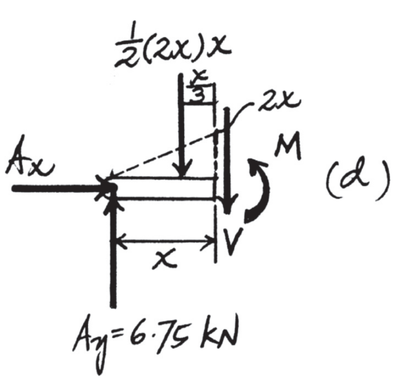
$$\circlearrowleft + \Sigma M_F = 0; \qquad 9(4)(2) - E_y(4) = 0 \qquad E_y = 18.0 \text{ kN}$$

Also, for member AB, Fig. *a*

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(4.5) - \frac{1}{2}(9)(4.5)(3) = 0 \qquad B_y = 13.5 \text{ kN}$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad \frac{1}{2}(9)(4.5)(1.5) - A_y(4.5) = 0 \qquad A_y = 6.75 \text{ kN}$$





7–87. Continued

Finally, for member $BCDE$, Fig. b

$\circlearrowleft + \Sigma M_C = 0;\quad N_D(2) + 13.5(2) - 9(6)(1) - 18(4) = 0 \qquad N_D = 49.5 \text{ kN}$

$\circlearrowleft + \Sigma M_D = 0;\quad 9(6)(1) + 13.5(4) - 18.0(2) - N_C(2) = 0 \quad N_C = 36.0 \text{ kN}$

Shear And Moment Functions. Referring to the FBD of the left segment of member AB sectioned at an distance x, Fig. d,

$+\uparrow \Sigma F_y = 0;\qquad 6.75 - \frac{1}{2}(2x)(x) - V = 0 \qquad V = \{6.75 - x^2\} \text{ kN}$

$\circlearrowleft + \Sigma M_O = 0;\qquad M + \left[\frac{1}{2}(2x)(x)\right]\left(\frac{x}{3}\right) - 6.75x = 0$

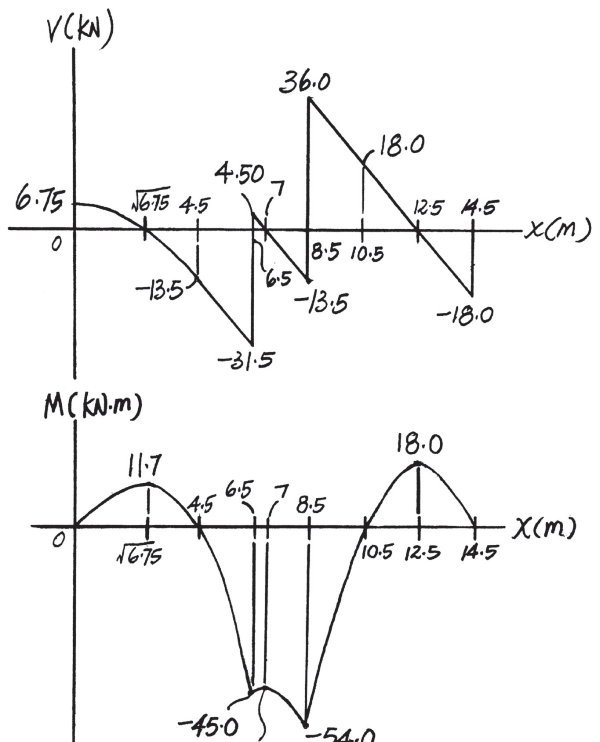
$$M = \left\{6.75x - \frac{1}{3}x^3\right\} \text{kN} \cdot \text{m}$$

Set $V = 0$, then

$$0 = 6.75 - x^2 \quad x = \sqrt{6.75} \text{ m}$$

The corresponding moment is

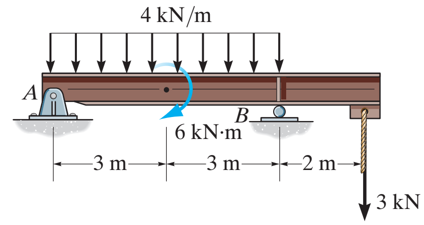
$$M = 6.75\left(\sqrt{6.75}\right) - \frac{1}{3}\left(\sqrt{6.75}\right)^3 = 11.69 \text{ kN} \cdot \text{m} = 11.7 \text{ kN} \cdot \text{m}$$



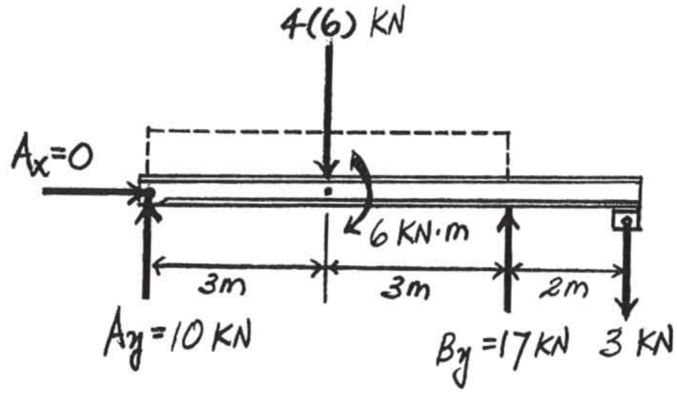
Ans:
$x = 6.5^-$
$V = -31.5 \text{ kN}$
$M = -45.0 \text{ kN} \cdot \text{m}$
$x = 8.5^+$
$V = 36.0 \text{ kN}$
$M = -54.0 \text{ kN} \cdot \text{m}$

***7–88.**

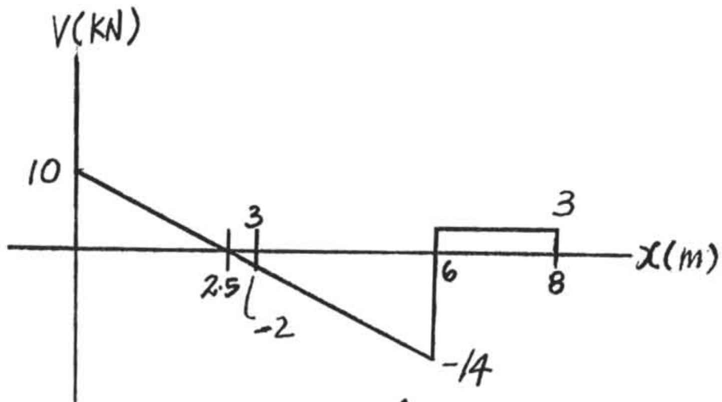
Draw the shear and moment diagrams for the overhang beam.



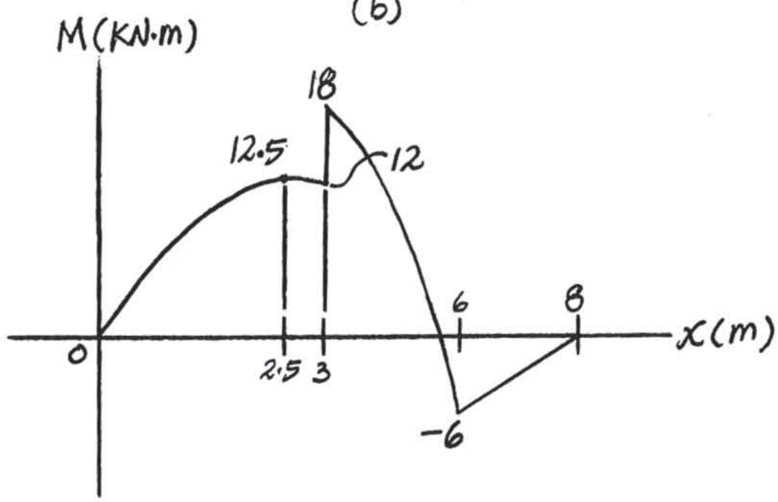
SOLUTION



(a)



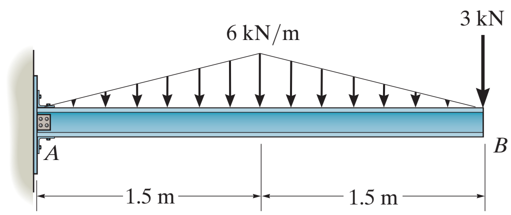
Shear diagram
(b)



Moment diagram
(c)

7–89.

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions. Referring to the *FBD* of the cantilevered beam shown in Fig. *a*.

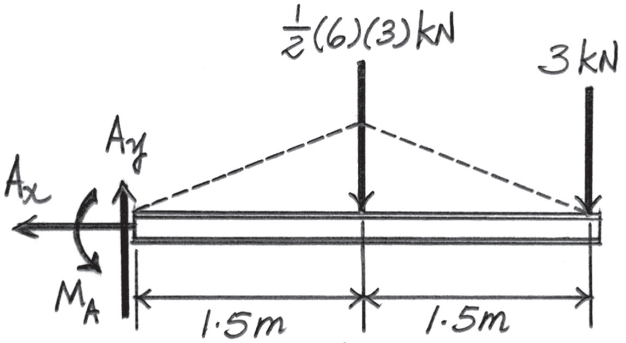
$$\zeta+\Sigma M_A = 0; \qquad M_A - \frac{1}{2}(6)(3)(1.5) - 3(3) = 0 \qquad M_A = 22.5\ \text{kN}\cdot\text{m}$$

$$+\uparrow\Sigma F_y = 0; \qquad A_y - \frac{1}{2}(6)(3) - 3 = 0 \qquad A_y = 12.0\ \text{kN}$$

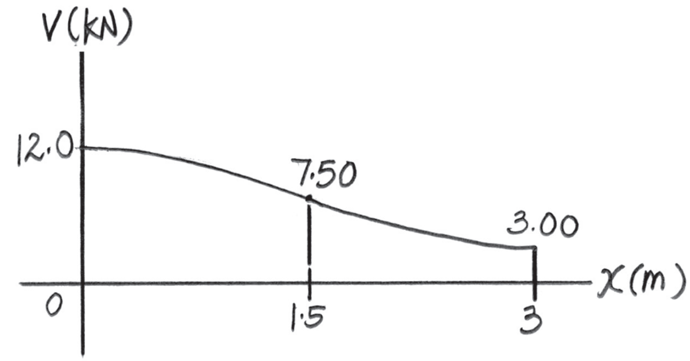
$$\overset{+}{\rightarrow}\Sigma F_x = 0 \qquad A_x = 0$$

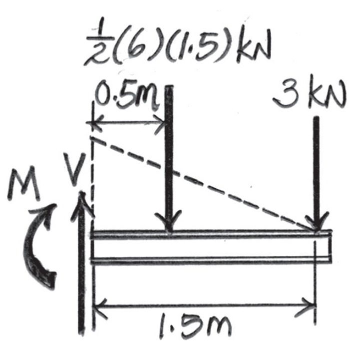
Internal Loadings. Referring to the *FBD* of the right segment of the beam sectioned at $x = 1.5$ m, the internal moment at this section is

$$\zeta+\Sigma M_O = 0; \qquad -M - \frac{1}{2}(6)(1.5)(0.5) - 3(1.5) = 0 \qquad M = -6.75\ \text{kN}\cdot\text{m}$$

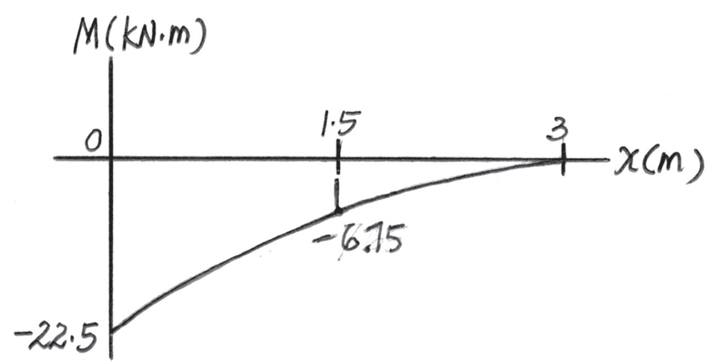


(a)





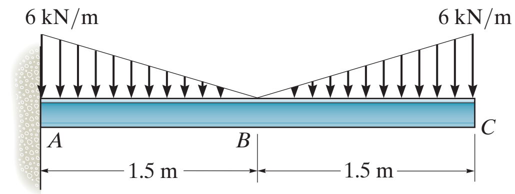
(b)



Ans:
$x = 1.5$
$V = 7.50$ kN
$M = -6.75\ \text{kN}\cdot\text{m}$

7–90.

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions. Referring to the *FBD* of the cantilevered beam shown in Fig. *a*

$$\zeta + \Sigma M_A = 0; \qquad M_A - \frac{1}{2}(6)(1.5)(0.5) - \frac{1}{2}(6)(1.5)(2.5) = 0$$

$$M_A = 13.5 \text{ kN} \cdot \text{m}$$

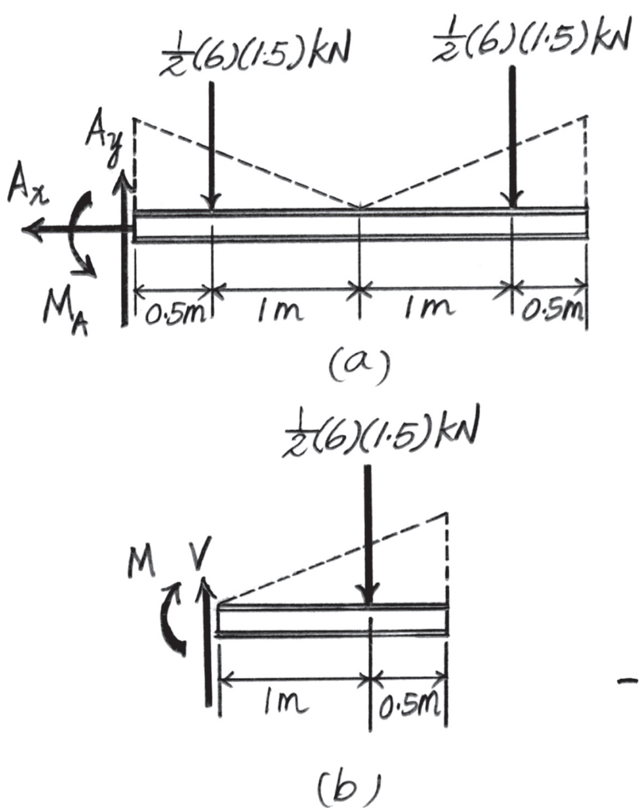
$$+\uparrow \Sigma F_y = 0; \qquad A_y - \frac{1}{2}(6)(1.5) - \frac{1}{2}(6)(1.5) = 0$$

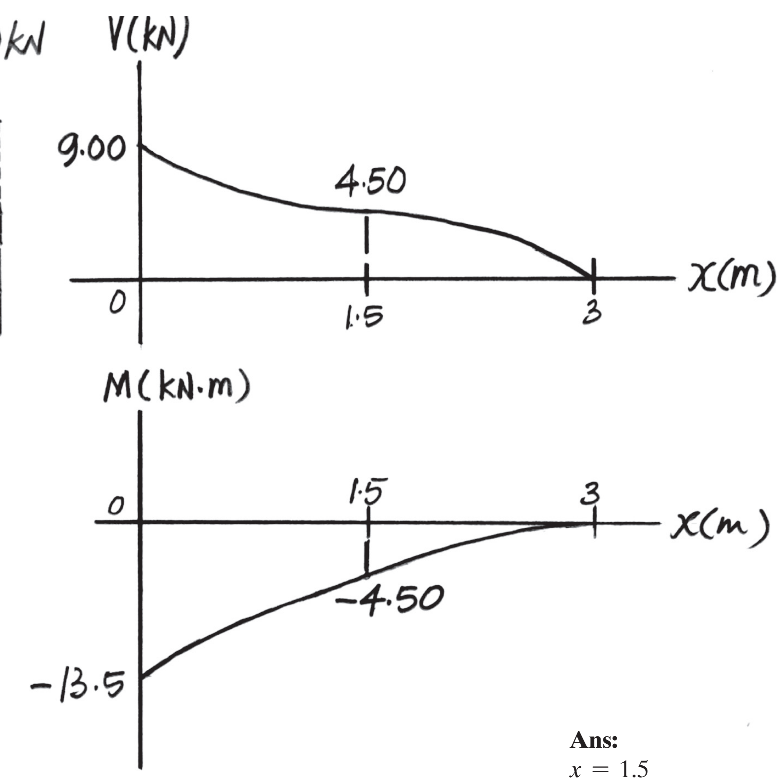
$$A_y = 9.00 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$$

Internal loadings. Referring to the *FBD* of the right segment of the beam sectioned at $x = 1.5$ m,

$$\zeta + \Sigma M_O = 0; \qquad -M - \frac{1}{2}(6)(1.5)(1) = 0 \qquad M = 4.50 \text{ kN} \cdot \text{m}$$

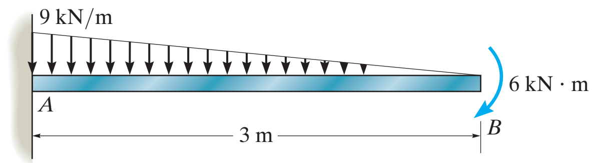




Ans:
$x = 1.5$
$V = 4.50$ kN
$M = -4.50$ kN · m

7–91.

Draw the shear and moment diagrams for the beam.



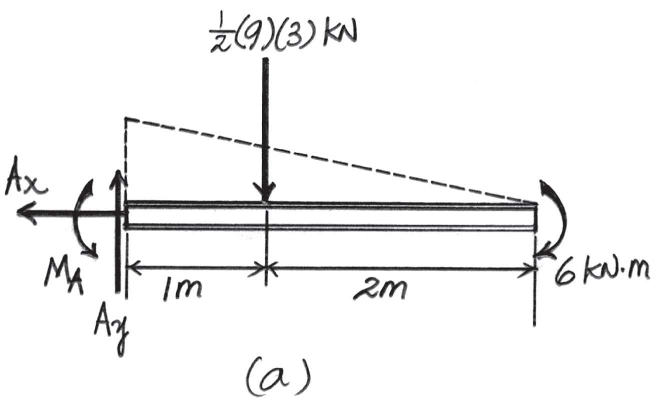
SOLUTION

Support Reactions. Referring to the *FBD* of the cantilevered beam shown in Fig. *a*,

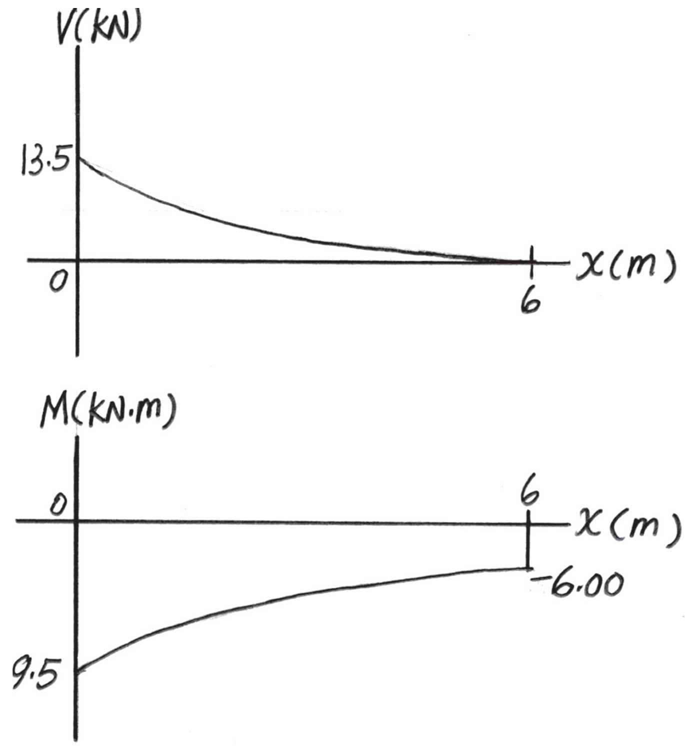
$$\circlearrowleft + \Sigma M_A = 0; \quad M_A - \frac{1}{2}(9)(3)(1) - 6 = 0 \quad M_A = 19.5 \text{ kN} \cdot \text{m}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - \frac{1}{2}(9)(3) = 0 \quad A_y = 13.5 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$$



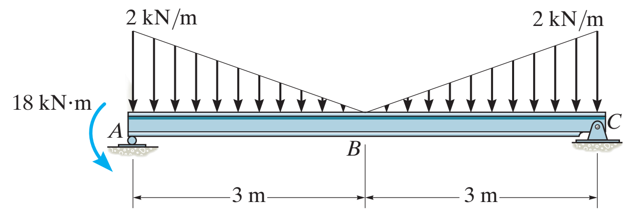
(a)



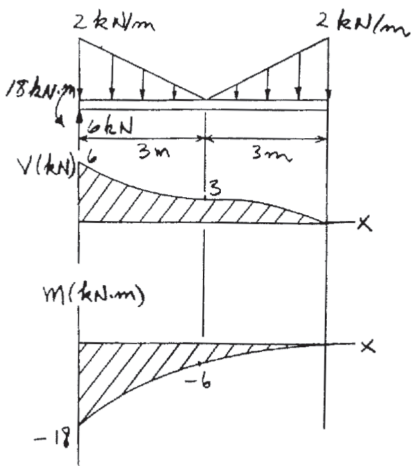
Ans:
$x = 0$
$V = 13.5$ kN
$M = -9.5$ kN · m

***7–92.**

Draw the shear and moment diagrams for the beam.

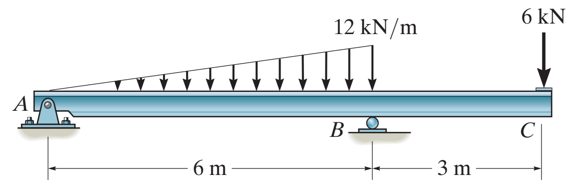


SOLUTION



7–93.

Draw the shear and moment diagrams for the beam.



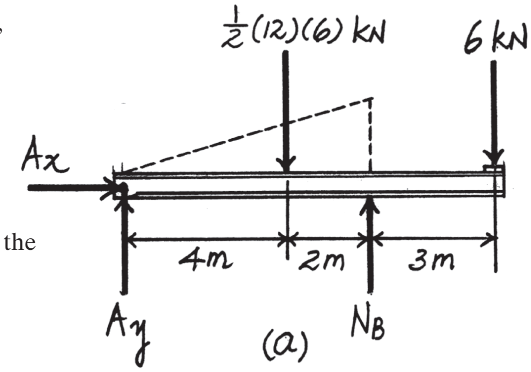
SOLUTION

Support Reactions. Referring to the *FBD* of the overhang beam shown in Fig. *a*,

$\zeta+\Sigma M_A = 0; \quad N_B(6) - \frac{1}{2}(12)(6)(4) - 6(9) = 0 \quad N_B = 33.0 \text{ kN}$

$\zeta+\Sigma M_B = 0; \quad \frac{1}{2}(12)(6)(2) - 6(3) - A_y(6) = 0 \quad A_y = 9.00 \text{ kN}$

$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad A_x = 0$



Shear And Moment Functions. Referring to the *FBD* of the left segment of the beam sectioned at a distance x, Fig. *b*

$+\uparrow\Sigma F_y = 0; \quad 9.00 - \frac{1}{2}(2x)(x) - V = 0 \quad V = \{9.00 - x^2\} \text{ kN}$

$\zeta+\Sigma M_O = 0; \quad M + \left[\frac{1}{2}(2x)(x)\right]\left(\frac{x}{3}\right) - 9.00x = 0$

$$M = \left\{9.00x - \frac{x^3}{3}\right\} \text{ kN}\cdot\text{m}$$

Set $V = 0$,

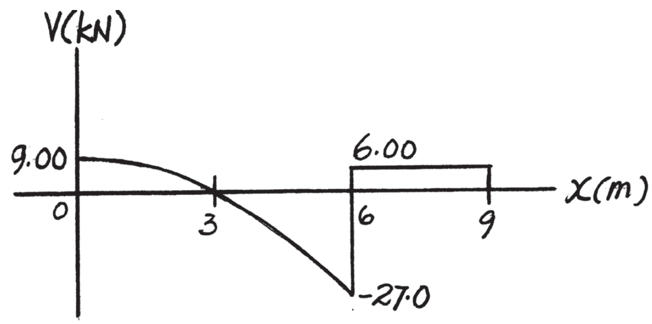
$$0 = 9.00 - x^2 \qquad x = 3 \text{ m}$$

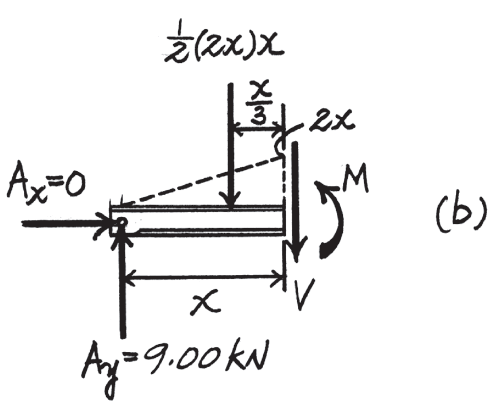
The corresponding moment is

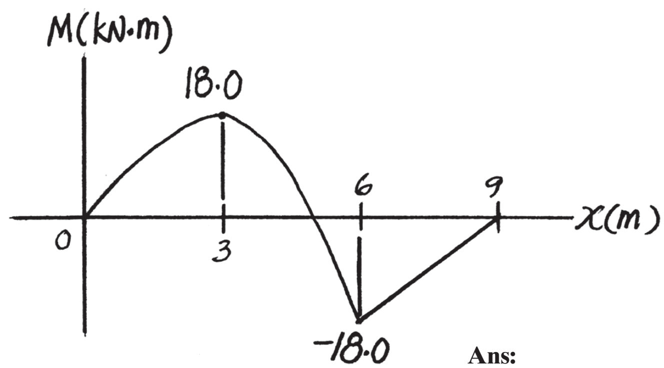
$$M = 9.00(3) - \frac{3^3}{3} = 18.0 \text{ kN}\cdot\text{m}$$

The moment at $x = 6$ m is

$$M = 9.00(6) - \frac{6^3}{3} = -18.0 \text{ kN}\cdot\text{m}$$

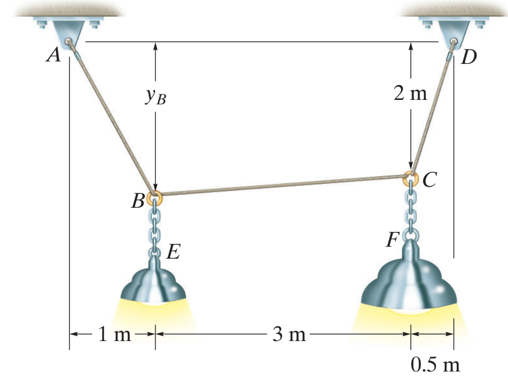






Ans:
$x = 3$
$V = 0$
$M = 18.0 \text{ kN}\cdot\text{m}$
$x = 6^-$
$V = -27.0 \text{ kN}$
$M = -18.0 \text{ kN}\cdot\text{m}$

7–94. Cable $ABCD$ supports the 10-kg lamp E and the 15-kg lamp F. Determine the maximum tension in the cable and the sag of point B.



SOLUTION

Given:

$M_1 = 10$ kg

$M_2 = 15$ kg

$a = 1$ m

$b = 3$ m

$c = 0.5$ m

$d = 2$ m

Guesses $\quad y_B = 1$ m $\quad T_{AB} = 1$ N $\quad T_{BC} = 1$ N $\quad T_{CD} = 1$ N

Given

$$\left(\frac{-a}{\sqrt{a^2 + y_B^2}}\right) T_{AB} + \left[\frac{b}{\sqrt{b^2 + (y_B - d)^2}}\right] T_{BC} = 0$$

$$\left(\frac{y_B}{\sqrt{a^2 + y_B^2}}\right) T_{AB} + \left[\frac{y_B - d}{\sqrt{b^2 + (y_B - d)^2}}\right] T_{BC} - M_1 g = 0$$

$$\left[\frac{-b}{\sqrt{b^2 + (y_B - d)^2}}\right] T_{BC} + \left(\frac{c}{\sqrt{c^2 + d^2}}\right) T_{CD} = 0$$

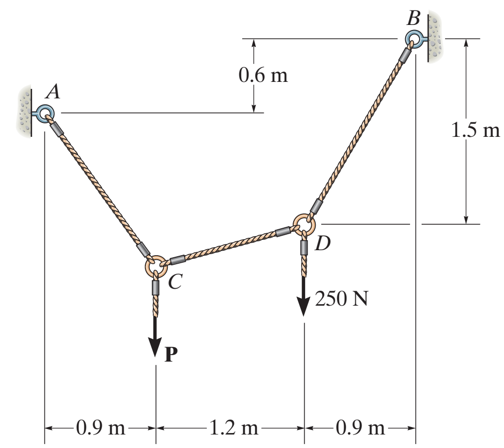
$$\left[\frac{-(y_B - d)}{\sqrt{b^2 + (y_B - d)^2}}\right] T_{BC} + \left(\frac{d}{\sqrt{c^2 + d^2}}\right) T_{CD} - M_2 g = 0$$

$$\begin{pmatrix} y_B \\ T_{AB} \\ T_{BC} \\ T_{CD} \end{pmatrix} = \text{Find}(y_B, T_{AB}, T_{BC}, T_{CD}) \qquad \begin{pmatrix} T_{AB} \\ T_{BC} \\ T_{CD} \end{pmatrix} = \begin{pmatrix} 100.163 \\ 38.524 \\ 157.243 \end{pmatrix} \text{N}$$

$T_{max} = \max(T_{AB}, T_{BC}, T_{CD})$ $\qquad T_{max} = 157.2$ N $\qquad y_B = 2.43$ m $\quad$ **Ans.**

Ans:
$T_{max} = 157.2$ N
$y_B = 2.43$ m

7–95. Determine the tension in each segment of the cable and the cable's total length. Set $P = 400$ N.



SOLUTION

From FBD (a)

$\circlearrowleft + \Sigma M_A = 0;\quad T_{BD} \cos 59.04° (0.9) + T_{BD} \sin 59.04° (2.1) - 250 (2.1) - 400 (0.9) = 0$

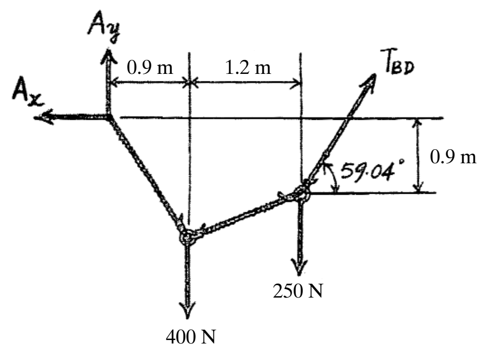
$T_{BD} = 390.935 \text{ N} = 390.9 \text{ N}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;\quad 390.935 \cos 59.04° - A_x = 0$

$A_x = 210.112 \text{ N}$

$+\uparrow \Sigma F_y = 0;\quad A_y + 390.935 \sin 59.04° - 400 - 250 = 0$

$A_y = 314.763 \text{ N}$



(a)

Joint A :

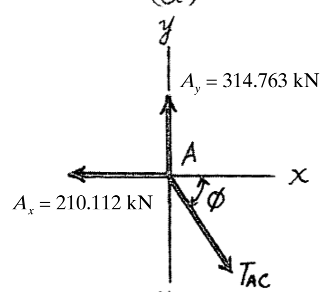
$\xrightarrow{+} \Sigma F_x = 0;\quad T_{AC} \cos \phi - 210.112 = 0$ (1)

$+\uparrow \Sigma F_y = 0;\quad -T_{AC} \sin \phi + 314.763 = 0$ (2)

Solving Eqs. (1) and (2) yields :

$\phi = 56.28°$

$T_{AC} = 378.4 \text{ N}$ **Ans.**



(b)

Joint D :

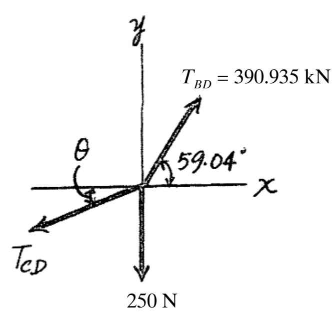
$\xrightarrow{+} \Sigma F_x = 0;\quad 390.935 \cos 59.04° - T_{CD} \cos \theta = 0$ (3)

$+\uparrow \Sigma F_y = 0;\quad 390.935 \sin 59.04° - T_{CD} \sin \theta - 250 = 0$ (4)

Solving Eqs. (3) and (4) yields :

$\theta = 22.97°$

$T_{CD} = 218.4 \text{ N}$ **Ans.**

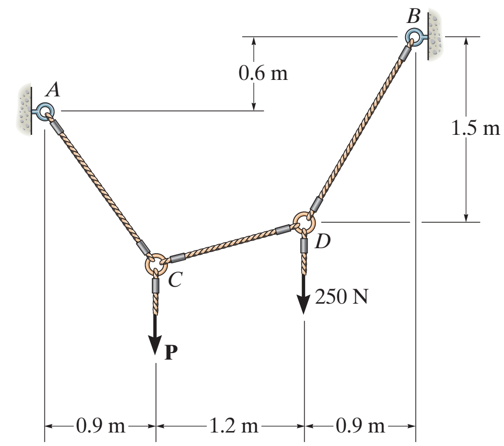


Total length of the cable:

$$l_T = \frac{1.5}{\sin 59.04°} + \frac{1.2}{\cos 22.97°} + \frac{0.9}{\cos 56.28°} = 4.674 \text{ m}$$ **Ans.**

Ans:
$T_{BD} = 390.9$ N, $T_{AC} = 378.4$ kN
$T_{CD} = 218.4$ N, $l_T = 4.674$ m

***7–96.** If each cable segment can support a maximum tension of 375 N, determine the largest load P that can be applied.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0;$ $\quad -T_{BD}\,(\cos 59.04°)\,(0.6) + T_{BD}\,(\sin 59.04°)\,(3) - 250\,(2.1) - P\,(0.9) = 0$

$T_{BD} = 0.39754P + 231.899$

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad -A_x + T_{BD}\cos 59.04° = 0$

$+\uparrow\Sigma F_y = 0;$ $\quad A_y - P - 250 + T_{BD}\sin 59.04° = 0$

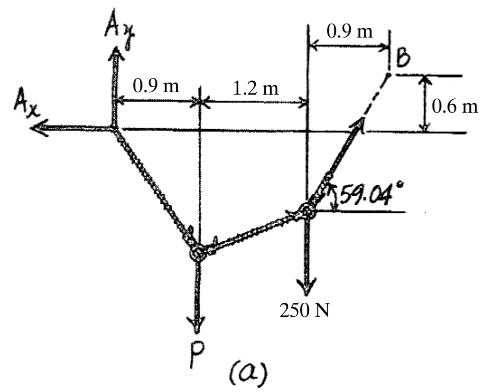
Assume maximum tension is in cable BD,

$T_{BD} = 375$ N

$P = 359.966$ N

$A_x = 192.915$ N

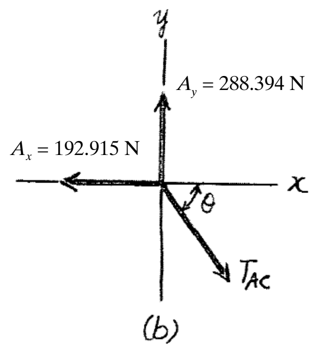
$A_y = 288.394$ N



Pin A :

$T_{AC} = \sqrt{(192.915)^2 + (288.394)^2} = 346.969 \text{ N} < 375 \text{ N} \quad \text{OK}$

$\theta = \tan^{-1}\left(\dfrac{288.394}{192.915}\right) = 56.22°$



Joint C :

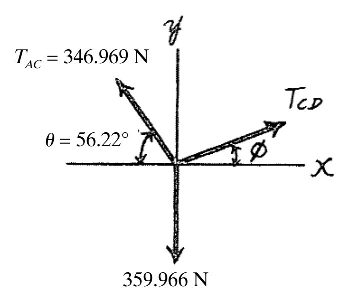
$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad T_{CD}\cos\phi - 346.969\cos 56.22° = 0$

$+\uparrow\Sigma F_y = 0;$ $\quad T_{CD}\sin\phi + 346.969\sin 56.22° - 359.966 = 0$

$T_{CD} = 205.766 \text{ N} < 375 \text{ N} \quad \text{OK}$

$\phi = 20.355°$

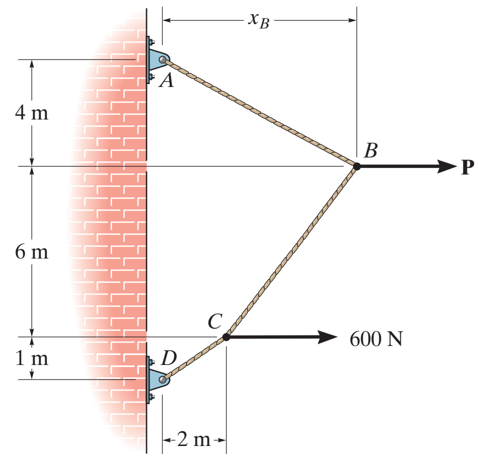
Thus, $\quad P = 360$ N **Ans.**



Ans:
$P = 360$ N

7–97.

The cable supports the loading shown. Determine the distance x_B the force at B acts from A. Set $P = 800$ N.



SOLUTION

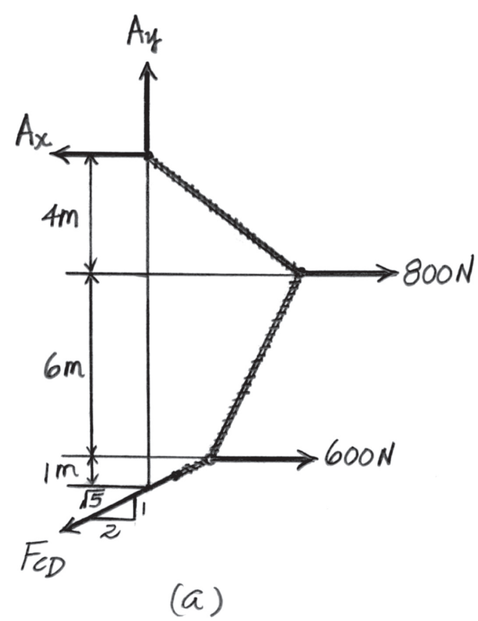
Support Reactions. Referring to the *FBD* of the cable system sectioned through cable *CD*, Fig. *a*

$$\zeta + \Sigma M_A = 0; \qquad 800(4) + 600(10) - F_{CD}\left(\frac{2}{\sqrt{5}}\right)(11) = 0$$

$$F_{CD} = 935.08 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 800 + 600 - 935.08\left(\frac{2}{\sqrt{5}}\right) - A_x = 0 \qquad A_x = 563.64 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 935.08\left(\frac{1}{\sqrt{5}}\right) = 0 \qquad A_y = 418.18 \text{ N}$$



Method of Joints. Consider the equilibrium of joint *A*, Fig. *b*

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AB}\left(\frac{x_B}{\sqrt{x_B^2 + 16}}\right) - 563.64 = 0$$

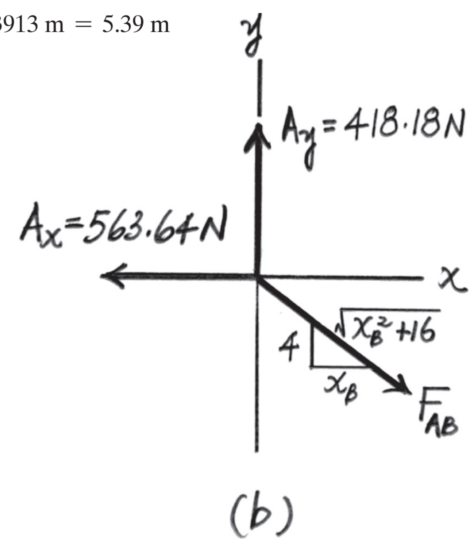
$$F_{AB}\left(\frac{x_B}{\sqrt{x_B^2 + 16}}\right) = 563.64 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad 418.18 - F_{AB}\left(\frac{4}{\sqrt{x_B^2 + 16}}\right) = 0$$

$$F_{AB}\left(\frac{4}{\sqrt{x_B^2 + 16}}\right) = 418.18 \qquad \textbf{(2)}$$

Divide Eq (1) by (2)

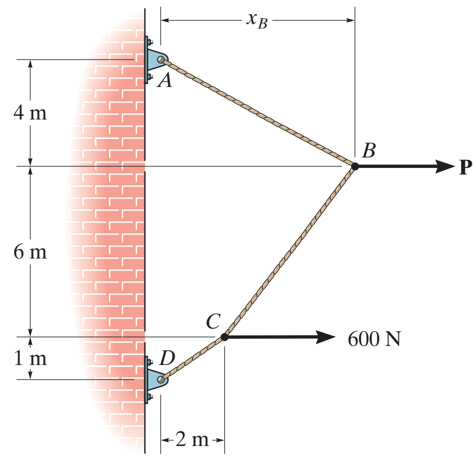
$$x_B = 5.3913 \text{ m} = 5.39 \text{ m} \qquad \textbf{Ans.}$$



Ans:
$x_B = 5.39$ m

7–98.

The cable supports the loading shown. Determine the magnitude of the horizontal force **P** so that $x_B = 5$ m.



SOLUTION

Support Reactions. Referring to the *FBD* of the cable system sectioned through cable *AB*, Fig. *a*

$$\zeta + \Sigma M_D = 0; \qquad F_{AB} = \left(\frac{5}{\sqrt{41}}\right)(11) - P(7) - 600(1) = 0$$

$$\frac{55}{\sqrt{41}} F_{AB} - 7P = 600 \qquad \textbf{(1)}$$

Method of Joints. Consider the equilibrium of joint *B*, Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\left(\frac{4}{\sqrt{41}}\right) - F_{BC}\left(\frac{2}{\sqrt{5}}\right) = 0 \qquad F_{BC} = \frac{2\sqrt{5}}{\sqrt{41}} F_{AB}$$

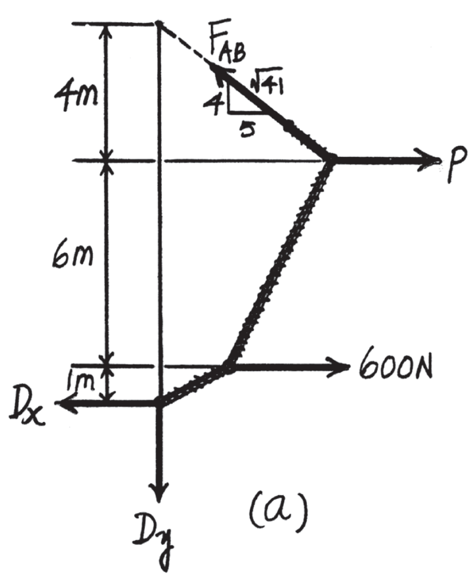
$$\xrightarrow{+} \Sigma F_x = 0; \qquad P - F_{AB}\left(\frac{5}{\sqrt{41}}\right) - \left(\frac{2\sqrt{5}}{\sqrt{41}} F_{AB}\right)\left(\frac{1}{\sqrt{5}}\right) = 0$$

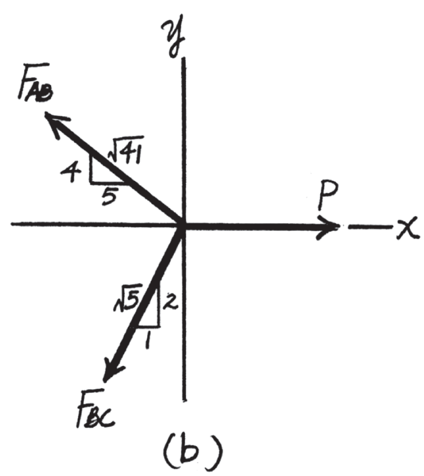
$$F_{AB} = \frac{\sqrt{41}}{7} P \qquad \textbf{(2)}$$

Substituting Eq. (2) into (1)

$$\frac{55}{\sqrt{41}}\left(\frac{\sqrt{41}}{7} P\right) - 7P = 600$$

$$P = 700 \text{ N} \qquad \textbf{Ans.}$$

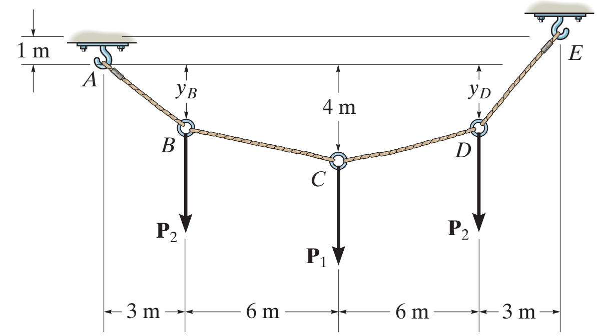




Ans:
$P = 700$ N

7–99.

The cable supports the three loads shown. Determine the sags y_B and y_D of B and D. Take $P_1 = 800$ N, $P_2 = 500$ N.



SOLUTION

Support Reactions. Referring to the *FBD* of the cable system sectioned through cable AB shown in Fig. a,

$$\circlearrowleft + \Sigma M_E = 0; \quad 500(3) + 800(9) + 500(15) - F_{AB}\left(\frac{y_B}{\sqrt{y_B^2 + 9}}\right)(15)$$

$$- F_{AB}\left(\frac{3}{\sqrt{y_B^2 + 9}}\right)(y_B + 1) = 0$$

$$F_{AB}\left(\frac{18y_B + 3}{\sqrt{y_B^2 + 9}}\right) = 16200 \qquad \textbf{(1)}$$

Also, referring to the *FBD* of the cable segment sectioned through cables AB and CD, shown Fig. b,

$$\circlearrowleft + \Sigma M_C = 0; \quad 500(6) + F_{AB}\left(\frac{3}{\sqrt{y_B^2 + 9}}\right)(4 - y_B) - F_{AB}\left(\frac{y_B}{\sqrt{y_B^2 + 9}}\right)(6) = 0$$

$$F_{AB}\left(\frac{9y_B - 12}{\sqrt{y_B^2 + 9}}\right) = 3000 \qquad \textbf{(2)}$$

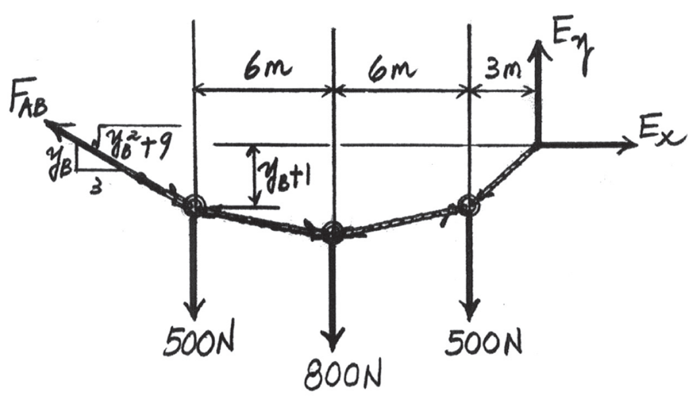
Divide Eq. (1) by (2)

$$y_B = 2.216 \text{ m} = 2.22 \text{ m} \qquad \textbf{Ans.}$$

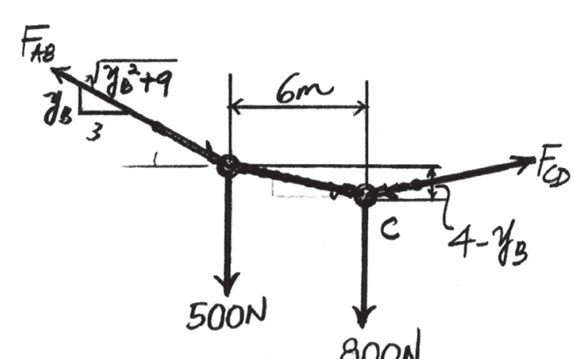
Substituting this result into Fig. (1)

$$F_{AB}\left[\frac{18(2.216) + 3}{\sqrt{2.216^2 + 9}}\right] = 16200$$

$$F_{AB} = 1408.93 \text{ N}$$



(a)



(b)

7–99. Continued

Method of joints. Perform the joint equilibrium analysis first for joint B and then joint C.

Joint *B*. Fig. *c*

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_{BC} \cos 16.56° - 1408.93 \cos 36.45° = 0$

$$F_{BC} = 1182.39 \text{ N}$$

$+\uparrow \Sigma F_y = 0;$ $\quad 1408.93 \sin 36.45 - 1182.39 \sin 16.56° - 500 = 0$ **(Check!)**

Joint *C*. Fig. *d*

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_{CD}\left(\dfrac{6}{\sqrt{(4 - y_D)^2 + 36}}\right) - 1182.39 \cos 16.56° = 0$

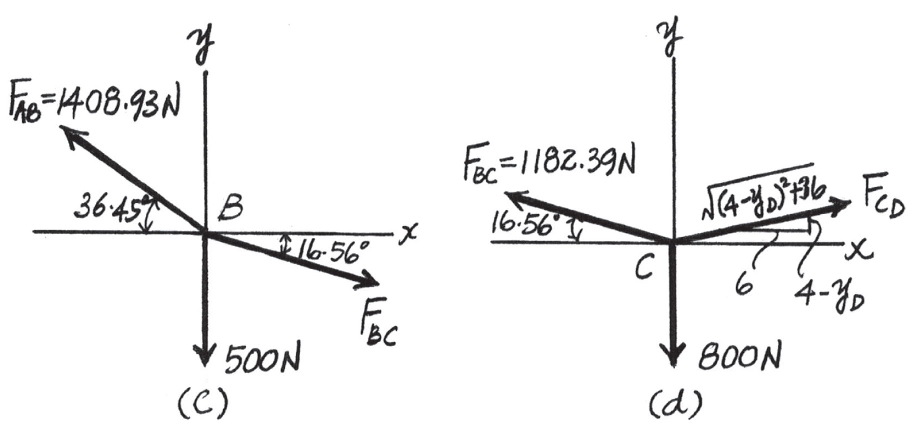
$$\frac{6}{\sqrt{(4 - y_D)^2 + 36}} F_{CD} = 1133.33 \qquad \textbf{(3)}$$

$+\uparrow \Sigma F_y = 0;$ $\quad F_{CD}\left(\dfrac{4 - y_D}{\sqrt{(4 - y_D)^2 + 36}}\right) + 1182.39 \sin 16.56° - 800 = 0$

$$\frac{4 - y_D}{\sqrt{(4 - y_D)^2 + 36}} F_{CD} = 462.96 \qquad \textbf{(4)}$$

Divide Eq (4) by (3)

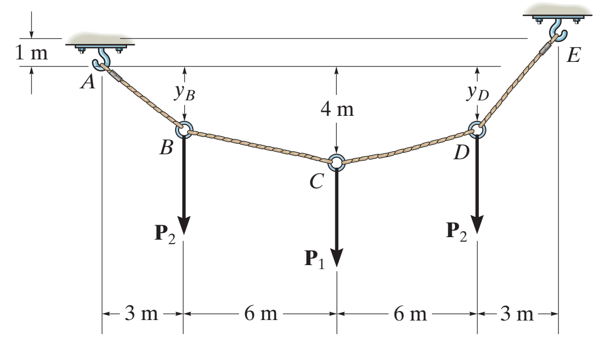
$$\frac{4 - y_D}{6} = 0.4085 \qquad y_D = 1.549 \text{ m} = 1.55 \text{ m} \qquad \textbf{Ans.}$$



Ans:
$y_B = 2.22$ m
$y_D = 1.55$ m

***7–100.**

The cable supports the three loads shown. Determine the magnitude of $\mathbf{P}_1$ if $P_2 = 600$ N and $y_B = 3$ m. Also find sag y_D.



SOLUTION

Support Reactions. Referring to the *FBD* of the cable system sectioned through cable *BC*, Fig. *a*

$$\circlearrowleft + \Sigma M_E = 0; \qquad 600(3) + P_1(9) - F_{BC}\left(\frac{6}{\sqrt{37}}\right)(5) - F_{BC}\left(\frac{1}{\sqrt{37}}\right)(9) = 0$$

$$\frac{39}{\sqrt{37}}F_{BC} - 9P_1 = 1800 \qquad \textbf{(1)}$$

Method of Joints. Perform the joint equilibrium analysis for joint *B* first, Fig. *b*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{BC}\left(\frac{6}{\sqrt{37}}\right) - F_{AB}\left(\frac{1}{\sqrt{2}}\right) = 0 \qquad \textbf{(2)}$$

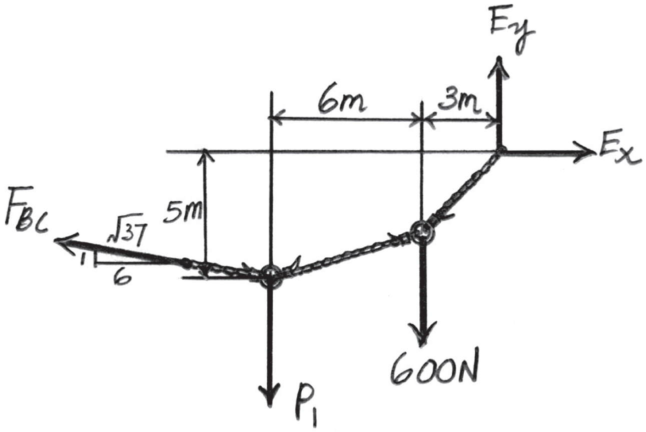
$$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\left(\frac{1}{\sqrt{2}}\right) - F_{BC}\left(\frac{1}{\sqrt{37}}\right) - 600 = 0 \qquad \textbf{(3)}$$

Solving Eqs (2) and (3),

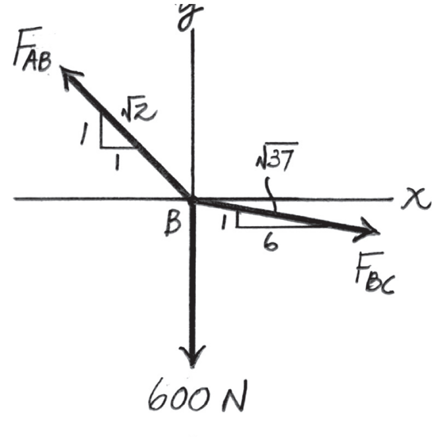
$$F_{BC} = 120\sqrt{37}\text{ N} \qquad F_{AB} = 720\sqrt{2}\text{ N}$$

Substitute the result of *FBC* into Fig. (1),

$$P_1 = 320\text{ N} \qquad \textbf{Ans.}$$



(a)



(b)

7–100. Continued

Next. Consider the equilibrium of joint C, Fig. c,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{CD}\left(\frac{6}{\sqrt{(4-y_D)^2+36}}\right) - \left(120\sqrt{37}\right)\left(\frac{6}{\sqrt{37}}\right) = 0$$

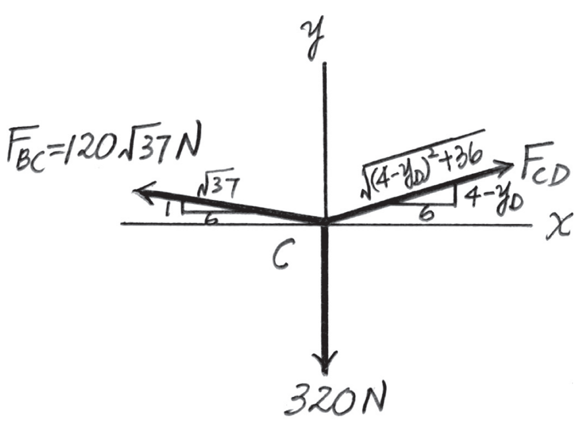
$$\frac{6}{\sqrt{(4-y_D)^2+36}}F_{CD} = 720 \qquad \textbf{(4)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{CD}\left(\frac{4-y_D}{\sqrt{(4-y_D)^2+36}}\right) + \left(120\sqrt{37}\right)\left(\frac{1}{\sqrt{37}}\right) - 320 = 0$$

$$\frac{4-y_D}{\sqrt{(4-y_D)^2+36}}F_{CD} = 200 \qquad \textbf{(5)}$$

Divide Eq (5) by (4)

$$\frac{4-y_D}{6} = \frac{5}{18} \qquad y_D = 2.3333\text{ m} = 2.33\text{ m} \qquad \textbf{Ans.}$$

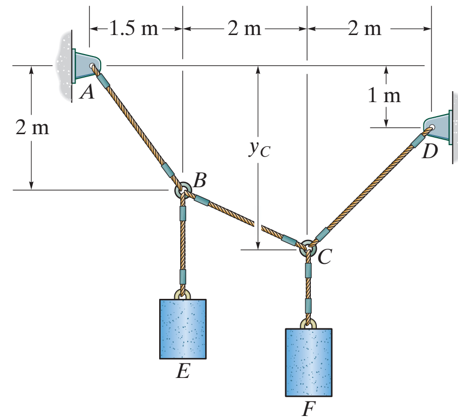


(C)

Ans:
$P_1 = 320$ N
$y_D = 2.33$ m

7–101.

If cylinders E and F have a mass of 20 kg and 40 kg, respectively, determine the tension developed in each cable and the sag y_C.



SOLUTION

First, $\mathbf{T}_{AB}$ will be obtained by considering the equilibrium of the free-body diagram shown in Fig. *a*. Subsequently, the result of T_{AB} will be used to analyze the equilibrium of joint B followed by joint C. Referring to Fig. *a*, we have

$$\circlearrowleft + \Sigma M_D = 0; \qquad 40(9.81)(2) + 20(9.81)(4) - T_{AB}\left(\frac{3}{5}\right)(1) - T_{AB}\left(\frac{4}{5}\right)(4) = 0$$

$$T_{AB} = 413.05 \text{ N} = 413 \text{ N} \qquad \textbf{Ans.}$$

Using the free-body diagram shown in Fig. *b*, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{BC} \cos\theta - 413.05\left(\frac{3}{5}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad 413.05\left(\frac{4}{5}\right) - 20(9.81) - T_{BC} \sin\theta = 0$$

Solving,

$$\theta = 28.44^\circ$$

$$T_{BC} = 281.85 \text{ N} = 282 \text{ N} \qquad \textbf{Ans.}$$

Using the result of θ and the geometry of the cable, y_C is given by

$$\frac{y_C - 2}{2} = \tan\theta = 28.44^\circ$$

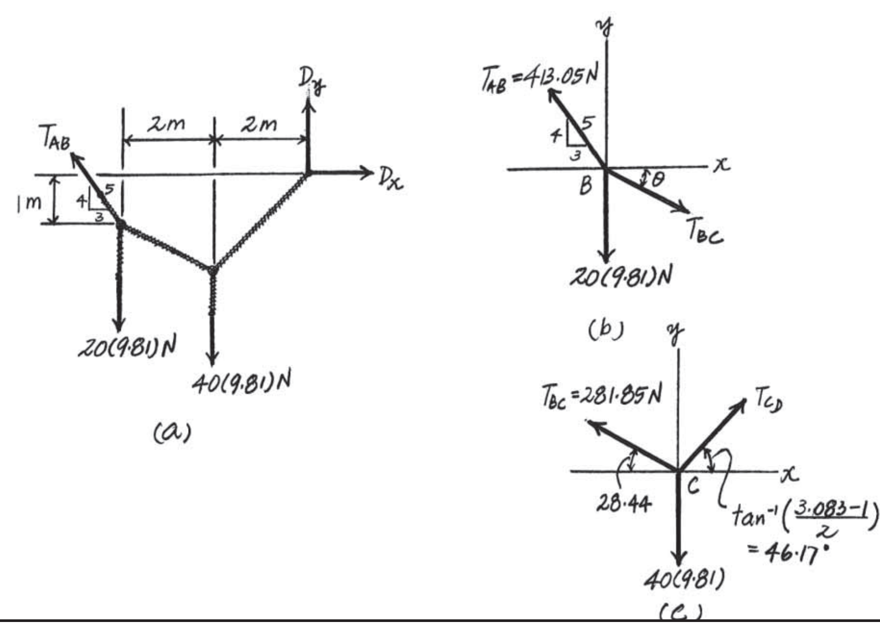
$$y_C = 3.083 \text{ m} = 3.08 \text{ m} \qquad \textbf{Ans.}$$

Using the results of y_C, θ, and T_{BC} and analyzing the equilibrium of joint C, Fig. *c*, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{CD} \cos 46.17^\circ - 281.85 \cos 28.44^\circ = 0$$

$$T_{CD} = 357.86 \text{ N} = 358 \text{ N} \qquad \textbf{Ans.}$$

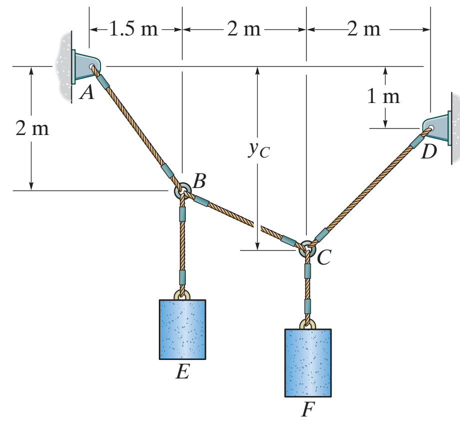
$$+\uparrow \Sigma F_y = 0; \qquad 281.85 \sin 28.44^\circ + 357.86 \sin 46.17^\circ - 40(9.81) = 0 \qquad \textbf{(\textit{Check!})}$$



Ans:
$T_{AB} = 413$ N
$T_{BC} = 282$ N
$y_C = 3.08$ m
$T_{CD} = 358$ N

7–102.

If cylinder E has a mass of 20 kg and each cable segment can sustain a maximum tension of 400 N, determine the largest mass of cylinder F that can be supported. Also, what is the sag y_C?

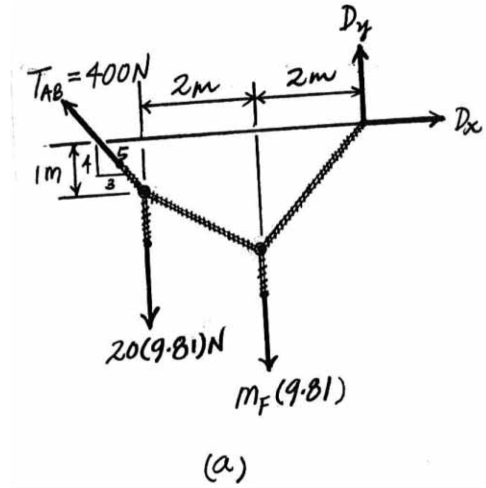


SOLUTION

We will assume that cable AB is subjected to the greatest tension, i.e., $T_{AB} = 400$ N. Based on this assumption, M_F can be obtained by considering the equilibrium of the free-body diagram shown in Fig. *a*. We have

$$\zeta + \Sigma M_D = 0; \qquad M_F(9.81)(2) + 20(9.81)(4) - 400\left(\frac{3}{5}\right)(1) - 400\left(\frac{4}{5}\right)(4) = 0$$

$$M_F = 37.47 \text{ kg} \qquad \textbf{Ans.}$$



Analyzing the equilibrium of joint B and referring to the free-body diagram shown in Fig. *b*, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{BC}\cos\theta - 400\left(\frac{3}{5}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad 400\left(\frac{4}{5}\right) - 20(9.81) - T_{BC}\sin\theta = 0$$

Solving,

$$\theta = 27.29°$$

$$T_{BC} = 270.05 \text{ N}$$

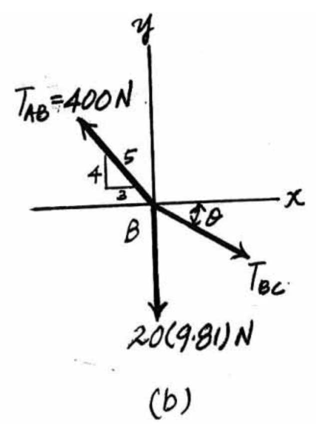
Using these results and analyzing the equilibrium of joint C,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{CD}\cos\phi - 270.05\cos 27.29° = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad T_{CD}\sin\phi + 270.05\sin 27.29° - 37.47(9.81) = 0$$

Solving,

$$\phi = 45.45° \qquad T_{CD} = 342.11 \text{ N}$$



By comparing the above results, we realize that cable AB is indeed subjected to the greatest tension. Thus,

$$M_F = 37.5 \text{ kg} \qquad \textbf{Ans.}$$

Using the result of either θ or ϕ, the geometry of the cable gives

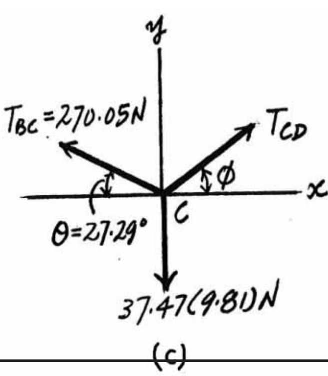
$$\frac{y_C - 2}{2} = \tan\theta = \tan 27.29°$$

$$y_C = 3.03 \text{ m} \qquad \textbf{Ans.}$$

or

$$\frac{y_C - 1}{2} = \tan\phi = \tan 45.45°$$

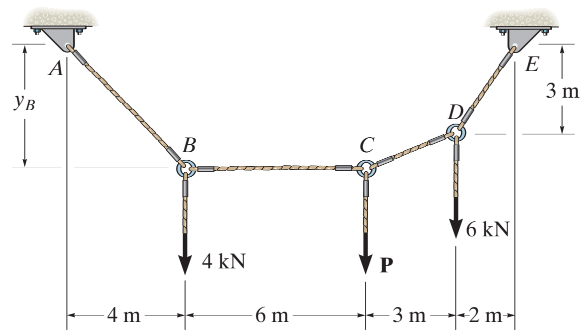
$$y_C = 3.03 \text{ m} \qquad \textbf{Ans.}$$



Ans:
$Mp = 37.47$ kg
$Mp = 37.5$ kg
$yc = 3.03$ m

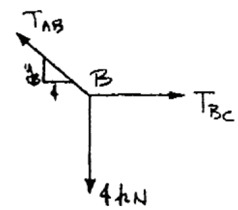
7–103.

Determine the force P needed to hold the cable in the position shown, i.e., so segment BC remains horizontal. Also, compute the sag y_B and the maximum tension in the cable.



SOLUTION

Joint B:

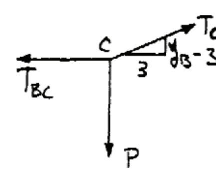


$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{BC} - \frac{4}{\sqrt{y_B^2 + 16}} T_{AB} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{y_B}{\sqrt{y_B^2 + 16}} T_{AB} - 4 = 0$$

$$y_B T_{BC} = 16 \qquad \textbf{(1)}$$

Joint C:



$$\xrightarrow{+} \Sigma F_x = 0; \qquad \frac{3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} - T_{BC} = 0 \qquad \textbf{(2)}$$

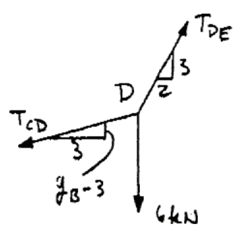
$$+\uparrow \Sigma F_y = 0; \qquad \frac{y_B - 3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} - P = 0$$

$$(y_B - 3) T_{BC} = 3P \qquad \textbf{(3)}$$

Combining Eqs. (1) and (2):

$$\frac{3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} = \frac{16}{y_B} \qquad \textbf{(4)}$$

Joint D:



$$\xrightarrow{+} \Sigma F_x = 0; \qquad \frac{2}{\sqrt{13}} T_{DE} - \frac{3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{3}{\sqrt{13}} T_{DE} - \frac{y_B - 3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} - 6 = 0$$

$$\frac{15 - 2\,y_B}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} = 12 \qquad \textbf{(5)}$$

From Eqs. (1) and (3): $\quad 3\,y_B P - 16\,y_B + 48 = 0$

From Eqs. (4) and (5): $\quad y_B = 3.53 \text{ m}$ **Ans.**

$P = 0.8 \text{ kN}$ **Ans.**

$T_{AB} = 6.05 \text{ kN}$

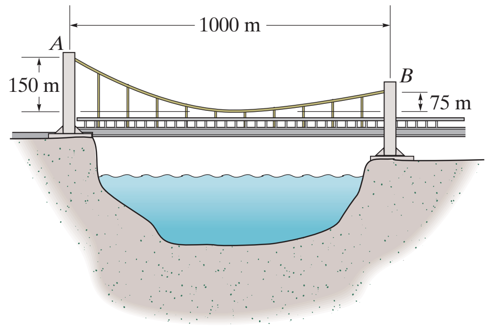
$T_{BC} = 4.53 \text{ kN}$

$T_{CD} = 4.60 \text{ kN}$

$T_{max} = T_{DE} = 8.17 \text{ kN}$ **Ans.**

Ans:
$y_B = 3.53 \text{ m}$
$P = 0.8 \text{ kN}$
$T_{\max} = T_{DE} = 8.17 \text{ kN}$

***7–104.** The bridge deck has a weight per unit length of 80 kN/m. It is supported on each side by a cable. Determine the tension in each cable at the piers A and B.



SOLUTION

As shown in Fig. a, the origin of the x, y coordinate system is set at the lowest point of the cable. Since the bridge deck is supported by two cables, $w(x) = \dfrac{80}{2} = 40$ kN/m.

$$\frac{d^2y}{dx^2} = \frac{w(x)}{F_H} = \frac{40(10^3)}{F_H}$$

Integrating,

$$\frac{dy}{dx} = \frac{40(10^3)}{F_H} + C_1$$

Applying the boundary condition $\dfrac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\frac{dy}{dx} = \frac{40(10^3)}{F_H}x \qquad \textbf{(1)}$$

Integrating,

$$y = \frac{20(10^3)}{F_H}x^2 + C_2$$

Applying the boundary condition $y = 0$ at $x = 0$ results in $C_2 = 0$. Thus,

$$y = \frac{20(10^3)}{F_H}x^2$$

Applying two other boundary conditions $y = 75$ m at $x = x_0$ and $y = 150$ m at $x = -(1000 - x_0)$,

$$75 = \frac{20(10^3)}{F_H}x_0^2$$

$$150 = \frac{20(10^3)}{F_H}[-(1000 - x_0)]^2$$

Solving these equations

$$x_0 = 414.21 \text{ m} \qquad F_H = 45.75(10^6) \text{ N}$$

Substituting the result for $\mathbf{F}_H$ into Eq. (1),

$$\frac{dy}{dx} = \frac{40(10^3)}{45.75(10^6)}x = 0.8743(10^{-3})x$$

7–104 (Continued)

Thus, the angles the cables make with the horizontal at A and B are

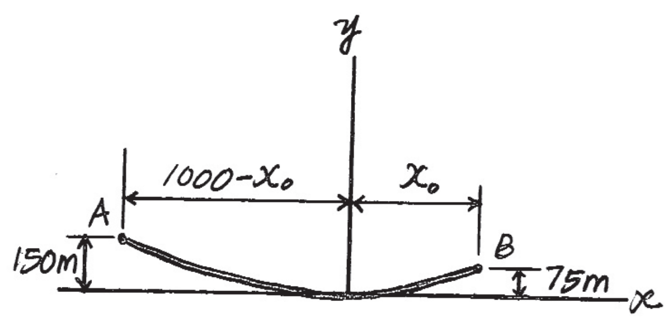
$$\theta_B = \left|\tan^{-1}\left(\frac{dy}{dx}\bigg|_{x_B}\right)\right| = \left|\tan^{-1}[0.8743(10^{-3})(414.21)]\right| = 19.91°$$

$$\theta_A = \left|\tan^{-1}\left(\frac{dy}{dx}\bigg|_{x_A}\right)\right| = \left|\tan^{-1}\{0.8743(10^{-3})[-(1000-414.21)]\}\right| = 27.12°$$

Thus,

$$T_B = \frac{F_H}{\cos\theta_B} = \frac{45.75(10^6)}{\cos 19.91°} = 48.66(10^6)\text{ N} = 48.7\text{ MN} \qquad \textbf{Ans.}$$

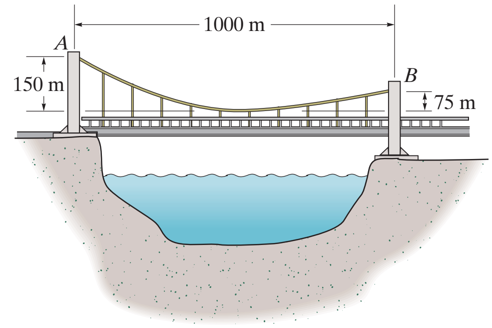
$$T_A = \frac{F_H}{\cos\theta_A} = \frac{45.75(10^6)}{\cos 27.12°} = 51.40(10^6)\text{ N} = 51.4\text{ MN} \qquad \textbf{Ans.}$$



(a)

Ans:
$T_B = 48.7$ MN
$T_A = 51.4$ MN

7–105. If each of the two side cables that support the bridge deck can sustain a maximum tension of 50 MN, determine the allowable uniform distributed load w_0 caused by the weight of the bridge deck.



SOLUTION

As shown in Fig. *a*, the origin of the *x*, *y* coordinate system is set at the lowest point of the cable. Since the bridge deck is supported by two cables, $w(x) = \frac{w_0}{2}$.

$$\frac{d^2y}{dx^2} = \frac{w_0/2}{F_H} = \frac{w_0}{2F_H}$$

Integrating,

$$\frac{dy}{dx} = \frac{w_0}{2F_H} + C_1$$

Applying the boundary condition $\frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\frac{dy}{dx} = \frac{w_0}{2F_H}x \qquad \textbf{(1)}$$

Integrating,

$$y = \frac{w_0}{4F_H}x^2 + C_2$$

Applying the boundary condition $y = 0$ at $x = 0$ results in $C_2 = 0$. Thus,

$$y = \frac{w_0}{4F_H}x^2$$

Applying two other boundary conditions $y = 75$ m at $x = x_0$ and $y = 150$ m at $x = -(1000–x_0)$,

$$75 = \frac{w_0}{4F_H}x^2$$

$$150 = \frac{w_0}{4F_H}[-(1000 - x_0)^2]$$

Solving these equations

$$x_0 = 414.21\text{m} \qquad F_H = 571.91w_0$$

Substituting the result for $\mathbf{F}_H$ into Eq. (1),

$$\frac{dy}{dx} = \frac{w_0}{2(571.91w_0)}x = 0.8743(10^{-3})x$$

7–105 (Continued)

By observation, the angle the cable makes with the horizontal at $A(\theta_A)$ is greater than that at $B(\theta_B)$. Thus, the cable tension at A is the greatest.

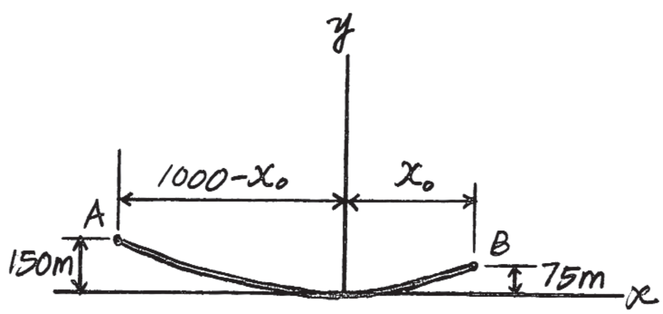
$$\theta_A = \left| \tan^{-1}\left(\frac{dy}{dx}\bigg|_{x_A}\right) \right| = \left| \tan^{-1}\{0.8743(10^{-3})[-(1000 - 414.21)]\} \right| = 27.12°$$

By setting $T_A = 50(10^6)$ N,

$$T_A = \frac{F_H}{\cos\theta_A}$$

$$50(10^6) = \frac{571.91w_0}{\cos 27.12°}$$

$$w_0 = 77.82(10^3)\text{ N/m} = 77.8\text{ kN/m}$$ **Ans.**

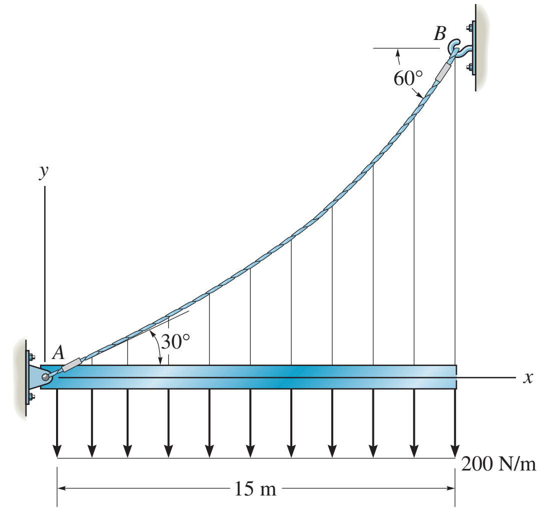


(a)

Ans:
$w_0 = 77.8$ kN/m

7–106.

The cable AB is subjected to a uniform loading of 200 N/m. If the weight of the cable is neglected and the slope angles at points A and B are 30° and 60°, respectively, determine the curve that defines the cable shape and the maximum tension developed in the cable.



SOLUTION

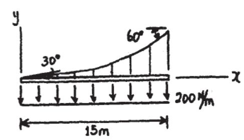
$$y = \frac{1}{F_H}\int\left(\int 200\,dx\right)dx$$

$$y = \frac{1}{F_H}(100x^2 + C_1x + C_2)$$

$$\frac{dy}{dx} = \frac{1}{F_H}(200x + C_1)$$

At $x = 0$, $\quad y = 0; \quad C_2 = 0$

At $x = 0$, $\quad \frac{dy}{dx} = \tan 30°; \quad C_1 = F_H \tan 30°$



$$y = \frac{1}{F_H}(100x^2 + F_H \tan 30° x)$$

At $x = 15$ m, $\quad \frac{dy}{dx} = \tan 60°; \quad F_H = 2598$ N

$$y = (38.5x^2 + 577x)(10^{-3})\text{ m}$$ **Ans.**

$$\theta_{max} = 60°$$

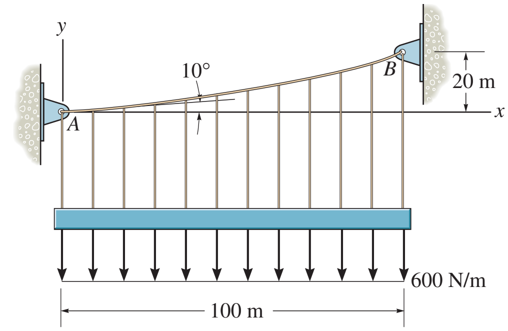
$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{2598}{\cos 60°} = 5196\text{ N}$$

$$T_{max} = 5.20\text{ kN}$$ **Ans.**

Ans:
$y = (38.5x^2 + 577x)(10^{-3})$ m
$T_{max} = 5.20$ kN

7–107.

Determine the maximum tension developed in the cable if it is subjected to a uniform load of 600 N/m.



SOLUTION

The Equation of The Cable:

$$y = \frac{1}{F_H}\int \left(\int w(x)dx\right) dx$$

$$= \frac{1}{F_H}\left(\frac{w_0}{2}x^2 + C_1x + C_2\right) \quad \textbf{(1)}$$

$$\frac{dy}{dx} = \frac{1}{F_H}(w_0x + C_1) \quad \textbf{(2)}$$

Boundary Conditions:

$y = 0$ at $x = 0$, then from Eq. (1) $0 = \frac{1}{F_H}(C_2) \qquad C_2 = 0$

$\frac{dy}{dx} = \tan 10°$ at $x = 0$, then from Eq. (2) $\tan 10° = \frac{1}{F_H}(C_1) \qquad C_1 = F_H \tan 10°$

Thus,
$$y = \frac{w_0}{2F_H}x^2 + \tan 10° x \quad \textbf{(3)}$$

$y = 20$ m at $x = 100$ m, then from Eq. (3)

$$20 = \frac{600}{2F_H}(100^2) + \tan 10°(100) \qquad F_H = 1\,267\,265.47 \text{ N}$$

and

$$\frac{dy}{dx} = \frac{w_0}{F_H}x + \tan 10°$$

$$= \frac{600}{1\,267\,265.47}x + \tan 10°$$

$$= 0.4735(10^{-3})x + \tan 10°$$

$\theta = \theta_{\max}$ at $x = 100$ m and the maximum tension occurs when $\theta = \theta_{\max}$.

$$\tan \theta_{\max} = \left.\frac{dy}{dx}\right|_{x=100\text{ m}} = 0.4735(10^{-3})(100) + \tan 10°$$

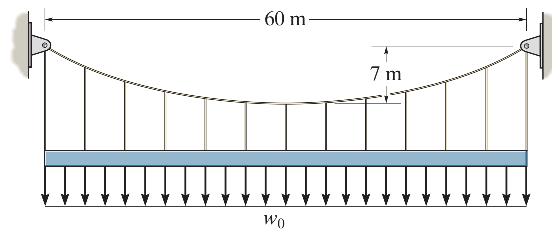
$$\theta_{\max} = 12.61°$$

The maximum tension in the cable is

$$T_{\max} = \frac{F_H}{\cos \theta_{\max}} = \frac{1\,267\,265.47}{\cos 12.61°} = 1\,298\,579.01 \text{ N} = 1.30 \text{ MN} \qquad \textbf{Ans.}$$

Ans:
$T_{max} = 1.30$ MN

***7–108.** Determine the maximum uniform distributed loading w_0 N/m that the cable can support if it is capable of sustaining a maximum tension of 60 kN.



SOLUTION

The Equation of The Cable:

$$y = \frac{1}{F_H}\int \left(\int w(x)dx\right)dx$$

$$= \frac{1}{F_H}\left(\frac{w_0}{2}x^2 + C_1x + C_2\right) \qquad \textbf{[1]}$$

$$\frac{dy}{dx} = \frac{1}{F_H}(w_0x + C_1) \qquad \textbf{[2]}$$

Boundary Conditions:

$y = 0$ at $x = 0$, then from Eq.[1] $0 = \frac{1}{F_H}(C_2) C_2 = 0$

$\frac{dy}{dx} = 0$ at $x = 0$, then from Eq.[2] $0 = \frac{1}{F_H}(C_1) C_1 = 0$

Thus,

$$y = \frac{w_0}{2F_H}x^2 \qquad \textbf{[3]}$$

$$\frac{dy}{dx} = \frac{w_0}{F_H}x \qquad \textbf{[4]}$$

$y = 7$ m at $x = 30$ m, then from Eq.[3] $7 = \frac{w_0}{2F_H}(30^2) F_H = \frac{450}{7}w_0$

$\theta = \theta_{max}$ at $x = 30$ m and the maximum tension occurs when $\theta = \theta_{max}$. From Eq.[4]

$$\tan\theta_{max} = \left.\frac{dy}{dx}\right|_{x=30\text{ m}} = \frac{w_0}{\frac{450}{7}w_0}x = 0.01556(30) = 0.4667$$

$$\theta_{max} = 25.02°$$

The maximum tension in the cable is

$$T_{max} = \frac{F_H}{\cos\theta_{max}}$$

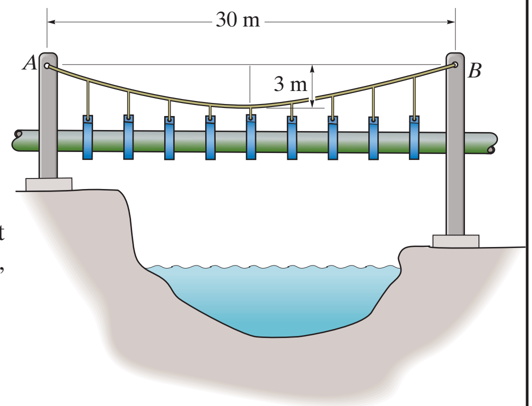
$$60 = \frac{\frac{450}{7}w_0}{\cos 25.02°}$$

$$w_0 = 0.846 \text{ kN/m} \qquad \textbf{Ans.}$$

Ans:
$w_0 = 0.846$ kN/m

7–109.

If the pipe has a mass per unit length of 1500 kg/m, determine the maximum tension developed in the cable.



SOLUTION

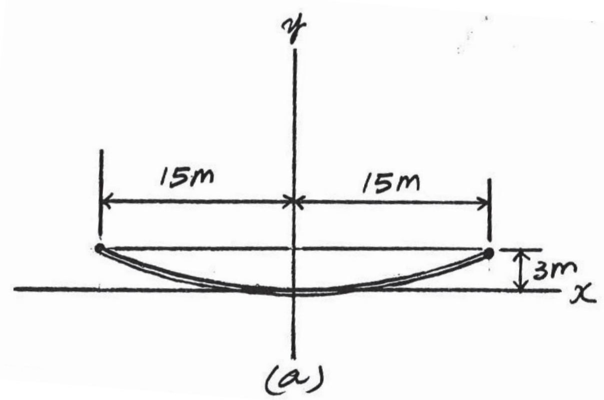
As shown in Fig. a, the origin of the x, y coordinate system is set at the lowest point of the cable. Here, $w(x) = w_0 = 1500(9.81) = 14.715(10^3)$ N/m. Using Eq. 7–12, we can write

$$y = \frac{1}{F_H}\int\left(\int w_0 dx\right)dx$$

$$= \frac{1}{F_H}\left(\frac{14.715(10^3)}{2}x^2 + c_1 x + c_2\right)$$

Applying the boundary condition $\frac{dy}{dx} = 0$ at $x = 0$, results in $c_1 = 0$.
Applying the boundary condition $y = 0$ at $x = 0$ results in $c_2 = 0$. Thus,

$$y = \frac{7.3575(10^3)}{F_H}x^2$$



Applying the boundary condition $y = 3$ m at $x = 15$ m, we have

$$3 = \frac{7.3575(10^3)}{F_H}(15)^2 \qquad F_H = 551.81(10^3)\text{ N}$$

Substituting this result into Eq. (1), we have

$$\frac{dy}{dx} = 0.02667x$$

The maximum tension occurs at either points at A or B where the cable has the greatest angle with the horizontal. Here,

$$\theta_{max} = \tan^{-1}\left(\left.\frac{dy}{dx}\right|_{15\text{ m}}\right) = \tan^{-1}[0.02667(15)] = 21.80°$$

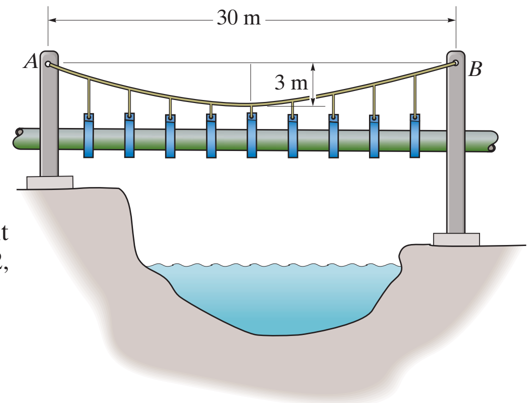
Thus,

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{551.8(10^3)}{\cos 21.80°} = 594.32(10^3)\text{ N} = 594\text{ kN}$$

Ans:
$T_{max} = 594$ kN

7–110.

If the pipe has a mass per unit length of 1500 kg/m, determine the minimum tension developed in the cable.



SOLUTION

As shown in Fig. *a*, the origin of the x, y coordinate system is set at the lowest point of the cable. Here, $w(x) = w_0 = 1500(9.81) = 14.715(10^3)$ N/m. Using Eq. 7–12, we can write

$$y = \frac{1}{F_H}\int\left(\int w_0 dx\right)dx$$

$$= \frac{1}{F_H}\left(\frac{14.715(10^3)}{2}x^2 + c_1x + c_2\right)$$

Applying the boundary condition $\dfrac{dy}{dx} = 0$ at $x = 0$, results in $c_1 = 0$.
Applying the boundary condition $y = 0$ at $x = 0$ results in $c_2 = 0$. Thus,

$$y = \frac{7.3575(10^3)}{F_H}x^2$$

Applying the boundary condition $y = 3$ m at $x = 15$ m, we have

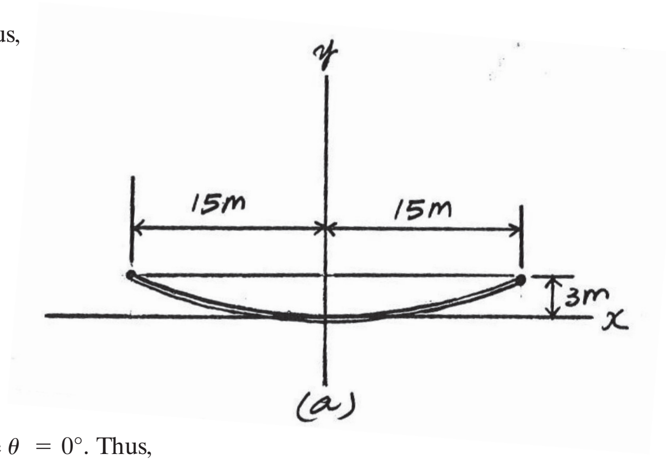
$$3 = \frac{7.3575(10^3)}{F_H}(15)^2 \qquad F_H = 551.81(10^3)\text{ N}$$

Substituting this result into Eq. (1), we have

$$\frac{dy}{dx} = 0.02667x$$

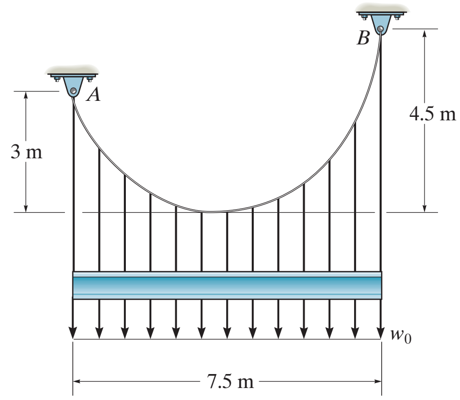
The minimum tension occurs at the lowest point of the cable, where $\theta = 0°$. Thus,

$$T_{min} = F_H = 551.81(10^3)\text{ N} = 552\text{ kN}$$



Ans:
$T_{min} = 552$ kN

7–111. The cable supports the uniform distributed load of $w_0 = 12$ kN/m. Determine the tension in the cable at each support A and B.



SOLUTION

Use the equations of Example 7.12.

$$y = \frac{w_0}{2\,F_H}x^2$$

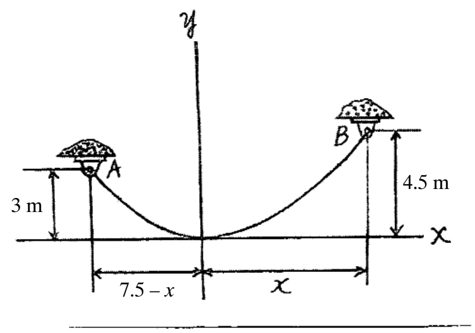
$$4.5 = \frac{12}{2\,F_H}x^2$$

$$3 = \frac{12}{2\,F_H}(7.5 - x)^2$$

$$\frac{12}{2\,(4.5)}x^2 = \frac{12}{2\,(3)}(7.5 - x)^2$$

$$x^2 = 1.5\,(56.26 - 15x + x^2)$$

$$0.5x^2 - 22.5x + 84.375 = 0$$



Choose root < 7.5 m

$$x = 4.1288 \text{ m}$$

$$F_H = \frac{w_0}{2\,y}x^2 = \frac{12}{2\,(4.5)}\,(4.1288)^2 = 22.729 \text{ kN}$$

At B:

$$y = \frac{w_0}{2\,F_H}x^2 = \frac{12}{2\,(22.729)}x^2$$

$$\frac{dy}{dx} = \tan\theta_B = 0.52796\,x\big|_{x = 4.1288} = 2.180$$

$$\theta_B = 65.36^\circ$$

$$T_B = \frac{F_H}{\cos\theta_B} = \frac{22.729}{\cos 65.36^\circ} = 54.52 \text{ kN}$$ **Ans.**

At A:

$$y = \frac{w_0}{2\,F_H}x^2 = \frac{12}{2\,(22.729)}x^2$$

$$\frac{dy}{dx} = \tan\theta_A = 0.52796\,x\big|_{x = (7.5 - 4.1288)} = 1.780$$

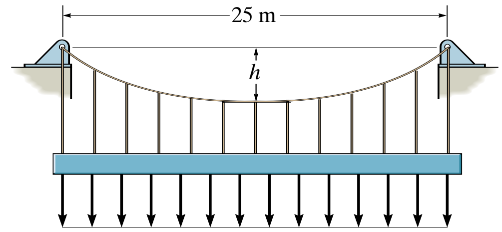
$$\theta_A = 60.67^\circ$$

$$T_A = \frac{F_H}{\cos\theta_A} = \frac{22.729}{\cos 60.67^\circ} = 46.40 \text{ kN}$$ **Ans.**

Ans:
$T_B = 54.52$ kN
$T_A = 46.40$ kN

***7–112.**

The cable will break when the maximum tension reaches $T_{\max} = 10$ kN. Determine the minimum sag h if it supports the uniform distributed load of $w = 600$ N/m.



SOLUTION

The Equation of The Cable:

$$y = \frac{1}{F_H}\int\left(\int w(x)dx\right)dx$$

$$= \frac{1}{F_H}\left(\frac{w_0}{2}x^2 + C_1x + C_2\right) \qquad \textbf{[1]}$$

$$\frac{dy}{dx} = \frac{1}{F_H}(w_0x + C_1) \qquad \textbf{[2]}$$

Boundary Conditions:

$y = 0$ at $x = 0$, then from Eq.[1] $0 = \frac{1}{F_H}(C_2) \qquad C_2 = 0$

$\frac{dy}{dx} = 0$ at $x = 0$, then from Eq.[2] $0 = \frac{1}{F_H}(C_1) \qquad C_1 = 0$

Thus,

$$y = \frac{w_0}{2F_H}x^2 \qquad \textbf{[3]}$$

$$\frac{dy}{dx} = \frac{w_0}{F_H}x \qquad \textbf{[4]}$$

$y = h$, at $x = 12.5$ m, then from Eq.[3] $h = \frac{w_0}{2F_H}(12.5^2) \qquad F_H = \frac{78.125}{h}w_0$

$\theta = \theta_{max}$ at $x = 12.5$ m and the maximum tension occurs when $\theta = \theta_{max}$. From Eq.[4] $\tan\theta_{max} = \left.\frac{dy}{dx}\right|_{x-12.5\text{m}} = \frac{w_0}{\frac{18.125}{h}w_0}x = 0.0128h(12.5) = 0.160h$

Thus, $\cos\theta_{max} = \frac{1}{\sqrt{0.0256h^2 + 1}}$

The maximum tension in the cable is

$$T_{max} = \frac{F_H}{\cos\theta_{max}}$$

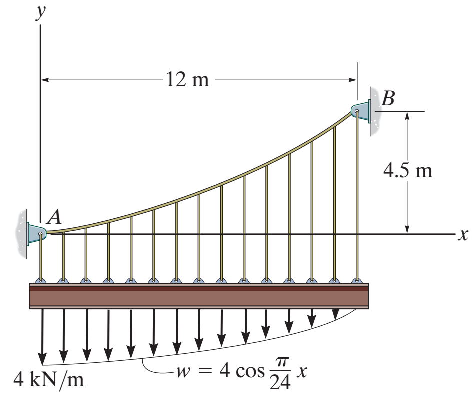
$$10 = \frac{\frac{18.125}{h}(0.6)}{\frac{1}{\sqrt{0.0256h^2 + 1}}}$$

$$h = 7.09 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$h = 7.09$ m

7–113.

If the slope of the cable at support A is zero, determine the deflection curve $y = f(x)$ of the cable and the maximum tension developed in the cable.



SOLUTION

Using Eq. 7–12,

$$y = \frac{1}{F_H}\int\left(\int w(x)dx\right)dx$$

$$y = \frac{1}{F_H}\int\left(\int 4 \cos \frac{\pi}{24} \times dx\right)dx$$

$$y = \frac{1}{F_H}\int \frac{24}{\pi}\left[4(10^3)\right] \sin \frac{\pi}{24}x + C_1$$

$$y = -\frac{24}{\pi}\left[\frac{96(10^3)}{\pi F_H} \cos \frac{\pi}{24}x\right] + C_1 x + C_2$$

Applying the boundary condition $\dfrac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$.

Applying the boundary condition $y = 0$ at $x = 0$, we have

$$0 = -\frac{24}{\pi}\left[\frac{96(10^3)}{\pi F_H} \cos 0°\right] + C_2$$

$$C_2 = \frac{2304(10^3)}{\pi^2 F_H}$$

Thus,

$$y = \frac{2304(10^3)}{\pi^2 F_H}\left[1 - \cos \frac{\pi}{24}x\right]$$

Applying the boundary condition $y = 4.5$ m at $x = 12$ m, we have

$$4.5 = \frac{2304(10^3)}{\pi^2 F_H}\left[1 - \cos \frac{\pi}{24}(12)\right]$$

$$F_H = 51.876(10^3)\text{ N}$$

Substituting this result into Eqs. (1) and (2), we obtain

$$\frac{dy}{dx} = \frac{96(10^3)}{\pi(51.876)(10^3)} \sin \frac{\pi}{24}x$$

$$= 0.5890 \sin \frac{\pi}{24}x$$

and

$$y = \frac{2304(10^3)}{\pi^2(51.876)(10^3)}\left[1 - \cos \frac{\pi}{24}x\right]$$

$$= 4.5\left(1 - \cos \frac{\pi}{24}x\right)\text{ m}$$ **Ans.**

The maximum tension occurs at point B where the cable makes the greatest angle with the horizontal. Here,

$$\theta_{\max} = \tan^{-1}\left(\left.\frac{dy}{dx}\right|_{x = 12\text{ m}}\right) = \tan^{-1}\left[0.5890 \sin\left(\frac{\pi}{24}(12)\right)\right] = 30.50°$$

Thus,

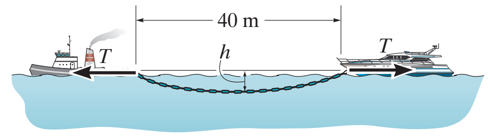
$$T_{\max} = \frac{F_H}{\cos \theta_{\max}} = \frac{51.876(10^3)}{\cos 30.50°} = 60.207(10^3)\text{ N} = 60.2\text{ kN}$$ **Ans.**

Ans:

$y = 4.5\left(1 - \cos \frac{\pi}{24}x\right)$ m

$T_{max} = 60.2$ kN

7–114. If the horizontal towing force is $T = 20$ kN and the chain has a mass per unit length of 15 kg/m, determine the maximum sag h. Neglect the buoyancy effect of the water on the chain. The boats are stationary.



SOLUTION

As shown in Fig. a, the origin of the x, y coordinates system is set at the lowest point of the chain. Here, $F_H = T = 20(10^3)$ N and

$w(s) = 15(9.81)\text{ N/m} = 147.15\text{ N/m}$.

$$\frac{d^2y}{dx^2} = \frac{147.15}{20(10^3)}\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = 7.3575(10^{-3})\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

Set $u = \frac{dy}{dx}$, then $\frac{du}{dx} = \frac{d^2y}{dx^2}$. Thus,

$$\frac{du}{\sqrt{1 + u^2}} = 7.3575(10^{-3})dx$$

Integrating,

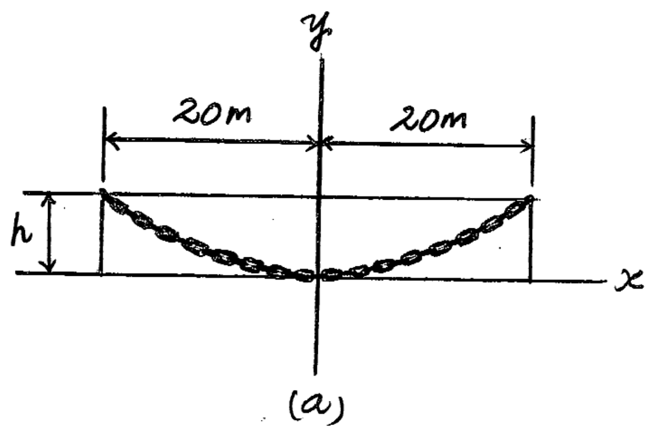
$$\ln\left(u + \sqrt{1 + u^2}\right) = 7.3575(10^{-3})x + C_1$$

Applying the boundary condition $u = \frac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\ln\left(u + \sqrt{1 + u^2}\right) = 7.3575(10^{-3})x$$

$$u + \sqrt{1 + u^2} = e^{7.3575(10^{-3})x}$$

$$\frac{dy}{dx} = u = \frac{e^{7.3575(10^{-3})x} - e^{-7.3575(10^{-3})x}}{2}$$



Since $\sin h\, x = \frac{e^x - e^{-x}}{2}$, then

$$\frac{dy}{dx} = \sin h\, 7.3575(10^{-3})x$$

Integrating,

$$y = 135.92 \cos h\, 7.3575(10^{-3})x + C_2$$

Applying the boundary equation $y = 0$ at $x = 0$ results in $C_2 = -135.92$. Thus,

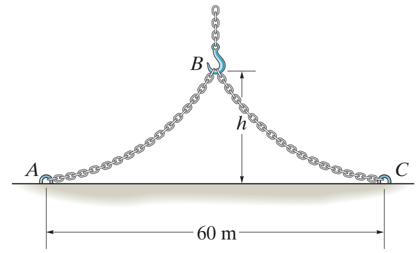
$$y = 135.92[\cos h\, 7.3575(10^{-3})x - 1]$$

Applying the boundary equation $y = h$ at $x = 20$ m,

$$h = 135.92[\cos h\, 7.3575(10^{-3})(20) - 1] = 1.47\text{ m} \qquad \textbf{Ans.}$$

Ans:
$h = 1.47$ m

7–115. The 80-m-long chain is fixed at its ends and hoisted at its midpoint B using a crane. If the chain has a weight of 0.5 kN/m, determine the minimum height h of the hook in order to lift the chain *completely* off the ground. What is the horizontal force at pin A or C when the chain is in this position? *Hint:* When h is a minimum, the slope at A and C is zero.



SOLUTION

Given:

$L = 80 \text{ m}$

$d = 60 \text{ m}$

$\text{w} = 0.5 \text{ kN/m}$

Guesses $\quad F_H = 10 \text{ kN} \qquad h = 1 \text{ m}$

Given $\quad h = \dfrac{F_H}{w}\cdot\left(\cos h\left(\dfrac{w}{F_H}\cdot\dfrac{d}{2}\right) - 1\right) \qquad \dfrac{L}{2} = \dfrac{F_H}{w}\cdot\sin h\left(\dfrac{w}{F_H}\cdot\dfrac{d}{2}\right)$

$\begin{pmatrix} h \\ F_H \end{pmatrix} = \text{Find}\,(h, F_H) \qquad F_A = F_H \qquad F_C = F_H \qquad \begin{pmatrix} F_A \\ F_C \end{pmatrix} = \begin{pmatrix} 11.1 \\ 11.1 \end{pmatrix} \text{kN}$ **Ans.**

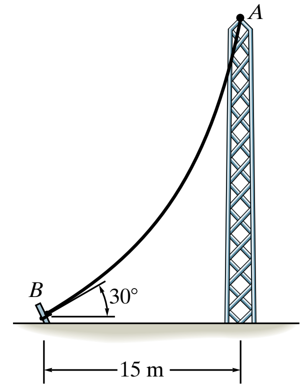
$h = 23.5 \text{ m}$ **Ans.**

Ans:

$F_A = 11.1 \text{ kN}$
$F_C = 11.1 \text{ kN}$
$h = 23.5 \text{ m}$

***7–116.**

The cable has a mass of 0.5 kg/m, and is 25 m long. Determine the vertical and horizontal components of force it exerts on the top of the tower.

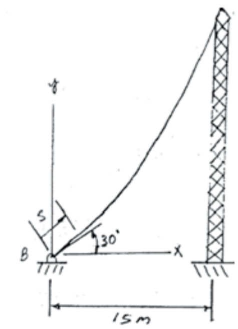


SOLUTION

$$x = \int \frac{ds}{\left\{1 + \frac{1}{F_H^2}(w_0\, ds)^2\right\}^{\frac{1}{2}}}$$

Performing the integration yields:

$$x = \frac{F_H}{4.905}\left\{\sinh^{-1}\left[\frac{1}{F_H}(4.905s + C_1)\right] + C_2\right\} \quad \textbf{(1)}$$



From Eq. 7–14

$$\frac{dy}{dx} = \frac{1}{F_H}\int w_0\, ds$$

$$\frac{dy}{dx} = \frac{1}{F_H}(4.905s + C_1)$$

At $s = 0$; $\dfrac{dy}{dx} = \tan 30°$. Hence $C_1 = F_H \tan 30°$

$$\frac{dy}{dx} = \frac{4.905s}{F_H} + \tan 30° \quad \textbf{(2)}$$

Applying boundary conditions at $x = 0$; $s = 0$ to Eq.(1) and using the result $C_1 = F_H \tan 30°$ yields $C_2 = -\sinh^{-1}(\tan 30°)$. Hence

$$x = \frac{F_H}{4.905}\left\{\sinh^{-1}\left[\frac{1}{F_H}(4.905s + F_H \tan 30°)\right] - \sinh^{-1}(\tan 30°)\right\} \quad \textbf{(3)}$$

At $x = 15$ m; $s = 25$ m. From Eq.(3)

$$15 = \frac{F_H}{4.905}\left\{\sinh^{-1}\left[\frac{1}{F_H}(4.905(25) + F_H \tan 30°)\right] - \sinh^{-1}(\tan 30°)\right\}$$

By trial and error $F_H = 73.94$ N

At point A, $s = 25$ m From Eq.(2)

$$\tan\theta_A = \left.\frac{dy}{dx}\right|_{s=25\text{ m}} = \frac{4.905(25)}{73.94} + \tan 30° \qquad \theta_A = 65.90°$$

$$(F_v)_A = F_H \tan\theta_A = 73.94 \tan 65.90° = 165 \text{ N} \qquad \textbf{Ans.}$$

$$(F_H)_A = F_H = 73.9 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$(F_v)_A = 165$ N
$(F_H)_A = 73.9$ N

7–117.

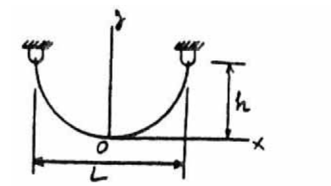
A uniform cord is suspended between two points having the same elevation. Determine the sag-to-span ratio so that the maximum tension in the cord equals the cord's total weight.

SOLUTION

From Example 7.13.

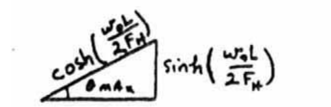
$$s = \frac{F_H}{w_0}\sinh\left(\frac{w_0}{F_H}x\right)$$

$$y = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$



At $x = \dfrac{L}{2}$,

$$\left.\frac{dy}{dx}\right|_{max} = \tan\theta_{max} = \sinh\left(\frac{w_0L}{2F_H}\right)$$



$$\cos\theta_{max} = \frac{1}{\cosh\left(\dfrac{w_0L}{2F_H}\right)}$$

$$T_{max} = \frac{F_H}{\cos\theta_{max}}$$

$$w_0(2s) = F_H\cosh\left(\frac{w_0L}{2F_H}\right)$$

$$2F_H\sinh\left(\frac{w_0L}{2F_H}\right) = F_H\cosh\left(\frac{w_0L}{2F_H}\right)$$

$$\tanh\left(\frac{w_0L}{2F_H}\right) = \frac{1}{2}$$

$$\frac{w_0L}{2F_H} = \tanh^{-1}(0.5) = 0.5493$$

when $x = \dfrac{L}{2}$, $y = h$

$$h = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$

$$h = \frac{F_H}{w_0}\left\{\frac{1}{\sqrt{1 - \tanh^2\left(\dfrac{w_{0L}}{2F_H}\right)}} - 1\right\} = 0.1547\left(\frac{F_H}{w_0}\right)$$

$$\frac{0.1547\,L}{2h} = 0.5493$$

$$\frac{h}{L} = 0.141 \qquad \textbf{Ans.}$$

Ans:

$$\frac{h}{L} = 0.141$$

7–118. A 50-m cable is suspended between two points a distance of 15 m apart and at the same elevation. If the minimum tension in the cables is 200 kN, determine the total weight of the cable and the maximum tension developed in the cable.

SOLUTION

Given:

$L = 50\text{ m} \quad d = 15\text{ m} \quad T_{min} = 200\text{ kN}$

$T_{min} = F_H \quad F_H = T_{min} \quad F_H = 200\text{ kN}$

From Example 7.13:

$$s = \frac{F_H}{w_0}\sin h\left(\frac{w_0\,x}{F_H}\right)$$

Guess $\quad w_0 = 1\text{ kN/m}$

Given $\quad \dfrac{L}{2} = \dfrac{F_H}{w_0}\sin h\left(\dfrac{w_0}{F_H}\dfrac{d}{2}\right) \quad w_0 = \text{Find}(w_0) \quad w_0 = 79.93\text{ kN/m}$

Total weight $= w_0\,L \quad$ Total weight $= 4.00\text{ MN}$ **Ans.**

$$\tan(\theta_{max}) = \frac{w_0}{F_H}\frac{L}{2} \quad \theta_{max} = a\tan\left[\frac{w_0\left(\frac{L}{2}\right)}{F_H}\right] \quad \theta_{max} = 84.284°$$

Then,

$$T_{max} = \frac{F_H}{cos(\theta_{max})} \quad T_{max} = 2.01\text{ MN}$$ **Ans.**

Ans:
Total weight $= 4.00$ MN
$T_{max} = 2.01$ MN

7–119.

Show that the deflection curve of the cable discussed in Example 7.13 reduces to Eq. 4 in Example 7.12 when the *hyperbolic cosine function* is expanded in terms of a series and only the first two terms are retained. (The answer indicates that the *catenary* may be replaced by a parabola in the analysis of problems in which the sag is small. In this case, the cable weight is assumed to be uniformly distributed along the horizontal.)

SOLUTION

$$\cosh x = 1 + \frac{x^2}{2!} + \cdots$$

Substituting into

$$y = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$

$$= \frac{F_H}{w_0}\left[1 + \frac{w_0^2 x^2}{2F_H^2} + \cdots - 1\right]$$

$$= \frac{w_0 x^2}{2F_H}$$

Using Eq. (3) in Example 7.12,

$$F_H = \frac{w_0 L^2}{8h}$$

We get $y = \dfrac{4h}{L^2}x^2$ **QED**

***7–120.** A cable has a weight of 30 N/m and is supported at points that are 25 m apart and at the same elevation. If it has a length of 26 m, determine the sag.

SOLUTION

Given:

$\gamma = 30\ \text{N/m} \qquad d = 25\ \text{m} \qquad L = 26\ \text{m}$

Guess $\quad F_H = 1000\ \text{N}$

Given $\quad \dfrac{L}{2} - \left[\dfrac{F_H}{\gamma}\sin h\left[\dfrac{\gamma}{F_H}\left(\dfrac{d}{2}\right)\right]\right] = 0 \qquad F_H = \text{Find}(F_H) \qquad F_H = 770\ \text{N}$

$h = \dfrac{F_H}{\gamma}\left(\cos h\left(\dfrac{1}{2}\dfrac{\gamma}{F_H}d\right) - 1\right) \qquad h = 3.104\ \text{m}$ **Ans.**

Ans:
$h = 3.104\ \text{m}$

7–121. A wire has a weight of 2 N/m. If it can span 10 m and has a sag of 1.2 m, determine the length of the wire. The ends of the wire are supported from the same elevation.

SOLUTION

Given :

$\gamma = 2\ \text{N/m} \quad d = 10\ \text{m} \quad h = 1.2\ \text{m}$

From Eq. (5) of Example 7.13 :

$$h = \frac{F_H}{\gamma}\left[\frac{\left(\frac{\gamma}{2}\frac{d}{F_H}\right)^2}{2}\right] \quad F_H = \frac{1}{8}\gamma\frac{d^2}{h} \quad F_H = 20.83\ \text{N}$$

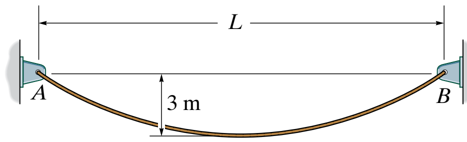
From Eq. (3) of Example 7.13 :

$$\frac{L}{2} = \frac{F_H}{\gamma}\sin h\left[\frac{\gamma}{F_H}\left(\frac{d}{2}\right)\right] \quad L = 2\frac{F_H}{\gamma}\sin h\left(\frac{1}{2}\gamma\frac{d}{F_H}\right) \quad L = 10.39\ \text{m}$$ **Ans.**

Ans:
$L = 10.39\ \text{m}$

7–122.

The 10 kg m cable is suspended between the supports A and B. If the cable can sustain a maximum tension of 1.5 kN and the maximum sag is 3 m, determine the maximum distance L between the supports



SOLUTION

The origin of the x, y coordinate system is set at the lowest point of the cable. Here $w_0 = 10(9.81)\text{ N/m} = 98.1\text{ N/m}$. Using Eq. (4) of Example 7.13,

$$y = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$

$$y = \frac{F_H}{98.1}\left[\cosh\left(\frac{98.1x}{F_H}\right) - 1\right]$$

Applying the boundary equation $y = 3$ m at $x = \frac{L}{2}$, we have

$$3 = \frac{F_H}{98.1}\left[\cosh\left(\frac{49.05L}{F_H}\right) - 1\right]$$

The maximum tension occurs at either points A or B where the cable makes the greatest angle with the horizontal. From Eq. (1),

$$\tan\theta_{\max} = \sinh\left(\frac{49.05L}{F_H}\right)$$

By referring to the geometry shown in Fig. b, we have

$$\cos\theta_{\max} = \frac{1}{\sqrt{1 + \sinh^2\left(\frac{49.05L}{F_H}\right)}} = \frac{1}{\cosh\left(\frac{49.05L}{F_H}\right)}$$

Thus,

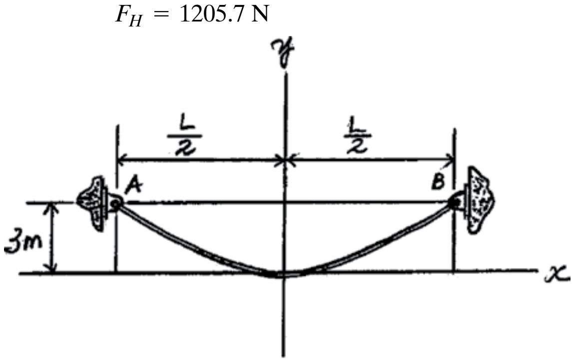
$$T_{\max} = \frac{F_H}{\cos\theta_{\max}}$$

$$1500 = F_H\cosh\left(\frac{49.05L}{F_H}\right) \qquad \textbf{(3)}$$

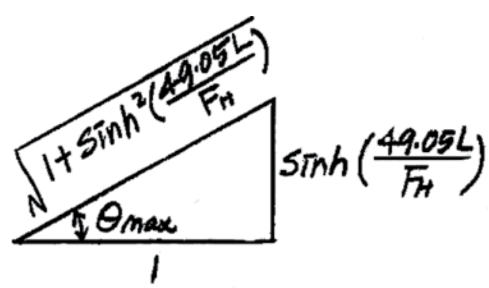
Solving Eqs. (2) and (3) yields

$L = 16.8\text{ m}$ **Ans.**

$F_H = 1205.7\text{ N}$



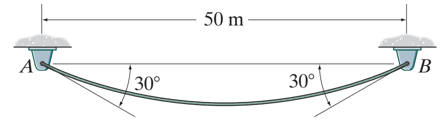
(a)



(b)

Ans:
$L = 16.8\text{ m}$

7–123. A cable having a weight per unit length of 0.1 kN/m is suspended between supports A and B. Determine the equation of the catenary curve of the cable and the cable's length.



SOLUTION

As shown in Fig. a, the origin of the x, y coordinate system is set at the lowest point of the cable. Here, $w(s) = 0.1$ kN/m.

$$\frac{d^2y}{dx^2} = \frac{0.1}{F_H} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

If we set $u = \dfrac{dy}{dx}$, then $\dfrac{du}{dx} = \dfrac{d^2y}{dx^2}$. Substituting these two values into the equation,

$$\frac{du}{\sqrt{1+u^2}} = \frac{0.1}{F_H}dx$$

Integrating,

$$\ln\left(u + \sqrt{1+u^2}\right) = \frac{0.1}{F_H}x + C_1$$

Applying the boundary condition $u = \dfrac{dy}{dx} = 0$ at $x = 0$ results in $C_1 = 0$. Thus,

$$\ln(u + \sqrt{1+u^2}) = \frac{0.1}{F_H}x$$

$$u + \sqrt{1+u^2} = e^{\frac{0.1}{F_H}x}$$

$$\frac{dy}{dx} = u = \frac{e^{\frac{0.1}{F_H}x} - e^{-\frac{0.1}{F_H}x}}{2}$$

Since $\sinh x = \dfrac{e^x - e^{-x}}{2}$, then

$$\frac{dy}{dx} = \sinh\frac{0.1}{F_H}x \qquad (1)$$

Applying the boundary equation $\dfrac{dy}{dx} = \tan 30°$ at $x = 25$ m,

$$\tan 30° = \sinh\left[\frac{0.1}{F_H}(25)\right]$$

$$F_H = 4.5512 \text{ kN}$$

Substituting this result into Eq. (1),

$$\frac{dy}{dx} = \sinh[(0.0219722)x] \qquad (2)$$

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Integrating,

$$y = 45.512 \cosh[(0.0219722)x] + C_2$$

Applying the boundary equation $y = 0$ at $x = 0$ results in $C_2 = -45.512$. Thus,

$$y = 45.512 \{\cosh[(0.0219722)x] - 1\} \text{ m} \qquad \textbf{Ans.}$$

If we write the force equation of equilibrium along the x and y axes by referring to the free – body diagram shown in Fig. b, we have

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad T \cos\theta - 4.5512 = 0$

$+\uparrow\Sigma F_y = 0;$ $\quad T \sin\theta - 0.1s = 0$

Eliminating T,

$$\frac{dy}{dx} = \tan\theta = 0.0219722 \qquad (3)$$

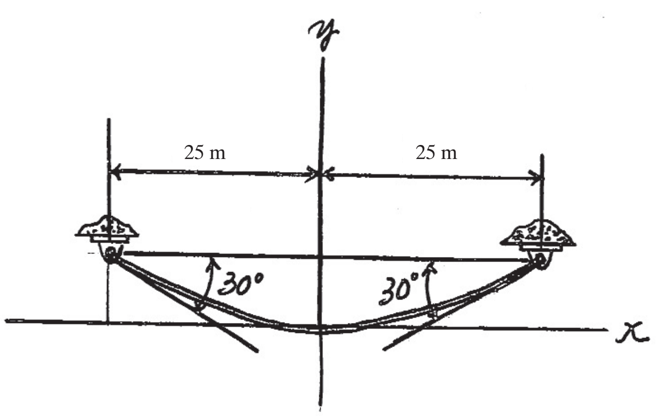
Equating Eqs. (2) and (3),

$$(0.0219722)s = \sinh[(0.0219722)x]$$

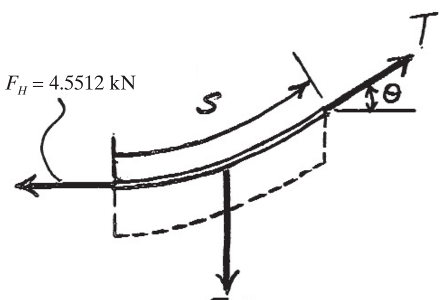
$$s = 45.512\ [(0.0219722)x] \text{ m}$$

Thus, the length of the cable is

$$L = 2\{45.512 \sinh[(0.0219722)\ (25)]\}$$
$$= 52.553 \text{ m} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$y = 45.512\ \{\cosh[(0.0219722)x] - 1\}$ m
$L = 52.553$ m