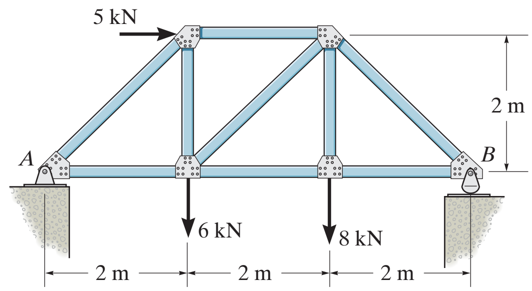
5–10.

Determine the reactions at the supports.



SOLUTION

Equations of Equilibrium. $\mathbf{A}_y$ and $\mathbf{N}_B$ can be determined by writing the moment equations of equilibrium about points B and A, respectively, by referring to the *FBD* of the truss shown in Fig. *a*.

$\zeta + \Sigma M_B = 0; \quad 8(2) + 6(4) - 5(2) - A_y(6) = 0$

$A_y = 5.00 \text{ kN}$ **Ans.**

$\zeta + \Sigma M_A = 0; \quad N_B(6) - 8(4) - 6(2) - 5(2) = 0$

$N_B = 9.00 \text{ kN}$ **Ans.**

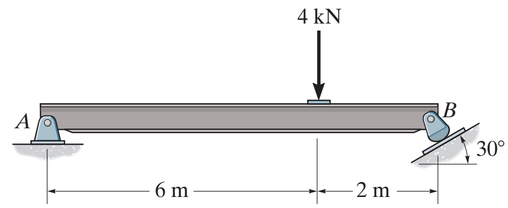
Also, $\mathbf{A}_x$ can be determined directly by writing the force equation of equilibrium along x axis.

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 5 - A_x = 0 \quad A_x = 5.00 \text{ kN}$ **Ans.**

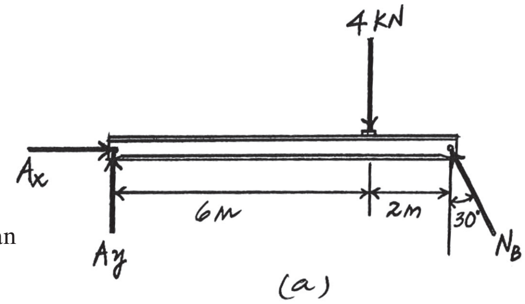
Ans:
$A_y = 5.00 \text{ kN}$
$N_B = 9.00 \text{ kN}$
$A_x = 5.00 \text{ kN}$

5–11.

Determine the horizontal and vertical components of reaction at the pin A and the reaction of the rocker B on the beam.



SOLUTION



Equations of Equilibrium: From the free-body diagram of the beam, Fig. *a*, N_B can be obtained by writing the moment equation of equilibrium about point A.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad N_B \cos 30°(8) - 4(6) = 0$

$N_B = 3.464 \text{ kN} = 3.46 \text{ kN}$ **Ans.**

Using this result and writing the force equations of equilibrium along the x and y axes, we have

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x - 3.464 \sin 30° = 0$

$A_x = 1.73 \text{ kN}$ **Ans.**

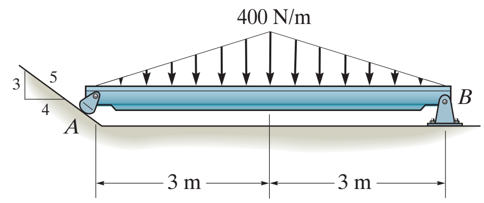
$+\uparrow \Sigma F_y = 0;$ $\quad A_y + 3.464 \cos 30° - 4 = 0$

$A_y = 1.00 \text{ kN}$ **Ans.**

Ans:
$N_B = 3.46$ kN
$A_x = 1.73$ kN
$A_y = 1.00$ kN

***5–12.**

Determine the reactions at the supports.



SOLUTION

Equations of Equilibrium. $\mathbf{N}_A$ and $\mathbf{B}_y$ can be determined directly by writing the moment equations of equilibrium about points B and A, respectively, by referring to the beam's *FBD* shown in Fig. *a*.

$$\zeta+\Sigma M_B = 0; \qquad \frac{1}{2}(400)(6)(3) - N_A\left(\frac{4}{5}\right)(6) = 0$$

$$N_A = 750 \text{ N} \qquad \textbf{Ans.}$$

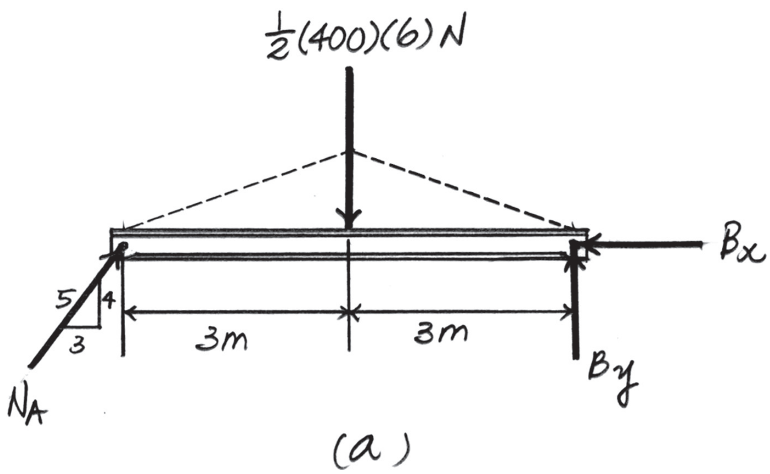
$$\zeta+\Sigma M_A = 0; \qquad B_y(6) - \frac{1}{2}(400)(6)(3) = 0$$

$$B_y = 600 \text{ N} \qquad \textbf{Ans.}$$

Using the result of $\mathbf{N}_A$ to write the force equation of equilibrium along the x axis,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 750\left(\frac{3}{5}\right) - B_x = 0$$

$$B_x = 450 \text{ N} \qquad \textbf{Ans.}$$

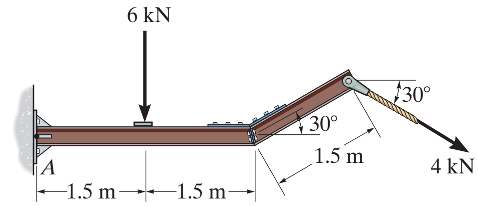


(a)

Ans:
$N_A = 750$ N
$B_y = 600$ N
$B_x = 450$ N

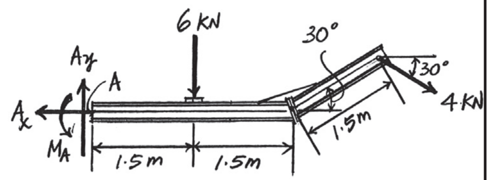
5–13.

Determine the components of the support reactions at the fixed support A on the cantilevered beam.



SOLUTION

Equations of Equilibrium: From the free-body diagram of the cantilever beam, Fig. *a*, A_x, A_y, and M_A can be obtained by writing the moment equation of equilibrium about point A.



(a)

$\xrightarrow{+} \Sigma F_x = 0; \qquad 4 \cos 30° - A_x = 0$

$A_x = 3.46 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y - 6 - 4 \sin 30° = 0$

$A_y = 8 \text{ kN}$ **Ans.**

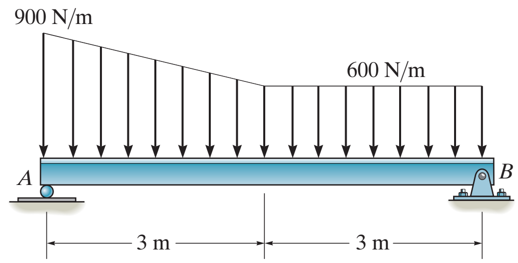
$\circlearrowleft + \Sigma M_A = 0; M_A - 6(1.5) - 4 \cos 30° (1.5 \sin 30°) - 4 \sin 30°(3 + 1.5 \cos 30°) = 0$

$M_A = 20.2 \text{ kN} \cdot \text{m}$ **Ans.**

Ans:
$A_x = 3.46 \text{ kN}$
$A_y = 8 \text{ kN}$
$M_A = 20.2 \text{ kN} \cdot \text{m}$

5–14.

Determine the reactions at the supports.



SOLUTION

Equations of Equilibrium. $\mathbf{N}_A$ and $\mathbf{B}_y$ can be determined directly by writing the moment equations of equilibrium about points B and A, respectively, by referring to the *FBD* of the beam shown in Fig. *a*.

$$\zeta+\Sigma M_B = 0; \qquad 600(6)(3) + \frac{1}{2}(300)(3)(5) - N_A(6) = 0$$

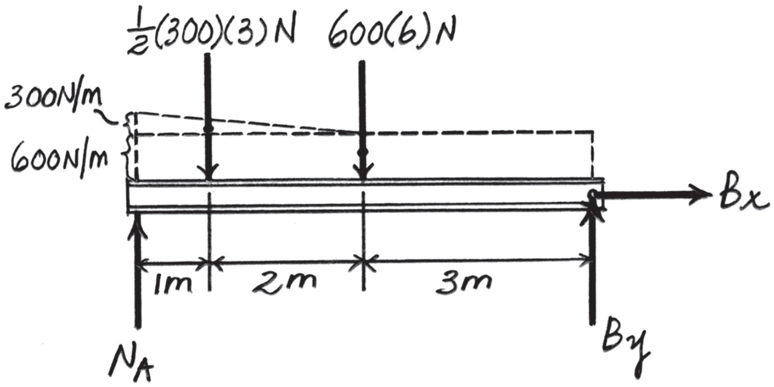
$$N_A = 2175 \text{ N} = 2.175 \text{ kN} \qquad \textbf{Ans.}$$

$$\zeta+\Sigma M_A = 0; \qquad B_y(6) - \frac{1}{2}(300)(3)(1) - 600(6)(3) = 0$$

$$B_y = 1875 \text{ N} = 1.875 \text{ kN} \qquad \textbf{Ans.}$$

Also, $\mathbf{B}_x$ can be determined directly by writing the force equation of equilibrium along the x axis.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0 \qquad \textbf{Ans.}$$

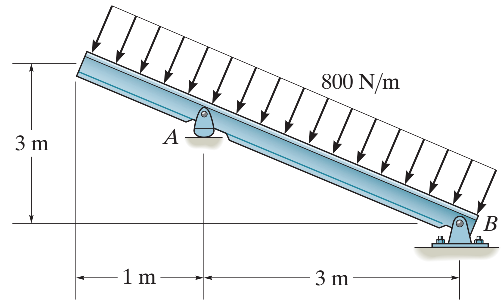


(a)

Ans:
$N_A = 2.175$ kN
$B_y = 1.875$ kN
$B_x = 0$

5–15.

Determine the reactions at the supports.



SOLUTION

Equations of Equilibrium. $\mathbf{N}_A$ can be determined directly by writing the moment equation of equilibrium about point B by referring to the FBD of the beam shown in Fig. a.

$\circlearrowleft + \Sigma M_B = 0; \qquad 800(5)(2.5) - N_A(3) = 0$

$$N_A = 3333.33 \text{ N} = 3.33 \text{ kN} \qquad \textbf{Ans.}$$

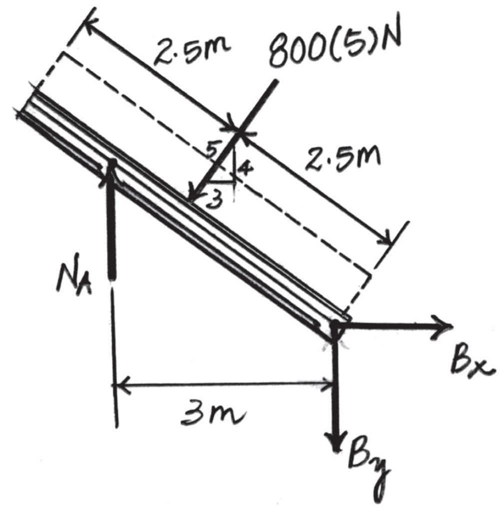
Using this result to write the force equations of equilibrium along the x and y axes,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x - 800(5)\left(\frac{3}{5}\right) = 0$

$$B_x = 2400 \text{ N} = 2.40 \text{ kN} \qquad \textbf{Ans.}$$

$+\uparrow \Sigma F_y = 0; \qquad 3333.33 - 800(5)\left(\frac{4}{5}\right) - B_y = 0$

$$B_y = 133.33 \text{ N} = 133 \text{ N} \qquad \textbf{Ans.}$$

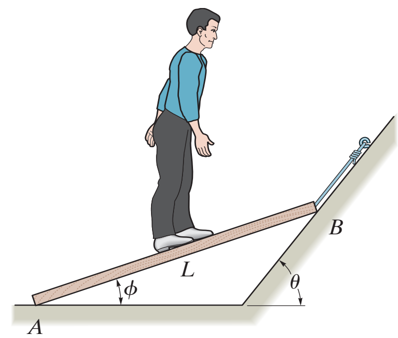


(a)

Ans:
$N_A = 3.33$ kN
$B_x = 2.40$ kN
$B_y = 133$ N

***5–16.**

The man has a weight W and stands at the center of the plank. If the planes at A and B are smooth, determine the tension in the cord in terms of W and θ.



SOLUTION

$\zeta + \Sigma M_B = 0;\qquad W\left(\frac{L}{2}\cos\phi\right) - N_A(L\cos\phi) = 0 \qquad N_A = \frac{W}{2}$

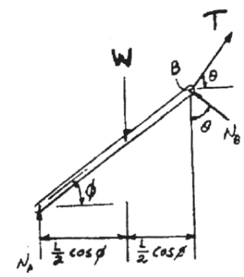
$\xrightarrow{+} \Sigma F_x = 0;\qquad T\cos\theta - N_B\sin\theta = 0 \qquad$ **(1)**

$+\uparrow \Sigma F_y = 0;\qquad T\sin\theta + N_B\cos\theta + \frac{W}{2} - W = 0 \qquad$ **(2)**

Solving Eqs. (1) and (2) yields:

$$T = \frac{W}{2}\sin\theta$$ **Ans.**

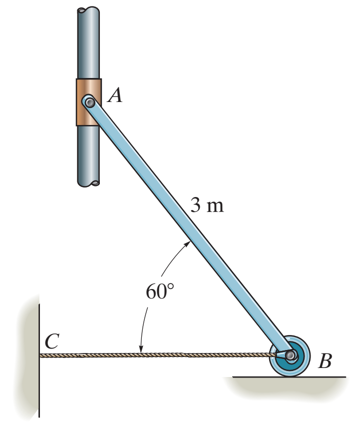
$$N_B = \frac{W}{2}\cos\theta$$



Ans:
$T = \frac{W}{2}\sin\theta$

5–17.

The uniform rod AB has a mass of 40 kg. Determine the force in the cable when the rod is in the position shown. There is a smooth collar at A.

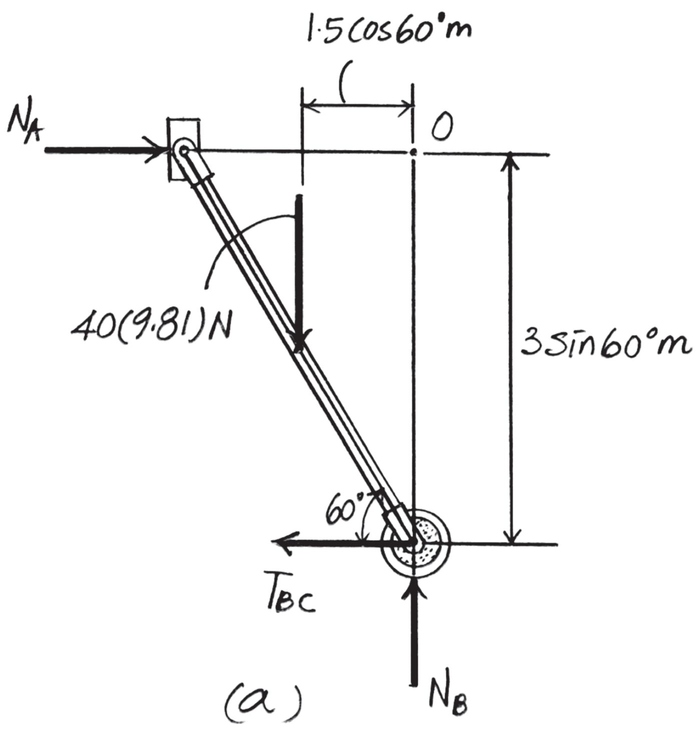


SOLUTION

Equations of Equilibrium. $\mathbf{T}_{BC}$ can be determined by writing the moment equation of equilibrium about point O by referring to the *FBD* of the rod shown in Fig. *a*.

$$\circlearrowleft + \Sigma M_O = 0; \qquad 40(9.81)(1.5 \cos 600°) - T_{BC}(3 \sin 60°) = 0$$

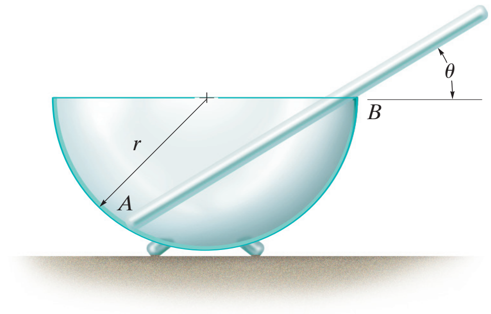
$$T_{BC} = 113.28 \text{ N} = 113 \text{ N} \qquad \textbf{Ans.}$$



Ans:
$T_{BC} = 113 \text{ N}$

5–18.

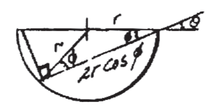
A uniform glass rod having a length L is placed in the smooth hemispherical bowl having a radius r. Determine the angle of inclination θ for equilibrium.



SOLUTION

By observation $\phi = \theta$.

Equilibrium:



$$\circlearrowleft + \Sigma M_A = 0; \qquad N_B(2r\cos\theta) - W\left(\frac{L}{2}\cos\theta\right) = 0 \qquad N_B = \frac{WL}{4r}$$

$$+\nearrow \Sigma F_x = 0; \qquad N_A\cos\theta - W\sin\theta = 0 \qquad N_A = W\tan\theta$$

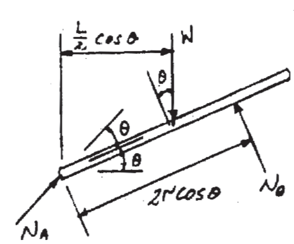
$$+\nwarrow \Sigma F_y = 0; \qquad (W\tan\theta)\sin\theta + \frac{WL}{4r} - W\cos\theta = 0$$

$$\sin^2\theta - \cos^2\theta + \frac{L}{4r}\cos\theta = 0$$

$$(1 - \cos^2\theta) - \cos^2\theta + \frac{L}{4r}\cos\theta = 0$$

$$2\cos^2\theta - \frac{L}{4r}\cos\theta - 1 = 0$$

$$\cos\theta = \frac{L \pm \sqrt{L^2 + 128r^2}}{16r}$$



Take the positive root

$$\cos\theta = \frac{L + \sqrt{L^2 + 128r^2}}{16r}$$

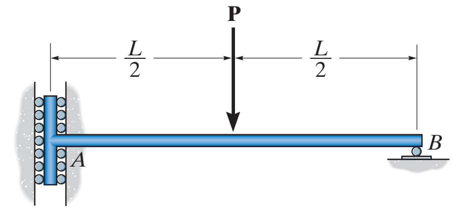
$$\theta = \cos^{-1}\left(\frac{L + \sqrt{L^2 + 128r^2}}{16r}\right) \qquad \textbf{Ans.}$$

Ans:

$$\theta = \cos^{-1}\left(\frac{L + \sqrt{L^2 + 128r^2}}{16r}\right)$$

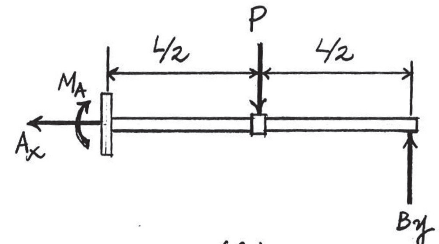
5–19.

Determine the components of reaction at the supports A and B on the rod.



SOLUTION

Equations of Equilibrium: Since the roller at A offers no resistance to vertical movement, the vertical component of reaction at support A is equal to zero. From the free-body diagram, A_x, B_y, and M_A can be obtained by writing the force equations of equilibrium along the x and y axes and the moment equation of equilibrium about point B, respectively.



(a)

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad B_y - P = 0$

$$B_y = P$$ **Ans.**

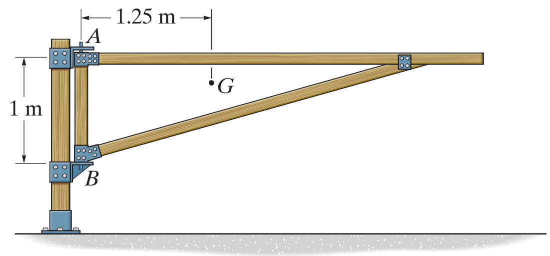
$\circlearrowleft + \Sigma M_B = 0;$ $\quad P\left(\frac{L}{2}\right) - M_A = 0$

$$M_A = \frac{PL}{2}$$ **Ans.**

Ans:
$A_x = 0$
$B_y = P$
$M_A = \frac{PL}{2}$

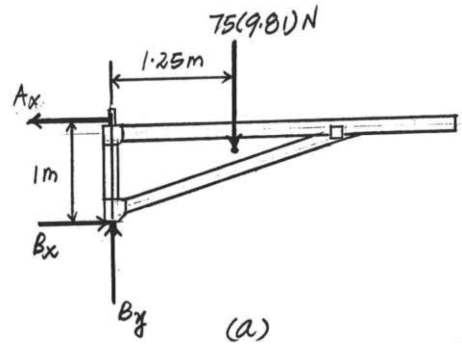
***5–20.**

The 75-kg gate has a center of mass located at G. If A supports only a horizontal force and B can be assumed as a pin, determine the components of reaction at these supports.



SOLUTION

Equations of Equilibrium: From the free-body diagram of the gate, Fig. *a*, B_y and A_x can be obtained by writing the force equation of equilibrium along the y axis and the moment equation of equilibrium about point B.



(a)

$+\uparrow \Sigma F_y = 0; \qquad B_y - 75(9.81) = 0$

$B_y = 735.75 \text{ N} = 736 \text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_B = 0; \qquad A_x(1) - 75(9.81)(1.25) = 0$

$A_x = 919.69 \text{ N} = 920 \text{ N}$ **Ans.**

Using the result $A_x = 919.69$ N and writing the force equation of equilibrium along the x axis, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x - 919.69 = 0$

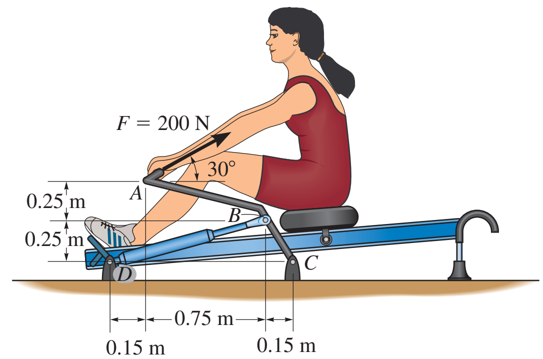
$B_x = 919.69 \text{ N} = 920 \text{ N}$ **Ans.**

Ans:

$B_y = 736$ N
$A_x = 920$ N
$B_x = 920$ N

5–21.

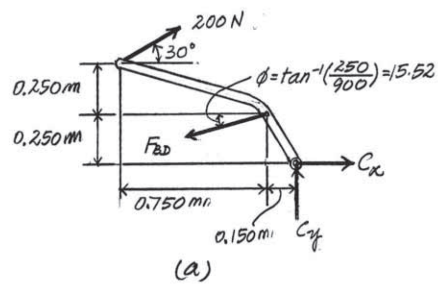
The woman exercises on the rowing machine. If she exerts a holding force of $F = 200$ N on handle ABC, determine the horizontal and vertical components of reaction at pin C and the force developed along the hydraulic cylinder BD on the handle.



SOLUTION

Equations of Equilibrium: Since the hydraulic cylinder is pinned at both ends, it can be considered as a two-force member and therefore exerts a force $\mathbf{F}_{BD}$ directed along its axis on the handle, as shown on the free-body diagram in Fig. *a*. From the free-body diagram, F_{BD} can be obtained by writing the moment equation of equilibrium about point C.

$$\circlearrowleft + \Sigma M_C = 0; \quad F_{BD} \cos 15.52°(250) + F_{BD} \sin 15.52°(150) - 200 \cos 30°(250 + 250)$$
$$-200 \sin 30°(750 + 150) = 0$$
$$F_{BD} = 628.42 \text{ N} = 628 \text{ N} \qquad \textbf{Ans.}$$



Using the above result and writing the force equations of equilibrium along the x and y axes, we have

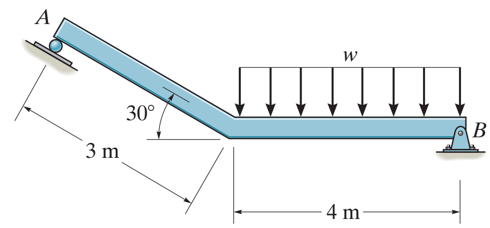
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad C_x + 200 \cos 30° - 628.42 \cos 15.52° = 0$$
$$C_x = 432.29 \text{ N} = 432 \text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 200 \sin 30° - 628.42 \sin 15.52° + C_y = 0$$
$$C_y = 68.19 \text{ N} = 68.2 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$F_{BD} = 628$ N
$C_x = 432$ N
$C_y = 68.2$ N

5–22.

If the intensity of the distributed load acting on the beam is $w = 3$ kN/m, determine the reactions at the roller A and pin B.



SOLUTION

Equations of Equilibrium. $\mathbf{N}_A$ can be determined directly by writing the moment equation of equilibrium about point B by referring to the *FBD* of the beam shown in Fig. *a*.

$\circlearrowleft + \Sigma M_B = 0;\quad 3(4)(2) - N_A \sin 30°\,(3 \sin 30°) - N_A \cos 30°\,(3 \cos 30° + 4) = 0$

$$N_A = 3.713 \text{ kN} = 3.71 \text{ kN} \qquad \textbf{Ans.}$$

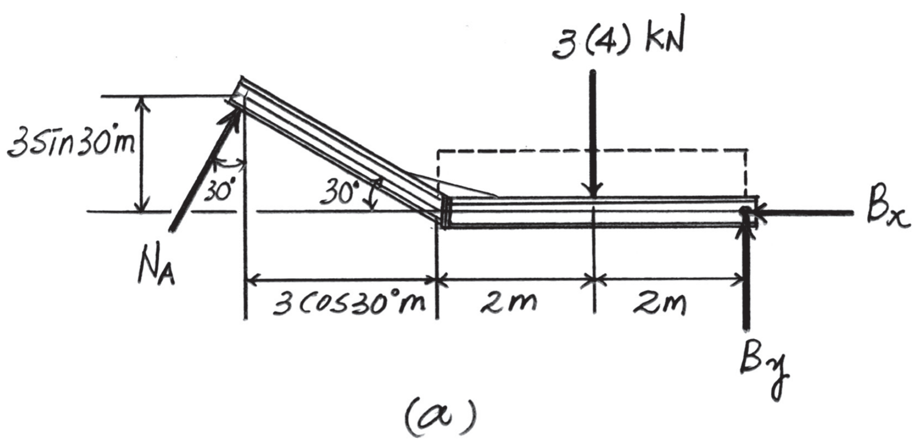
Using this result to write the force equation of equilibrium along the x and y axes,

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 3.713 \sin 30° - B_x = 0$

$$B_x = 1.856 \text{ kN} = 1.86 \text{ kN} \qquad \textbf{Ans.}$$

$+\uparrow \Sigma F_y = 0;\quad B_y + 3.713 \cos 30° - 3(4) = 0$

$$B_y = 8.7846 \text{ kN} = 8.78 \text{ kN} \qquad \textbf{Ans.}$$

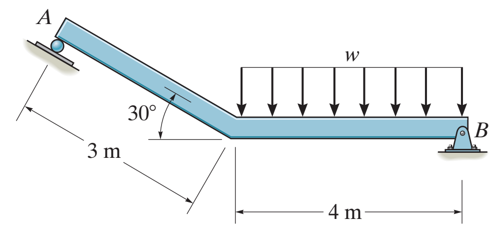


(a)

Ans:
$N_A = 3.71$ kN
$B_x = 1.86$ kN
$B_y = 8.78$ kN

5–23.

If the roller at A and the pin at B can support a load up to 4 kN and 8 kN, respectively, determine the maximum intensity of the distributed load w, measured in kN/m, so that failure of the supports does not occur.



SOLUTION

Equations of Equilibrium. $\mathbf{N}_A$ can be determined directly by writing the moment equation of equilibrium about point B by referring to the *FBD* of the beam shown in Fig. *a*.

$$\circlearrowleft + \Sigma M_B = 0; \quad w(4)(2) - N_A \sin 30° \,(3 \sin 30°) - N_A \cos 30° \,(3 \cos 30° + 4) = 0$$

$$N_A = 1.2376\, w$$

Using this result to write the force equation of equilibrium along x and y axes,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 1.2376 w \sin 30° - B_x = 0 \qquad B_x = 0.6188 w$$

$$+\uparrow \Sigma F_y = 0; \quad B_y + 1.2376 w \cos 30° - w(4) = 0 \qquad B_y = 2.9282 w$$

Thus,

$$F_B = \sqrt{B_x^2 + B_y^2} = \sqrt{(0.6188\, w)^2 + (2.9282\, w)^2} = 2.9929\, w$$

It is required that

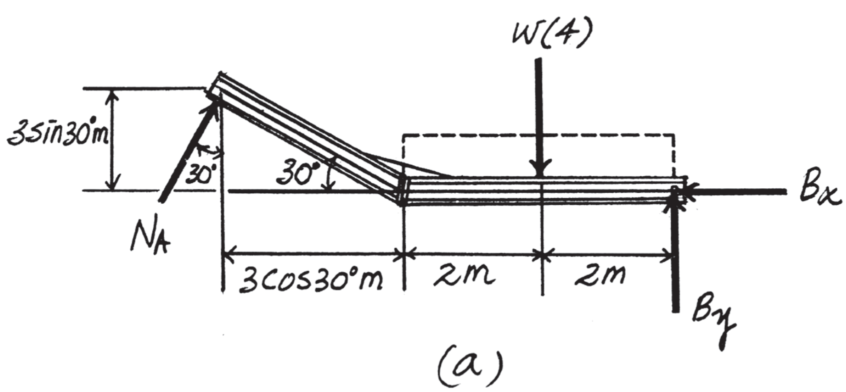
$$F_B < 8 \text{ kN}; \quad 2.9929\, w < 8 \qquad w < 2.673 \text{ kN/m}$$

And

$$N_A < 4 \text{ kN}; \quad 1.2376\, w < 4 \qquad w < 3.232 \text{ kN/m}$$

Thus, the maximum intensity of the distributed load is

$$w = 2.673 \text{ kN/m} = 2.67 \text{ kN/m} \qquad \textbf{Ans.}$$

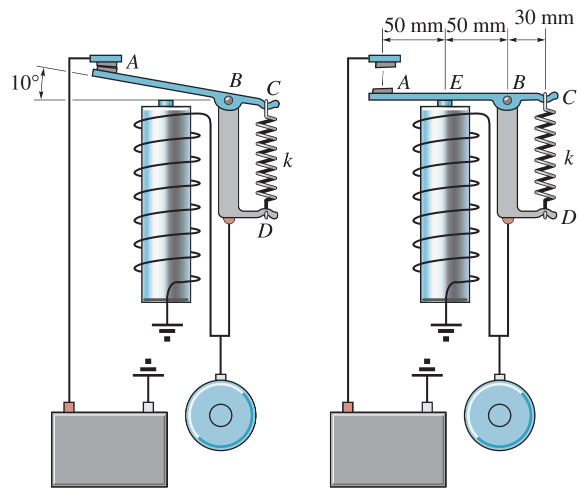


(a)

Ans:
$w = 2.67$ kN/m

***5–24.**

The relay regulates voltage and current. Determine the force in the spring CD, which has a stiffness of $k = 120$ N/m, so that it will allow the armature to make contact at A in figure (a) with a vertical force of 0.4 N. Also, determine the force in the spring when the coil is energized and attracts the armature to E, figure (b), thereby breaking contact at A.



(a) (b)

SOLUTION

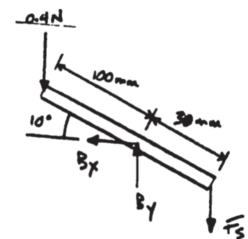
From Fig. (a):

$\circlearrowleft + \Sigma M_B = 0; \qquad 0.4(100 \cos 10°) - F_s(30 \cos 10°) = 0$

$F_s = 1.333 \text{ N} = 1.33 \text{ N}$ **Ans.**

$F_s = kx; \qquad 1.333 = 120\, x$

$x = 0.01111 \text{ m} = 11.11 \text{ mm}$



From Fig (b), energizing the coil requires the spring to be stretched an additional amount

$\Delta x = 30 \sin 10° = 5.209 \text{ mm}.$

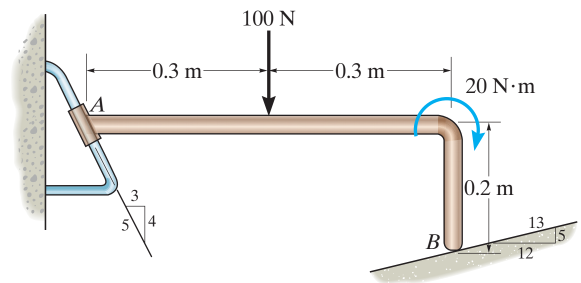
Thus

$x' = 11.11 + 5.209 = 16.32 \text{ mm}$

$F_s = 120\,(0.01632) = 1.96 \text{ N}$ **Ans.**

Ans:
$F_s = 1.33$ N
$F_s = 1.96$ N

5–25. Determine the reactions on the bent rod which is supported by a smooth surface at B and by a collar at A, which is fixed to the rod and is free to slide over the fixed inclined rod.

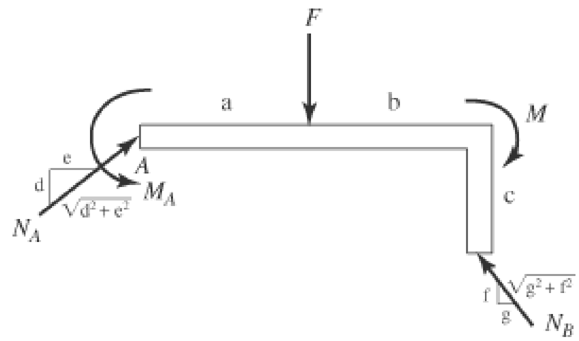


SOLUTION

Initial Guesses:

$N_A = 20\text{ N} \quad N_B = 10\text{ N} \quad M_A = 30\text{ N}\cdot\text{m}$

Given:



$F = 100\text{ N}$

$M = 20\text{ N}\cdot\text{m}$

$a = 0.3\text{ m}$

$b = 0.3\text{ m}$

$c = 0.2\text{ m}$

$d = 3$

$e = 4$

$f = 12$

$g = 5$

Given

$$\Sigma M_A = 0; \quad M_A - F\cdot a - M + N_B\cdot\frac{f}{\sqrt{f^2+g^2}}\cdot(a+b) - N_B\cdot\frac{g}{\sqrt{f^2+g^2}}\cdot c = 0$$

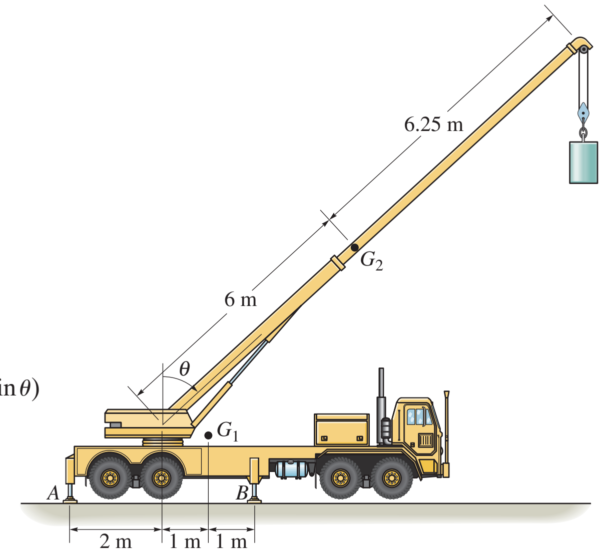
$$\Sigma F_x = 0; \quad N_A\cdot\frac{e}{\sqrt{e^2+d^2}} - N_B\cdot\frac{g}{\sqrt{f^2+g^2}} = 0$$

$$\Sigma F_y = 0; \quad N_A\cdot\frac{d}{\sqrt{e^2+d^2}} + N_B\cdot\frac{f}{\sqrt{f^2+g^2}} - F = 0$$

$$\begin{pmatrix} N_A \\ N_B \\ M_A \end{pmatrix} = \text{Find}(N_A, N_B, M_A) \quad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 39.7 \\ 82.5 \end{pmatrix}\text{N} \quad M_A = 10.6\text{ N}\cdot\text{m}$$ **Ans.**

Ans:
$N_A = 39.7\text{ N}$
$N_B = 82.5\text{ N}$
$M_A = 10.6\text{ N}\cdot\text{m}$

5–26. The mobile crane is symmetrically supported by two outriggers at A and two at B in order to relieve the suspension of the truck upon which it rests and to provide greater stability. If the crane boom and truck have a mass of 18 Mg and center of mass at G_1, and the boom has a mass of 1.8 Mg and a center of mass at G_2, determine the vertical reactions at each of the four outriggers as a function of the boom angle θ when the boom is supporting a load having a mass of 1.2 Mg. Plot the results measured from $\theta = 0°$ to the critical angle where tipping starts to occur.



SOLUTION

$+\Sigma M_B = 0;\quad -N_A(4) + 18(10^3)(9.81)(1) + 1.8(10^3)(9.81)(2 - 6\sin\theta)$

$+ 1.2(10^3)(9.81)(2 - 12.25\sin\theta) = 0$

$N_A = 58\,860 - 62\,539\sin\theta$

Tipping occurs when $N_A = 0$, or

$\theta = 70.3°$ **Ans.**

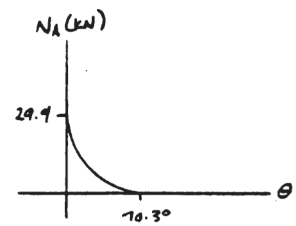
$+\uparrow\Sigma F_y = 0;\quad N_B + 58\,860 - 62\,539\sin\theta - (18 + 1.8 + 1.2)(10^3)(9.81) = 0$

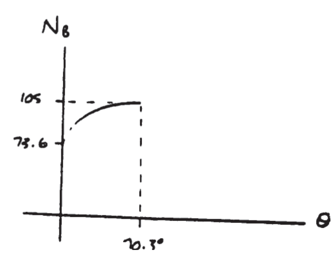
$N_B = 147\,150 + 62\,539\sin\theta$

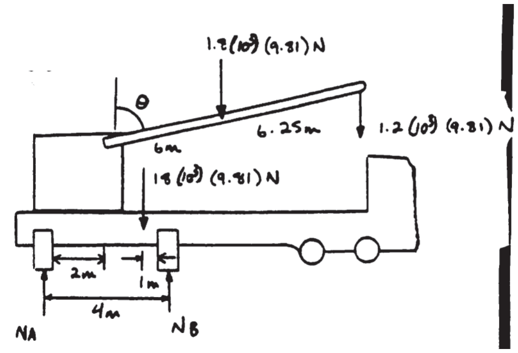
Since there are two outriggers on each side of the crane,

$$N'_A = \frac{N_A}{2} = (29.4 - 31.3\sin\theta)\text{ kN}$$ **Ans.**

$$N'_B = \frac{N_B}{2} = (73.6 + 31.3\sin\theta)\text{ kN}$$ **Ans.**



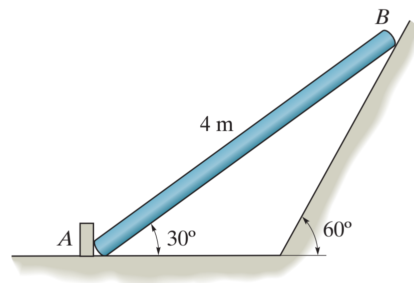




Ans:
$\theta = 70.3°$
$N'_A = (29.4 - 31.3\sin\theta)$ kN
$N'_B = (73.6 + 31.3\sin\theta)$ kN

5–27.

Determine the reactions acting on the smooth uniform bar, which has a mass of 20 kg.



SOLUTION

Equations of Equilibrium. $\mathbf{N}_B$ can be determined directly by writing the moment equation of equilibrium about point A by referring to the *FBD* of the bar shown in Fig. *a*.

$$\circlearrowleft + \Sigma M_A = 0; \qquad N_B \cos 30^\circ(4) - 20(9.81)\cos 30^\circ(2) = 0$$

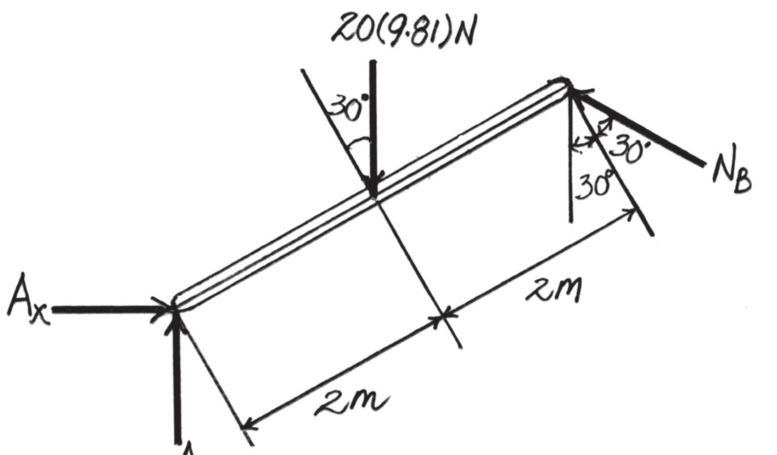
$$N_B = 98.1 \text{ N} \qquad \textbf{Ans.}$$

Using this result to write the force equation of equilibrium along the x and y axes,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - 98.1 \sin 60^\circ = 0 \qquad A_x = 84.96 \text{ N} = 85.0 \text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + 98.1 \cos 60^\circ - 20(9.81) = 0$$

$$A_y = 147.15 \text{ N} = 147 \text{ N} \qquad \textbf{Ans.}$$

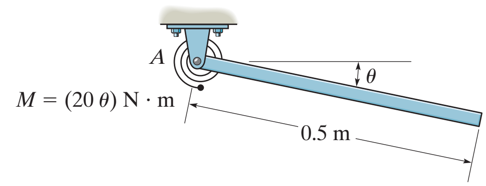


(a)

Ans:
$N_B = 98.1$ N
$A_x = 85.0$ N
$A_y = 147$ N

***5–28.**

A linear *torsional spring* deforms such that an applied couple moment M is related to the spring's rotation θ in radians by the equation $M = (20\,\theta)$ N · m. If such a spring is attached to the end of a pin-connected uniform 10-kg rod, determine the angle θ for equilibrium. The spring is undeformed when $\theta = 0^\circ$.

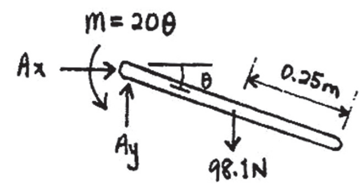


SOLUTION

$\zeta + \Sigma M_A = 0; \qquad -98.1\,(0.25\cos\theta) + 20(\theta) = 0$

Solving for θ,

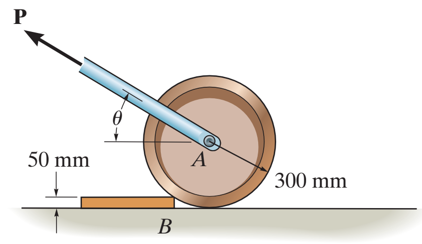
$$\theta = 47.5^\circ \qquad \textbf{Ans.}$$



Ans:
$\theta = 47.5^\circ$

5–29.

Determine the force P needed to pull the 50-kg roller over the smooth step. Take $\theta = 30^\circ$.

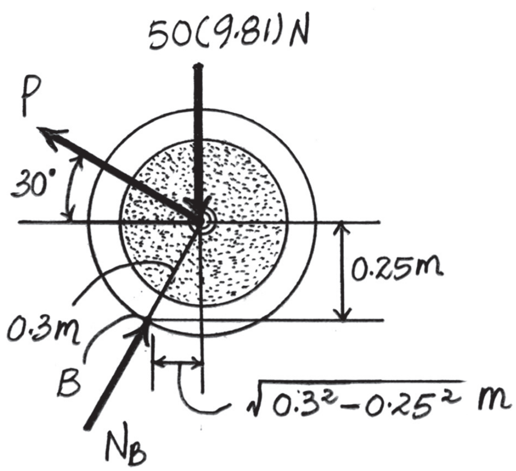


SOLUTION

Equations of Equilibrium. P can be determined directly by writing the moment equation of Equilibrium about point B, by referring to the FBD of the roller shown in Fig. a.

$$\circlearrowleft + \Sigma M_B = 0; \qquad P\cos 30^\circ(0.25) + P\sin 30^\circ\left(\sqrt{0.3^2 - 0.25^2}\right) - 50(9.81)\sqrt{0.3^2 - 0.25^2} = 0$$

$$P = 271.66 \text{ N} = 272 \text{ N} \qquad \textbf{Ans.}$$

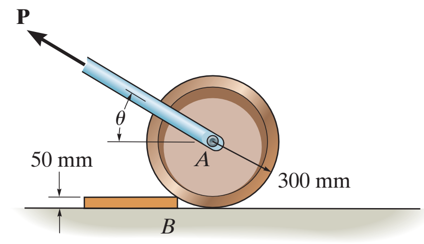


(a)

Ans:
$P = 272$ N

5–30.

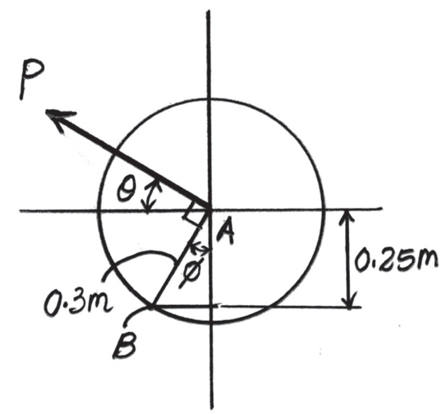
Determine the magnitude and direction θ of the minimum force P needed to pull the 50-kg roller over the smooth step.



SOLUTION

Equations of Equilibrium. P will be minimum if its orientation produces the greatest moment about point B. This happens when it acts perpendicular to AB as shown in Fig. a. Thus

$$\theta = \phi = \cos^{-1}\left(\frac{0.25}{0.3}\right) = 33.56° = 33.6° \qquad \textbf{Ans.}$$

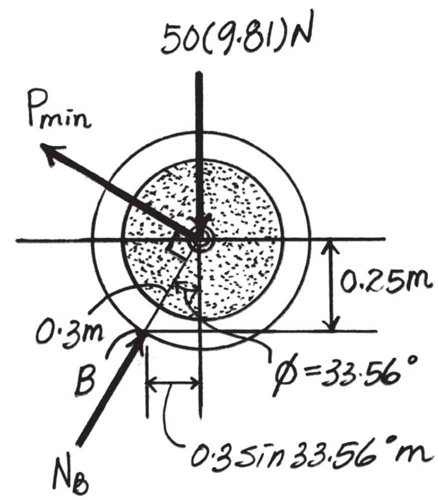


(a)

P_{min} can be determined by writing the moment equation of equilibrium about point B by referring to the FBD of the roller shown in Fig. b.

$$\circlearrowleft + \Sigma M_B = 0; \qquad P_{\text{min}}(0.3) - 50(9.81)(0.3 \sin 33.56°) = 0$$

$$P_{\text{min}} = 271.13 \text{ N} = 271 \text{ N} \qquad \textbf{Ans.}$$

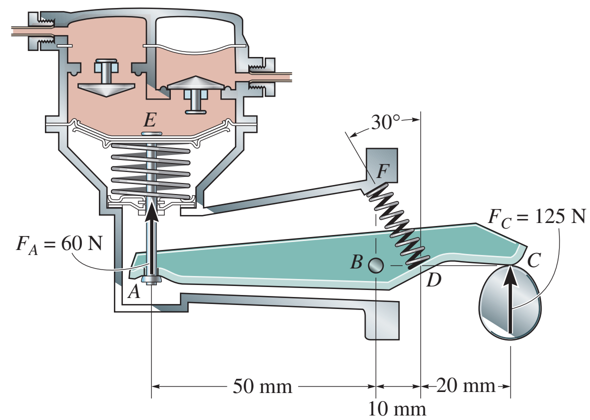


(b)

Ans:
$P_{\text{min}} = 271$ N

5–31.

The operation of the fuel pump for an automobile depends on the reciprocating action of the rocker arm ABC, which is pinned at B and is spring loaded at A and D. When the smooth cam C is in the position shown, determine the horizontal and vertical components of force at the pin and the force along the spring DF for equilibrium. The vertical force acting on the rocker arm at A is $F_A = 60$ N, and at C it is $F_C = 125$ N.



SOLUTION

$\circlearrowleft + \Sigma M_B = 0; \qquad -60(50) - F_B \cos 30°(10) + 125(30) = 0$

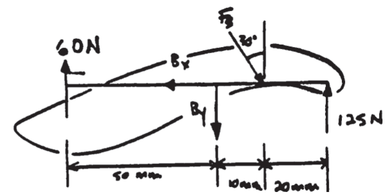
$F_B = 86.6025 = 86.6$ N **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \qquad -B_x + 86.6025 \sin 30° = 0$

$B_x = 43.3$ N **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad 60 - B_y - 86.6025 \cos 30° + 125 = 0$

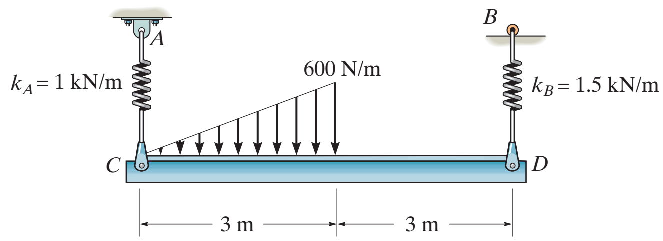
$B_y = 110$ N **Ans.**



Ans:
$F_B = 86.6$ N
$B_x = 43.3$ N
$B_y = 110$ N

***5–32.**

The beam of negligible weight is supported horizontally by two springs. If the beam is horizontal and the springs are unstretched when the load is removed, determine the angle of tilt of the beam when the load is applied.



SOLUTION

Equations of Equilibrium. $\mathbf{F}_A$ and $\mathbf{F}_B$ can be determined directly by writing the moment equations of equilibrium about points B and A, respectively, by referring to the *FBD* of the beam shown in Fig. *a*.

Assuming that the angle of tilt is small,

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_B(6) - \frac{1}{2}(600)(3)(2) = 0 \quad F_B = 300 \text{ N}$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad \frac{1}{2}(600)(3)(4) - F_A(6) = 0 \quad F_A = 600 \text{ N}$$

Thus, the stretches of springs A and B can be determined from

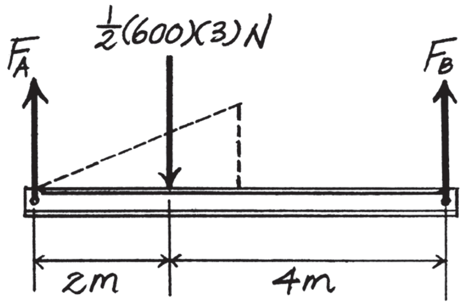
$$F_A = k_A x_A; \qquad 600 = 1000 x_A \qquad x_A = 0.6 \text{ m}$$

$$F_B = k_B x_B; \qquad 300 = 1500 x_B \qquad x_B = 0.2 \text{ m}$$

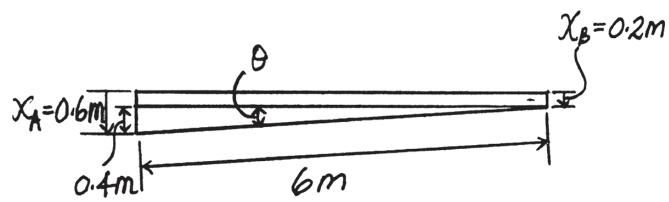
From the geometry shown in Fig. *b*

$$\theta = \sin^{-1}\left(\frac{0.4}{6}\right) = 3.82° \qquad \textbf{Ans.}$$

The assumption of small θ is confirmed.



(a)

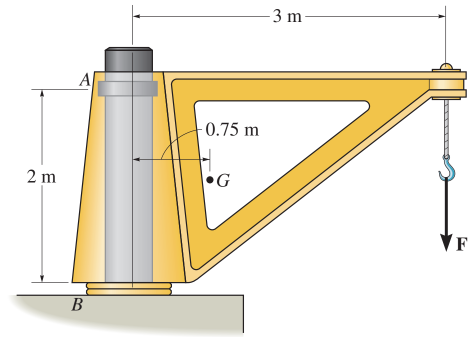


(b)

Ans:
$\theta = 3.82°$

5–33.

The dimensions of a jib crane, which is manufactured by the Basick Co., are given in the figure. If the crane has a mass of 800 kg and a center of mass at G, and the maximum rated force at its end is $F = 15$ kN, determine the reactions at its bearings. The bearing at A is a journal bearing and supports only a horizontal force, whereas the bearing at B is a thrust bearing that supports both horizontal and vertical components.



SOLUTION

$\circlearrowleft + \Sigma M_B = 0; \qquad A_x(2) - 800\,(9.81)\,(0.75) - 15\,000(3) = 0$

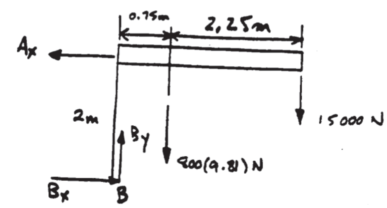
$A_x = 25.4\text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad B_y - 800\,(9.81) - 15\,000 = 0$

$B_y = 22.8\text{ kN}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x - 25.4 = 0$

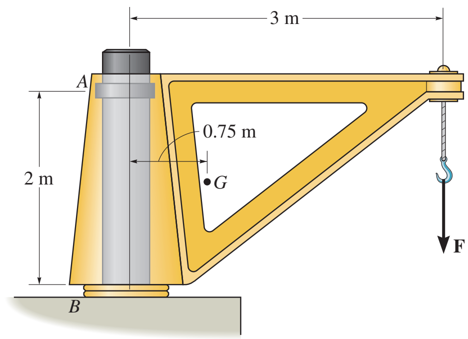
$B_x = 25.4\text{ kN}$ **Ans.**



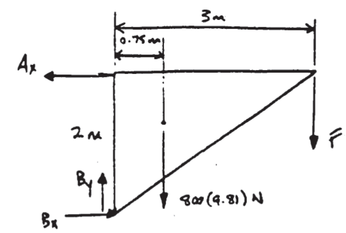
Ans:
$A_x = 25.4$ kN
$B_y = 22.8$ kN
$B_x = 25.4$ kN

5–34.

The dimensions of a jib crane, which is manufactured by the Basick Co., are given in the figure. The crane has a mass of 800 kg and a center of mass at G. The bearing at A is a journal bearing and can support a horizontal force, whereas the bearing at B is a thrust bearing that supports both horizontal and vertical components. Determine the maximum load F that can be suspended from its end if the selected bearings at A and B can sustain a maximum resultant load of 24 kN and 34 kN, respectively.



SOLUTION



$\circlearrowleft + \Sigma M_B = 0; \qquad A_x(2) - 800(9.81)(0.75) - F(3) = 0$

$+\uparrow \Sigma F_y = 0; \qquad B_y - 800(9.81) - F = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x - A_x = 0$

Assume $A_x = 24\,000$ N.

Solving,

$B_x = 24$ kN

$B_y = 21.9$ kN

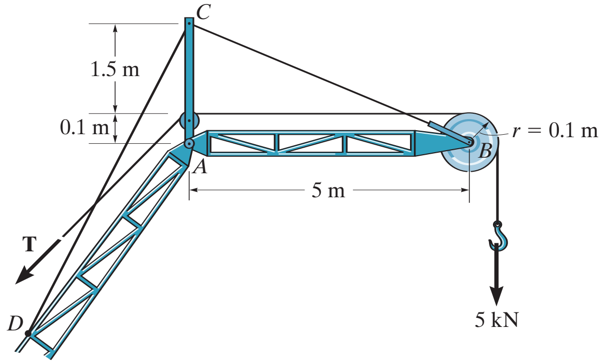
$F = 14.0$ kN **Ans.**

$F_B = \sqrt{(24)^2 + (21.9)^2} = 32.5 \text{ kN} < 34 \text{ kN}$ **OK**

Ans:
$F = 14.0$ kN

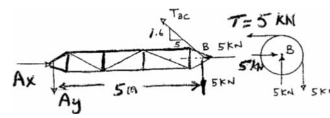
5–35.

The upper portion of the crane boom consists of the jib AB, which is supported by the pin at A, the guy line BC, and the backstay CD, each cable being separately attached to the mast at C. If the 5-kN load is supported by the hoist line, which passes over the pulley at B, determine the magnitude of the resultant force the pin exerts on the jib at A for equilibrium, the tension in the guy line BC, and the tension T in the hoist line. Neglect the weight of the jib. The pulley at B has a radius of 0.1 m.



SOLUTION

From pulley, tension in the hoist line is



$$\circlearrowleft + \Sigma M_B = 0; \qquad T(0.1) - 5(0.1) = 0;$$

$$T = 5\text{ kN} \qquad \textbf{Ans.}$$

From the jib,

$$\circlearrowleft + \Sigma M_A = 0; \qquad -5(5) + T_{BC}\left(\frac{1.6}{\sqrt{27.56}}\right)(5) = 0$$

$$T_{BC} = 16.4055 = 16.4\text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad -A_y + (16.4055)\left(\frac{1.6}{\sqrt{27.56}}\right) - 5 = 0$$

$$A_y = 0$$

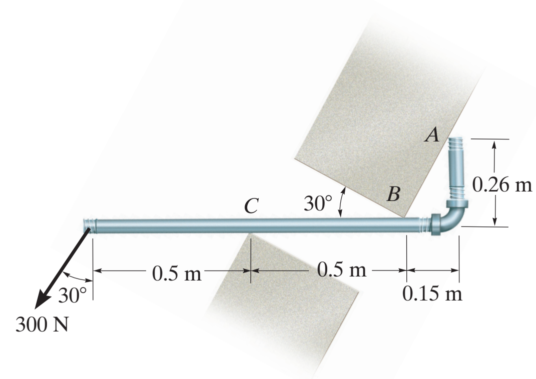
$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - 16.4055\left(\frac{5}{\sqrt{27.56}}\right) - 5 = 0$$

$$F_A = F_x = 20.6\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$T = 5$ kN
$T_{BC} = 16.4$ kN
$F_A = F_x = 20.6$ kN

***5–36.**

The smooth pipe rests against the opening at the points of contact A, B, and C. Determine the reactions at these points needed to support the force of 300 N. Neglect the pipe's thickness in the calculation.



SOLUTION

Equations of Equilibrium. $\mathbf{N}_A$ can be determined directly by writing the force equation of equilibrium along the x axis by referring to the *FBD* of the pipe shown in Fig. *a*.

$\xrightarrow{+} \Sigma F_x = 0; \qquad N_A \cos 30° - 300 \sin 30° = 0 \qquad N_A = 173.21\text{ N} = 173\text{ N}$ **Ans.**

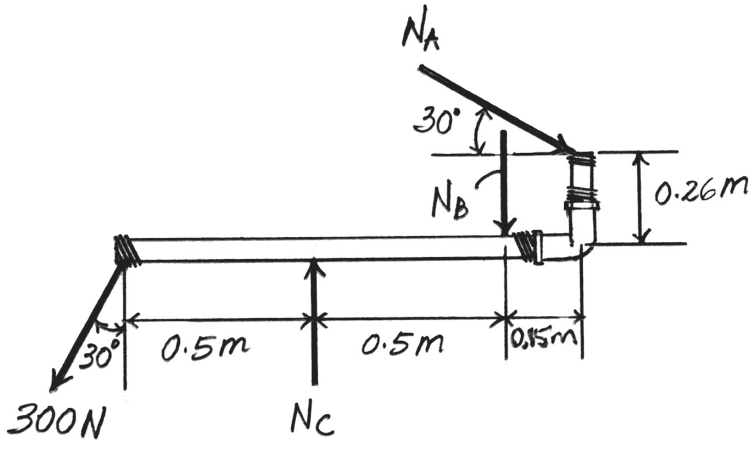
Using this result to write the moment equations of equilibrium about points B and C,

$\zeta + \Sigma M_B = 0; \qquad 300 \cos 30°(1) - 173.21 \cos 30°(0.26) - 173.21 \sin 30°(0.15) - N_C(0.5) = 0$

$N_C = 415.63\text{ N} = 416\text{ N}$ **Ans.**

$\zeta + \Sigma M_C = 0; \qquad 300 \cos 30°(0.5) - 173.21 \cos 30°(0.26) - 173.21 \sin 30°(0.65) - N_B(0.5) = 0$

$N_B = 69.22\text{ N} = 69.2\text{ N}$ **Ans.**

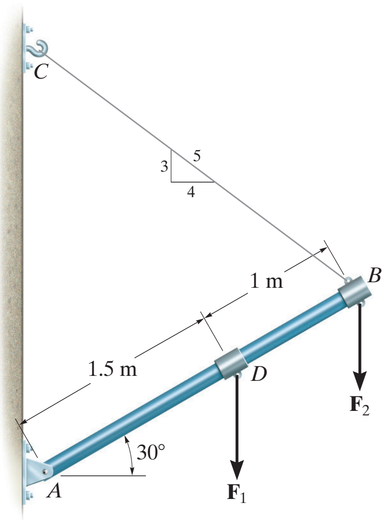


(a)

Ans:
$N_A = 173\text{ N}$
$N_C = 416\text{ N}$
$N_B = 69.2\text{ N}$

5–37.

The boom supports the two vertical loads. Neglect the size of the collars at D and B and the thickness of the boom, and compute the horizontal and vertical components of force at the pin A and the force in cable CB. Set $F_1 = 800$ N and $F_2 = 350$ N.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0;$ $\quad -800(1.5\cos 30°) - 350(2.5\cos 30°)$

$$+\frac{4}{5}F_{CB}(2.5\sin 30°) + \frac{3}{5}F_{CB}(2.5\cos 30°) = 0$$

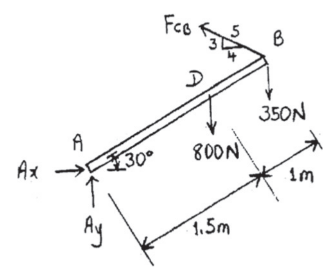
$$F_{CB} = 781.6 = 782 \text{ N}$$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x - \frac{4}{5}(781.6) = 0$

$$A_x = 625 \text{ N}$$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 800 - 350 + \frac{3}{5}(781.6) = 0$

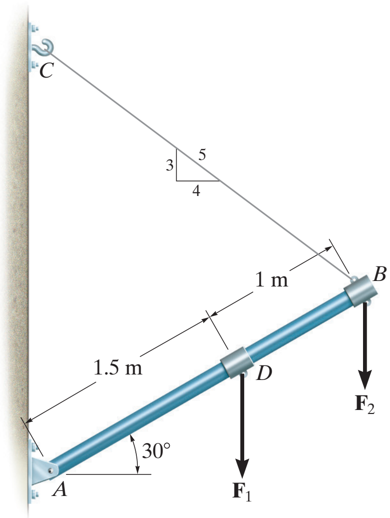
$$A_y = 681 \text{ N}$$ **Ans.**



Ans:
$F_{CB} = 782$ N
$A_x = 625$ N
$A_y = 681$ N

5–38.

The boom is intended to support two vertical loads, $\mathbf{F}_1$ and $\mathbf{F}_2$. If the cable CB can sustain a maximum load of 1500 N before it fails, determine the critical loads if $F_1 = 2F_2$. Also, what is the magnitude of the maximum reaction at pin A?



SOLUTION

$\zeta + \Sigma M_A = 0;\qquad -2F_2(1.5\cos 30^\circ) - F_2(2.5\cos 30^\circ)$

$$+\frac{4}{5}(1500)(2.5\sin 30^\circ) + \frac{3}{5}(1500)(2.5\cos 30^\circ) = 0$$

$F_2 = 724\text{ N}$ **Ans.**

$F_1 = 2F_2 = 1448\text{ N}$

$F_1 = 1.45\text{ kN}$ **Ans.**

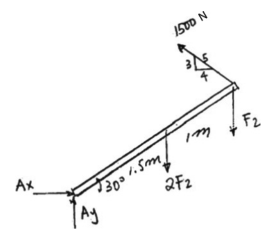
$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad A_x - \frac{4}{5}(1500) = 0$

$A_x = 1200\text{ N}$

$+\uparrow\Sigma F_y = 0;\qquad A_y - 724 - 1448 + \frac{3}{5}(1500) = 0$

$A_y = 1272\text{ N}$

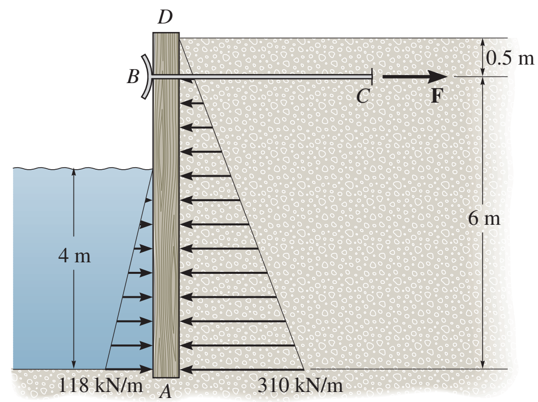
$F_A = \sqrt{(1200)^2 + (1272)^2} = 1749\text{ N} = 1.75\text{ kN}$ **Ans.**



Ans:
$F_2 = 724\text{ N}$
$F_1 = 1.45\text{ kN}$
$F_A = 1.75\text{ kN}$

5–39.

The bulk head AD is subjected to both water and soil-backfill pressures. Assuming AD is "pinned" to the ground at A, determine the horizontal and vertical reactions there and also the required tension in the ground anchor BC necessary for equilibrium. The bulk head has a mass of 800 kg.



SOLUTION

Equations of Equilibrium: The force in ground anchor BC can be obtained directly by summing moments about point A.

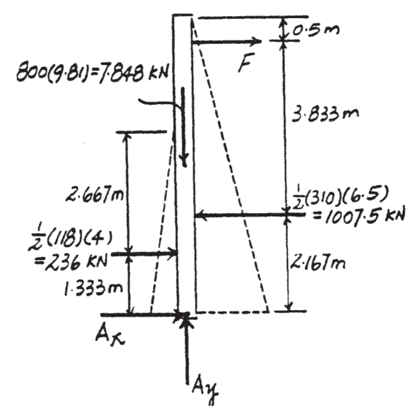
$$\circlearrowleft + \Sigma M_A = 0; \qquad 1007.5(2.167) - 236(1.333) - F(6) = 0$$

$$F = 311.375 \text{ kN} = 311 \text{ kN} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x + 311.375 + 236 - 1007.5 = 0$$

$$A_x = 460 \text{ kN} \qquad \textbf{Ans.}$$

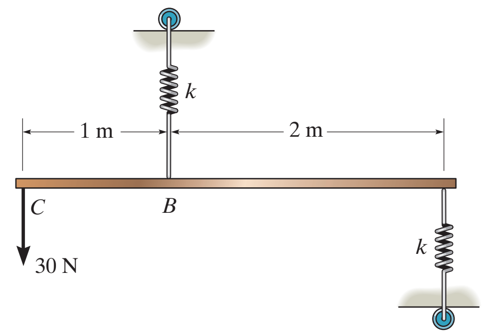
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 7.848 = 0 \qquad A_y = 7.85 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
$F = 311$ kN
$A_x = 460$ kN
$A_y = 7.85$ kN

***5–40.**

The bar of negligible weight is supported by two springs, each having a stiffness $k = 100$ N/m. If the springs are originally unstretched, and the force is vertical as shown, determine the angle θ the bar makes with the horizontal, when the 30-N force is applied to the bar.



SOLUTION

Equations of Equilibrium. $\mathbf{F}_A$ and $\mathbf{F}_B$ can be determined directly by writing the moment equation of equilibrium about points B and A respectively by referring to the *FBD* of the bar shown in Fig. *a*.

$$\circlearrowleft + \Sigma M_B = 0; \quad 30(1) - F_A(2) = 0 \quad F_A = 15 \text{ N}$$

$$\circlearrowleft + \Sigma M_A = 0; \quad 30(3) - F_B(2) = 0 \quad F_B = 45 \text{ N}$$

Thus, the stretches of springs A and B can be determined from

$$F_A = kx_A; \quad 15 = 100x_A \quad x_A = 0.15 \text{ m}$$

$$F_B = kx_B; \quad 45 = 100x_B \quad x_B = 0.45 \text{ m}$$

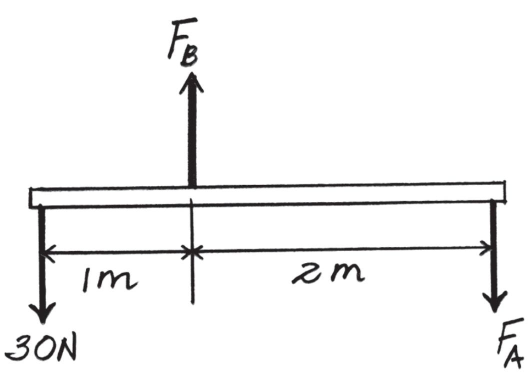
From the geometry shown in Fig. *b*,

$$\frac{d}{0.45} = \frac{2-d}{0.15}; \quad d = 1.5 \text{ m}$$

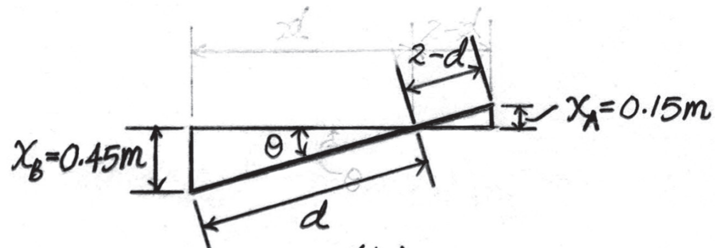
Thus

$$\theta = \sin^{-1}\left(\frac{0.45}{1.5}\right) = 17.46° = 17.5° \qquad \textbf{Ans.}$$

Note: The moment equations are set up assuming small θ, but even with non-small θ the reactions come out with the same F_A, F_B, and then the rest of the solution goes through as before.



(a)

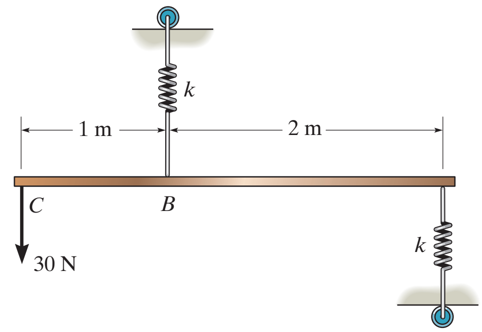


(b)

Ans:
$\theta = 17.5°$

5–41.

Determine the stiffness k of each spring so that the 30-N force causes the bar to tip $\theta = 15°$ when the force is applied. Originally the bar is horizontal and the springs are unstretched. Neglect the weight of the bar.



SOLUTION

Equations of Equilibrium. $\mathbf{F}_A$ and $\mathbf{F}_B$ can be determined directly by writing the moment equation of equilibrium about points B and A respectively by referring to the *FBD* of the bar shown in Fig. *a*.

$$\zeta+\Sigma M_B = 0; \qquad 30(1) - F_A(2) = 0 \qquad F_A = 15\text{ N}$$

$$\zeta+\Sigma M_A = 0; \qquad 30(3) - F_B(2) = 0 \qquad F_B = 45\text{ N}$$

Thus, the stretches of springs A and B can be determined from

$$F_A = kx_A; \qquad 15 = kx_A \qquad x_A = \frac{15}{k}$$

$$F_B = kx_B; \qquad 45 = kx_B \qquad x_B = \frac{45}{k}$$

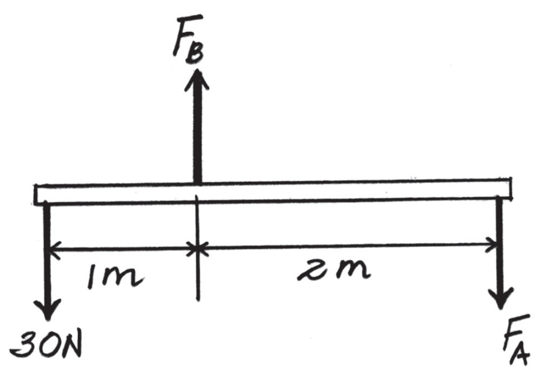
From the geometry shown in Fig. *b*

$$\frac{d}{45/k} = \frac{2-d}{15/k}; \qquad d = 1.5\text{ m}$$

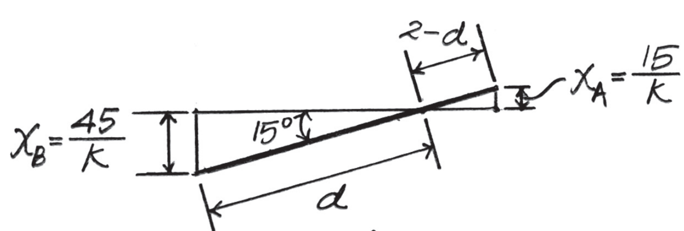
Thus,

$$\sin 15° = \frac{45/k}{1.5} \qquad k = 115.91\text{ N/m} = 116\text{ N/m} \qquad \textbf{Ans.}$$

Note: The moment equations are set up assuming small θ, but even with non-small θ the reactions come out with the same F_A, F_B, and then the rest of the solution goes through as before.



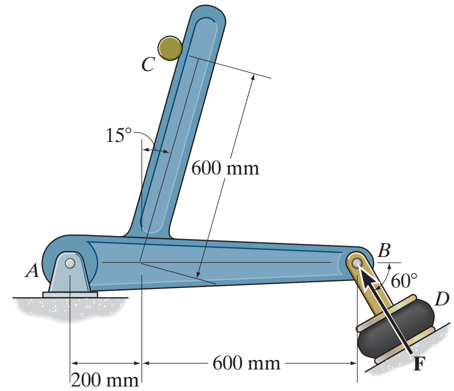
(a)



(b)

Ans:
$k = 116\text{ N/m}$

5–42. The airstroke actuator at D is used to apply a force of $F = 200$ N on the member at B. Determine the horizontal and vertical components of reaction at the pin A and the force of the smooth shaft at C on the member.



SOLUTION

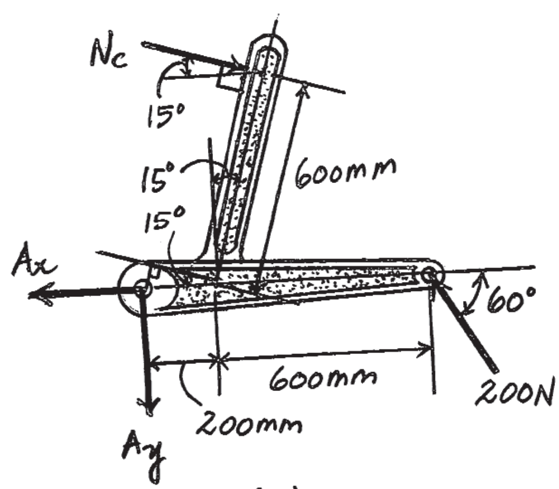
Equations of Equilibrium: From the free-body diagram of member ABC, Fig. a, N_C can be obtained by writing the moment equation of equilibrium about point A.

$$\zeta + \Sigma M_A = 0; \quad 200 \sin 60° (800) - N_C(600 + 200 \sin 15°) = 0$$
$$N_C = 212.60 \text{ N} = 213 \text{ N} \qquad \textbf{Ans.}$$

Using this result and writing the force equations of equilibrium along the x and y axes,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -A_x + 212.60 \sin 15° - 200 \sin 60° = 0$$
$$A_x = 105 \text{ N} \qquad \textbf{Ans.}$$

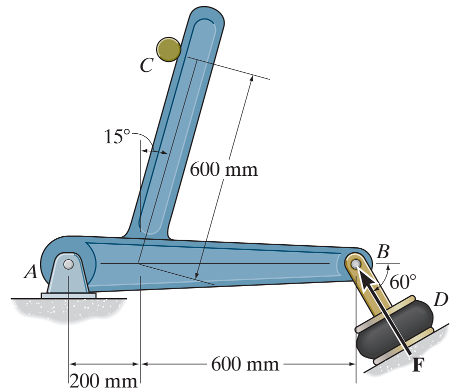
$$+\uparrow \Sigma F_y = 0; \quad -A_y - 212.60 \cos 15° + 200 \cos 60° = 0$$
$$A_y = 118 \text{ N} \qquad \textbf{Ans.}$$



(a)

Ans:
$N_C = 213$ N
$A_x = 105$ N
$A_y = 118$ N

5–43. The airstroke actuator at D is used to apply a force of $\mathbf{F}$ on the member at B. The normal reaction of the smooth shaft at C on the member is 300 N. Determine the magnitude of $\mathbf{F}$ and the horizontal and vertical components of reaction at pin A.



SOLUTION

Equations of Equilibrium: From the free-body diagram of member ABC, Fig. a, force F can be obtained by writing the moment equation of equilibrium about point A.

$\circlearrowleft + \Sigma M_A = 0;\quad F \sin 60^\circ(800) - 300(600 + 200 \sin 15^\circ) = 0$

$F = 282.22 \text{ N} = 282 \text{ N}$ **Ans.**

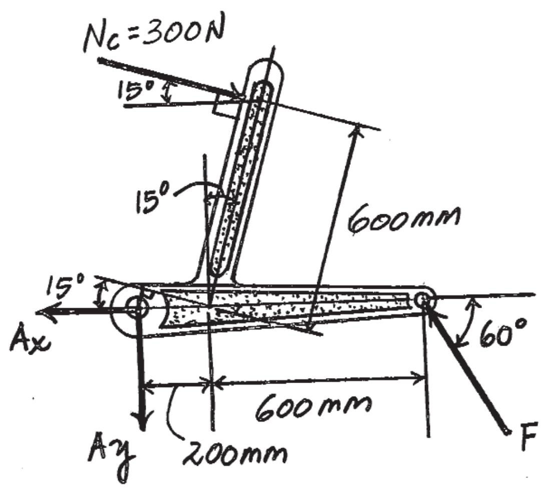
Using this result and writing the force equations of equilibrium along the x and y axes,

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad -A_x + 300 \cos 15^\circ - 282.22 \cos 60^\circ = 0$

$A_x = 149 \text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad -A_y - 300 \sin 15^\circ + 282.22 \sin 60^\circ = 0$

$A_y = 167 \text{ N}$ **Ans.**

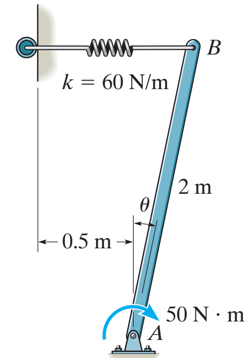


(a)

Ans:
$F = 282 \text{ N}$
$A_x = 149 \text{ N}$
$A_y = 167 \text{ N}$

***5–44.**

The 10-kg uniform rod is pinned at end A. If it is also subjected to a couple moment of 50 N·m, determine the smallest angle θ for equilibrium. The spring is unstretched when $\theta = 0$, and has a stiffness of $k = 60$ N/m.



SOLUTION

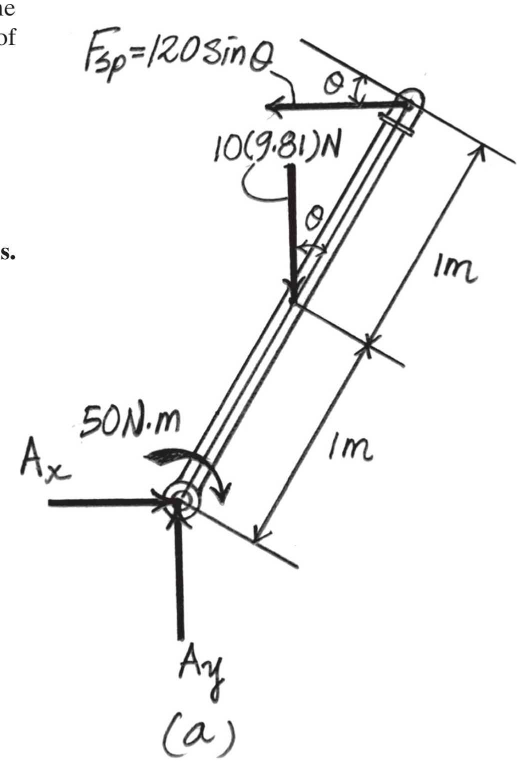
Equations of Equilibrium. Here the spring stretches $x = 2 \sin \theta$. The force in the spring is $F_{sp} = kx = 60\,(2 \sin \theta) = 120 \sin \theta$. Write the moment equation of equilibrium about point A by referring to the *FBD* of the rod shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad 120 \sin \theta \cos \theta\,(2) - 10(9.81) \sin \theta\,(1) - 50 = 0$$

$$240 \sin \theta \cos \theta - 98.1 \sin \theta - 50 = 0$$

Solve numerically

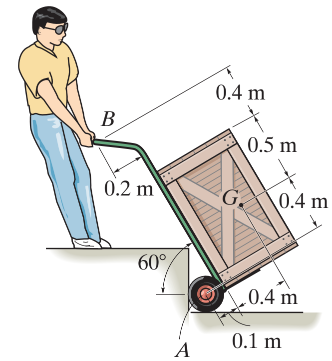
$$\theta = 24.598° = 24.6° \qquad \textbf{Ans.}$$



Ans:
$\theta = 24.6°$

5–45.

The man uses the hand truck to move material up the step. If the truck and its contents have a mass of 50 kg with center of gravity at G, determine the normal reaction on both wheels and the magnitude and direction of the minimum force required at the grip B needed to lift the load.



SOLUTION

Equations of Equilibriums. $\mathbf{P}_y$ can be determined directly by writing the force equation of equilibrium along y axis by referring to the *FBD* of the hand truck shown in Fig. *a*.

$+\uparrow \Sigma F_y = 0; \qquad P_y - 50(9.81) = 0 \qquad P_y = 490.5 \text{ N}$

Using this result to write the moment equation of equilibrium about point A,

$$\circlearrowleft + \Sigma M_A = 0; \quad \begin{aligned} &P_x \sin 60°(1.3) - P_x \cos 60°(0.1) - 490.5 \cos 30°(0.1) \\ &-490.5 \sin 30°(1.3) - 50(9.81) \sin 60°(0.5) \\ &+50(9.81) \cos 60°(0.4) = 0 \\ &P_x = 442.07 \text{ N} \end{aligned}$$

Thus, the magnitude of minimum force $\boldsymbol{P}$, Fig. b, is

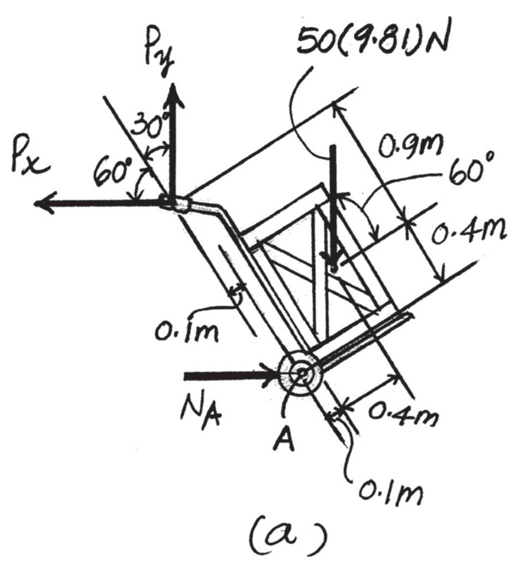
$P = \sqrt{P_x^2 + P_y^2} = \sqrt{442.07^2 + 490.5^2} = 660.32 \text{ N} = 660 \text{ N}$ **Ans.**

and the angle is

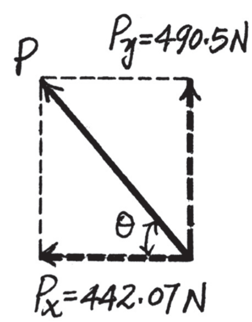
$\theta = \tan^{-1}\left(\dfrac{490.5}{442.07}\right) = 47.97° = 48.0°$ ⦪ **Ans.**

Write the force equation of equilibrium along x axis,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_A - 442.07 = 0 \qquad N_A = 442.07 \text{ N} = 442 \text{ N}$ **Ans.**



(a)

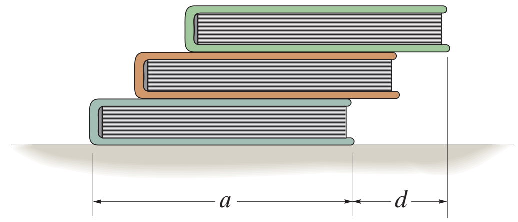


(b)

Ans:
$P = 660$ N
$N_A = 442$ N
$\theta = 48.0°$ ⦪

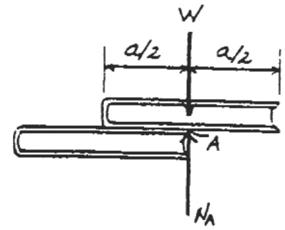
5–46.

Three uniform books, each having a weight W and length a, are stacked as shown. Determine the maximum distance d that the top book can extend out from the bottom one so the stack does not topple over.



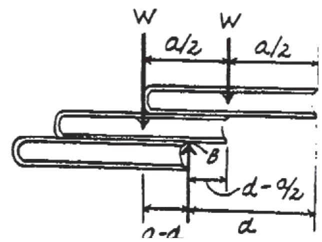
SOLUTION

Equilibrium: For top two books, the upper book will topple when the center of gravity of this book is to the right of point A. Therefore, the maximum distance from the right edge of this book to point A is $a/2$.



Equation of Equilibrium: For the entire three books, the top two books will topple about point B.

$$\circlearrowleft +\Sigma M_B = 0; \qquad W(a-d) - W\left(d - \frac{a}{2}\right) = 0$$

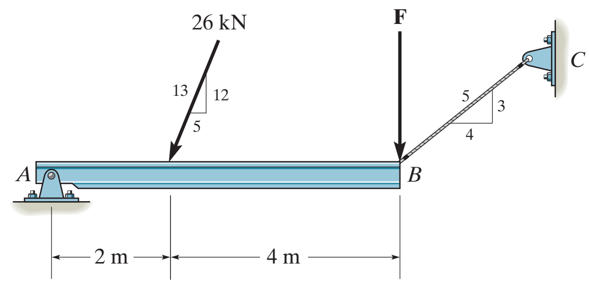


$$d = \frac{3a}{4}$$ **Ans.**

Ans:
$d = \dfrac{3a}{4}$

5–47.

Determine the reactions at the pin A and the tension in cord BC. Set $F = 40$ kN. Neglect the thickness of the beam.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \quad -26\left(\frac{12}{13}\right)(2) - 40(6) + \frac{3}{5}F_{BC}(6) = 0$

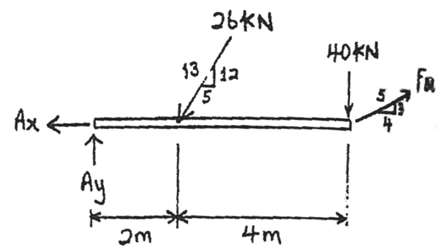
$F_{BC} = 80$ kN **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \quad 80\left(\frac{4}{5}\right) - A_x - 26\left(\frac{5}{13}\right) = 0$

$A_x = 54$ kN **Ans.**

$+\uparrow \Sigma F_y = 0; \quad A_y - 26\left(\frac{12}{13}\right) - 40 + 80\left(\frac{3}{5}\right) = 0$

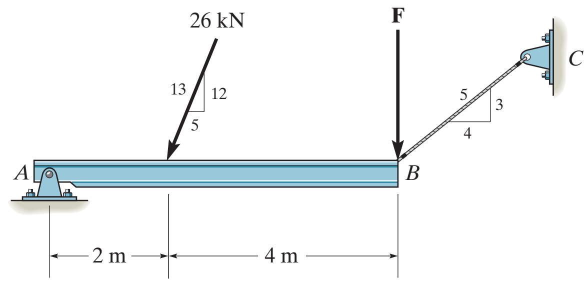
$A_y = 16$ kN **Ans.**



Ans:
$F_{BC} = 80$ kN
$A_x = 54$ kN
$A_y = 16$ kN

***5–48.**

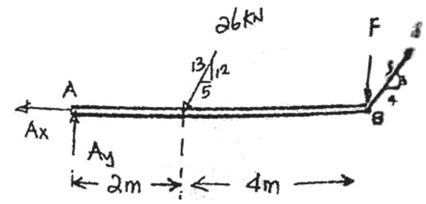
If rope BC will fail when the tension becomes 50 kN, determine the greatest vertical load F that can be applied to the beam at B. What is the magnitude of the reaction at A for this loading? Neglect the thickness of the beam.



SOLUTION

$\zeta + \Sigma M_A = 0;\qquad -26\left(\frac{12}{13}\right)(2) - F(6) + \frac{3}{5}(50)(6) = 0$

$F = 22 \text{ kN}$ **Ans.**



$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad 50\left(\frac{4}{5}\right) - A_x - 26\left(\frac{5}{13}\right) = 0$

$A_x = 30 \text{ kN}$ **Ans.**

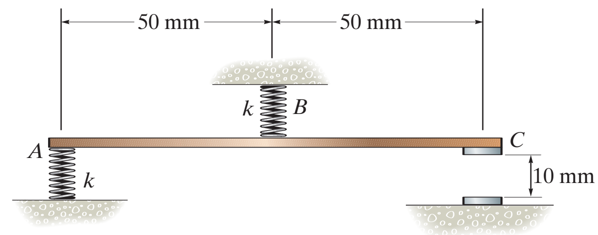
$+\uparrow \Sigma F_y = 0;\qquad A_y - 26\left(\frac{12}{13}\right) - 22 + 50\left(\frac{3}{5}\right) = 0$

$A_y = 16 \text{ kN}$ **Ans.**

Ans:
$F = 22$ kN
$A_x = 30$ kN
$A_y = 16$ kN

5–49.

The rigid metal strip of negligible weight is used as part of an electromagnetic switch. If the stiffness of the springs at A and B is $k = 5$ N/m, and the strip is originally horizontal when the springs are unstretched, determine the smallest force needed to close the contact gap at C.

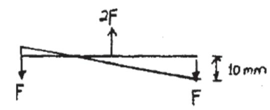


SOLUTION

$\Sigma M_B = 0; \qquad F_A = F_C = F$

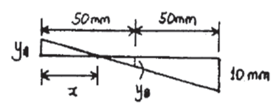
$\Sigma F_y = 0; \qquad F_B = 2F$

$$\frac{x}{y_A} = \frac{50 - x}{y_B} \qquad \textbf{(1)}$$



$$\frac{2F}{F} = \frac{k y_B}{k y_A}$$

$$2y_A = y_B \qquad \textbf{(2)}$$



Substituting into Eq. (1):

$$\frac{x}{y_A} = \frac{50 - x}{2y_A}$$

$$2x = 50 - x$$

$$x = \frac{50}{3} = 16.67 \text{ mm}$$

$$\frac{x}{y_A} = \frac{100 - x}{10}$$

Set $x = 16.67$, then

$$y_A = 2 \text{ mm}$$

From Eq. (2),

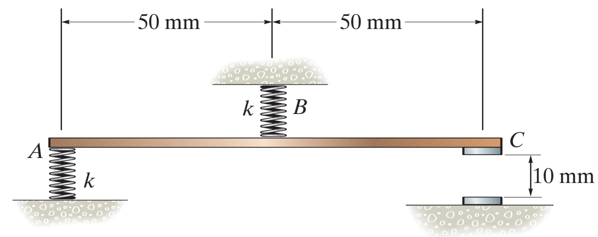
$$y_B = 4 \text{ mm}$$

$$F_C = F_A = k y_A = (5)(0.002) = 10 \text{ mN} \qquad \textbf{Ans.}$$

Ans:
$F_C = 10$ mN

5–50.

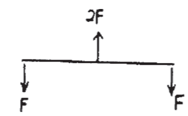
The rigid metal strip of negligible weight is used as part of an electromagnetic switch. Determine the maximum stiffness k of the springs at A and B so that the contact at C closes when the vertical force developed there is 0.5 N. Originally the strip is horizontal as shown.



SOLUTION

$\Sigma M_B = 0; \qquad F_A = F_C = F$

$\Sigma F_y = 0; \qquad F_B = 2F$

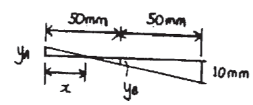


$$\frac{x}{y_A} = \frac{50 - x}{y_B} \tag{1}$$

$$\frac{2F}{F} = \frac{k y_B}{k y_A}$$

$$2y_A = y_B \tag{2}$$

Substituting into Eq. (1):



$$\frac{x}{y_A} = \frac{50 - x}{2y_A}$$

$$2x = 50 - x$$

$$x = \frac{50}{3} = 16.67 \text{ mm}$$

$$\frac{x}{y_A} = \frac{100 - x}{10}$$

Set $x = 16.67$, then

$$y_A = 2 \text{ mm}$$

From Eq. (2),

$$y_B = 4 \text{ mm}$$

$$F_C = F_A = k y_A$$

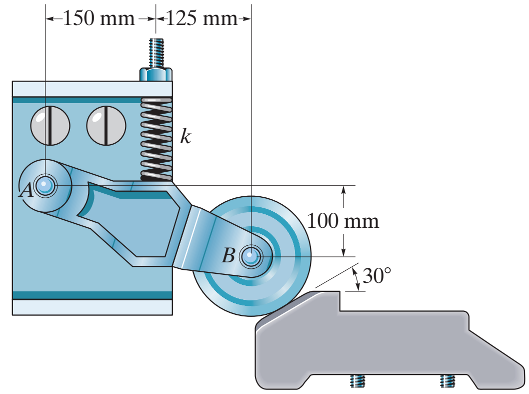
$$0.5 = k(0.002)$$

$k = 250$ N/m **Ans.**

Ans:
$k = 250$ N/m

5–51.

The device is used to hold an elevator door open. If the spring has a stiffness of $k = 40$ N/m and it is compressed 0.2 m, determine the horizontal and vertical components of reaction at the pin A and the resultant force at the wheel bearing B.



SOLUTION

$F_s = ks = (40)(0.2) = 8\ \text{N}$

$\circlearrowleft + \Sigma M_A = 0; \quad -(8)(150) + F_B(\cos 30°)(275) - F_B(\sin 30°)(100) = 0$

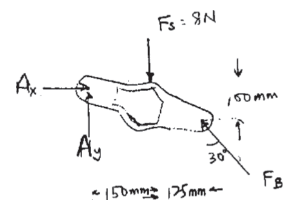
$F_B = 6.37765\ \text{N} = 6.38\ \text{N}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 6.37765 \sin 30° = 0$

$A_x = 3.19\ \text{N}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad A_y - 8 + 6.37765\ \cos 30° = 0$

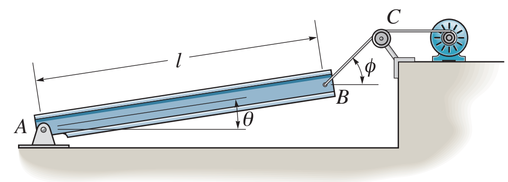
$A_y = 2.48\ \text{N}$ **Ans.**



Ans:
$F_B = 6.38$ N
$A_x = 3.19$ N
$A_y = 2.48$ N

***5–52.**

The uniform beam has a weight W and length l and is supported by a pin at A and a cable BC. Determine the horizontal and vertical components of reaction at A and the tension in the cable necessary to hold the beam in the position shown.

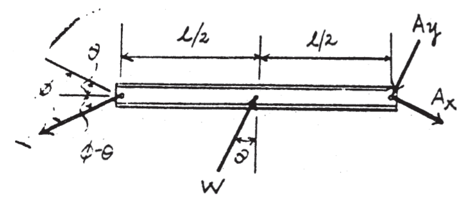


SOLUTION

Equations of Equilibrium: The tension in the cable can be obtained directly by summing moments about point A.

$$\circlearrowleft + \Sigma M_A = 0; \qquad T\sin(\phi - \theta)l - W\cos\theta\left(\frac{l}{2}\right) = 0$$

$$T = \frac{W\cos\theta}{2\sin(\phi - \theta)} \qquad \textbf{Ans.}$$



Using the result $T = \dfrac{W\cos\theta}{2\sin(\phi - \theta)}$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad \left(\frac{W\cos\theta}{2\sin(\phi - \theta)}\right)\cos\phi - A_x = 0$$

$$A_x = \frac{W\cos\phi\cos\theta}{2\sin(\phi - \theta)} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + \left(\frac{W\cos\theta}{2\sin(\phi - \theta)}\right)\sin\phi - W = 0$$

$$A_y = \frac{W(\sin\phi\cos\theta - 2\cos\phi\sin\theta)}{2\sin(\phi - \theta)} \qquad \textbf{Ans.}$$

Ans:

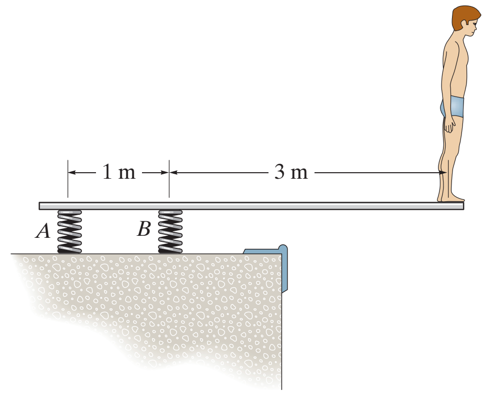
$$T = \frac{W\cos\theta}{2\sin(\phi - \theta)}$$

$$A_x = \frac{W\cos\phi\cos\theta}{2\sin(\phi - \theta)}$$

$$A_y = \frac{W(\sin\phi\cos\theta - 2\cos\phi\sin\theta)}{2\sin(\phi - \theta)}$$

5–53.

A boy stands out at the end of the diving board, which is supported by two springs A and B, each having a stiffness of $k = 15\text{kN/m}$. In the position shown the board is horizontal. If the boy has a mass of 40 kg, determine the angle of tilt which the board makes with the horizontal after he jumps off. Neglect the weight of the board and assume it is rigid.

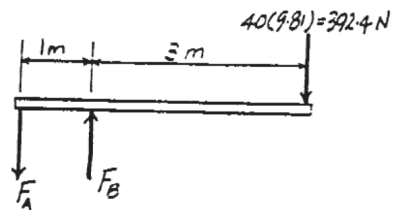


SOLUTION

Equations of Equilibrium: The spring force at A and B can be obtained directly by summing moments about points B and A, respectively.

$$\circlearrowleft +\Sigma M_B = 0; \qquad F_A(1) - 392.4(3) = 0 \qquad F_A = 1177.2\text{ N}$$

$$\circlearrowleft +\Sigma M_A = 0; \qquad F_B(1) - 392.4(4) = 0 \qquad F_B = 1569.6\text{ N}$$

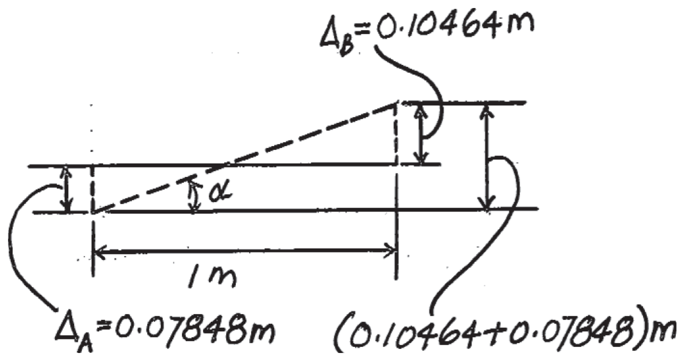


Spring Formula: Applying $\Delta = \dfrac{F}{k}$, we have

$$\Delta_A = \frac{1177.2}{15(10^3)} = 0.07848\text{ m} \qquad \Delta_B = \frac{1569.6}{15(10^3)} = 0.10464\text{ m}$$

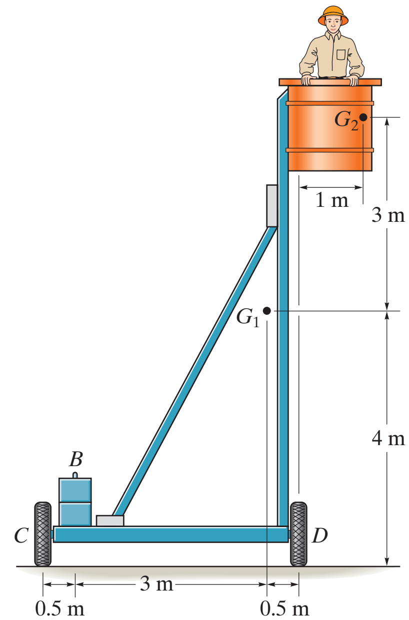
Geometry: The angle of tilt α is

$$\alpha = \tan^{-1}\left(\frac{0.10464 + 0.07848}{1}\right) = 10.4^\circ$$ **Ans.**



Ans:
$\alpha = 10.4^\circ$

5–54. The platform assembly has a weight of 1000 N ($\approx$ 100 kg) and center of gravity at G_1. If it is intended to support a maximum load of 1600 N placed, at point G_2, determine the smallest counterweight W that should be placed at B in order to prevent the platform from tipping over.



SOLUTION

Given:

$W_1 = 1000$ N

$W_2 = 1600$ N

$a = 0.5$ m

$b = 3$ m

$c = 0.5$ m

$d = 4$ m

$e = 3$ m

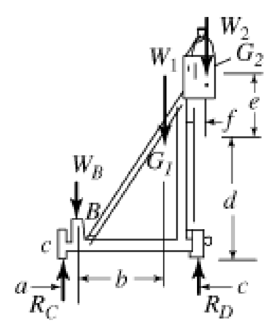
$f = 1$ m

When tipping occurs, $R_c = 0$

$$\circlearrowleft + \Sigma M_D = 0; \quad -W_2 \cdot f + W_1 \cdot c + W_B \cdot (b + c) = 0$$

$$W_B = \frac{W_2 \cdot f - W_1 \cdot c}{b + c}$$

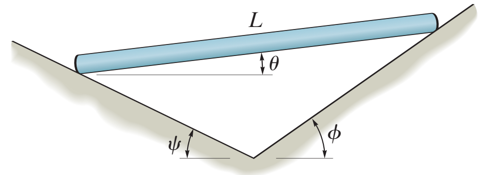
$$W_B = 314 \text{ N}$$ **Ans.**



Ans:
$W_B = 314$ N

5–55.

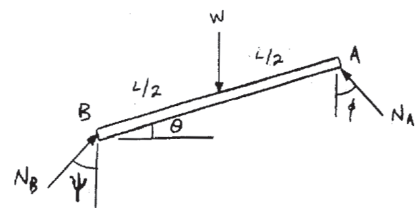
The uniform rod of length L and weight W is supported on the smooth planes. Determine its position θ for equilibrium. Neglect the thickness of the rod.



SOLUTION

$\zeta + \Sigma M_B = 0;\quad -W\left(\frac{L}{2}\cos\theta\right) + N_A\cos\phi\,(L\cos\theta) + N_A\sin\phi\,(L\sin\theta) = 0$

$$N_A = \frac{W\cos\theta}{2\cos(\phi - \theta)} \qquad \textbf{(1)}$$



$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad N_B\sin\psi - N_A\sin\phi = 0 \qquad \textbf{(2)}$

$+\uparrow\Sigma F_y = 0;\quad N_B\cos\psi + N_A\cos\phi - W = 0$

$$N_B = \frac{W - N_A\cos\phi}{\cos\psi} \qquad \textbf{(3)}$$

Substituting Eqs. (1) and (3) into Eq. (2):

$$\left(W - \frac{W\cos\theta\cos\phi}{2\cos(\phi - \theta)}\right)\tan\psi - \frac{W\cos\theta\sin\phi}{2\cos(\phi - \theta)} = 0$$

$2\cos(\phi - \theta)\tan\psi - \cos\theta\tan\psi\cos\phi - \cos\theta\sin\phi = 0$

$\sin\theta\,(2\sin\phi\tan\psi) - \cos\theta\,(\sin\phi - \cos\phi\tan\psi) = 0$

$\tan\theta = \dfrac{\sin\phi - \cos\phi\tan\psi}{2\sin\phi\tan\psi}$

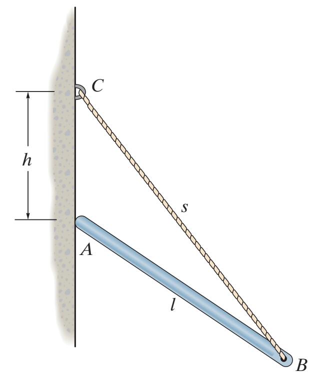
$\theta = \tan^{-1}\left(\frac{1}{2}\cot\psi - \frac{1}{2}\cot\phi\right)$ **Ans.**

Ans:

$\theta = \tan^{-1}\left(\frac{1}{2}\cot\psi - \frac{1}{2}\cot\phi\right)$

***5–56.**

The uniform rod has a length l and weight W. It is supported at one end A by a smooth wall and the other end by a cord of length s which is attached to the wall as shown. Determine the placement h for equilibrium.

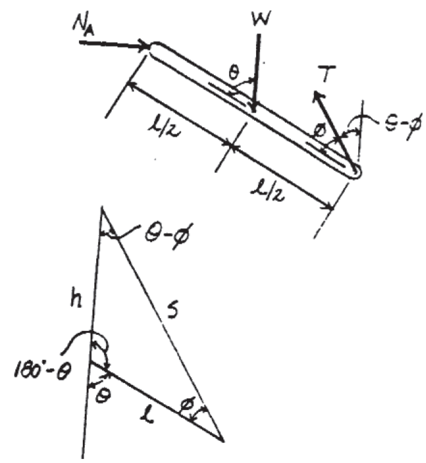


SOLUTION

Equations of Equilibrium: The tension in the cable can be obtained directly by summing moments about point A.

$$\zeta +\Sigma M_A = 0; \qquad T\sin\phi(l) - W\sin\theta\left(\frac{l}{2}\right) = 0$$

$$T = \frac{W\sin\theta}{2\sin\phi}$$



Using the result $T = \dfrac{W\sin\theta}{2\sin\phi}$,

$$+\uparrow\Sigma F_y = 0; \qquad \frac{W\sin\theta}{2\sin\phi}\cos(\theta - \phi) - W = 0$$

$$\sin\theta\cos(\theta - \phi) - 2\sin\phi = 0 \qquad \textbf{(1)}$$

Geometry: Applying the sine law with $\sin(180° - \theta) = \sin\theta$, we have

$$\frac{\sin\phi}{h} = \frac{\sin\theta}{s} \qquad \sin\phi = \frac{h}{s}\sin\theta \qquad \textbf{(2)}$$

Substituting Eq. (2) into (1) yields

$$\cos(\theta - \phi) = \frac{2h}{s} \qquad \textbf{(3)}$$

Using the cosine law,

$$l^2 = h^2 + s^2 - 2hs\cos(\theta - \phi)$$

$$\cos(\theta - \phi) = \frac{h^2 + s^2 - l^2}{2hs} \qquad \textbf{(4)}$$

Equating Eqs. (3) and (4) yields

$$\frac{2h}{s} = \frac{h^2 + s^2 - l^2}{2hs}$$

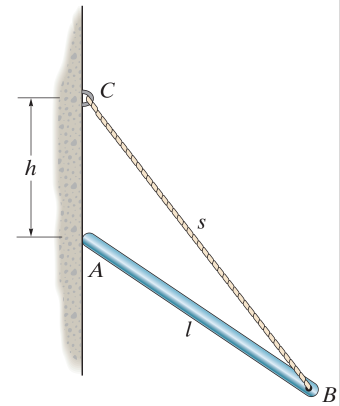
$$h = \sqrt{\frac{s^2 - l^2}{3}} \qquad \textbf{Ans.}$$

Ans:

$$h = \sqrt{\frac{s^2 - l^2}{3}}$$

5–57.

The 30-N uniform rod has a length of $l = 1$ m. If $s = 1.5$ m, determine the distance h of placement at the end A along the smooth wall for equilibrium.



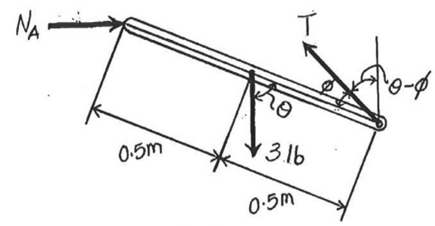
SOLUTION

Equations of Equilibrium: Referring to the FBD of the rod shown in Fig. a, write the moment equation of equilibrium about point A.

$$\circlearrowleft + \Sigma M_A = 0; \qquad T \sin \phi(1) - 3 \sin \theta(0.5) = 0$$

$$T = \frac{1.5 \sin \theta}{\sin \phi}$$

Using this result to write the force equation of equilibrium along y axis,



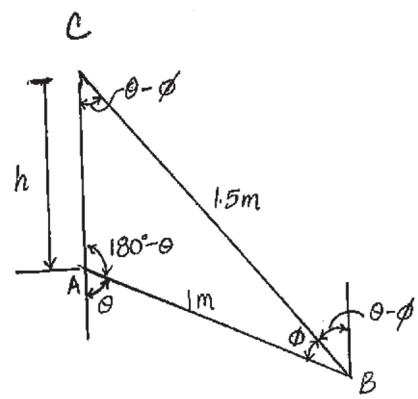
(a)

$$+\uparrow \Sigma F_y = 0; \qquad \left(\frac{15 \sin \theta}{\sin \phi}\right) \cos(\theta - \phi) - 3 = 0$$

$$\sin \theta \cos(\theta - \phi) - 2 \sin \phi = 0 \qquad \textbf{(1)}$$

Geometry: Applying the sine law with $\sin(180° - \theta) = \sin \theta$ by referring to Fig. b,

$$\frac{\sin \phi}{h} = \frac{\sin \theta}{1.5}; \qquad \sin \theta = \left(\frac{h}{1.5}\right) \sin \theta \qquad \textbf{(2)}$$



Substituting Eq. (2) into (1) yields

$$\sin \theta[\cos(\theta - \phi) - \frac{4}{3}h] = 0$$

since $\sin \theta \neq 0$, then

$$\cos(\theta - \phi) - (4/3)h \qquad \cos(\theta - \phi) = (4/3)h \qquad \textbf{(3)}$$

Again, applying law of cosine by referring to Fig. b,

$$l^2 = h^2 + 1.5^2 - 2(h)(1.5)\cos(\theta - \phi)$$

$$\cos(\theta - \phi) = \frac{h^2 + 1.25}{3h} \qquad \textbf{(4)}$$

Equating Eqs. (3) and (4) yields

$$\frac{4}{3}h = \frac{h^2 + 1.25}{3h}$$

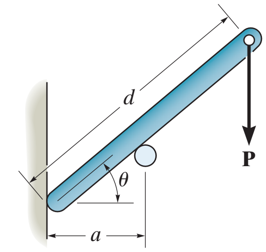
$$3h^2 = 1.25$$

$$h = 0.645 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$h = 0.645$ m

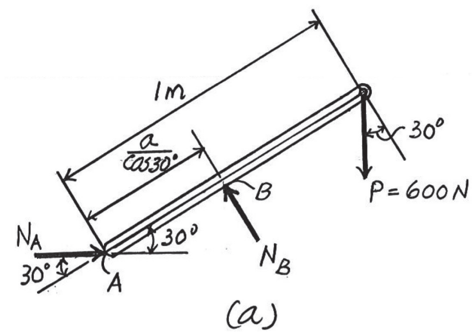
5–58.

If $d = 1$ m, and $\theta = 30°$, determine the normal reaction at the smooth supports and the required distance a for the placement of the roller if $P = 600$ N. Neglect the weight of the bar.



SOLUTION

Equations of Equilibrium: Referring to the FBD of the rod shown in Fig. *a*,



$\zeta + \Sigma M_A = 0;\qquad N_B = \left(\dfrac{a}{\cos 30°}\right) - 600 \cos 30°(1) = 0$

$$N_B = \frac{450}{a} \qquad \textbf{(1)}$$

$\nwarrow^{+} \Sigma F_{y'} = 0;\qquad N_B - N_A \sin 30° - 600 \cos 30° = 0$

$$N_B - 0.5N_A = 600 \cos 30° \qquad \textbf{(2)}$$

$\overset{+}{\nearrow} \Sigma F_{x'} = 0;\qquad N_A \cos 30° - 600 \sin 30° = 0$

$$N_A = 346.41 \text{ N} = 346 \text{ N} \qquad \textbf{Ans.}$$

Substitute this result into Eq (2),

$$N_B - 0.5(346.41) = 600 \cos 30°$$

$$N_B = 692.82 \qquad N = 693 \text{ N} \qquad \textbf{Ans.}$$

Substitute this result into Eq (1),

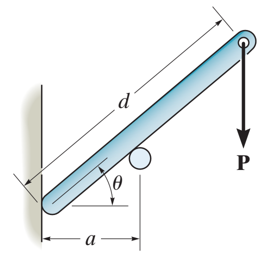
$$692.82 = \frac{450}{a}$$

$$a = 0.6495 \text{ m} \qquad a = 0.650 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$N_A = 346$ N
$N_B = 693$ N
$a = 0.650$ m

5–59.

Determine the distance d for placement of the load **P** for equilibrium of the smooth bar in the position θ as shown. Neglect the weight of the bar.



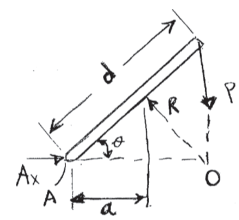
SOLUTION

$+\uparrow \Sigma F_y = 0;$ $\qquad R\cos\theta - P = 0$

$\zeta + \Sigma M_A = 0;$ $\qquad -P(d\cos\theta) + R\left(\dfrac{a}{\cos\theta}\right) = 0$

$$Rd\cos^2\theta = R\left(\frac{a}{\cos\theta}\right)$$

$$d = \frac{a}{\cos^3\theta} \qquad \textbf{Ans.}$$



Also;

Require forces to be concurrent at point O.

$$AO = d\cos\theta = \frac{a/\cos\theta}{\cos\theta}$$

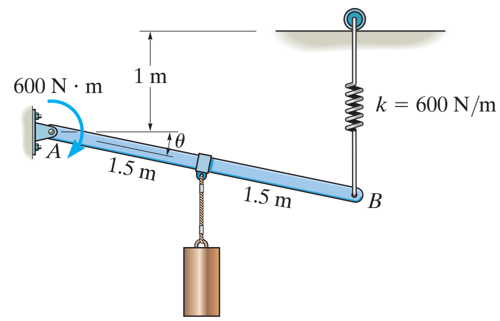
Thus,

$$d = \frac{a}{\cos^3\theta} \qquad \textbf{Ans.}$$

Ans:

$d = \dfrac{a}{\cos^3\theta}$

***5–60.** The rod supports a cylinder of mass 50 kg and is pinned at its end A. If it is also subjected to a couple moment of 600 N·m, determine the angle θ for equilibrium. The spring has an unstretched length of 1 m and a stiffness of $k = 600$ N/m.



SOLUTION

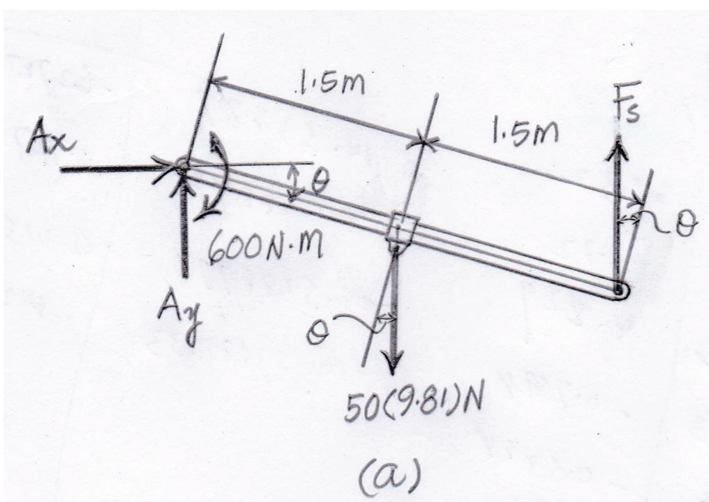
Equation of Equilibrium: At equilibrium position, the spring stretches $x = 3 \sin\theta$ m. Thus, the force in the spring is $F_s = kx = 600\,(3\sin\theta) = 1800\sin\theta$ N. Write the moment equation of equilibrium about point A by referring to the *FBD* of the rod, Fig. *a*,

$$\circlearrowleft +\Sigma M_A = 0; \quad 1800\sin\theta\cos\theta(3) - [50(9.81)\cos\theta](1.5) - 600 = 0$$

$$5400\sin\theta\cos\theta - 735.75\cos\theta - 600 = 0$$

The numerical solution gives

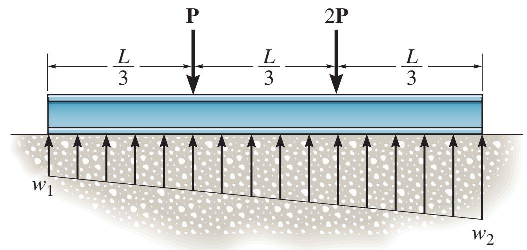
$$\theta = 14.54° = 14.5° \quad \text{and} \quad \theta = 82.54 = 82.5° \qquad \textbf{Ans.}$$



Ans:
$\theta = 14.5°$
$\theta = 85.2°$

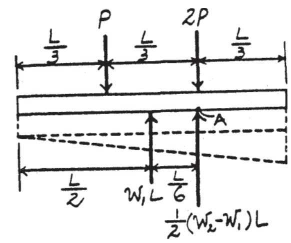
5–61.

The beam is subjected to the two concentrated loads. Assuming that the foundation exerts a linearly varying load distribution on its bottom, determine the load intensities w_1 and w_2 for equilibrium in terms of the parameters shown.



SOLUTION

Equations of Equilibrium: The load intensity w_1 can be determined directly by summing moments about point A.



$\circlearrowleft + \Sigma M_A = 0;\qquad P\left(\frac{L}{3}\right) - w_1 L\left(\frac{L}{6}\right) = 0$

$$w_1 = \frac{2P}{L}$$ **Ans.**

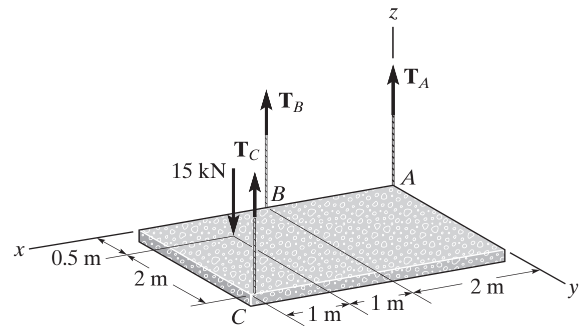
$+\uparrow \Sigma F_y = 0;\qquad \frac{1}{2}\left(w_2 - \frac{2P}{L}\right)L + \frac{2P}{L}(L) - 3P = 0$

$$w_2 = \frac{4P}{L}$$ **Ans.**

Ans:
$w_1 = \frac{2P}{L}, w_2 = \frac{4P}{L}$

5–62.

The uniform concrete slab has a mass of 2400 kg. Determine the tension in each of the three parallel supporting cables when the slab is held in the horizontal plane as shown.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the slab shown in Fig. *a*, we notice that T_C can be obtained directly by writing the moment equation of equilibrium about the x axis.

$\Sigma M_x = 0;\quad T_C(2.5) - 2400(9.81)(1.25) - 15(10^3)(0.5) = 0$

$T_C = 14{,}772 \text{ N} = 14.8 \text{ kN}$ **Ans.**

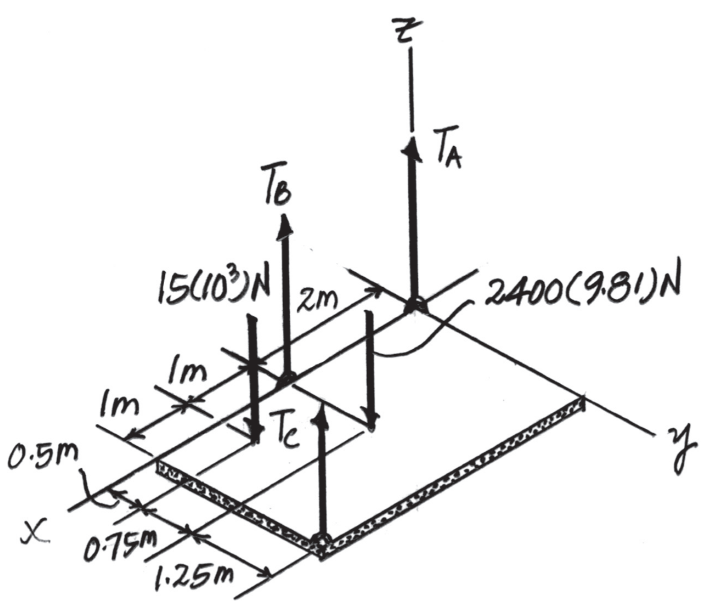
Using this result to write moment equation of equilibrium about y axis and force equation of equilibrium along z axis,

$\Sigma M_y = 0;\quad T_B(2) + 14{,}772(4) - 2400(9.81)(2) - 15(10^3)(3) = 0$

$T_B = 16{,}500 \text{ N} = 16.5 \text{ kN}$ **Ans.**

$\Sigma F_z = 0;\quad T_A + 16{,}500 + 14{,}772 - 2400(9.81) - 15(10^3) = 0$

$T_A = 7272 \text{ N} = 7.27 \text{ kN}$ **Ans.**

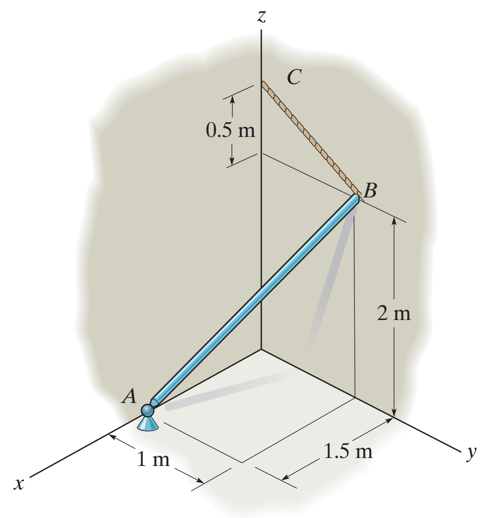


(a)

Ans:
$T_C = 14.8$ kN
$T_B = 16.5$ kN
$T_A = 7.27$ kN

5–63.

The smooth uniform rod AB is supported by a ball-and-socket joint at A, the wall at B, and cable BC. Determine the components of reaction at A, the tension in the cable, and the normal reaction at B if the rod has a mass of 20 kg.



SOLUTION

Force And Position Vectors. The coordinates of points A, B and G are $A(1.5, 0, 0)$ m, $B(0, 1, 2)$ m, $C(0, 0, 2.5)$ m and $G(0.75, 0.5, 1)$ m

$$F_A = -A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{T}_{BC} = T_{BC}\left(\frac{\mathbf{r}_{BC}}{r_{BC}}\right) = T_{BC}\left[\frac{(0-1)\mathbf{j} + (2.5-2)k}{\sqrt{(0-1)^2 + (2.5-2)^2}}\right] = -\frac{1}{\sqrt{1.25}}T_{BC}\mathbf{j} + \frac{0.5}{\sqrt{1.25}}T_{BC}\,\mathbf{k}$$

$$\mathbf{N}_B = N_B\mathbf{i}$$

$$\mathbf{W} = \{-20(9.81)\mathbf{k}\}\ \text{N}$$

$$\mathbf{r}_{AG} = (0.75 - 1.5)\mathbf{i} + (0.5 - 0)\mathbf{j} + (1 - 0)\mathbf{k} = \{-0.75\mathbf{i} + 0.5\mathbf{j} + \mathbf{k}\}\ \text{m}$$

$$\mathbf{r}_{AB} = (0 - 1.5)\mathbf{i} + (1 - 0)\mathbf{j} + (2 - 0)\mathbf{k} = \{-1.5\mathbf{i} + \mathbf{j} + 2\mathbf{k}\}\ \text{m}$$

Equations of Equilibrium. Referring to the *FBD* of the rod shown in Fig. *a*, the force equation of equilibrium gives

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_A + \mathbf{T}_{BC} + \mathbf{N}_B + \mathbf{W} = 0$$

$$(-A_x + N_B)\mathbf{i} + \left(A_y - \frac{1}{\sqrt{1.25}}T_{BC}\right)\mathbf{j} + \left[A_z + \frac{0.5}{\sqrt{1.25}}T_{BC} - 20\,(9.81)\right]\mathbf{k} = 0$$

Equating **i**, **j** and **k** components,

$$-A_x + N_B = 0 \qquad \textbf{(1)}$$

$$A_y - \frac{1}{\sqrt{1.25}}T_{BC} = 0 \qquad \textbf{(2)}$$

$$A_z + \frac{0.5}{\sqrt{1.25}}T_{BC} - 20(9.81) = 0 \qquad \textbf{(3)}$$

The moment equation of equilibrium gives

$$\Sigma\mathbf{M}_A = 0; \qquad \mathbf{r}_{AG} \times \mathbf{W} + \mathbf{r}_{AB} \times (\mathbf{T}_{BC} + \mathbf{N}_B) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.75 & 0.5 & 1 \\ 0 & 0 & -20(9.81) \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1.5 & 1 & 2 \\ N_B & -\frac{1}{\sqrt{1.25}}T_{BC} & \frac{0.5}{\sqrt{1.25}}T_{BC} \end{vmatrix} = 0$$

$$\left(\frac{0.5}{\sqrt{1.25}}T_{BC} + \frac{2}{\sqrt{1.25}}T_{BC} - 98.1\right)\mathbf{i} + \left(\frac{0.75}{\sqrt{1.25}}T_{BC} + 2N_B - 147.15\right)\mathbf{j} + \left(\frac{1.5}{\sqrt{1.25}}T_{BC} - N_B\right)\mathbf{k} = 0$$

5–63. Continued

Equating **i**, **j** and **k** Components

$$\frac{0.5}{\sqrt{1.25}} T_{BC} + \frac{2}{\sqrt{1.25}} T_{BC} - 98.1 = 0 \qquad \textbf{(4)}$$

$$\frac{0.75}{\sqrt{1.25}} T_{BC} + 2N_B - 147.15 = 0 \qquad \textbf{(5)}$$

$$\frac{1.5}{\sqrt{1.25}} T_{BC} - N_B = 0 \qquad \textbf{(6)}$$

Solving Eqs. (1) to (6)

$T_{BC} = 43.87 \text{ N} = 43.9 \text{ N}$ **Ans.**

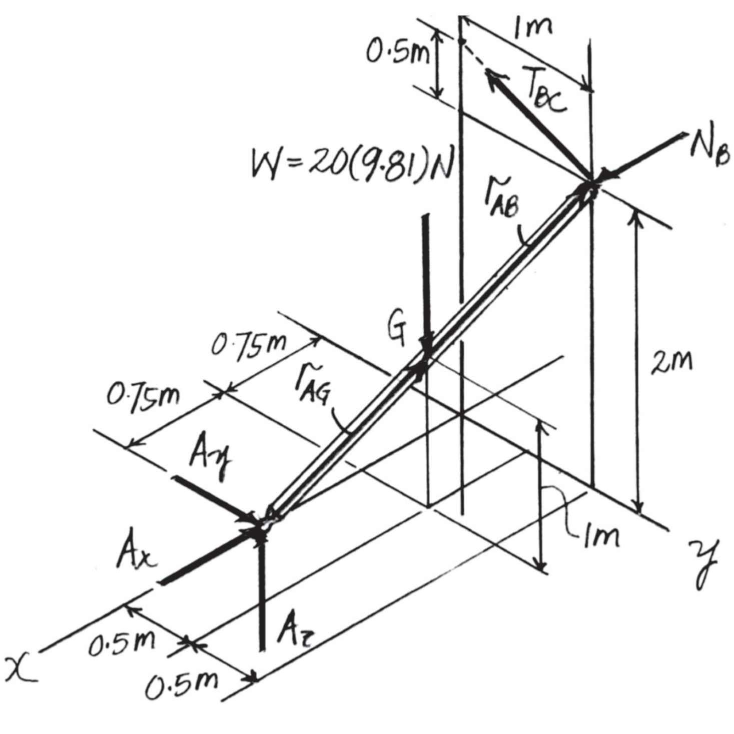
$N_B = 58.86 \text{ N} = 58.9 \text{ N}$ **Ans.**

$A_x = 58.86 \text{ N} = 58.9 \text{ N}$ **Ans.**

$A_y = 39.24 \text{ N} = 39.2 \text{ N}$ **Ans.**

$A_z = 176.58 \text{ N} = 177 \text{ N}$ **Ans.**

Note: One of the equations (4), (5) and (6) is redundant that will be satisfied automatically.

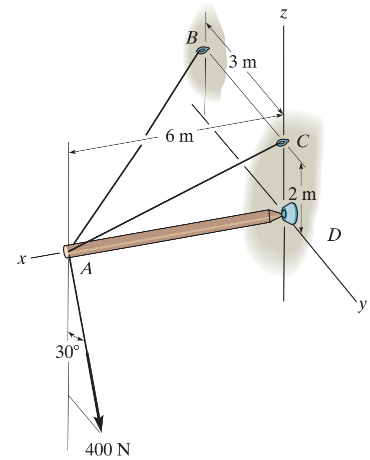


(a)

Ans:
$T_{BC} = 43.9$ N
$N_B = 58.9$ N
$A_x = 58.9$ N
$A_y = 39.2$ N
$A_z = 177$ N

***5–64.**

Determine the tension in each cable and the components of reaction at D needed to support the load.



SOLUTION

Force And Position Vectors. The coordinates of points A, B, and C are $A(6, 0, 0)$ m, $B(0, -3, 2)$ m and $C(0, 0, 2)$ m respectively.

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = F_{AB}\left[\frac{(0-6)\mathbf{i} + (-3-0)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(0-6)^2 + (-3-0)^2 + (2-0)^2}}\right] = -\frac{6}{7}F_{AB}\mathbf{i} - \frac{3}{7}F_{AB}\mathbf{j} + \frac{2}{7}F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = F_{AC}\left[\frac{(0-6)\mathbf{i} + (2-0)\mathbf{k}}{\sqrt{(0-6)^2 + (2-0)^2}}\right] = -\frac{6}{\sqrt{40}}F_{AC}\mathbf{i} + \frac{2}{\sqrt{40}}F_{AC}\mathbf{k}$$

$$\mathbf{F} = 400\,(\sin 30°\mathbf{j} - \cos 30°\mathbf{k}) = \{200\mathbf{j} - 346.41\mathbf{k}\}\text{N}$$

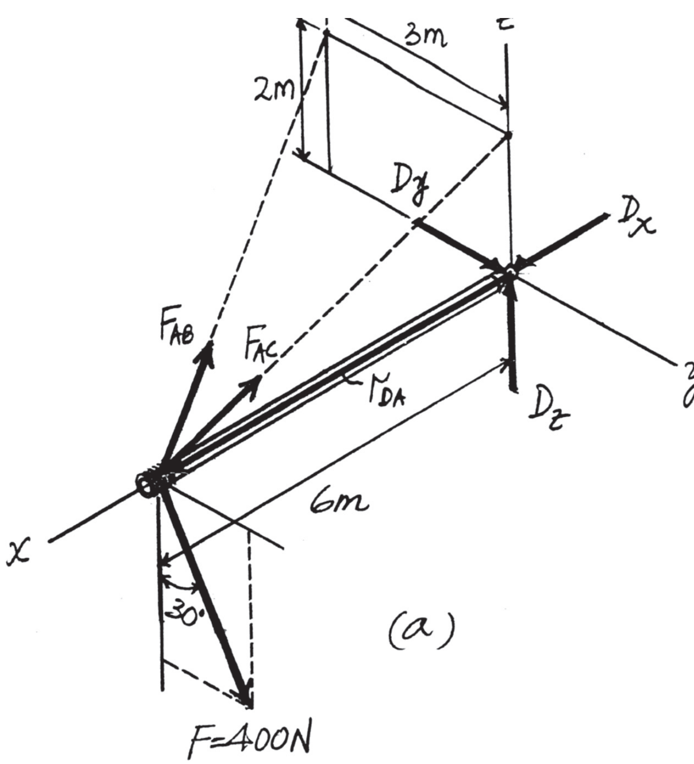
$$F_D = D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}$$

$$\mathbf{r}_{DA} = \{6\mathbf{i}\}\text{ m}$$

Referring to the *FBD* of the rod shown in Fig. *a*, the force equation of equilibrium gives

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F} + \mathbf{F}_D = 0$$

$$\left(-\frac{6}{7}F_{AB} - \frac{6}{\sqrt{40}}F_{AC} + D_x\right)\mathbf{i} + \left(-\frac{3}{7}F_{AB} + D_y + 200\right)\mathbf{j}$$
$$+\left(\frac{2}{7}F_{AB} + \frac{2}{\sqrt{40}}F_{AC} + D_z - 346.41\right)\mathbf{k} = 0$$



(a)

5–64. Continued

Equating **i**, **j** and **k** components,

$$-\frac{6}{7}F_{AB} - \frac{6}{\sqrt{40}}F_{AC} + D_x = 0 \qquad \textbf{(1)}$$

$$-\frac{3}{7}F_{AB} + D_y + 200 = 0 \qquad \textbf{(2)}$$

$$\frac{2}{7}F_{AB} + \frac{2}{\sqrt{40}}F_{AC} + D_z - 346.41 = 0 \qquad \textbf{(3)}$$

Moment equation of equilibrium gives

$\Sigma\mathbf{M}_D = 0; \qquad \mathbf{r}_{DA} \times (\mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}) = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 0 & 0 \\ \left(-\frac{6}{7}F_{AB} - \frac{6}{\sqrt{40}}F_{AC}\right) & \left(-\frac{3}{7}F_{AB} + 200\right) & \left(\frac{2}{7}F_{AB} + \frac{2}{\sqrt{40}}F_{AC} - 346.41\right) \end{vmatrix} = 0$$

$$-6\left(\frac{2}{7}F_{AB} + \frac{2}{\sqrt{40}}F_{AC} - 346.41\right)\mathbf{j} + 6\left(-\frac{3}{7}F_{AB} + 200\right)\mathbf{k} = 0$$

Equating **j** and **k** Components,

$$-6\left(\frac{2}{7}F_{AB} + \frac{2}{\sqrt{40}}F_{AC} - 346.41\right) = 0 \qquad \textbf{(4)}$$

$$6\left(-\frac{3}{7}F_{AB} + 200\right) = 0 \qquad \textbf{(5)}$$

Solving Eqs. (1) to (5)

$F_{AB} = 466.67\text{ N} = 467\text{ N}$ **Ans.**

$F_{AC} = 673.81\text{ N} = 674\text{ N}$ **Ans.**

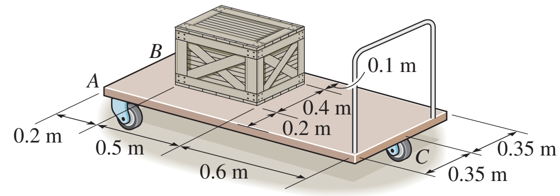
$D_x = 1039.23\text{ N} = 1.04\text{ kN}$ **Ans.**

$D_y = 0$ **Ans.**

$D_z = 0$ **Ans.**

Ans:
$F_{AB} = 467\text{ N}$
$F_{AC} = 674\text{ N}$
$D_x = 1.04\text{ kN}$
$D_y = 0$
$D_z = 0$

5–65. The cart supports the uniform crate having a mass of 85 kg. Determine the vertical reactions on the three casters at A, B, and C. The caster at B is not shown. Neglect the mass of the cart.



SOLUTION

Equations of Equilibrium: The normal reaction N_C can be obtained directly by Sigming moments about x axis.

$\Sigma M_x = 0;\quad N_C(1.3) - 833.85(0.45) = 0$

$N_C = 288.64\text{ N} = 289\text{ N}$ **Ans.**

$\Sigma M_y = 0;\quad 833.85(0.3) - 288.64(0.35) - N_A(0.7) = 0$

$N_A = 213.04\text{ N} = 213\text{ N}$ **Ans.**

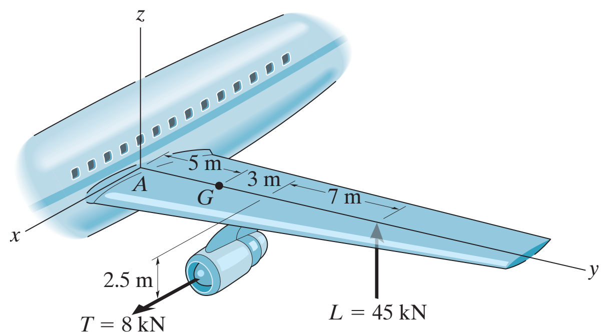
$\Sigma F_z = 0;\quad N_B + 288.64 + 213.04 - 833.85 = 0$

$N_B = 332\text{ N}$ **Ans.**

Ans:
$N_C = 289\text{ N}$
$N_A = 213\text{ N}$
$N_B = 332\text{ N}$

5–66.

The wing of the jet aircraft is subjected to a thrust of $T = 8$ kN from its engine and the resultant lift force $L = 45$ kN. If the mass of the wing is 2.1 Mg and the mass center is at G, determine the x, y, z components of reaction where the wing is fixed to the fuselage at A.



SOLUTION

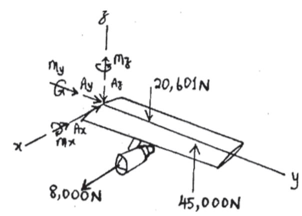
$\Sigma F_x = 0; \qquad -A_x + 8000 = 0$

$A_x = 8.00$ kN **Ans.**

$\Sigma F_y = 0; \qquad A_y = 0$ **Ans.**

$\Sigma F_z = 0; \qquad -A_z - 20\,601 + 45\,000 = 0$

$A_z = 24.4$ kN **Ans.**



$\Sigma M_y = 0; \qquad M_y - 2.5(8000) = 0$

$M_y = 20.0$ kN · m **Ans.**

$\Sigma M_x = 0; \qquad 45\,000(15) - 20\,601(5) - M_x = 0$

$M_x = 572$ kN · m **Ans.**

$\Sigma M_z = 0; \qquad M_z - 8000(8) = 0$

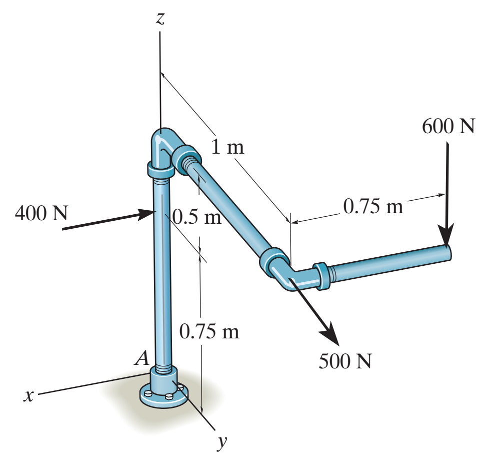
$M_z = 64.0$ kN · m **Ans.**

Ans:

$A_x = 8.00$ kN
$A_y = 0$
$A_z = 24.4$ kN
$M_y = 20.0$ kN · m
$M_x = 572$ kN · m
$M_z = 64.0$ kN · m

5–67.

Determine the components of reaction at the fixed support A. The 400 N, 500 N, and 600 N forces are parallel to the x, y, and z axes, respectively.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the rod shown in Fig. *a*

$\Sigma F_x = 0; \quad A_x - 400 = 0 \quad A_x = 400\text{ N}$ **Ans.**

$\Sigma F_y = 0; \quad 500 - A_y = 0 \quad A_y = 500\text{ N}$ **Ans.**

$\Sigma F_z = 0; \quad A_z - 600 = 0 \quad A_z = 600\text{ N}$ **Ans.**

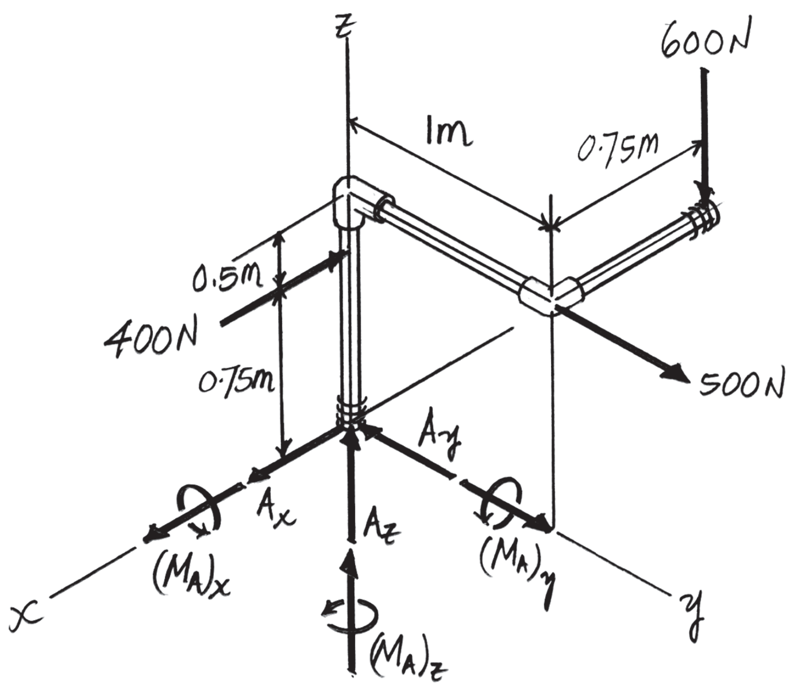
$\Sigma M_x = 0; \quad (M_A)_x - 500(1.25) - 600(1) = 0$

$(M_A)_x = 1225\text{ N}\cdot\text{m} = 1.225\text{ kN}\cdot\text{m}$ **Ans.**

$\Sigma M_y = 0; \quad (M_A)_y - 400(0.75) - 600(0.75) = 0$

$(M_A)_y = 750\text{ N}\cdot\text{m}$ **Ans.**

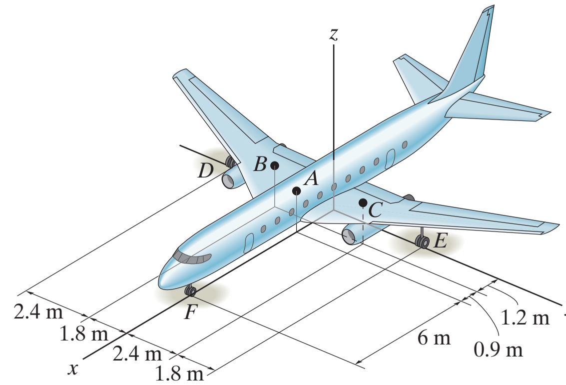
$\Sigma M_z = 0; \quad (M_A)_z = 0$ **Ans.**



(a)

Ans:
$A_x = 400\text{ N}$
$A_y = 500\text{ N}$
$A_z = 600\text{ N}$
$(M_A)_x = 1.225\text{ kN}\cdot\text{m}$
$(M_A)_y = 750\text{ N}\cdot\text{m}$
$(M_A)_z = 0$

***5–68.** Due to an unequal distribution of fuel in the wing tanks, the centers of gravity for the airplane fuselage A and wings B and C are located as shown. If these components have weights $W_A = 225$ kN, $W_B = 40$ kN, and $W_C = 30$ kN, determine the normal reactions of the wheels D, E, and F on the ground.



SOLUTION

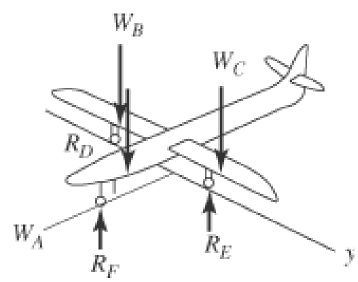
Given:

$W_A = 225$ kN

$W_B = 40$ kN

$W_C = 30$ kN

$a = 2.4$ m $e = 6$ m
$b = 1.8$ m $f = 1.2$ m
$c = 2.4$ m $g = 0.9$ m
$d = 1.8$ m



$\Sigma M_x = 0;\quad W_B \cdot b - R_D \cdot (a + b) - W_C \cdot c + R_E \cdot (c + d) = 0$
$\Sigma M_y = 0;\quad W_B \cdot f + W_A \cdot (g + f) + W_C f - R_F \cdot (e + g + f) = 0$
$\Sigma F_z = 0;\quad R_D + R_E + R_F - W_A - W_B - W_C = 0$

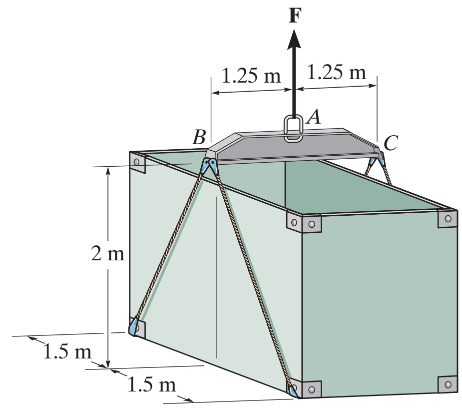
$$\begin{pmatrix} R_D \\ R_E \\ R_F \end{pmatrix} = \text{Find}(R_D, R_E, R_F) \quad \begin{pmatrix} R_D \\ R_E \\ R_F \end{pmatrix} = \begin{pmatrix} 113.1 \\ 113.1 \\ 68.7 \end{pmatrix} \text{kN}$$

Ans.

Ans:
$R_D = 113.1$ kN
$R_E = 113.1$ kN
$R_F = 68.7$ kN

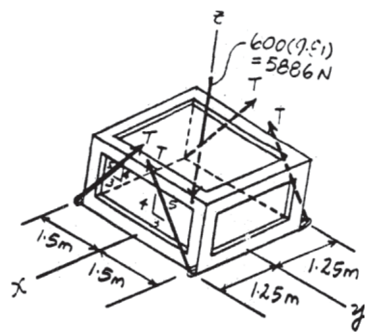
5–69.

The uniform load has a mass of 600 kg and is lifted using a uniform 30-kg strongback beam and four wire ropes as shown. Determine the tension in each segment of rope and the force that must be applied to the sling at A.



SOLUTION

Equations of Equilibrium: Due to symmetry, all wires are subjected to the same tension. This condition statisfies moment equilibrium about the x and y axes and force equilibrium along y axis.



$\Sigma F_z = 0;$ $$4T\left(\frac{4}{5}\right) - 5886 = 0$$

$$T = 1839.375\text{ N} = 1.84\text{ kN}$$ **Ans.**

The force **F** applied to the sling A must support the weight of the load and strongback beam. Hence

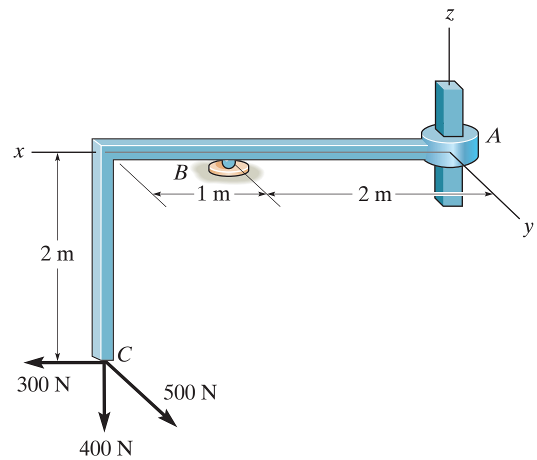
$\Sigma F_z = 0;$ $$F - 600(9.81) - 30(9.81) = 0$$

$$F = 6180.3\text{ N} = 6.18\text{ kN}$$ **Ans.**

Ans:
$T = 1.84$ kN
$F = 6.18$ kN

5–70.

The member is supported by a square rod which fits loosely through the smooth square hole of the attached collar at A and by a roller at B. Determine the components of reaction at these supports when the member is subjected to the loading shown.



SOLUTION

Force And Position Vectors. The coordinates of points B and C are $B(2,0,0)$ m and $C(3,0,-2)$ m.

$$\mathbf{F}_A = -A_x\,\mathbf{i} - A_y\,\mathbf{j}$$

$$F = \{300\mathbf{i} + 500\mathbf{j} - 400\mathbf{k}\}\text{ N}$$

$$\mathbf{N}_B = N_B\,\mathbf{k}$$

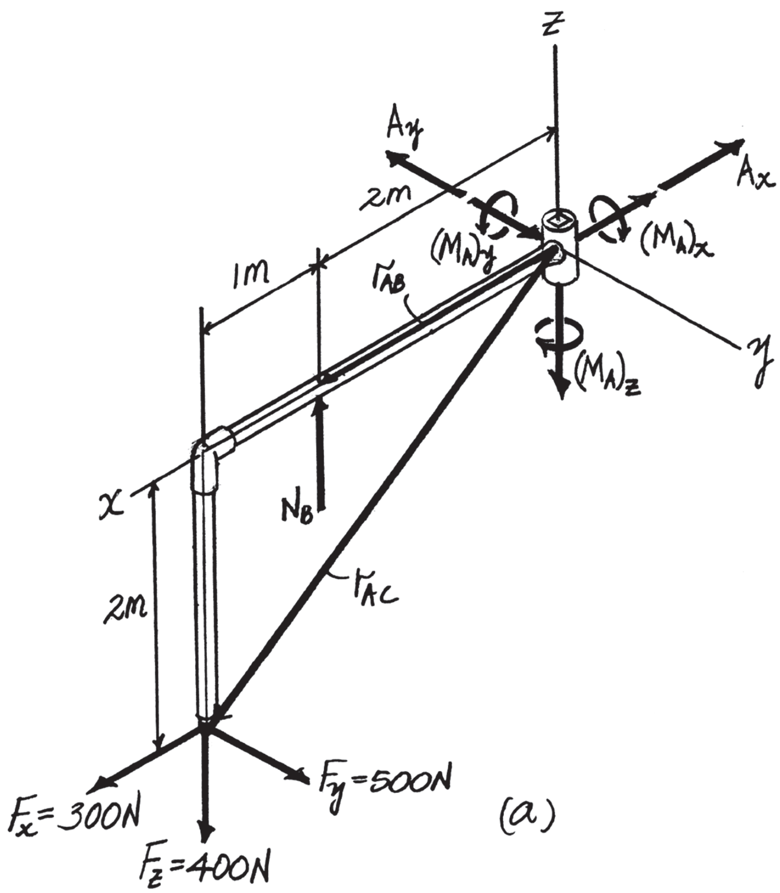
$$\mathbf{M}_A = -(M_A)_x\,\mathbf{i} + (M_A)_y\,\mathbf{j} - (M_A)_z\mathbf{k}$$

$$\mathbf{r}_{AB} = \{2\mathbf{i}\}\text{ m} \qquad \mathbf{r}_{AC} = \{3\mathbf{i} - 2\mathbf{k}\}\text{ m}$$

Equations of Equilibrium. Referring to the *FBD* of the member shown in Fig. *a*, the force equation of equilibrium gives

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_A + \mathbf{F} + \mathbf{N}_B = 0$$

$$(300 - A_x)\mathbf{i} + (500 - A_y)\mathbf{j} + (N_B - 400)\mathbf{k} = 0$$



5–70. Continued

Equating **i**, **j** and **k** components,

$300 - A_x = 0 \qquad A_x = 300\text{ N}$ **Ans.**

$500 - A_y = 0 \qquad A_y = 500\text{ N}$ **Ans.**

$N_B - 400 = 0 \qquad N_B = 400\text{ N}$ **Ans.**

The moment equation of equilibrium gives

$$\Sigma \mathbf{M}_A = 0; \qquad \mathbf{M}_A + \mathbf{r}_{AB} \times \mathbf{N}_B + \mathbf{r}_{AC} \times \mathbf{F} = 0$$

$$-(M_A)_x\mathbf{i} + (M_A)_y\mathbf{j} - (M_A)_z\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 0 \\ 0 & 0 & 400 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & -2 \\ 300 & 500 & -400 \end{vmatrix} = 0$$

$$[1000 - (M_A)_x]\mathbf{i} + [(M_A)_y - 200]\mathbf{j} + [1500 - (M_A)_z]\mathbf{k} = 0$$

Equating **i**, **j** and **k** components,

$1000 - (M_A)_x = 0 \qquad (M_A)_x = 1000\text{ N}\cdot\text{m} = 1.00\text{ kN}\cdot\text{m}$ **Ans.**

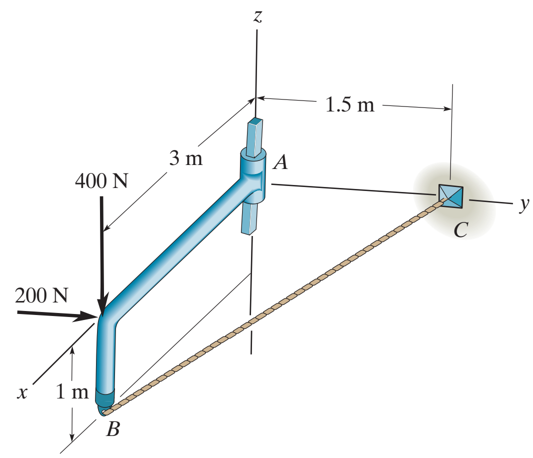
$(M_A)_y - 200 = 0 \qquad (M_A)_y = 200\text{ N}\cdot\text{m}$ **Ans.**

$1500 - (M_A)_z = 0 \qquad (M_A)_z = 1500\text{ N}\cdot\text{m} = 1.50\text{ kN}\cdot\text{m}$ **Ans.**

Ans:
$A_x = 300\text{ N}$
$A_y = 500\text{ N}$
$N_B = 400\text{ N}$
$(M_A)_x = 1.00\text{ kN}\cdot\text{m}$
$(M_A)_y = 200\text{ N}\cdot\text{m}$
$(M_A)_z = 1.50\text{ kN}\cdot\text{m}$

5–71.

Member AB is supported by a cable BC and at A by a *square* rod which fits loosely through the square hole in the collar fixed to the member as shown. Determine the components of reaction at A and the tension in the cable needed to hold the rod in equilibrium.



SOLUTION

Force And Position Vectors. The coordinates of points B and C are $B(3, 0, -1)$ m $C(0, 1.5, 0)$ m, respectively.

$$\mathbf{T}_{BC} = \mathbf{T}_{BC}\left(\frac{\mathbf{r}_{BC}}{\mathbf{r}_{BC}}\right) = T_{BC}\left\{\frac{(0-3)\mathbf{i} + (1.5-0)\mathbf{j} + [0-(-1)]\mathbf{k}}{\sqrt{(0-3)^2 + (1.5-0)^2 + [0-(-1)]^2}}\right\}$$

$$= -\frac{6}{7}T_{BC}\mathbf{i} + \frac{3}{7}T_{BC}\mathbf{j} + \frac{2}{7}T_{BC}\mathbf{k}$$

$$\mathbf{F} = \{200\mathbf{j} - 400\mathbf{k}\}\text{ N}$$

$$F_A = A_x\mathbf{i} + A_y\mathbf{j}$$

$$\mathbf{M}_A = (M_A)_x\mathbf{i} + (M_A)_y\mathbf{j} + (M_A)_z\mathbf{k}$$

$\mathbf{r}_1\{3\,\mathbf{i}\}$ m $\quad r_2 = \{1.5\,\mathbf{j}\}$ m

Equations of Equilibrium. Referring to the *FBD* of member AB shown in Fig. *a*, the force equation of equilibrium gives

$$\Sigma\mathbf{F} = 0; \quad \mathbf{T}_{BC} + \mathbf{F} + \mathbf{F}_A = 0$$

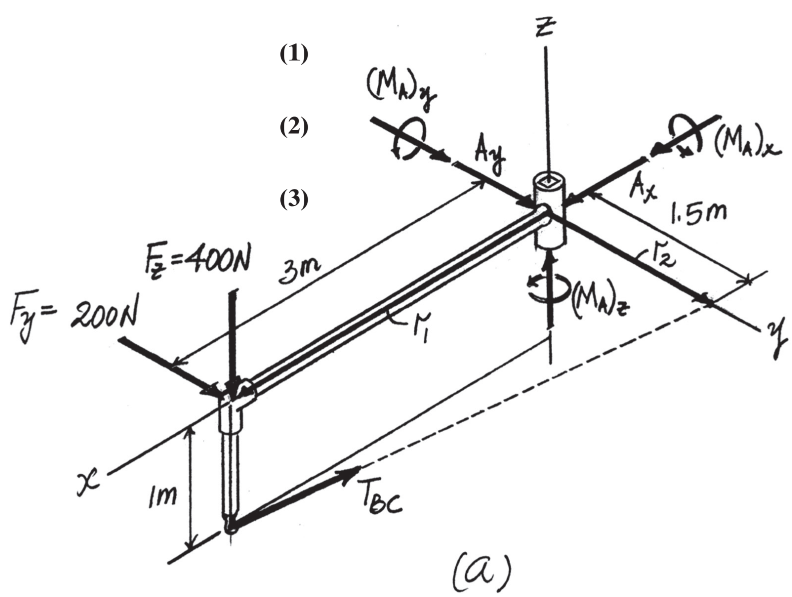
$$\left(-\frac{6}{7}T_{BC} + A_x\right)\mathbf{i} + \left(\frac{3}{7}T_{BC} + 200 + A_y\right)\mathbf{j} + \left(\frac{2}{7}T_{BC} - 400\right)\mathbf{k} = 0$$

Equating **i**, **j** and **k** components

$$-\frac{6}{7}T_{BC} + A_x = 0 \quad \textbf{(1)}$$

$$\frac{3}{7}T_{BC} + 200 + A_y = 0 \quad \textbf{(2)}$$

$$\frac{2}{7}T_{BC} - 400 = 0 \quad \textbf{(3)}$$



5–71. Continued

The moment equation of equilibrium gives

$\Sigma M_A = \mathbf{O}$; $\quad \mathbf{M}_A + \mathbf{r}_1 \times \mathbf{F} + r_2 \times \mathbf{T}_{BC} = 0$

$$(M_A)_x\mathbf{i} + (M_A)_y\mathbf{j} + (M_A)_z\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ 0 & 200 & -400 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.5 & 0 \\ -\frac{6}{7}T_{BC} & \frac{3}{7}T_{BC} & \frac{2}{7}T_{BC} \end{vmatrix} = 0$$

$$\left[(M_A)_x + \frac{3}{7}T_{BC}\right]\mathbf{i} + \left[(M_A)_y + 1200\right]\mathbf{j} + \left[(M_A)_z + \frac{9}{7}T_{BC} + 600\right]\mathbf{k} = 0$$

Equating **i**, **j**, and **k** components,

$$(M_A)_x + \frac{3}{7}T_{BC} = 0 \qquad \textbf{(4)}$$

$$(M_A)_y + 1200 = 0 \qquad \textbf{(5)}$$

$$(M_A)_z + \frac{9}{7}T_{BC} + 600 = 0 \qquad \textbf{(6)}$$

Solving Eqs. (1) to (6),

$T_{BC} = 1400\text{ N} = 1.40\text{ kN}$ **Ans.**

$A_y = 800\text{ N}$ **Ans.**

$A_x = 1200\text{ N} = 1.20\text{ kN}$ **Ans.**

$(M_A)_x = 600\text{ N}\cdot\text{m}$ **Ans.**

$(M_A)_y = -1200\text{ N}\cdot\text{m} = 1.20\text{ kN}\cdot\text{m}$ **Ans.**

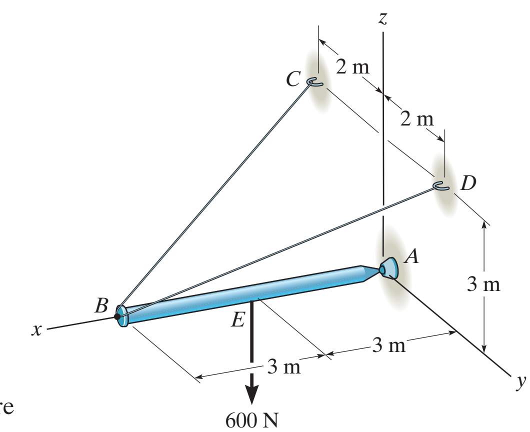
$(M_A)_z = -2400\text{ N}\cdot\text{m} = 2.40\text{ kN}\cdot\text{m}$ **Ans.**

The negative signs indicate that $\mathbf{A}_y$, $(\mathbf{M}_A)_x$, $(\mathbf{M}_A)_y$ and $(\mathbf{M}_A)_z$ are directed in sense opposite to those shown in *FBD*.

Ans:
$T_{BC} = 1.40\text{ kN}$
$A_y = 800\text{ N}$
$A_x = 1.20\text{ kN}$
$(M_A)_x = 600\text{ N}\cdot\text{m}$
$(M_A)_y = 1.20\text{ kN}\cdot\text{m}$
$(M_A)_z = 2.40\text{ kN}\cdot\text{m}$

***5–72.**

Determine the components of reaction at the ball-and-socket joint A and the tension in each cable necessary for equilibrium of the rod.



SOLUTION

Force And Position Vectors. The coordinates of points A, B, C, D and E are $A(0, 0, 0)$, $B(6, 0, 0)$, $C(0, -2, 3)$ m, $D(0, 2, 3)$ m and $E(3, 0, 0)$ m respectively.

$$\mathbf{F}_{BC} = F_{BC}\left(\frac{\mathbf{r}_{BC}}{r_{BC}}\right) = F_{BC}\left[\frac{(0-6)\mathbf{i} + (-2-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-6)^2 + (-2-0)^2 + (3-0)^2}}\right] = -\frac{6}{7}F_{BC}\mathbf{i} - \frac{2}{7}F_{BC}\mathbf{j} + \frac{3}{7}F_{BC}\mathbf{k}$$

$$\mathbf{F}_{BD} = F_{BD}\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) = F_{BD}\left[\frac{(0-6)\mathbf{i} + (2-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-6)^2 + (2-0)^2 + (3-0)^2}}\right] = -\frac{6}{7}F_{BD}\mathbf{i} + \frac{2}{7}F_{BD}\mathbf{j} + \frac{3}{7}F_{BD}\mathbf{k}$$

$$F_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

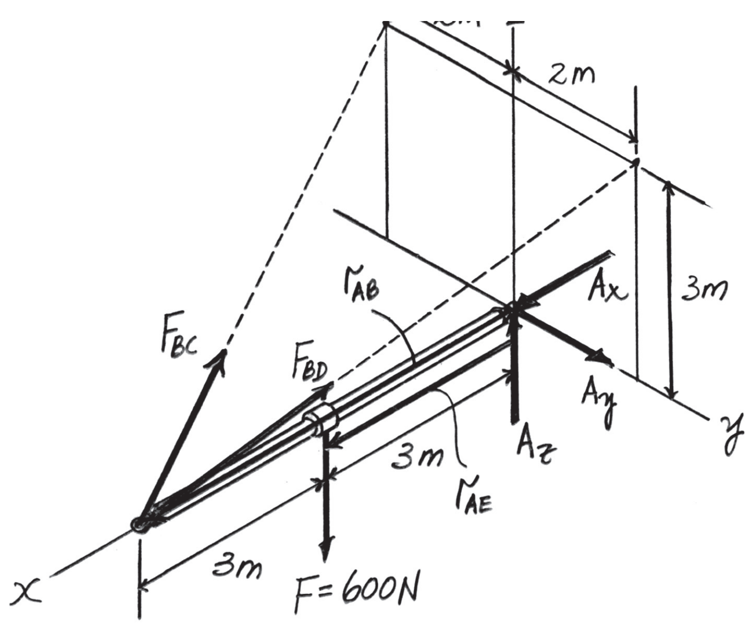
$$F = \{-600\mathbf{k}\}\text{ N}$$

$$r_{AB} = \{6\mathbf{i}\}\text{ m} \qquad r_{AE} = \{3\mathbf{i}\}\text{ m}$$

Equations of Equilibrium. Referring to the FBD of the rod shown in Fig. a, the force equation of equilibrium gives

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_{BC} + \mathbf{F}_{BD} + \mathbf{F}_A + \mathbf{F} = 0$$

$$\left(-\frac{6}{7}F_{BC} - \frac{6}{7}F_{BD} + A_x\right)\mathbf{i} + \left(\frac{2}{7}F_{BD} - \frac{2}{7}F_{BC} + A_y\right)\mathbf{j} + \left(\frac{3}{7}F_{BC} + \frac{3}{7}F_{BD} + A_z - 600\right)\mathbf{k} = 0$$



(a)

5–72. Continued

Equating **i**, **j** and **k** components,

$$-\frac{6}{7}F_{BC} - \frac{6}{7}F_{BD} + A_x = 0 \qquad \textbf{(1)}$$

$$\frac{2}{7}F_{BD} - \frac{2}{7}F_{BC} + A_y = 0 \qquad \textbf{(2)}$$

$$\frac{3}{7}F_{BC} + \frac{3}{7}F_{BD} + A_z - 600 = 0 \qquad \textbf{(3)}$$

The moment equation of equilibrium gives

$\Sigma \mathbf{M}_A = 0; \qquad \mathbf{r}_{AE} \times \mathbf{F} + \mathbf{r}_{AB} \times (\mathbf{F}_{BC} + \mathbf{F}_{BD}) = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ 0 & 0 & -600 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 0 & 0 \\ -\frac{6}{7}(F_{BC} + F_{BD}) & \frac{2}{7}(F_{BD} - F_{BC}) & \frac{3}{7}(F_{BC} + F_{BD}) \end{vmatrix} = 0$$

$$\left[1800 - \frac{18}{7}(F_{BC} + F_{BD})\right]\mathbf{j} + \frac{12}{7}(F_{BD} - F_{BC})\mathbf{k} = 0$$

Equating **j** and **k** components,

$$1800 - \frac{18}{7}(F_{BC} + F_{BD}) = 0 \qquad \textbf{(4)}$$

$$\frac{12}{7}(F_{BD} - F_{BC}) = 0 \qquad \textbf{(5)}$$

Solving Eqs. (1) to (5),

$F_{BD} = F_{BC} = 350 \text{ N}$ **Ans.**

$A_x = 600 \text{ N}$ **Ans.**

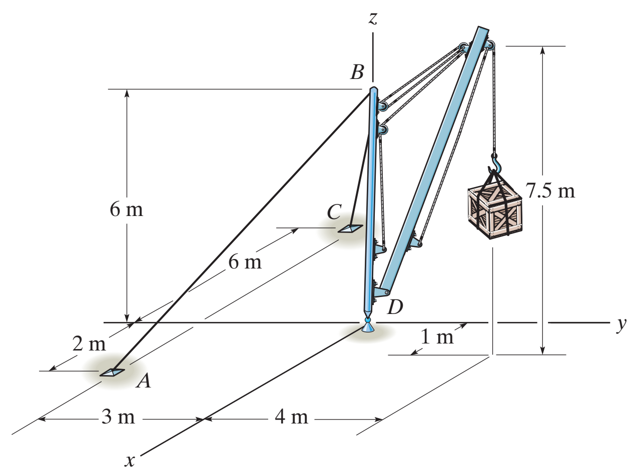
$A_y = 0$ **Ans.**

$A_z = 300 \text{ N}$ **Ans.**

Ans:
$F_{BD} = F_{BC} = 350 \text{ N}$
$A_x = 600 \text{ N}$
$A_y = 0$
$A_z = 300 \text{ N}$

5–73.

The stiff-leg derrick used on ships is supported by a ball-and-socket joint at D and two cables BA and BC. The cables are attached to a smooth collar ring at B, which allows rotation of the derrick about z axis. If the derrick supports a crate having a mass of 200 kg, determine the tension in the cables and the x, y, z components of reaction at D.



SOLUTION

$$\Sigma F_x = 0; \qquad D_x + \frac{2}{7}T_{BA} - \frac{6}{9}T_{BC} = 0$$

$$\Sigma F_y = 0; \qquad D_y - \frac{3}{7}T_{BA} - \frac{3}{9}T_{BC} = 0$$

$$\Sigma F_z = 0; \qquad D_z - \frac{6}{7}T_{BA} - \frac{6}{9}T_{BC} - 200(9.81) = 0$$

$$\Sigma M_x = 0; \qquad \frac{3}{7}T_{BA}(6) + \frac{3}{9}T_{BC}(6) - 200(9.81)(4) = 0$$

$$\Sigma M_y = 0; \qquad \frac{2}{7}T_{BA}(6) - \frac{6}{9}T_{BC}(6) + 200(9.81)(1) = 0$$

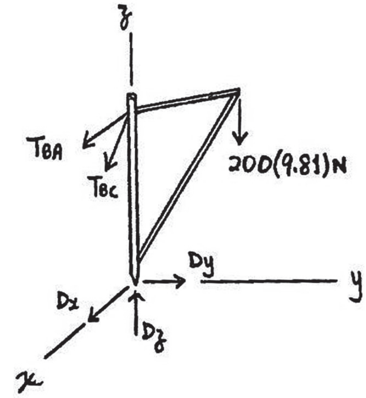
$T_{BA} = 2.00$ kN **Ans.**

$T_{BC} = 1.35$ kN **Ans.**

$D_x = 0.327$ kN **Ans.**

$D_y = 1.31$ kN **Ans.**

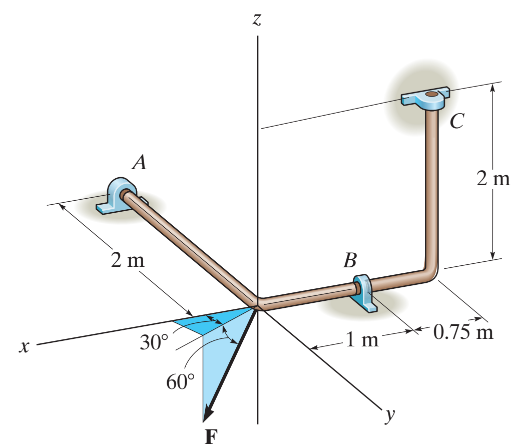
$D_z = 4.58$ kN **Ans.**



Ans:
$T_{BA} = 2.00$ kN
$T_{BC} = 1.35$ kN
$D_x = 0.327$ kN
$D_y = 1.31$ kN
$D_z = 4.58$ kN

5–74.

The bent rod is supported at A, B, and C by smooth journal bearings. Determine the components of reaction at the bearings if the rod is subjected to the force $F = 800$ N. The bearings are in proper alignment and exert only force reactions on the rod.



SOLUTION

Equations of Equilibrium. The x, y and z components of force F are

$F_x = 800 \cos 60° \cos 30° = 346.41$ N

$F_y = 800 \cos 60° \sin 30° = 200$ N

$F_z = 800 \sin 60° = 692.82$ N

Referring to the FBD of the bent rod shown in Fig. a,

$$\Sigma M_x = 0; \quad -C_y(2) + B_z(2) - 692.82\,(2) = 0 \tag{1}$$

$$\Sigma M_y = 0; \quad B_z(1) + C_x(2) = 0 \tag{2}$$

$$\Sigma M_z = 0; \quad -C_y(1.75) - C_x(2) - B_y(1) - 346.41(2) = 0 \tag{3}$$

$$\Sigma F_x = 0; \quad A_x + C_x + 346.41 = 0 \tag{4}$$

$$\Sigma F_y = 0; \quad 200 + B_y + C_y = 0 \tag{5}$$

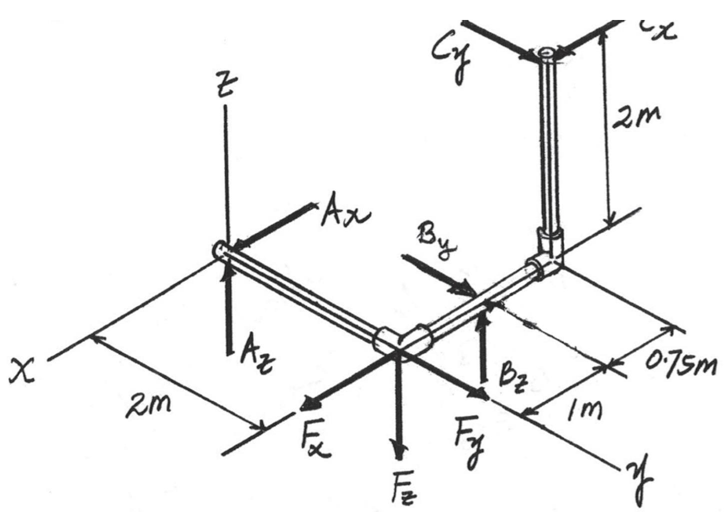
$$\Sigma F_z = 0; \quad A_z + B_z - 692.82 = 0 \tag{6}$$

Solving Eqs. (1) to (6)

$C_y = 800$ N $\quad B_z = -107.18$ N $= 107$ N $\quad B_y = 600$ N **Ans.**

$C_x = 53.59$ N $= 53.6$ N $\quad A_x = 400$ N $\quad A_z = 800$ N **Ans.**

The negative signs indicate that $\mathbf{C}_y$, $\mathbf{B}_z$ and $\mathbf{A}_z$ are directed in the senses opposite to those shown in FBD.

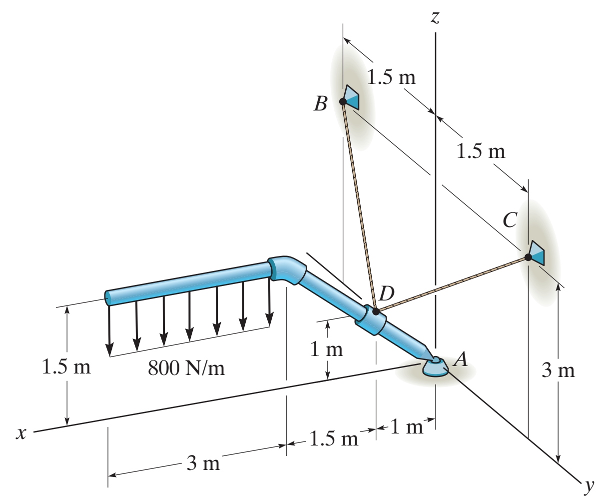


(a)

Ans:
$C_y = 800$ N, $B_z = 107$ N, $B_y = 600$ N,
$C_x = 53.6$ N, $A_x = 400$ N, $A_z = 800$ N

5–75.

Determine the components of reaction at the ball-and-socket joint A and the tension in the supporting cables DB and DC.



SOLUTION

Force And Position Vectors. The coordinates of points A, B, C, and D are $A(0, 0, 0)$, $B(0, -1.5, 3)$ m, $C(0, 1.5, 3)$ m and $D(1, 0, 1)$ m, respectively.

$$\mathbf{F}_{DC} = F_{DC}\left(\frac{\mathbf{r}_{DC}}{r_{DC}}\right) = F_{DC}\left[\frac{(0-1)\mathbf{i} + (1.5-0)\mathbf{j} + (3-1)\mathbf{k}}{\sqrt{(0-1)^2 + (1.5-0)^2 + (3-1)^2}}\right]$$

$$= -\frac{1}{\sqrt{7.25}}F_{CD}\mathbf{i} + \frac{1.5}{\sqrt{7.25}}F_{DC}\mathbf{j} + \frac{2}{\sqrt{7.25}}F_{DC}\mathbf{k}$$

$$\mathbf{F}_{DB} = F_{DB}\left(\frac{\mathbf{r}_{DB}}{r_{DB}}\right) = F_{DB}\left[\frac{(0-1)\mathbf{i} + (-1.5-0)\mathbf{j} + (3-1)\mathbf{k}}{\sqrt{(0-1)^2 + (-1.5-0)^2 + (3-1)^2}}\right]$$

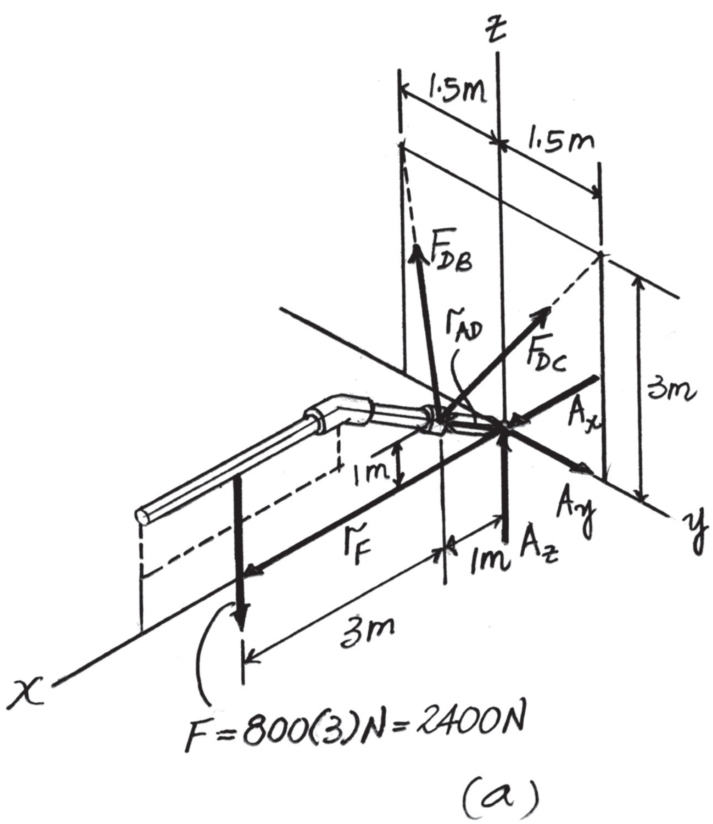
$$= -\frac{1}{\sqrt{7.25}}F_{DB}\mathbf{i} + \frac{1.5}{\sqrt{7.25}}F_{DB}\mathbf{j} + \frac{2}{\sqrt{7.25}}F_{DB}\mathbf{k}$$

$$F_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$F = \{-2400\mathbf{k}\}\text{ N}$$

$$\mathbf{r}_{AD} = (1-0)\mathbf{i} + (1-0)\mathbf{k} = \{\mathbf{i} + \mathbf{k}\}\text{ m}$$

$$\mathbf{r}_F = \{4\mathbf{i}\}\text{ m}$$



5–75. Continued

Equations of Equilibrium. Referring to the *FBD* of the assembly shown in Fig. *a*. Force equation of equilibrium gives

$\Sigma \mathbf{F} = 0; \qquad \mathbf{F}_{DC} + \mathbf{F}_{DB} + \mathbf{F}_A + \mathbf{F} = 0$

$$\left(-\frac{1}{\sqrt{7.25}} F_{DC} - \frac{1}{\sqrt{7.25}} F_{DB} + A_x\right)\mathbf{i} + \left(\frac{1.5}{\sqrt{7.25}} F_{DC} - \frac{1.5}{\sqrt{7.25}} F_{DB} + A_y\right)\mathbf{j}$$

$$+ \left(\frac{2}{\sqrt{7.25}} F_{DC} + \frac{2}{\sqrt{7.25}} F_{DB} + A_z - 2400\right)\mathbf{k} = 0$$

Equating **i**, **j** and **k** components,

$$-\frac{1}{\sqrt{7.25}} F_{DC} - \frac{1}{\sqrt{7.25}} F_{DB} + A_x = 0 \qquad \textbf{(1)}$$

$$\frac{1.5}{\sqrt{7.25}} F_{DC} - \frac{1.5}{7.25} F_{DB} + A_y = 0 \qquad \textbf{(2)}$$

$$\frac{2}{\sqrt{7.25}} F_{DC} + \frac{2}{\sqrt{7.25}} F_{DB} + A_z - 2400 = 0 \qquad \textbf{(3)}$$

Moment equation of equilibrium gives

$\Sigma M_A = 0; \qquad \mathbf{r}_F \times \mathbf{F} + \mathbf{r}_{AD} \times (\mathbf{F}_{DB} + \mathbf{F}_{DC}) = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 0 \\ 0 & 0 & -2400 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ -\frac{1}{\sqrt{7.25}}(F_{DB} + F_{DC}) & \frac{1.5}{\sqrt{7.25}}(F_{DC} - F_{DB}) & \frac{2}{\sqrt{7.25}}(F_{DC} + F_{DB}) \end{vmatrix} = 0$$

$$-\frac{1.5}{\sqrt{7.25}} (F_{DC} - F_{DB})\mathbf{i} + \left[9600 - \frac{3}{\sqrt{7.25}} (F_{DC} + F_{DB})\right]\mathbf{j} + \frac{1.5}{\sqrt{7.25}} (F_{DC} + F_{DB})\mathbf{k} = 0$$

Equating **i**, **j** and **k** Components

$$-\frac{1.5}{\sqrt{7.25}} (F_{DC} - F_{DB}) = 0 \qquad \textbf{(4)}$$

$$9600 - \frac{3}{\sqrt{7.25}} (F_{DC} + F_{DB}) = 0 \qquad \textbf{(5)}$$

$$\frac{1.5}{\sqrt{7.25}} (F_{DC} - F_{DB}) = 0 \qquad \textbf{(6)}$$

Solving Eqs. (1) to (6)

$F_{DC} = F_{DB} = 4308.13 \text{ N} = 4.31 \text{ kN}$ **Ans.**

$A_x = 3200 \text{ N} = 3.20 \text{ kN}$ **Ans.**

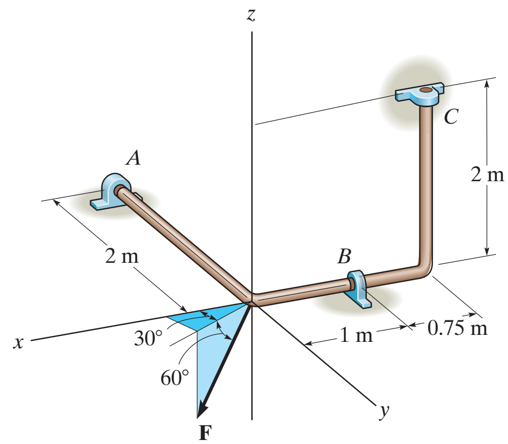
$A_y = 0$ **Ans.**

$A_z = -4000 \text{ N} = -4 \text{ kN}$ **Ans.**

Negative sign indicates that A_z directed in the sense opposite to that shown in *FBD*.

Ans:
$F_{DC} = F_{DB} = 4.31 \text{ kN}$
$A_x = 3.20 \text{ kN}$
$A_y = 0$
$A_z = -4 \text{ kN}$

***5–76.** The bent rod is supported at A, B, and C by smooth journal bearings. Determine the magnitude of **F** which will cause the positive x component of reaction at the bearing C to be $C_x = 50$ N. The bearings are in proper alignment and exert only force reactions on the rod.



SOLUTION

Equations of Equilibrium. The x, y and z components of force **F** are

$$F_x = F\cos 60^\circ \cos 30^\circ = 0.4330\,F$$

$$F_y = F\cos 60^\circ \sin 30^\circ = 0.25\,F$$

$$F_z = F\sin 60^\circ = 0.8660\,F$$

Here, it is required that $C_x = 50$. Thus, by referring to the *FBD* of the beat rod shown in Fig. *a*,

$\Sigma M_x = 0;\quad -C_y(2) + B_z(2) - 0.8660\,F(2) = 0$ **(1)**

$\Sigma M_y = 0;\quad B_z(1) + 50(2) = 0$ **(2)**

$\Sigma M_z = 0;\quad -C_y(1.75) - 50(2) - B_y(1) - 0.4330\,F(2) = 0$ **(3)**

$\Sigma F_y = 0;\quad 0.25\,F + B_y + C_y = 0$ **(4)**

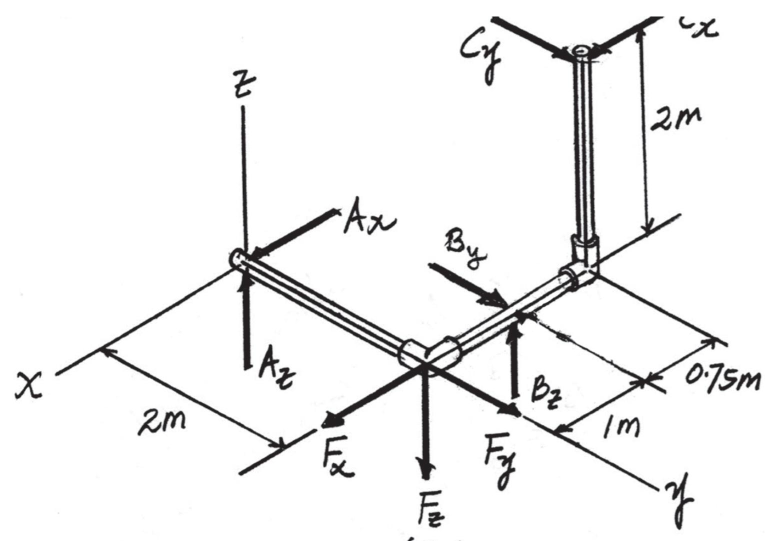
Solving Eqs. (1) to (4)

$F = 746.41\text{ N} = 746\text{ N}$ **Ans.**

$C_y = -746.41$ N

$B_z = -100$ N

$B_y = 559.81$ N

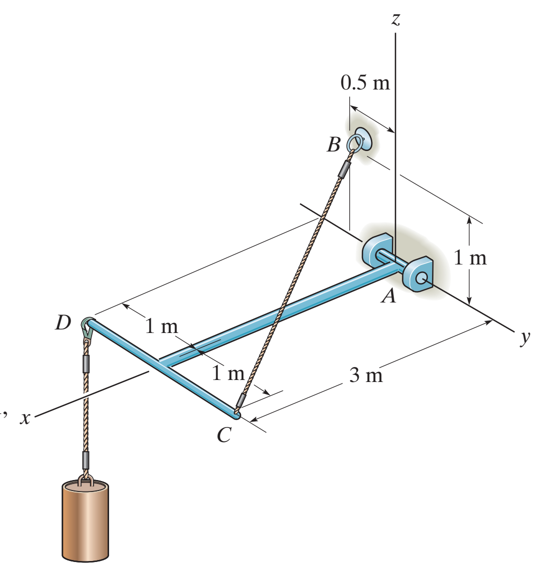


(a)

Ans:
$F = 746$ N

5–77.

The member is supported by a pin at A and cable BC. Determine the components of reaction at these supports if the cylinder has a mass of 40 kg.



SOLUTION

Force And Position Vectors. The coordinates of points B, C and D are $B(0, -0.5, 1)$ m, $C(3, 1, 0)$ m and $D(3, -1, 0)$ m, respectively.

$$\mathbf{F}_{CB} = F_{CB}\left(\frac{\mathbf{r}_{CB}}{r_{CB}}\right) = F_{CB}\left[\frac{(0-3)\mathbf{i} + (-0.5-1)\mathbf{j} + (1-0)\mathbf{k}}{\sqrt{(0-3)^2 + (-0.5-1)^2 + (1-0)^2}}\right]$$

$$= -\frac{6}{7}F_{CB}\mathbf{i} - \frac{3}{7}F_{CB}\mathbf{j} + \frac{2}{7}F_{CB}\mathbf{k}$$

$$\mathbf{W} = \{-40(9.81)\mathbf{k}\}\text{ N} = \{-392.4\mathbf{k}\}\text{ N}.$$

$$F_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{M}_A = (M_A)_x\mathbf{i} + (M_A)_z\mathbf{k}$$

$$\mathbf{r}_{AC} = \{3\mathbf{i} + \mathbf{j}\}\text{ m} \quad \mathbf{r}_{AD} = \{3\mathbf{i} - \mathbf{j}\}\text{ m}$$

Equations of Equilibrium. Referring to the FBD of the assembly shown in Fig. a. the force equation of equilibrium gives

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_{CB} + \mathbf{W} + \mathbf{F}_A = 0;$$

$$\left(-\frac{6}{7}F_{CB} + A_x\right)\mathbf{i} + \left(-\frac{3}{7}F_{CB} + A_y\right)\mathbf{j} + \left(\frac{2}{7}F_{CB} + A_z - 392.4\right)\mathbf{k} = 0$$

Equating **i**, **j** and **k** components

$$-\frac{6}{7}F_{CB} + A_x = 0 \qquad \textbf{(1)}$$

$$-\frac{3}{7}F_{CB} + A_y = 0 \qquad \textbf{(2)}$$

$$\frac{2}{7}F_{CB} + A_z - 392.4 = 0 \qquad \textbf{(3)}$$

The moment equation of equilibrium gives

$$\Sigma\mathbf{M}_A = 0; \qquad \mathbf{r}_{AC} \times F_{CB} + \mathbf{r}_{AD} \times \mathbf{W} + \mathbf{M}_A = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & 0 \\ -\frac{6}{7}F_{CB} & -\frac{3}{7}F_{CB} & \frac{2}{7}F_{CB} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 0 \\ 0 & 0 & -392.4 \end{vmatrix} + (M_A)_x\mathbf{i} + (M_A)_Z\mathbf{k} = 0$$

$$\left[\frac{2}{7}F_{CB} + 392.4 + (M_A)_x\right]\mathbf{i} + \left(-\frac{6}{7}\mathbf{F_{CB}} + \mathbf{1177.2}\right)\mathbf{j} + \left[-\frac{9}{7}F_{CB} + \frac{6}{7}F_{CB} + (M_A)_z\right]\mathbf{k} = 0$$

5–77. Continued

Equating **i**, **j** and **k** components,

$$\frac{2}{7}F_{CB} + 392.4 + (M_A)_x = 0 \qquad \textbf{(4)}$$

$$-\frac{6}{7}F_{CB} + 1177.2 = 0 \qquad \textbf{(5)}$$

$$-\frac{9}{7}F_{CB} + \frac{6}{7}F_{CB} + (M_A)_z = 0 \qquad \textbf{(6)}$$

Solving Eqs (1) to (6),

$F_{CB} = 1373.4 \text{ N} = 1.37 \text{ kN}$ **Ans.**

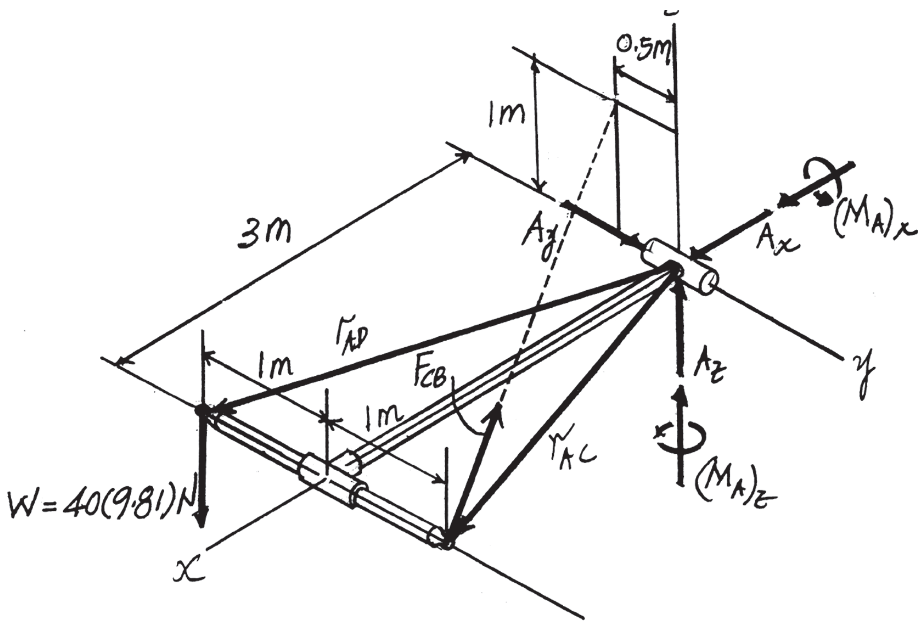
$(M_A)_x = -784.8 \text{ N}\cdot\text{m} = 785 \text{ N}\cdot\text{m}$ **Ans.**

$(M_A)_z = 588.6 \text{ N}\cdot\text{m} = 589 \text{ N}\cdot\text{m}$ **Ans.**

$A_x = 1177.2 \text{ N} = 1.18 \text{ kN}$ **Ans.**

$A_y = 588.6 \text{ N} = 589 \text{ N}$ **Ans.**

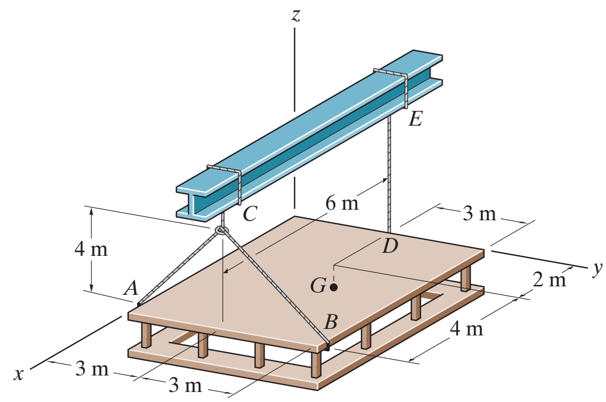
$A_z = 0$ **Ans.**



(a)

Ans:
$F_{CB} = 1.37$ kN
$(M_A)_x = 785 \text{ N}\cdot\text{m}$
$(M_A)_z = 589 \text{ N}\cdot\text{m}$
$A_x = 1.18$ kN
$A_y = 589$ N
$A_z = 0$

5–78. The platform has mass of 3 Mg and center of mass located at G. If it is lifted with constant velocity using the three cables, determine the force in each of these cables.



SOLUTION

Given:

$M = 3\ \text{Mg}$

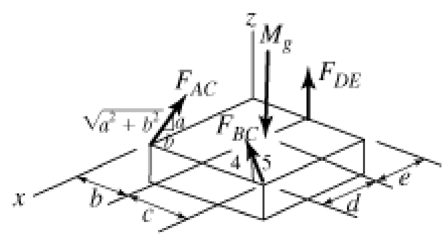
$a = 4\ \text{m}$

$b = 3\ \text{m}$

$c = 3\ \text{m}$

$d = 4\ \text{m}$

$e = 2\ \text{m}$



$$\frac{b \cdot F_{AC}}{\sqrt{a^2 + b^2}} - \frac{c \cdot F_{BC}}{\sqrt{a^2 + c^2}} = 0$$

$$M \cdot g \cdot e - F_{AC} \cdot a \cdot \frac{d + e}{\sqrt{a^2 + b^2}} - F_{BC} \cdot \frac{a \cdot (d + r)}{\sqrt{a^2 + c^2}} = 0$$

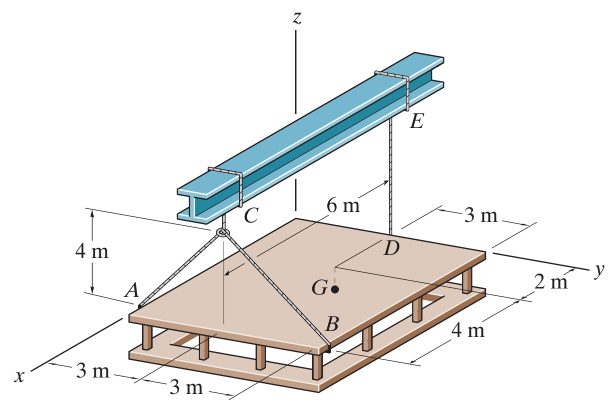
$$\frac{a}{\sqrt{a^2 + c^2}} \cdot F_{BC}(b + c) - M \cdot g \cdot b + F_{DE} \cdot b = 0$$

$$\begin{pmatrix} F_{AC} \\ F_{BC} \\ F_{DE} \end{pmatrix} = \text{Find}(F_{AC}, F_{BC}, F_{DE}) \quad \begin{pmatrix} F_{AC} \\ F_{BC} \\ F_{DE} \end{pmatrix} = \begin{pmatrix} 6.13 \\ 6.13 \\ 19.62 \end{pmatrix} \text{kN}$$

Ans.

Ans:
$F_{AC} = 6.13$ kN
$F_{BC} = 6.13$ kN
$F_{DE} = 19.62$ kN

5–79. The platform has a mass of 2 Mg and center of mass located at G. If it is lifted using the three cables, determine the force in each of the cables. Solve for each force by using a single moment equation of equilibrium.



SOLUTION

Given:

$M = 2\ \text{Mg}$ $\quad c = 3\ \text{m}$

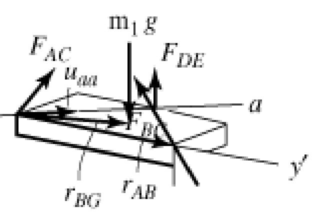
$a = 4\ \text{m}$ $\quad d = 4\ \text{m}$

$b = 3\ \text{m}$ $\quad e = 2\ \text{m}$

$$r_{BC} = \begin{pmatrix} 0 \\ -c \\ a \end{pmatrix} \qquad r_{AC} = \begin{pmatrix} 0 \\ b \\ a \end{pmatrix}$$

$$r_{AD} = \begin{pmatrix} -e-d \\ b \\ 0 \end{pmatrix} \qquad r_{BD} = \begin{pmatrix} -d-e \\ -c \\ 0 \end{pmatrix}$$

First find F_{DE}.



$$\Sigma M'_y = 0; \quad F_{DE} \cdot (d+e) - M \cdot g \cdot d = 0 \quad F_{DE} = \frac{M \cdot g \cdot d}{d+e} \quad F_{DE} = 13.1\ \text{kN}$$

Next find F_{BC}.

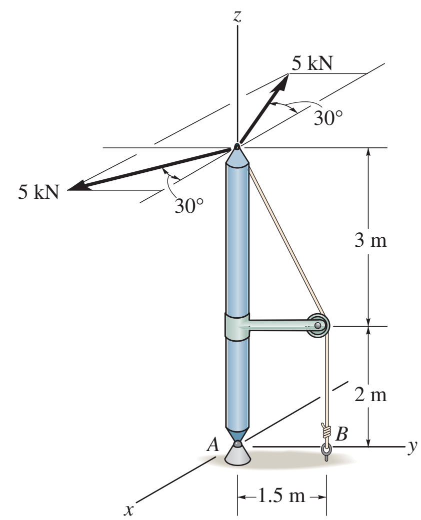
Guess $F_{BC} := 1\text{kN}$

$$\textbf{Given} \left[\begin{pmatrix} e \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -M \cdot g \end{pmatrix} \times \begin{pmatrix} e+d \\ c \\ 0 \end{pmatrix} \times \left(F_{BC} \cdot \frac{r_{BC}}{|r_{BC}|} \right) \right] \cdot r_{AD} = 0 \quad F_{BC} = \text{Find}(F_{BC})$$

$F_{BC} = 4.09\ \text{kN}$ **Ans.**

Ans:
$F_{BC} = 4.09\ \text{kN}$

***5–80.** The boom is supported by a ball-and-socket joint at A and a guy wire at B. If the 5-kN loads lie in a plane which is parallel to the x–y plane, determine the x, y, z components of reaction at A and the tension in the cable at B.



SOLUTION

Equations of Equilibrium:

$\Sigma M_x = 0;\quad 2[5 \sin 30°(5)] - T_B(1.5) = 0$

$T_B = 16.67 \text{ kN} = 16.7 \text{ kN}$ **Ans.**

$\Sigma M_y = 0;\quad 5 \cos 30°(5) - 5 \cos 30°(5) = 0$ (*Statisfied*!)

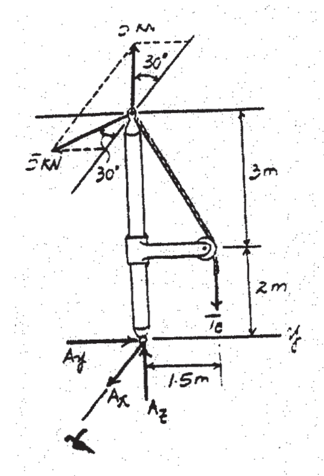
$\Sigma F_x = 0;\quad A_x + 5 \cos 30° - 5 \cos 30° = 0$

$A_x = 0$ **Ans.**

$\Sigma F_y = 0;\quad A_y - 2(5 \sin 30°) = 0$

$A_y = 5.00 \text{ kN}$ **Ans.**

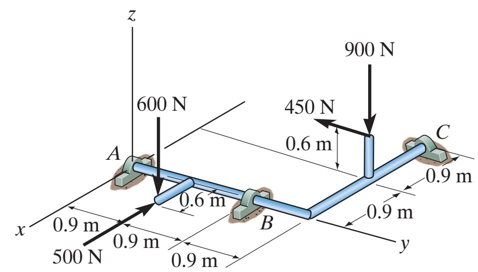
$\Sigma F_z = 0;\quad A_z - 16.67 = 0 \quad A_z = 16.7 \text{ kN}$ **Ans.**



Ans:

$T_B = 16.7 \text{ kN}$
$A_x = 0$
$A_z = 16.7 \text{ kN}$

5–81. The shaft is supported by three smooth journal bearings at A, B, and C. Determine the components of reaction at these bearings.



SOLUTION

Equations of Equilibrium: From the free-body diagram, Fig. a, C_y and C_z, can be obtained by writing the force equation of equilibrium along the y axis and the moment equation of equilibrium about the y axis.

$\Sigma F_y = 0;\quad C_y - 450 = 0$

$$C_y = 450 \text{ N} \qquad \textbf{Ans.}$$

$\Sigma M_y = 0;\quad C_z(0.9 + 0.9) - 900(0.9) + 600(0.6) = 0$

$$C_z = 250 \text{ N} \qquad \textbf{Ans.}$$

Using the above results

$\Sigma M_x = 0;\quad B_z(0.9 + 0.9) + 250(0.9 + 0.9 + 0.9) + 450(0.6) - 900(0.9 + 0.9 + 0.9) - 600(0.9) = 0$

$$B_z = 1125 \text{ N} = 1.125 \text{ kN} \qquad \textbf{Ans.}$$

$\Sigma M_{x'} = 0;\quad 600(0.9) + 450(0.6) - 900(0.9) + 250(0.9) - A_z(0.9 + 0.9) = 0$

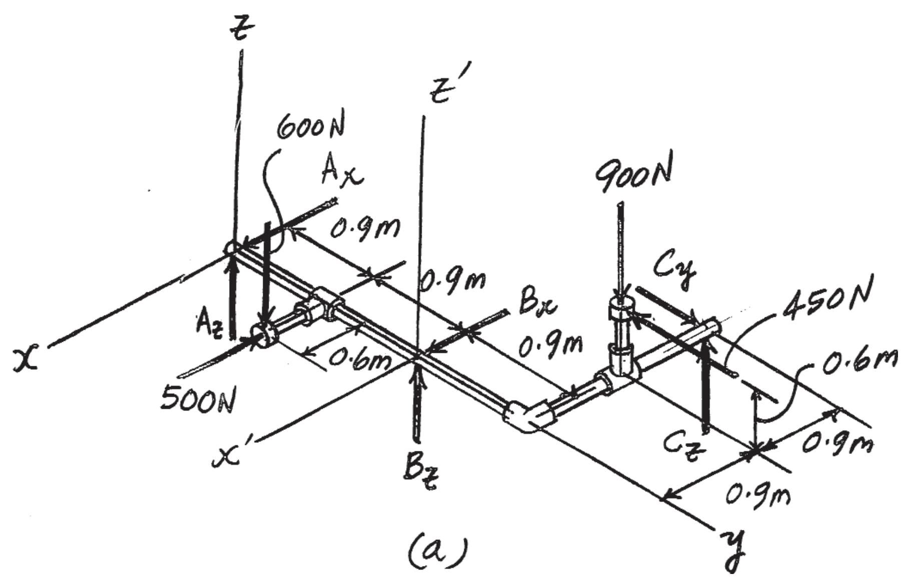
$$A_z = 125 \text{ N} \qquad \textbf{Ans.}$$

$\Sigma M_z = 0;\quad -B_x(0.9 + 0.9) + 500(0.9) + 450(0.9) - 450(0.9 + 0.9) = 0$

$$B_x = 25 \text{ N} \qquad \textbf{Ans.}$$

$\Sigma F_x = 0;\quad A_x + 25 - 500 = 0$

$$A_x = 475 \text{ N} \qquad \textbf{Ans.}$$

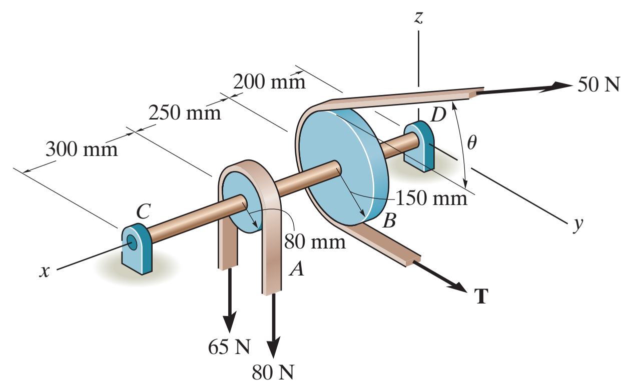


(a)

Ans:
$C_y = 450$ N, $C_z = 250$ N, $B_z = 1.125$ kN, $A_z = 125$ N, $B_x = 25$ N, $A_x = 475$ N

5–82.

Both pulleys are fixed to the shaft and as the shaft turns with constant angular velocity, the power of pulley A is transmitted to pulley B. Determine the horizontal tension $\mathbf{T}$ in the belt on pulley B and the x, y, z components of reaction at the journal bearing C and thrust bearing D if $\theta = 45°$. The bearings are in proper alignment and exert only force reactions on the shaft.



SOLUTION

Equations of Equilibrium:

$\Sigma M_x = 0;\quad 65(0.08) - 80(0.08) + T(0.15) - 50(0.15) = 0$

$T = 58.0\text{ N}$ **Ans.**

$\Sigma M_y = 0;\quad (65 + 80)(0.45) - 50 \sin 45°(0.2) - C_z(0.75) = 0$

$C_z = 77.57\text{ N} = 77.6\text{ N}$ **Ans.**

$\Sigma M_z = 0;\quad 58.0(0.2) + 50 \cos 45°(0.2) - C_y(0.75) = 0$

$C_y = 24.89\text{ N} = 24.9\text{ N}$ **Ans.**

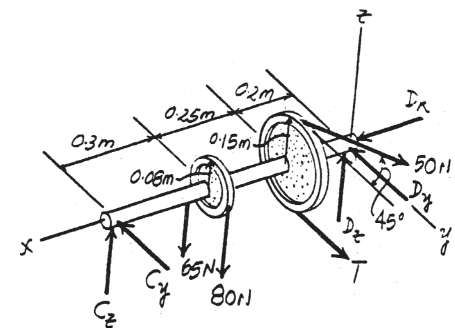
$\Sigma F_x = 0;\quad D_x = 0$ **Ans.**

$\Sigma F_y = 0;\quad D_y + 24.89 - 50 \cos 45° - 58.0 = 0$

$D_y = 68.5\text{ N}$ **Ans.**

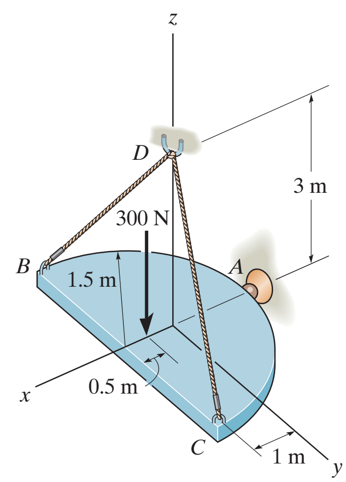
$\Sigma F_z = 0;\quad D_z + 77.57 + 50 \sin 45° - 80 - 65 = 0$

$D_z = 32.1\text{ N}$ **Ans.**



Ans:
$T = 58.0\text{ N}$
$C_z = 77.6\text{ N}$
$C_y = 24.9\text{ N}$
$D_y = 68.5\text{ N}$
$D_z = 32.1\text{ N}$

5–83. Determine the tension in cables BD and CD and the x, y, z components of reaction at the ball-and-socket joint at A.



SOLUTION

Given:

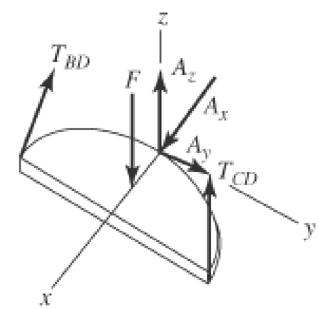
$F = 300\ \text{N}$

$a = 3\ \text{m}$

$b = 1\ \text{m}$

$c = 0.5\ \text{m}$

$d = 1.5\ \text{m}$



$$r_{BD} = \begin{pmatrix} -b \\ d \\ a \end{pmatrix}$$

$$r_{CD} = \begin{pmatrix} -b \\ -d \\ a \end{pmatrix}$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + T_{BD}\frac{r_{BD}}{|r_{BD}|} + T_{CD}\frac{r_{CD}}{|r_{CD}|} + \begin{pmatrix} 0 \\ 0 \\ -F \end{pmatrix} = 0$$

$$\begin{pmatrix} d \\ -d \\ 0 \end{pmatrix} \times \left(T_{BD}\frac{r_{BD}}{|r_{BD}|}\right) + \begin{pmatrix} d \\ d \\ 0 \end{pmatrix} \times \left(T_{CD}\frac{r_{CD}}{|r_{CD}|}\right) + \begin{pmatrix} d-c \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -F \end{pmatrix} = 0$$

$$\begin{pmatrix} T_{BD} \\ T_{CD} \\ A_x \\ A_y \\ A_z \end{pmatrix} = \text{Find}(T_{BD}, T_{CD}, A_x, A_y, A_z)$$

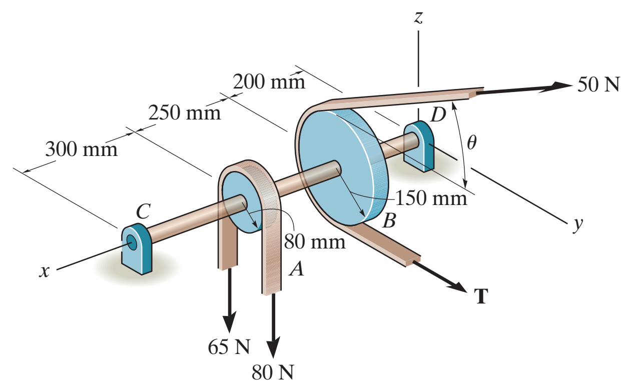
$$\begin{pmatrix} T_{BD} \\ T_{CD} \end{pmatrix} = \begin{pmatrix} 116.7 \\ 116.7 \end{pmatrix}\ \text{N}$$ **Ans.**

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} 66.7 \\ 0 \\ 100 \end{pmatrix}\ \text{N}$$ **Ans.**

Ans:
$T_{BD} = 116.7\ \text{N}$, $T_{CD} = 116.7\ \text{N}$
$A_x = 66.7\ \text{N}$, $A_y = 0\ \text{N}$, $A_z = 100\ \text{N}$

***5–84.**

Both pulleys are fixed to the shaft and as the shaft turns with constant angular velocity, the power of pulley A is transmitted to pulley B. Determine the horizontal tension **T** in the belt on pulley B and the x, y, z components of reaction at the journal bearing C and thrust bearing D if $\theta = 0°$. The bearings are in proper alignment and exert only force reactions on the shaft.



SOLUTION

Equations of Equilibrium:

$\Sigma M_x = 0;\qquad 65(0.08) - 80(0.08) + T(0.15) - 50(0.15) = 0$

$T = 58.0\text{ N}$ **Ans.**

$\Sigma M_y = 0;\qquad (65 + 80)(0.45) - C_z(0.75) = 0$

$C_z = 87.0\text{ N}$ **Ans.**

$\Sigma M_z = 0;\qquad (50 + 58.0)(0.2) - C_y(0.75) = 0$

$C_y = 28.8\text{ N}$ **Ans.**

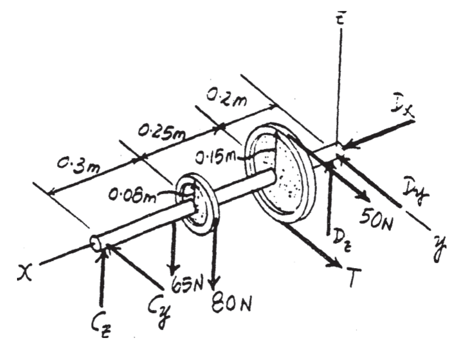
$\Sigma F_x = 0;\qquad D_x = 0$ **Ans.**

$\Sigma F_y = 0;\qquad D_y + 28.8 - 50 - 58.0 = 0$

$D_y = 79.2\text{ N}$ **Ans.**

$\Sigma F_z = 0;\qquad D_z + 87.0 - 80 - 65 = 0$

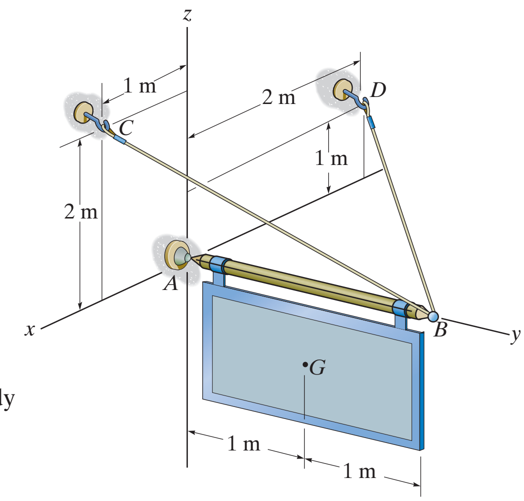
$D_z = 58.0\text{ N}$ **Ans.**



Ans:
$T = 58.0\text{ N}$
$C_z = 87.0\text{ N}$
$C_y = 28.8\text{ N}$
$D_x = 0$
$D_y = 79.2\text{ N}$
$D_z = 58.0\text{ N}$

5–85.

The sign has a mass of 100 kg with center of mass at G. Determine the x, y, z components of reaction at the ball-and-socket joint A and the tension in wires BC and BD.



SOLUTION

Equations of Equilibrium: Expressing the forces indicated on the free-body diagram, Fig. a, in Cartesian vector form, we have

$$\mathbf{F}_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{W} = \{-100(9.81)\mathbf{k}\}\text{ N} = \{-981\text{k}\}\text{ N}$$

$$\mathbf{F}_{BD} = F_{BD}\mathbf{u}_{BD} = F_{BD}\left[\frac{(-2-0)\mathbf{i} + (0-2)\mathbf{j} + (1-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (0-2)^2 + (1-0)^2}}\right] = \left(-\frac{2}{3}F_{BD}\mathbf{i} - \frac{2}{3}F_{BD}\mathbf{j} + \frac{1}{3}F_{BD}\mathbf{k}\right)$$

$$\mathbf{F}_{BC} = F_{BC}\mathbf{u}_{BC} = F_{BC}\left[\frac{(1-0)\mathbf{i} + (0-2)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(1-0)^2 + (0-2)^2 + (2-0)^2}}\right] = \left(\frac{1}{3}F_{BC}\mathbf{i} - \frac{2}{3}F_{BC}\mathbf{j} + \frac{2}{3}F_{BC}\mathbf{k}\right)$$

Applying the forces equation of equilibrium, we have

$$\Sigma\mathbf{F} = 0; \quad \mathbf{F}_A + \mathbf{F}_{BD} + \mathbf{F}_{BC} + \mathbf{W} = 0$$

$$(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) + \left(-\frac{2}{3}F_{BD}\mathbf{i} - \frac{2}{3}F_{BD}\mathbf{j} + \frac{1}{3}F_{BD}\mathbf{k}\right) + \left(\frac{1}{3}F_{BC}\mathbf{i} - \frac{2}{3}F_{BC}\mathbf{j} + \frac{2}{3}F_{BC}\mathbf{k}\right) + (-981\,k) = 0$$

$$\left(A_x - \frac{2}{3}F_{BD} + \frac{1}{3}F_{BC}\right)\mathbf{i} + \left(A_y - \frac{2}{3}F_{BD} - \frac{2}{3}F_{BC}\right)\mathbf{j} + \left(A_z + \frac{1}{3}F_{BD} + \frac{2}{3}F_{BC} - 981\right)\mathbf{k} = 0$$

Equating **i**, **j**, and **k** components, we have

$$A_x - \frac{2}{3}F_{BD} + \frac{1}{3}F_{BC} = 0 \qquad \textbf{(1)}$$

$$A_y - \frac{2}{3}F_{BD} - \frac{2}{3}F_{BC} = 0 \qquad \textbf{(2)}$$

$$A_z + \frac{1}{3}F_{BD} + \frac{2}{3}F_{BC} - 981 = 0 \qquad \textbf{(3)}$$

In order to write the moment equation of equilibrium about point A, the position vectors $\mathbf{r}_{AG}$ and $\mathbf{r}_{AB}$ must be determined first.

$$\mathbf{r}_{AG} = \{1\mathbf{j}\}\text{ m}$$

$$\mathbf{r}_{AB} = \{2\mathbf{j}\}\text{ m}$$

5–85. Continued

Thus,

$\Sigma \mathbf{M}_A = 0;\ \mathbf{r}_{AB} \times (\mathbf{F}_{BC} + \mathbf{F}_{BD}) + (\mathbf{r}_{AG} \times \mathbf{W}) = 0$

$$(2\mathbf{j}) \times \left[\left(\frac{1}{3}F_{BC} - \frac{2}{3}F_{BD}\right)\mathbf{i} - \left(\frac{2}{3}F_{BC} + \frac{2}{3}F_{BD}\right)\mathbf{j} + \left(\frac{2}{3}F_{BC} + \frac{1}{3}F_{BD}\right)\mathbf{k}\right] + (1\mathbf{j}) \times (-981\mathbf{k}) = 0$$

$$\left(\frac{4}{3}F_{BC} + \frac{2}{3}F_{BD} - 981\right)\mathbf{i} + \left(\frac{4}{3}F_{BD} - \frac{2}{3}F_{BC}\right)\mathbf{k} = 0$$

Equating **i**, **j**, and **k** components we have

$$\frac{4}{3}F_{BC} + \frac{2}{3}F_{BC} - 981 = 0 \qquad \textbf{(4)}$$

$$\frac{4}{3}F_{BC} - \frac{2}{3}F_{BC} = 0 \qquad \textbf{(5)}$$

Ans:
$F_{BD} = 294\text{ N}$
$F_{BC} = 589\text{ N}$
$A_x = 0$
$A_y = 589\text{ N}$
$A_z = 490.5\text{ N}$