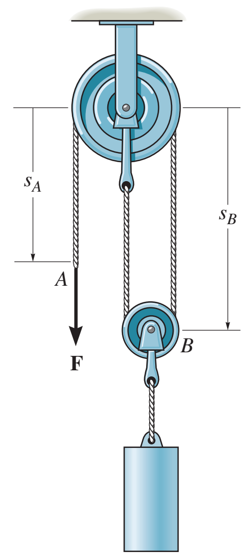
11–1. Determine the force F needed to lift the block having a mass of 50 kg. *Hint:* Note that the coordinates s_A and s_B can be related to the *constant* vertical length l of the cord.



SOLUTION

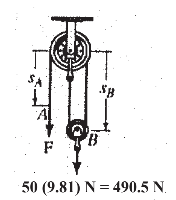
$I = s_A + 2s_B$

$\delta s_A = -2\delta s_B$

$\delta U = 0; \quad W\delta s_B + F\delta s_A = 0$

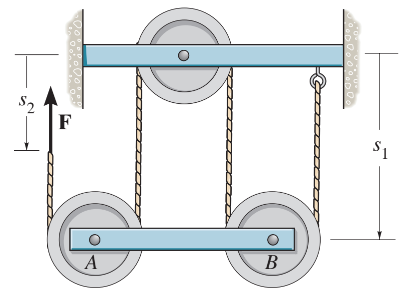
$$490.5\delta s_B + F(-2\delta s_B) = 0$$

$$F = 245.25 \text{ N}$$ **Ans.**



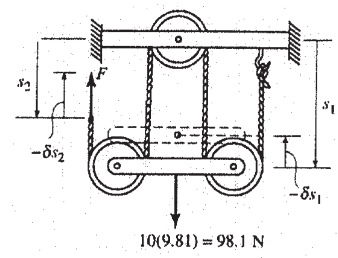
Ans:
$F = 245.25$ N

11–2. Determine the force **F** acting on the cord which is required to maintain equlibrium of the horizontal 10-kg bar AB. *Hint:* Express the total *constant vertical length l* of the cord in terms of position coordinates s_1 and s_2. The derivative of this equation yields a relationship between δ_1 and δ_2.



SOLUTION

Free—Body Diagram: Only force **F** and the weight of link AB (98.1 N) do work.



Virtual Displacements: Force **F** and the weight of link AB (98.1 N) are located from the top of the fixed link using position coordinates s_2 and s_1. Since the cord has a constant length, l, then

$$4s_1 - s_2 = l \quad 4\delta s_1 - \delta s_2 = 0 \qquad \textbf{[1]}$$

Virtual—Work Equation: When s_1 and s_2 undergo positive virtual displacements δs_1 and δs_2, the weight of link AB (98.1 N) and force **F** do positive work and negative work, respectively.

$$\delta U = 0; \quad 98.1(-\delta s_1) - F(-\delta s_2) = 0 \qquad \textbf{[2]}$$

Substituting into Eq. [2] into [1] yields

$$(-98.1 + 4F)\delta s_1 = 0$$

Since $\delta s_1 \neq 0$, then

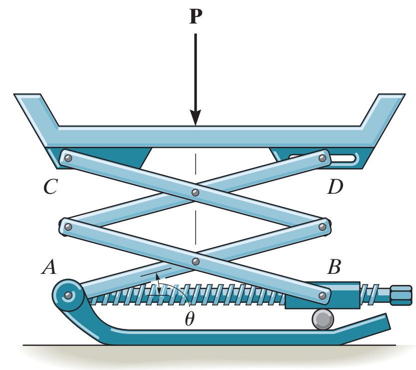
$$-98.1 + 4F = 0$$

$$F = 24.5\text{ N}$$ **Ans.**

Ans:
$F = 24.5$ N

11–3.

The scissors jack supports a load **P**. Determine the axial force in the screw necessary for equilibrium when the jack is in the position θ. Each of the four links has a length L and is pin-connected at its center. Points B and D can move horizontally.



SOLUTION

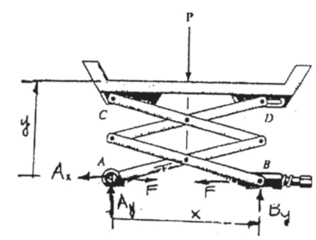
$x = L\cos\theta, \qquad \delta x = -L\sin\theta\,\delta\theta$

$y = 2L\sin\theta, \qquad \delta y = 2L\cos\theta\,\delta\theta$

$\delta U = 0; \qquad -P\delta y - F\delta x = 0$

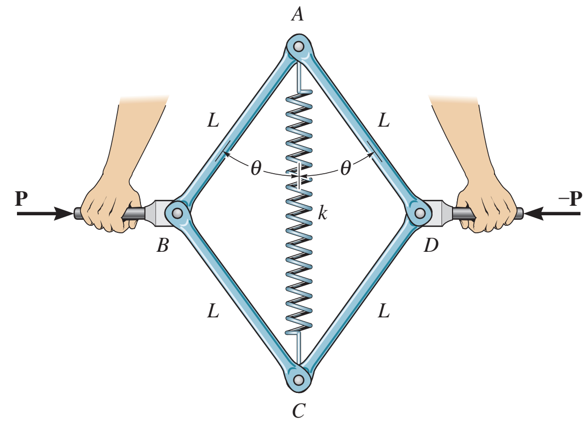
$-P(2L\cos\theta\,\delta\theta) - F(-L\sin\theta\,\delta\theta) = 0$

$F = 2P\cot\theta$ **Ans.**

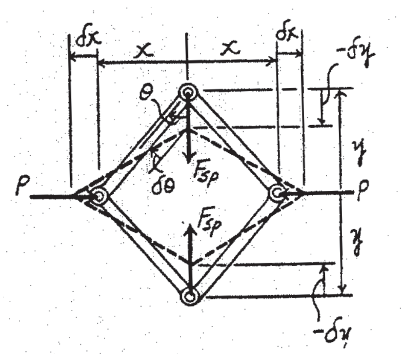


Ans:
$F = 2P\cot\theta$

***11–4.** The assembly is used for exercise. It consist of four pin-connected bars, each of length L, and a spring of stiffness k and unstretched length a ($<2L$). If horizontal forces **P** and $-$**P** are applied to the handles so that θ is slowly decreased, determine the angle θ at which the magnitude of **P** becomes a maximum.



SOLUTION



Free Body Diagram: The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, the spring force F_{sp} and force **P** do work.

Virtual Displacements: The spring force F_{ap} and force **P** are located from the fixed point D and A using position coordinates y and x, respectively.

$$y = L\cos\theta \quad \delta y = -L\sin\theta\delta\theta \qquad \textbf{[1]}$$

$$x = L\sin\theta \quad \delta x = L\cos\theta\delta\theta \qquad \textbf{[2]}$$

Virtual – Work Equation: When points A, C, B and D undergo positive virtual displacement δy and δx, the spring force F_{sp} and force **P** do negative work.

$$\delta U = 0; \quad -2F_{sp}\delta y - 2P\delta x = 0 \qquad \textbf{[3]}$$

Substituting Eqs. [1] and [2] into [3] yields

$$(2F_{sp}\sin\theta - 2P\cos\theta)L\delta\theta = 0 \qquad \textbf{[4]}$$

From the geometry, the spring stretches $x = 2L\cos\theta - a$. Then the spring force $F_{sp} = kx = k(2L\cos\theta - a) = 2kL\cos\theta - ka$. Substituting this value into Eq.[4] yields

$$(4kL\sin\theta\cos\theta - 2ka\sin\theta - 2P\cos\theta)L\delta\theta = 0$$

Since $L\delta\theta \neq 0$, then

$$4kL\sin\theta\cos\theta - 2ka\sin\theta - 2P\cos\theta = 0$$

$$P = k(2L\sin\theta - a\tan\theta)$$

In order to obtain maximum $P, \dfrac{dP}{d\theta} = 0$.

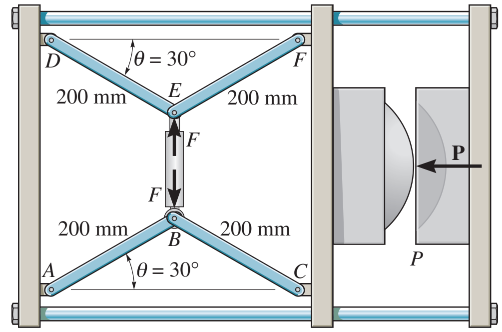
$$\frac{dP}{d\theta} = k(2L\cos\theta - a\sec^2\theta) = 0$$

$$\theta = \cos^{-1}\left(\frac{a}{2L}\right)^{\frac{1}{3}} \qquad \textbf{Ans.}$$

Ans:

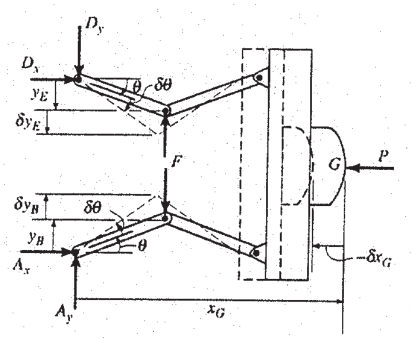
$\theta = \cos^{-1}\left(\dfrac{a}{2L}\right)^{\frac{1}{3}}$

11–5. The machine shown is used for forming metal plates. It consists of two toggles ABC and DEF, which are operated by hydraulic cylinder BE. The toggles push the moveable bar FC forward, pressing the plate p into the cavity. If the force which the plate exerts on the head is $P = 8$ kN, determine the force $\mathbf{F}$ in the hydraulic cylinder when $\theta = 30°$.



SOLUTION

Free—Body Diagram: The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the forces $\mathbf{F}$ and $\mathbf{P}$ do work.



Virtual Displacements: The spring force $\mathbf{F}$ acting on joints E and B and force $\mathbf{P}$ are located from the fixed points D and A using position coordinates y_E and y_B, respectively. The location for force $\mathbf{P}$ is measured from the fixed point A using position coordinate x_a.

$y_E = 0.2 \sin\theta \quad \delta y_E = 0.2\cos\,\theta\delta\theta$ **[1]**

$y_B = 0.2 \sin\theta \quad \delta y_B = 0.2\cos\,\theta\delta\theta$ **[2]**

$x_G = 2(0.2\cos\theta) + l \quad \delta x_G = -0.4\sin\,\theta\delta\theta$ **[3]**

Virtual – Work Equation: When points E, B and G undergo positive virtual displacements δ_{yE}, δ_{yB} and δ_{yG}, force $\mathbf{F}$ and $\mathbf{P}$ do negative work.

$\delta U = 0; \quad -F\delta y_E - F\delta y_B - P\delta x_G = 0$ **[4]**

Substituting Eqs. [1], [2] and [3] into [4] yields

$$(0.4P\sin\theta - 0.4F\cos\theta)\delta\theta = 0$$

Since $\delta\theta \neq 0$, then

$$0.4P\sin\theta - 0.4F\cos\theta = 0 \quad F = P\tan\theta$$

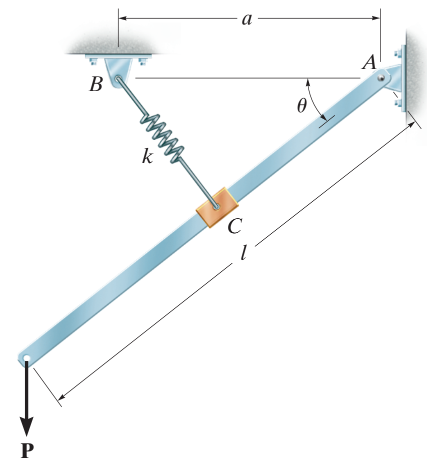
At equilibrium position $\theta = 30°$ set $P = 8$ kN, we have

$F = 8\tan 30° = 4.62$ kN **Ans.**

Ans:
$F = 4.62$ kN

11–6.

The bar is supported by the spring and smooth collar that allows the spring to be always perpendicular to the bar for any angle θ. If the unstretched length of the spring is l_0, determine the force P needed to hold the bar in the equilibrium position θ. Neglect the weight of the bar.



SOLUTION

$s = a \sin \theta, \qquad \delta s = a \cos \theta \, \delta\theta$

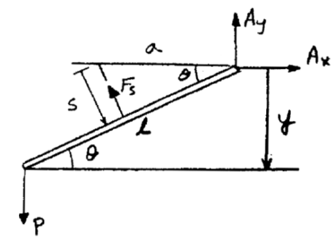
$y = l \sin \theta, \qquad \delta y = l \cos \theta \, \delta\theta$

$F_s = k(a \sin \theta - l_0)$

$\delta U = 0; \qquad P\delta y - F_s \delta s = 0$

$Pl \cos \; \delta\theta - k(a \sin \theta - l_0) \, a \cos \theta \, \delta\theta = 0$

$$P = \frac{ka\,(a \sin \theta - l_0)}{l}$$ **Ans.**

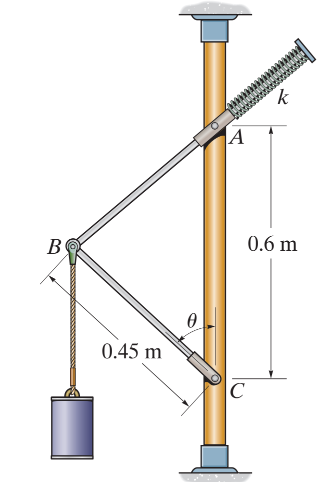


Ans:

$P = \dfrac{ka\,(a \sin \theta - l_0)}{l}$

11–7.

If the spring is unstretched when $\theta = 30°$, the mass of the cylinder is 25 kg, and the mechanism is in equilibrium when $\theta = 45°$, determine the stiffness k of the spring. Rod AB slides freely through the collar at A. Neglect the mass of the rods.



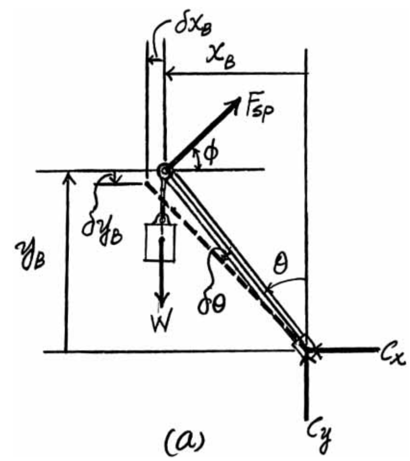
SOLUTION

Free-Body Diagram: When θ undergoes a positive virtual angular displacement of $\delta\theta$, the dash line configuration shown in Fig. *a* is formed. We observe that only the spring force $\mathbf{F}_{sp}$ and the weight $\mathbf{W}$ of the cylinder do work when the virtual displacement takes place.

Virtual Displacement: The position of the point B at which $\mathbf{W}_D$ and $\mathbf{F}_{sp}$ act is specified by the position coordinates x_B and y_B, measured from the fixed point C.

$$x_B = 0.45 \sin\theta \qquad \delta x_B = 0.45 \cos\theta\delta\theta \qquad \textbf{(1)}$$

$$y_B = 0.45 \cos\theta \qquad \delta y_B = -0.45 \sin\theta\delta\theta \qquad \textbf{(2)}$$



Virtual–Work Equation: In this case $\mathbf{F}_{sp}$ must be resolved into its horizontal and vertical component, i.e. $(\mathbf{F}_{sp})_x = F_{sp}\cos\phi$ and $(\mathbf{F}_{sp})_y = F_{sp}\sin\phi$. Since $(\mathbf{F}_{sp})_x$ and $\mathbf{W}$ and act towards the negative sense of their corresponding virtual displacements, their work is negative. However, $(\mathbf{F}_{sp})_y$ does positive work since it acts towards the positive sense of its corresponding virtual displacement. Thus,

$$\delta U = 0; \qquad -F_{sp}\cos\phi\delta x_B + F_{sp}\sin\phi\delta y_B + (-W\delta y_B) = 0 \qquad \textbf{(3)}$$

Substituting $W = 25(9.81) = 245.25$ N, Eqs. (1) and (2) into Eq. (3), we have

$$-F_{sp}\cos\phi(0.45\cos\theta\delta\theta) + F_{sp}\sin\phi(-0.45\sin\theta\delta\theta) - 245.25(-0.45\sin\theta\delta\theta) = 0$$

$$\delta\theta\big(110.3625\sin\theta - 0.45F_{sp}(\cos\theta\cos\phi + \sin\theta\sin\phi)\big) = 0$$

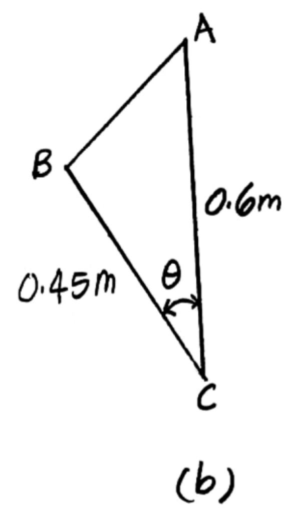
Using the indentity $\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$, the above equation can be rewritten as

$$\delta\theta\big(110.3625\sin\theta - 0.45F_{sp}\cos(\theta - \phi)\big) = 0$$

Since $\delta\theta \neq 0$, then

$$110.3625\sin\theta - 0.45F_{sp}\cos(\theta - \phi) = 0$$

$$F_{sp} = \frac{245.25\sin\theta}{\cos(\theta - \phi)} \qquad \textbf{(4)}$$

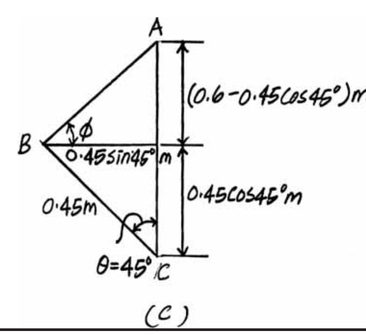


Applying the law of cosines to the geometry shown in Fig. *b*, we have

$$AB = \sqrt{0.45^2 + 0.6^2 - 2(0.45)(0.6)\cos\theta}$$
$$= \sqrt{0.5625 - 0.54\cos\theta}$$

Thus, the stretch of the spring is given by

$$x = \sqrt{0.5625 - 0.54\cos 45°} - \sqrt{0.5625 - 0.54\cos 30°}$$
$$= 0.1171 \text{ m}$$



11–7. (continued)

The magnitude of $\mathbf{F}_{sp}$ computed using the spring force formula is therefore

$$F_{sp} = kx = 0.1171\,k$$

The angle ϕ at $\theta = 45°$ can be obtained by referring to the geometry shown in Fig. *c*.

$$\tan\phi = \frac{0.6 - 0.45\cos 45°}{0.45\sin 45°}$$
$$\phi = 41.53°$$

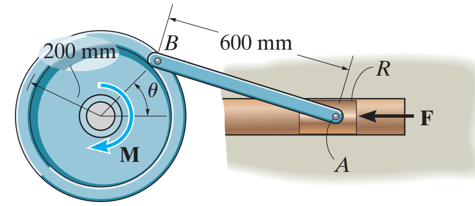
Substituting $\theta = 45°$ and the results for $\mathbf{F}_{sp}$ and ϕ into Eq. (4), we have

$$0.1171\,k = \frac{245.25\sin 45°}{\cos(45° - 41.53°)}$$
$$k = 1484\text{ N/m} = 1.48\text{ kN/m}$$ **Ans.**

Ans:
$k = 1.48$ kN/m

***11–8.**

The punch press consists of the ram R, connecting rod AB, and a flywheel. If a torque of $M = 75\ \text{N}\cdot\text{m}$ is applied to the flywheel, determine the force **F** applied at the ram to hold the rod in the position $\theta = 60°$.

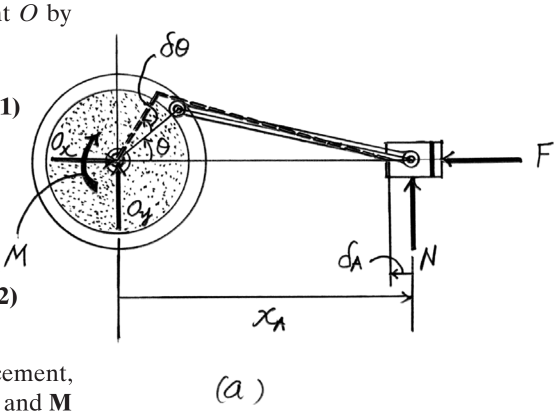


SOLUTION

Free Body Diagram. The system has only one degree of freedom, defined by the independent coordinate θ. When θ undergoes a positive angular displacement $\delta\theta$ as shown in Fig. a, only force **F** and couple moment **M** do work.

Virtual Displacement. The position of force **F** is measured from fixed point O by position coordinate x_A. Applying the law of cosines by referring to Fig. b,

$$0.6^2 = x_A^2 + 0.2^2 - 2x_A(0.2)\cos\theta \quad \textbf{(1)}$$



Differentiating the above expression,

$$0 = 2x_A\delta x_A + 0.4x_A\sin\theta\,\delta\theta - 0.4\cos\theta\,\delta x_A$$

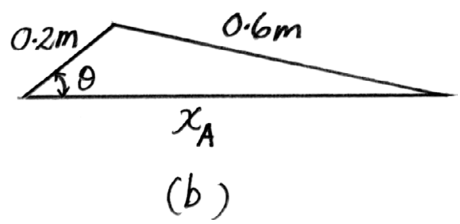
$$\delta x_A = \frac{0.4x_A\sin\theta}{0.4\cos\theta - 2x_A}\delta\theta \quad \textbf{(2)}$$

Virtual–Work Equation. When point A undergoes a positive virtual displacement, and the flywheel undergoes positive virtual angular displacement $\delta\theta$, both **F** and **M** do negative work.

$$\delta U = 0; \qquad -F\delta x_A - M\delta\theta = 0 \quad \textbf{(3)}$$

Substituting Eq. (2) into (3)

$$\left[\left(\frac{0.4\,x_A\sin\theta}{0.4\cos\theta - 2x_A}\right)F + M\right]\delta\theta = 0$$



Since $\delta\theta \neq 0$, then

$$\frac{0.4\,x_A\sin\theta}{0.4\cos\theta - 2x_A}F + M = 0$$

$$F = -\left(\frac{0.4\cos\theta - 2x_A}{0.4\,x_A\sin\theta}\right)M \quad \textbf{(4)}$$

At the equilibrium position $\theta = 60°$, Eq. (1) gives

$$0.6^2 = x_A^2 + 0.2^2 - 2x_A(0.2)\cos 60°$$

$$x_A = 0.6745\ \text{m}$$

Substitute $M = 75\ \text{N}\cdot\text{m}$, $\theta = 60°$ and this result into Eq. (4)

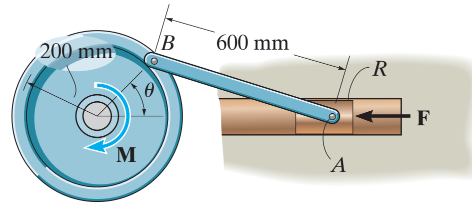
$$F = -\left[\frac{0.4\cos 60° - 2(0.6745)}{0.4(0.6745)\sin 60°}\right](75)$$

$$= 368.81\ \text{N} = 369\ \text{N} \qquad \textbf{Ans.}$$

Ans:
$F = 369\ \text{N}$

11–9.

The flywheel is subjected to a torque of $M = 75\ \text{N}\cdot\text{m}$. Determine the horizontal compressive force F and plot the result of F (ordinate) versus the equilibrium position θ (abscissa) for $0° \le \theta \le 180°$.



SOLUTION

Free Body Diagram. The system has only one degree of freedom, defined by the independent coordinate θ. When θ undergoes a positive angular displacement $\delta\theta$ as shown in Fig. a, only force **F** and couple moment **M** do work.

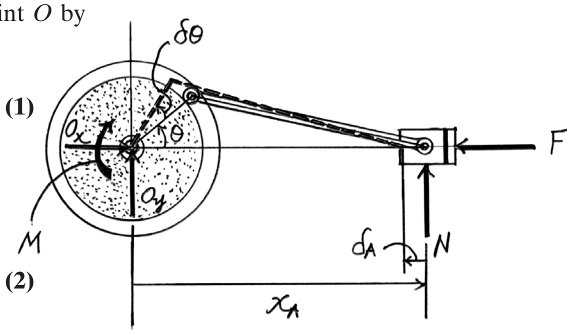
Virtual Displacement. The position of force **F** is measured from fixed point O by position coordinate x_A. Applying the law of cosines by referring to Fig. b,

$$0.6^2 = x_A^2 + 0.2^2 - 2x_A(0.2)\cos\theta \qquad \textbf{(1)}$$

Differentiating the above expression,

$$0 = 2x_A\delta x_A + 0.4x_A\sin\theta\,\delta\theta - 0.4\cos\theta\,\delta x_A$$

$$\delta x_A = \frac{0.4x_A\sin\theta}{0.4\cos\theta - 2x_A} \qquad \textbf{(2)}$$



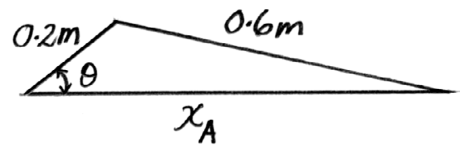
(a)

Virtual–Work Equation. When point A undergoes a positive virtual displacement and the flywheel undergoes positive virtual angular displacement $\delta\theta$, both **F** and **M** do negative work.

$$\delta U = 0; \qquad -F\delta x_A - M\delta\theta = 0 \qquad \textbf{(3)}$$

Substituting Eq. (2) into (3)

$$\left[\left(\frac{0.4\,x_A\sin\theta}{0.4\cos\theta - 2x_A}\right)F + M\right]\delta\theta = 0$$



(b)

Since $\delta\theta \ne 0$, then

$$\left(\frac{0.4\,x_A\sin\theta}{0.4\cos\theta - 2x_A}\right)F + M = 0$$

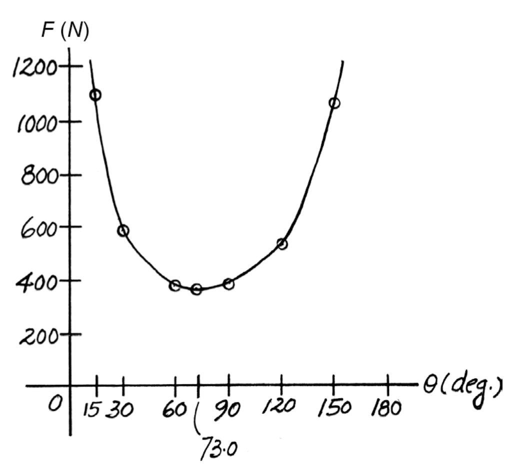
$$F = -M\left(\frac{0.4\cos\theta - 2x_A}{0.4\,x_A\sin\theta}\right)$$

Here, $M = 75\ \text{N}\cdot\text{m}$. Then

$$F = -75\left(\frac{0.4\cos\theta - 2x_A}{0.4\,x_A\sin\theta}\right) \qquad \textbf{(4)}$$

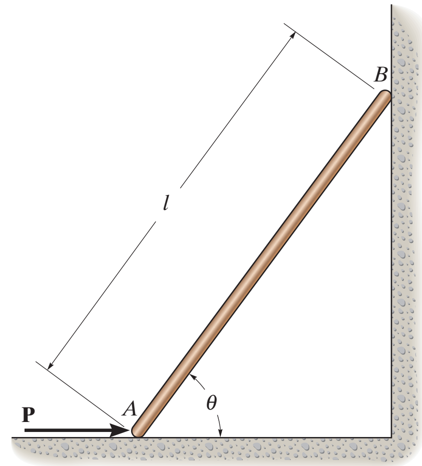
Using Eq. (1) and (4), the following tabulation can be computed. Subsequently, the graph of **F** vs θ shown in Fig. c can be plotted

θ(deg.)	0	30	60	90	120	150	180	15	73.0
x_A(m)	0.80	0.7648	0.6745	0.5657	0.4745	0.4184	0.4	0.7909	0.6272
$F(N)$	∞	580	369	375	524	1060	∞	1095	356



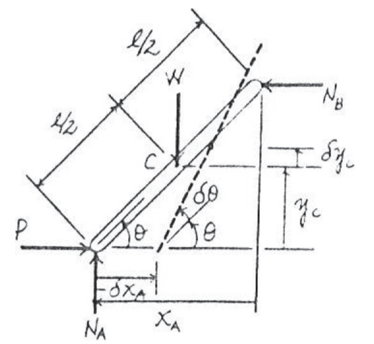
11–10.

The thin rod of weight W rest against the smooth wall and floor. Determine the magnitude of force **P** needed to hold it in equilibrium for a given angle θ.



SOLUTION

Free-Body Diagram: The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the weight of the rod W and force **P** do work.



Virtual Displacements: The weight of the rod W and force **P** are located from the fixed points A and B using position coordinates y_C and x_A, respectively

$$y_C = \frac{1}{2}\sin\theta \qquad \delta y_C = \frac{1}{2}\cos\theta\delta\theta \qquad \textbf{(1)}$$

$$x_A = l\cos\theta \qquad \delta x_A = -l\sin\theta\delta\theta \qquad \textbf{(2)}$$

Virtual-Work Equation: When points C and A undergo positive virtual displacements δy_C and δx_A, the weight of the rod W and force **F** do negative work.

$$\delta U = 0; -W\delta y_C - P\delta y_A = 0 \qquad \textbf{(3)}$$

Substituting Eqs. (1) and (2) into (3) yields

$$\left(Pl\sin\theta - \frac{Wl}{2}\cos\theta\right)\delta\theta = 0$$

Since $\delta\theta \neq 0$, then

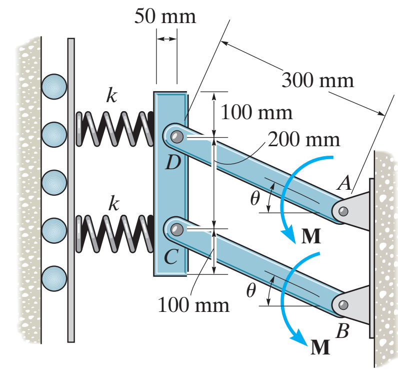
$$Pl\sin\theta - \frac{Wl}{2}\cos\theta = 0$$

$$P = \frac{W}{2}\cot\theta \qquad \textbf{Ans.}$$

Ans:
$P = \dfrac{W}{2}\cot\theta$

11–11.

When $\theta = 30°$, the 25-kg uniform block compresses the two horizontal springs 100 mm. Determine the magnitude of the applied couple moments **M** needed to maintain equilibrium. Take $k = 3$ kN/m and neglect the mass of the links.



SOLUTION

Free Body Diagram. The system has only one degree of freedom, defined by the independent coordinate θ. When θ undergoes a positive angular displacement $\delta\theta$ as shown in Fig. *a*, only spring force $\mathbf{F}_{sp}$, the weight of the block **W**, and couple moment **M** do work.

Virtual Displacement. The positions of $\mathbf{F}_{sp}$ and **W** are measured from fixed point **B** using position coordinates x and y respectively.

$$x = 0.3\cos\theta + 0.05 \qquad \delta x = -0.3\sin\theta\,\delta\theta \qquad \textbf{(1)}$$

$$y = 0.3\sin\theta + 0.1 \qquad \delta y = 0.3\cos\theta\delta\theta \qquad \textbf{(2)}$$

Virtual–Work Equation. When $\mathbf{F}_{sp}$, **W** and **M** undergo their respective positive virtual displacement, all of them do negative work. Thus

$$\delta U = 0; \qquad -2F_{sp}\,\delta x - W\delta y - 2M\delta\theta = 0 \qquad \textbf{(3)}$$

Substitute Eqs. (1) and (2) into (3),

$$-2F_{sp}(-0.3\sin\theta\delta\theta) - W(0.3\cos\theta\delta\theta) - 2M\delta\theta = 0$$

$$(0.6F_{sp}\sin\theta - 0.3\,W\cos\theta - 2M)\delta\theta = 0$$

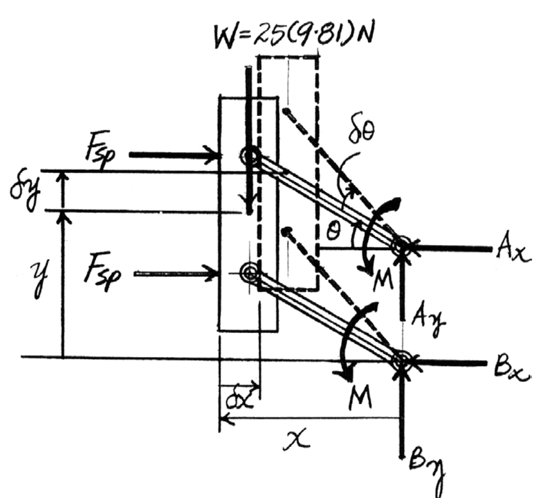
Since $\delta\theta \neq 0$, then

$$0.6\,F_{sp}\sin\theta - 0.3\,W\cos\theta - 2M = 0$$

$$M = 0.3\,F_{sp}\sin\theta - 0.15\,W\cos\theta \qquad \textbf{(4)}$$

When $\theta = 30°$, $F_{sp} = kx = 3000(0.1) = 300$ N. Also $W = 25(9.81) = 245.25$ N. Substitute these values into Eq. 4.

$$M = 0.3(300)\sin 30° - 0.15(245.25)\cos 30° = 13.14\text{ N}\cdot\text{m} = 13.1\text{ N}\cdot\text{m} \quad \textbf{Ans.}$$

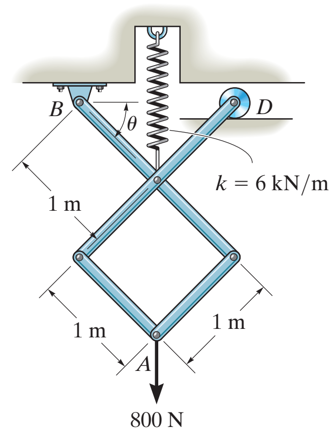


(a)

Ans:
$M = 13.1$ N·m

***11–12.**

The members of the mechanism are pin connected. If a vertical force of 800 N acts at A, determine the angle θ for equilibrium. The spring is unstretched when $\theta = 0°$. Neglect the mass of the links.



SOLUTION

Free Body Diagram. The system has only one degree of freedom, defined by the independent coordinate θ. When θ undergoes a positive angular displacement $\delta\theta$ as shown in Fig. *a*, only spring force $\mathbf{F}_{sp}$ and force $\mathbf{P}$ do work.

Virtual Displacement. The positions of $\mathbf{F}_{sp}$ and $\mathbf{P}$ are measured from fixed point B using position coordinates y_c and y_A respectively.

$$y_c = 1 \sin\theta \qquad \delta y_c = \cos\theta\,\delta\theta \qquad \textbf{(1)}$$

$$y_A = 3(1 \sin\theta) \qquad \delta y_A = 3\cos\theta\,\delta\theta \qquad \textbf{(2)}$$

Virtual Work Equation. When $\mathbf{F}_{sp}$ and $\mathbf{P}$ undergo their respective positive virtual displacement, $\mathbf{P}$ does positive work whereas $\mathbf{F}_{sp}$ does negative work.

$$\delta U = 0; \qquad -F_{sp}\delta y_c + P\delta y_A = 0 \qquad \textbf{(3)}$$

Substitute Eqs. (1) and (2) into (3)

$$-F_{sp}(\cos\theta\delta\theta) + P(3\cos\theta\delta\theta) = 0$$

$$(-F_{sp}\cos\theta + 3\text{P}\cos\theta)\delta\theta = 0$$

Since $\delta\theta \neq 0$, and assuming $\theta < 90°$, then

$$-F_{sp}\cos\theta + 3P\cos\theta = 0$$

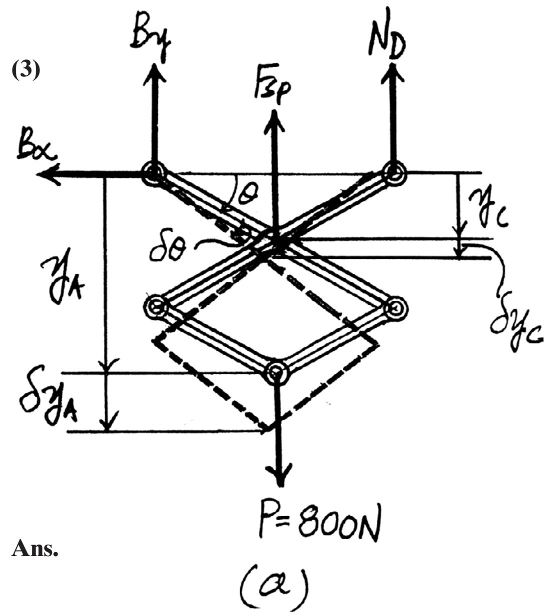
$$F_{sp} = 3P$$

Here $F_{sp} = kx = 6000(1 \sin\theta) = 6000 \sin\theta$ and $P = 800$ N, Then

$$6000 \sin\theta = 3(800)$$

$$\sin\theta = 0.4$$

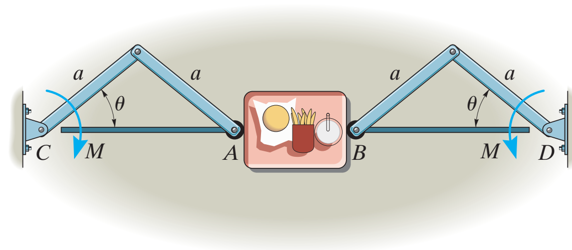
$$\theta = 23.58° = 23.6° \qquad \textbf{Ans.}$$

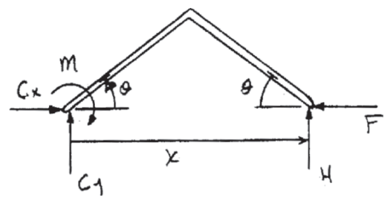


Ans:
$\theta = 23.6°$

11–13.

The service window at a fast-food restaurant consists of glass doors that open and close automatically using a motor which supplies a torque **M** to each door. The far ends, A and B, move along the horizontal guides. If a food tray becomes stuck between the doors as shown, determine the horizontal force the doors exert on the tray at the position θ.





SOLUTION

$$x = 2a\cos\theta, \qquad \delta x = -2a\sin\theta\,\delta\theta$$

$$\delta U = 0; \qquad -M\,\delta\theta - F\,\delta x = 0$$

$$-M\,\delta\theta + F\,(2a\sin\theta)\delta\theta = 0$$

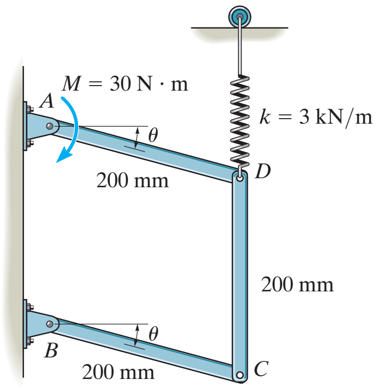
$$F = \frac{M}{2a\sin\theta} \qquad \textbf{Ans.}$$

Ans:

$$F = \frac{M}{2a\sin\theta}$$

11–14.

If each of the three links of the mechanism have a mass of 4 kg, determine the angle θ for equilibrium. The spring, which always remains vertical, is unstretched when $\theta = 0°$.



SOLUTION

Free Body Diagram. The system has only one degree of freedom, defined by the independent coordinate θ. When θ undergoes a positive angular displacement $\delta\theta$ as shown in Fig. *a*, only the weights $W_1 = W_2 = W_3 = W$, couple moment $\mathbf{M}$, and spring force $\mathbf{F}_{sp}$ do work.

Virtual Displacement. The positions of the weights W_1, W_2, W_3 and spring force $\mathbf{F}_{sp}$ are measured from fixed point A using position coordinates y_1, y_2, y_3 and y_4 respectively

$$y_1 = 0.1 \sin\theta \qquad \delta y_1 = 0.1 \cos\theta\, \delta\theta \qquad \textbf{(1)}$$

$$y_2 = 0.2 \sin\theta + 0.1 \qquad \delta y_2 = 0.2 \cos\theta\, \delta\theta \qquad \textbf{(2)}$$

$$y_3 = 0.1 \sin\theta + 0.2 \qquad \delta y_3 = 0.1 \cos\theta\, \delta\theta \qquad \textbf{(3)}$$

$$y_4 = 0.5 \sin\theta \qquad \delta y_4 = 0.2 \cos\theta\, \delta\theta \qquad \textbf{(4)}$$

Virtual Work Equation. When all the weights undergo positive virtual displacement, all of them do positive work. However, $\mathbf{F}_{sp}$ does negative work when its undergoes positive virtual displacement. Also, $\mathbf{M}$ does positive work when it undergoes positive virtual angular displacement.

$$\delta U = 0; \qquad W_1\delta y_1 + W_2\delta y_2 + W_3\delta y_3 - F_{sp}\delta y_4 + M\delta\theta = 0 \qquad \textbf{(5)}$$

Substitute Eqs (1), (2) and (3) into (5), using $W_1 = W_2 = W_3 = W$.

$$W(0.1 \cos\theta\, \delta\theta) + W(0.2 \cos\theta\, \delta\theta) + W(0.1 \cos\theta\, \delta\theta) - F_{sp}(0.2 \cos\theta\, \delta\theta) + M\delta\theta = 0$$

$$(0.4\, W \cos\theta - 0.2\, F_{sp} \cos\theta + M)\delta\theta = 0$$

Since $\delta \neq 0$, then

$$0.4\, W \cos\theta - 0.2\, F_{sp} \cos\theta + M = 0$$

Here $M = 30\text{ N}\cdot\text{m}$, $W = 4(9.81)\text{N} = 39.24\text{ N}$ and $F_{sp} = kx = 3000(0.2 \sin\theta) = 600 \sin\theta$. Substitute these results into this equation,

$$0.4(39.24) \cos\theta - 0.2(600 \sin\theta)\cos\theta + 30 = 0$$

$$15.696 \cos\theta - 120 \sin\theta \cos\theta + 30 = 0$$

11–14. Continued

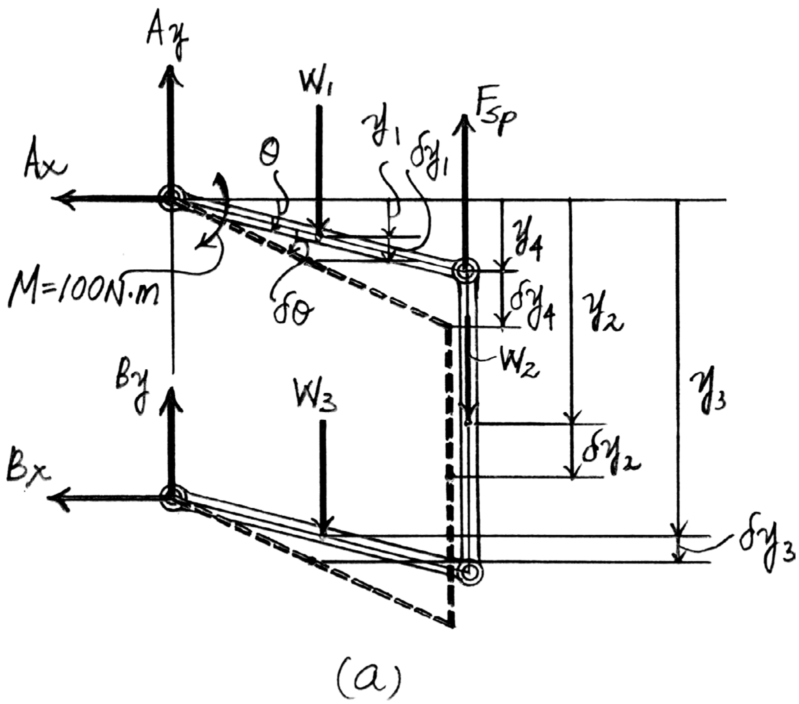
Solve numerically:

$$\theta = 23.8354° = 23.8° \qquad \textbf{Ans.}$$

or

$$\theta = 72.2895° = 72.3° \qquad \textbf{Ans.}$$

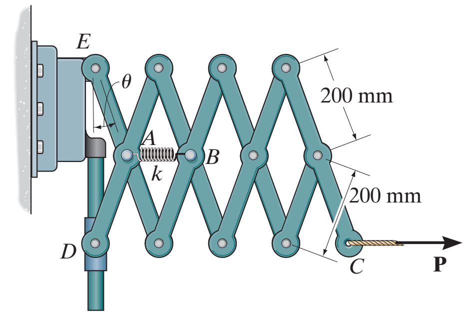
Note: $\theta = 23.8°$ is a stable equilibrium, while $\theta = 72.3°$ is an unstable one.



Ans:
$\theta = 23.8°$
$\theta = 72.3°$

11–15.

The "Nuremberg scissors" is subjected to a horizontal force of $P = 600$ N. Determine the angle θ for equilibrium. The spring has a stiffness of $k = 15$ kN/m and is unstretched when $\theta = 15°$.



SOLUTION

Free - Body Diagram: When θ undergoes a positive virtual angular displacement of $\delta\theta$, the dash line configuration shown in Fig. *a* is a formed. We observe that only the spring force $\mathbf{F}_{sp}$ acting at points A and B and the force $\mathbf{P}$ do work when the virtual displacements take place. The magnitude of $\mathbf{F}_{sp}$ can be computed using the spring force formula,

$$F_{sp} = kx = 15(10^3)\left[2(0.2 \sin \theta) - 2(0.2 \sin 15°)\right] = 6000(\sin \theta - 0.2588)\,\text{N}$$

Virtual Displacement: The position of points A and B at which $\mathbf{F}_{sp}$ acts and point C at which force $\mathbf{P}$ acts are specified by the position coordinates y_A, y_B, and y_C, measured from the fixed point E, respectively.

$$y_A = 0.2 \sin \theta \qquad \delta y_A = 0.2 \cos \theta \delta\theta \qquad \textbf{(1)}$$

$$y_B = 3(0.2 \sin \theta) \qquad \delta y_B = 0.6 \cos \theta \delta\theta \qquad \textbf{(2)}$$

$$y_C = 8(0.2 \sin \theta) \qquad \delta y_B = 1.6 \cos \theta \delta\theta \qquad \textbf{(3)}$$

Virtual Work Equation: Since $\mathbf{F}_{sp}$ at point A and force $\mathbf{P}$ acts towards the positive sense of its corresponding virtual displacement, their work is positive. The work of $\mathbf{F}_{sp}$ at point B is negative since it acts towards the negative sense of its corresponding virtual displacement. Thus,

$$\delta U = 0; \qquad F_{sp}\delta y_A + \left(-F_{sp}\delta y_B\right) + P\delta y_C = 0 \qquad \textbf{(4)}$$

Substituting $F_{sp} = 6000(\sin \theta - 0.2588)$, $P = 600$ N, Eqs. (1), (2), and (3) into Eq. (4),

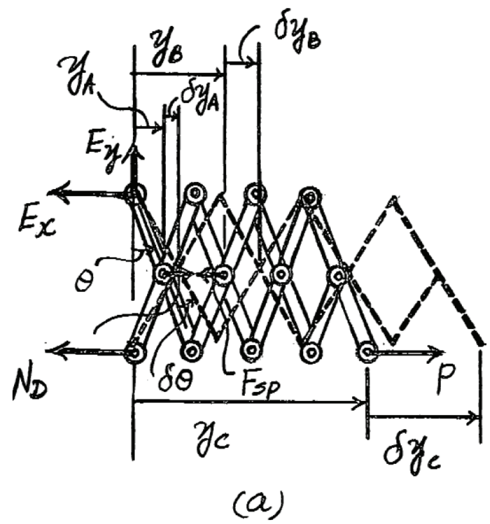
$$6000(\sin \theta - 0.2588)(0.2 \cos \theta\delta\theta - 0.6 \cos \theta\delta\theta) + 600(1.6 \cos \theta\delta\theta) = 0$$

$$\cos \theta\delta\theta\left[-2400(\sin \theta - 0.2588) + 960\right] = 0$$

Since $\cos \theta\delta\theta \neq 0$, then

$$-2400(\sin \theta - 0.2588) + 960 = 0$$

$$\theta = 41.2° \qquad \textbf{Ans.}$$

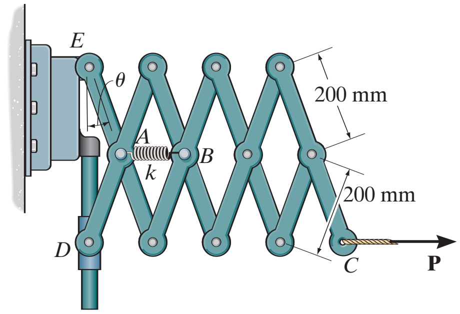


(a)

Ans:
$\theta = 41.2°$

***11–16.**

The "Nuremberg scissors" is subjected to a horizontal force of $P = 600$ N. Determine the stiffness k of the spring for equilibrium when $\theta = 60°$. The spring is unstretched when $\theta = 15°$.



SOLUTION

Free - Body Diagram: When θ undergoes a positive virtual angular displacement of $\delta\theta$, the dash line configuration shown in Fig. *a* is formed. We observe that only the spring force $\mathbf{F}_{sp}$ acting at points A and B and the force $\mathbf{P}$ do work when the virtual displacements take place. The magnitude of $\mathbf{F}_{sp}$ can be computed using the spring force formula.

$$F_{sp} = kx = k\left[2(0.2 \sin\theta) - 2(0.2 \sin 15°)\right] = (0.4)k(\sin\theta - 0.2588) \text{ N}$$

Virtual Displacement: The position of points A and B at which $\mathbf{F}_{sp}$ acts and point C at which force $\mathbf{P}$ acts are specified by the position coordinates y_A, y_B, and y_C, measured from the fixed point E, respectively.

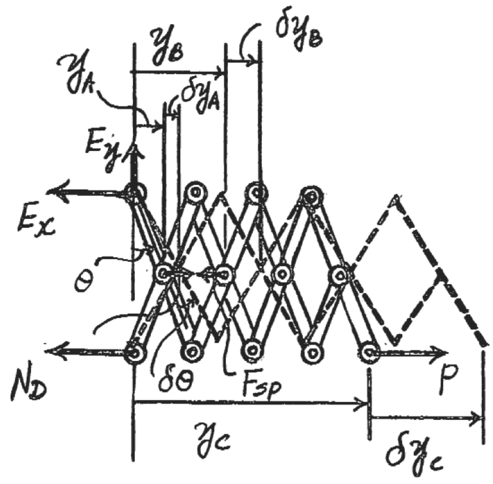
$$y_A = 0.2 \sin\theta \qquad \delta y_A = 0.2 \cos\theta\delta\theta \qquad \textbf{(1)}$$

$$y_B = 3(0.2 \sin\theta) \qquad \delta y_B = 0.6 \cos\theta\delta\theta \qquad \textbf{(2)}$$

$$y_C = 8(0.2 \sin\theta) \qquad \delta y_B = 0.6 \cos\theta\delta\theta \qquad \textbf{(3)}$$

Virtual Work Equation: Since $\mathbf{F}_{sp}$ at point A and force $\mathbf{P}$ acts towards the positive sense of its corresponding virtual displacement, their work is positive. The work of $\mathbf{F}_{sp}$ at point B is negative since it acts towards the negative sense of its corresponding virtual displacement. Thus,

$$\delta U = 0; \qquad F_{sp}\delta y_A + \left(-F_{sp}\delta y_B\right) + P\delta y_C = 0 \qquad \textbf{(4)}$$



(a)

Substituting $F_{sp} = k(\sin\theta - 0.2588)$, $P = 600$ N, Eqs. (1), (2), and (3) into Eq. (4),

$$(0.4)k(\sin\theta - 0.2588)(0.2 \cos\theta\delta\theta - 0.6 \cos\theta\delta\theta) + 600(1.6 \cos\theta\delta\theta) = 0$$

$$\cos\theta\delta\theta\left[-0.16k(\sin\theta - 0.2588) + 960\right] = 0$$

Since $\cos\theta\delta\theta \neq 0$, then

$$-0.16k(\sin\theta - 0.2588) + 960 = 0$$

$$k = \frac{6000}{\sin\theta - 0.2588}$$

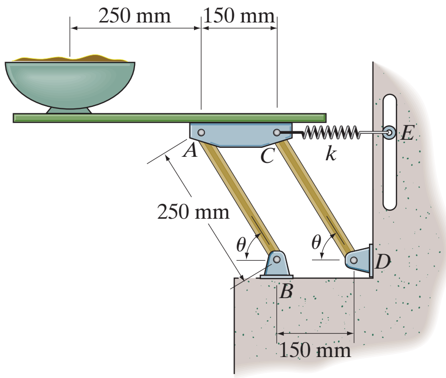
When $\theta = 60°$,

$$k = \frac{6000}{\sin 60° - 0.2588} = 9881 \text{N/m} = 9.88 \text{kN/m} \qquad \textbf{Ans.}$$

Ans:
$k = 9.88$ kN/m

11–17.

A 5-kg uniform serving table is supported on each side by pairs of two identical links, AB and CD, and springs CE. If the bowl has a mass of 1 kg, determine the angle θ where the table is in equilibrium. The springs each have a stiffness of $k = 200$ N/m and are unstretched when $\theta = 90°$. Neglect the mass of the links.



SOLUTION

Free-Body Diagram: When θ undergoes a positive virtual angular displacement of $\delta\theta$, the dash line configuration shown in Fig. *a* is formed. We observe that only the spring force $\mathbf{F}_{sp}$, the weight $\mathbf{W}_t$ of the table, and the weight $\mathbf{W}_b$ of the bowl do work when the virtual displacement takes place. The magnitude of $\mathbf{F}_{sp}$ can be computed using the spring force formula, $F_{sp} = kx = 200(0.25\cos\theta) = 50\cos\theta$ N.

Virtual Displacement: The position of points of application of $\mathbf{W}_b$, $\mathbf{W}_t$, and $\mathbf{F}_{sp}$ are specified by the position coordinates y_{G_b}, y_{G_t}, and x_C, respectively. Here, y_{G_b} and y_{G_t} are measured from the fixed point B while x_C is measured from the fixed point D.

$$y_{G_b} = 0.25\sin\theta + b \qquad \delta y_{G_b} = 0.25\cos\theta\delta\theta \qquad \textbf{(1)}$$

$$y_{G_t} = 0.25\sin\theta + a \qquad \delta y_{G_t} = 0.25\cos\theta\delta\theta \qquad \textbf{(2)}$$

$$x_C = 0.25\cos\theta \qquad \delta x_C = -0.25\sin\theta\delta\theta \qquad \textbf{(3)}$$

Virtual Work Equation: Since $\mathbf{W}_b$, $\mathbf{W}_t$, and $\mathbf{F}_{sp}$ act towards the negative sense of their corresponding virtual displacement, their work is negative. Thus,

$$\delta U = 0; \qquad -W_b\delta y_{G_b} + \left(-W_t\delta y_{G_t}\right) + \left(-F_{sp}\delta x_C\right) = 0 \qquad \textbf{(4)}$$

Substituting $W_b = \left(\frac{1}{2}\right)(9.81) = 4.905$ N, $W_t = \left(\frac{5}{2}\right)(9.81) = 24.525$ N,

$F_{sp} = 50\cos\theta$ N, Eqs. (1), (2), and (3) into Eq. (4), we have

$$-4.905(0.25\cos\theta\delta\theta) - 24.525(0.25\cos\theta\delta\theta) - 50\cos\theta(-0.25\sin\theta\delta\theta) = 0$$

$$\delta\theta\left(-7.3575\cos\theta + 12.5\sin\theta\cos\theta\right) = 0$$

Since $\delta\theta \neq 0$, then

$$-7.3575\cos\theta + 12.5\sin\theta\cos\theta = 0$$

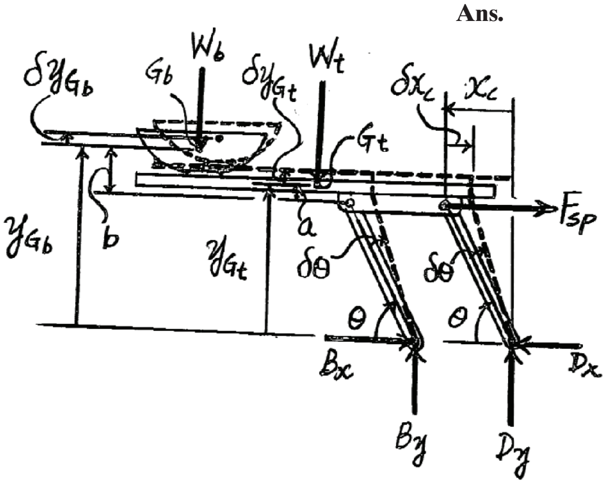
$$\cos\theta(-7.3575 + 12.5\sin\theta) = 0$$

Solving the above equation,

$$\cos\theta = 0 \quad \theta = 90° \qquad \textbf{Ans.}$$

$$-7.3575 + 12.5\sin\theta = 0$$

$$\theta = 36.1° \qquad \textbf{Ans.}$$

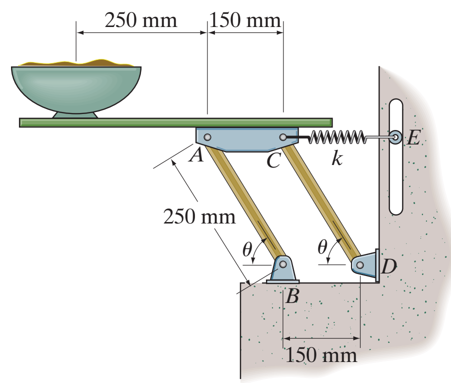


(a)

Ans:
$\theta = 90°$
$\theta = 36.1°$

11–18.

A 5-kg uniform serving table is supported on each side by two pairs of identical links, AB and CD, and springs CE. If the bowl has a mass of 1 kg and is in equilibrium when $\theta = 45°$, determine the stiffness k of each spring. The springs are unstretched when $\theta = 90°$. Neglect the mass of the links.



SOLUTION

Free-Body Diagram: When θ undergoes a positive virtual angular displacement of $\delta\theta$, the dash line configuration shown in Fig. *a* is formed. We observe that only the spring force $\mathbf{F}_{sp}$, the weight $\mathbf{W}_t$ of the table, and the weight $\mathbf{W}_b$ of the bowl do work when the virtual displacement takes place. The magnitude of $\mathbf{F}_{sp}$ can be computed using the spring force formula, $F_{sp} = kx = k(0.25\cos\theta) = 0.25\,k\cos\theta$.

Virtual Displacement: The position of points of application of $\mathbf{W}_b$, $\mathbf{W}_t$, and $\mathbf{F}_{sp}$ are specified by the position coordinates y_{G_b}, y_{G_t}, and x_C, respectively. Here, y_{G_b} and y_{G_t} are measured from the fixed point B while x_C is measured from the fixed point D.

$$y_{G_b} = 0.25\sin\theta + b \qquad \delta y_{G_b} = 0.25\cos\theta\delta\theta \qquad \textbf{(1)}$$

$$y_{G_t} = 0.25\sin\theta + a \qquad \delta y_{G_t} = 0.25\cos\theta\delta\theta \qquad \textbf{(2)}$$

$$x_C = 0.25\cos\theta \qquad \delta x_C = -0.25\sin\theta\delta\theta \qquad \textbf{(3)}$$

Virtual Work Equation: Since $\mathbf{W}_b$, $\mathbf{W}_t$, and $\mathbf{F}_{sp}$ act towards the negative sense of their corresponding virtual displacement, their work is negative. Thus,

$$\delta U = 0; \qquad -W_b\delta y_{G_b} + \left(-W_t\delta y_{G_t}\right) + \left(-F_{sp}\delta x_C\right) = 0 \qquad \textbf{(4)}$$

Substituting $W_b = \left(\frac{1}{2}\right)(9.81) = 4.905\text{ N}$, $W_t = \left(\frac{5}{2}\right)(9.81) = 24.525\text{ N}$,

$F_{sp} = 0.25k\cos\theta$ N, Eqs. (1), (2), and (3) into Eq. (4), we have

$$-4.905(0.25\cos\theta\delta\theta) - 24.525(0.25\cos\theta\delta\theta) - 0.25k\cos\theta(-0.25\sin\theta\delta\theta) = 0$$

$$\delta\theta\left(-7.3575\cos\theta + 0.0625k\sin\theta\cos\theta\right) = 0$$

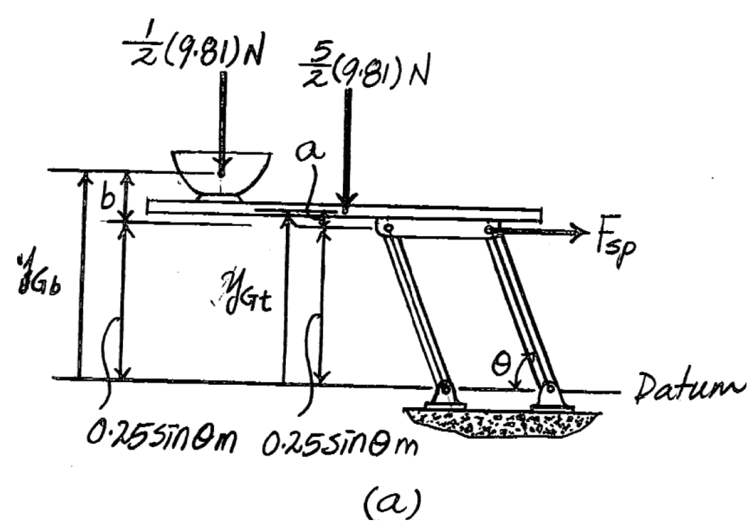
Since $\delta\theta \neq 0$, then

$$-7.3575\cos\theta + 0.0625k\sin\theta\cos\theta = 0$$

$$k = \frac{117.72}{\sin\theta}$$

When $\theta = 45°$, then

$$k = \frac{117.72}{\sin 45°} = 166\text{ N/m} \qquad \textbf{Ans.}$$

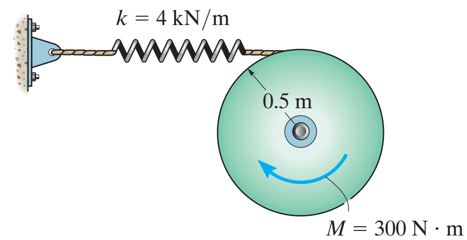


(a)

Ans:
$k = 166\text{ N/m}$

11–19.

The disk is subjected to a couple moment M. Determine the disk's rotation θ required for equilibrium. The end of the spring wraps around the periphery of the disk as the disk turns. The spring is originally unstretched.



SOLUTION

Free Body Diagram. The system has only one degree of freedom, defined by the independent coordinate θ. When θ undergoes a positive angular displacement $\delta\theta$ as shown in Fig. a, only the spring force $\mathbf{F}_{sp}$ and couple moment $\mathbf{M}$ do work.

Virtual Work Equation. When the disk undergoes a positive angular displacement $\delta\theta$, correspondingly point A undergoes a positive displacement of δx_A. As a result couple moment $\mathbf{M}$ does positive work whereas spring force $\mathbf{F}_{sp}$ does negative work.

$$\delta U = 0; \qquad M\delta\theta + (-F_{sp}\delta x_A) = 0 \qquad \textbf{(1)}$$

Here, $F_{sp} = kx_A = 4000(0.50) = 2000\theta$ and $\delta x_A = 0.5\delta\theta$. Substitute these results into Eq. (1)

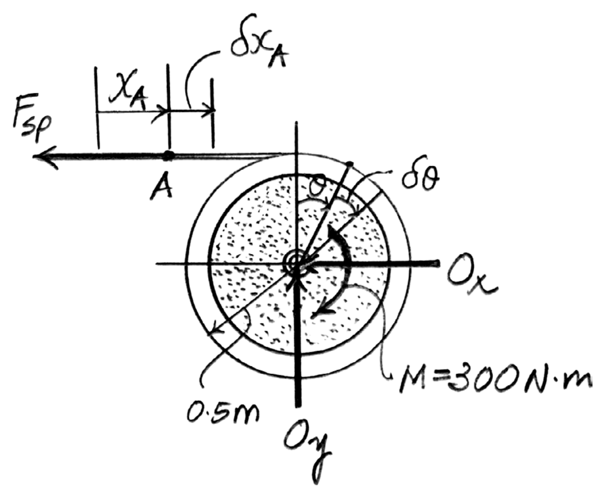
$$300\delta\theta - 2000\theta(0.5\delta\theta) = 0$$

$$(300 - 1000\theta)\delta\theta = 0$$

Since $\delta\theta \neq 0$,

$$300 - 1000\theta = 0$$

$$\theta = 0.3 \text{ rad} = 17.19^\circ = 17.2^\circ \qquad \textbf{Ans.}$$

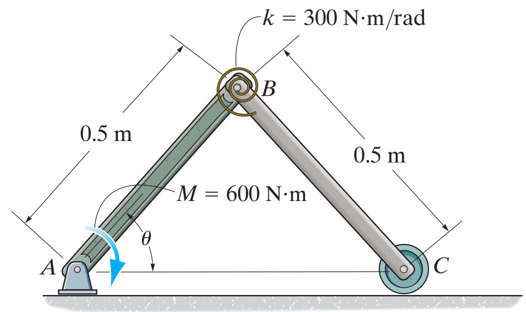


(a)

Ans:
$\theta = 17.2^\circ$

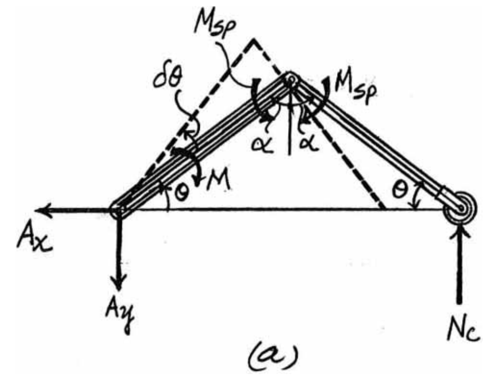
***11–20.**

If the spring has a torsional stiffness of $k = 300 \text{ N}\cdot\text{m/rad}$ and it is unstretched when $\theta = 90°$, determine the angle θ when the frame is in equilibrium.



SOLUTION

Free-Body Diagram: When θ undergoes a positive virtual angular displacement of $\delta\theta$, the dash line configuration shown in Fig. *a* is formed. We observe that only couple moment **M** and the torque $\mathbf{M}_{sp}$ developed in the torsional spring do work when the virtual displacement takes place. The magnitude of $\mathbf{M}_{sp}$ can be computed using the spring force formula,



$$M_{sp} = k(2\alpha) = 300\left[2\left(\frac{\pi}{2} - \theta\right)\right] = 300(\pi - 2\theta)$$

Virtual Displacement: Since $\alpha = \frac{\pi}{2} - \theta$, then

$$\delta\alpha = -\delta\theta \qquad \textbf{(1)}$$

Virtual–Work Equation: Since **M** and $\mathbf{M}_{sp}$ act towards the negative sense of their corresponding angular virtual displacements, their work is negative. Thus,

$\delta U = 0;$

$$-M\delta\theta + 2(-M_{sp}\delta\alpha) = 0 \qquad \textbf{(2)}$$

Substituting $M = 600 \text{ N}\cdot\text{m}$, $M_{sp} = 300(\pi - 2\theta)$, and Eq. (1) into Eq. (2), we have

$$-600\delta\theta - 2[300(\pi - 2\theta)](-\delta\theta) = 0$$
$$\delta\theta[-600 + 600(\pi - 2\theta)] = 0$$

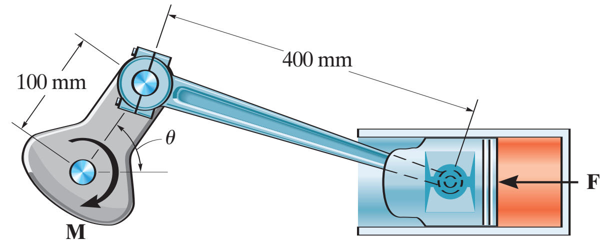
Since $\delta\theta \neq 0$, then

$$-600 + 600(\pi - 2\theta) = 0$$
$$\theta = 1.071 \text{ rad} = 61.4° \qquad \textbf{Ans.}$$

Ans:
$\theta = 61.4°$

11–21.

The crankshaft is subjected to a torque of $M = 50\text{ N}\cdot\text{m}$. Determine the horizontal compressive force F applied to the piston for equilibrium when $\theta = 60°$.

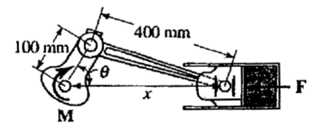


SOLUTION

$$(0.4)^2 = (0.1)^2 + x^2 - 2(0.1)(x)(\cos\theta)$$

$$0 = 0 + 2x\,\delta x + 0.2x\sin\theta\,\delta\theta - 0.2\cos\theta\,\delta x$$

$$\delta U = 0; \qquad -50\delta\theta - F\delta x = 0$$



For $\theta = 60°$, $\quad x = 0.4405\text{ m}$

$$\delta x = -0.09769\,\delta\theta$$ **Ans.**

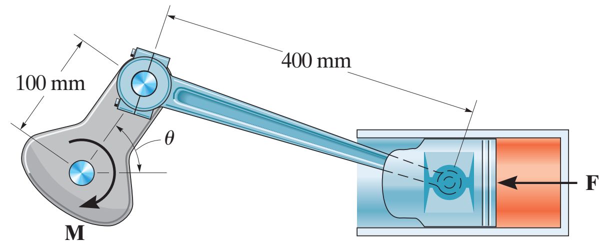
$$(-50 + 0.09769F)\,\delta\theta = 0$$

$$F = 512\text{ N}$$ **Ans.**

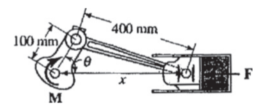
Ans:
$\delta x = -0.09769\,\delta\theta$
$F = 512\text{ N}$

11–22.

The crankshaft is subjected to a torque of $M = 50\ \text{N}\cdot\text{m}$. Determine the horizontal compressive force F and plot the result of F (ordinate) versus θ (abscissa) for $0° \leq \theta \leq 90°$.



SOLUTION



$$(0.4)^2 = (0.1)^2 + x^2 - 2(0.1)(x)(\cos\theta) \qquad \textbf{(1)}$$

$$0 = 0 + 2x\,\delta x + 0.2x\sin\theta\,\delta\theta - 0.2\cos\theta\,\delta x$$

$$\delta x = \left(\frac{0.2x\sin\theta}{0.2\cos\theta - 2x}\right)\delta\theta$$

$$\delta U = 0; \qquad -50\delta\theta - F\delta x = 0$$

$$-50\delta\theta - F\left(\frac{0.2x\sin\theta}{0.2\cos\theta - 2x}\right)\delta\theta = 0, \qquad \delta\theta \neq 0$$

$$F = \frac{50(2x - 0.2\cos\theta)}{0.2x\sin\theta}$$

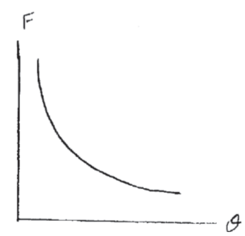
From Eq. (1)

$$x^2 - 0.2x\cos\theta - 0.15 = 0$$

$$x = \frac{0.2\cos\theta \pm \sqrt{0.04\cos^2\theta + 0.6}}{2}, \qquad \text{since}\sqrt{0.04\cos^2\theta + 0.6} > 0.2\cos\theta$$

$$x = \frac{0.2\cos\theta + \sqrt{0.04\cos^2\theta + 0.6}}{2}$$

$$F = \frac{500\sqrt{0.04\cos^2\theta + 0.6}}{(0.2\cos\theta + \sqrt{0.04\cos^2\theta + 0.6})\sin\theta} \qquad \textbf{Ans.}$$

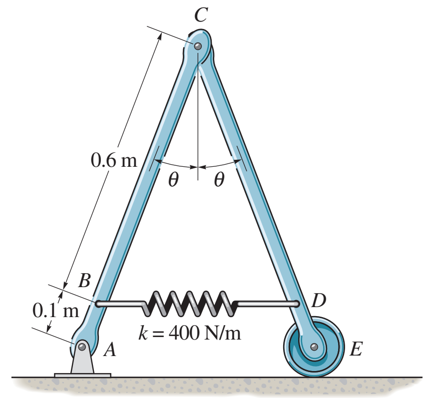


Ans:

$$F = \frac{500\sqrt{0.04\cos^2\theta + 0.6}}{(0.2\cos\theta + \sqrt{0.04\cos^2\theta + 0.6})\sin\theta}$$

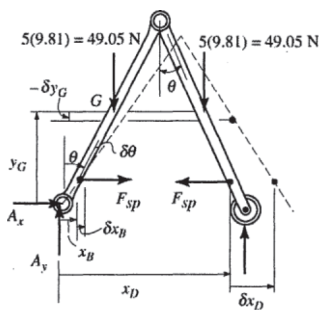
11–23.

The spring has an unstretched length of 0.3 m. Determine the angle θ for equilibrium if the uniform links each have a mass of 5 kg.



SOLUTION

Free Body Diagram: The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the spring force F_{sp} and the weights of the links (49.05 N) do work.



Virtual Displacements: The position of points *B, D* and *G* are measured from the fixed point *A* using position coordinates x_B, x_D and y_G, respectively.

$$x_B = 0.1 \sin\theta \quad \delta x_B = 0.1 \cos\theta\delta\theta \qquad \textbf{(1)}$$

$$x_D = 2(0.7 \sin\theta) - 0.1 \sin\theta = 1.3 \sin\theta \quad \delta x_D = 1.3 \cos\theta\delta\theta \qquad \textbf{(2)}$$

$$y_G = 0.35 \cos\theta \quad \delta y_G = -0.35 \sin\theta\delta\theta \qquad \textbf{(3)}$$

Virtual–Work Equation: When points *B, D* and *G* undergo positive virtual displacements δx_B, δx_D and δy_G, the spring force F_{sp} that acts at point *B* does positive work while the spring force F_{sp} that acts at point *D* and the weight of link *AC* and *CE* (49.05 N) do negative work.

$$\delta U = 0; \qquad 2(-49.05\delta y_G) + F_{sp}(\delta x_B - \delta x_D) = 0 \qquad \textbf{(4)}$$

Substituting Eqs. (1), (2) and (3) into (4) yields

$$(34.335 \sin\theta - 1.2F_{sp} \cos\theta)\,\delta\theta = 0 \qquad \textbf{(5)}$$

However, from the spring formula, $F_{sp} = kx = 400[2(0.6 \sin\theta) - 0.3] = 480 \sin\theta - 120$. Substituting this value into Eq. (5) yields

$$(34.335 \sin\theta - 576 \sin\theta\cos\theta + 144 \cos\theta)\,\delta\theta = 0$$

Since $\delta\theta \neq 0$, then

$$34.335 \sin\theta - 576 \sin\theta\cos\theta + 144 \cos\theta = 0$$

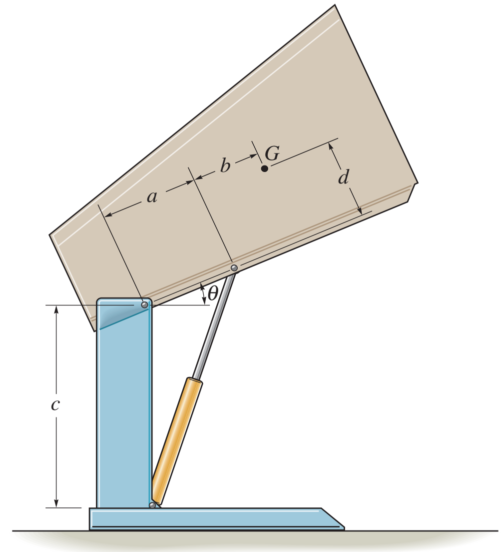
$$\theta = 15.5° \qquad \textbf{Ans.}$$

$$\text{and } \theta = 85.4° \qquad \textbf{Ans.}$$

Ans:
$\theta = 15.5°$
$\theta = 85.4°$

***11–24.**

The dumpster has a weight W and a center of gravity at G. Determine the force in the hydraulic cylinder needed to hold it in the general position θ.



SOLUTION

$$s = \sqrt{a^2 + c^2 - 2a\,c\cos(\theta + 90^\circ)}$$

$$= \sqrt{a^2 + c^2 + 2a\,c\sin\theta}$$

$$\delta s = (a^2 + c^2 + 2a\,c\sin\theta)^{-\frac{1}{2}}\,ac\cos\theta\,\delta\theta$$

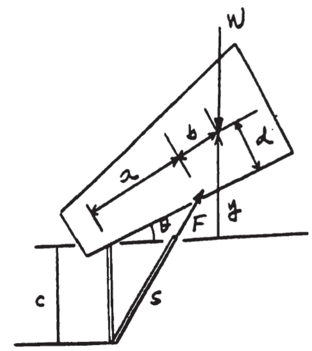
$$y = (a + b)\sin\theta + d\cos\theta$$

$$\delta y = (a + b)\cos\theta\,\delta\theta - d\sin\theta\,\delta\theta$$

$$\delta U = 0; \qquad F\delta s - W\delta y = 0$$

$$F(a^2 + c^2 + 2a\,c\sin\theta)^{-\frac{1}{2}}\,ac\cos\theta\,\delta\theta - W(a + b)\cos\theta\,\delta\theta + Wd\sin\theta\,\delta\theta = 0$$

$$F = \left(\frac{W(a + b - d\tan\theta)}{ac}\right)\sqrt{a^2 + c^2 + 2a\,c\sin\theta}$$ **Ans.**



Ans:

$$F = \left(\frac{W(a + b - d\tan\theta)}{ac}\right)\sqrt{a^2 + c^2 + 2a\,c\sin\theta}$$

11–25. If the potential energy for a conservative one-degree-of-freedom system is expressed by the relation $V = (3y^3 + 2y^2 - 4y + 50)$ J, where y is given in meters, determine the equilibrium positions and investigate the stability at each position.

SOLUTION

Potential Function:

$$V = 3y^3 + 2y^2 - 4y + 50$$

Equilibrium Configuration: Taking the first derivative of V,

$$\frac{dV}{d\theta} = 9y^2 + 4y - 4 = 0$$

Thus,

$y = 0.481$ m, $\quad y = -925$ m **Ans.**

Stability: The second derivative of V is

$$\frac{d^2V}{d\theta^2} = 18y + 4$$

At $y = 0.481$ m, $\frac{d^2V}{d\theta^2} = 12.7 > 0$ Stable **Ans.**

At $y = -925$ m, $\frac{d^2V}{d\theta^2} = -12.7 < 0$ Unstable **Ans.**

Ans:
$y = 0.481$ m Stable
$y = -925$ m Unstable

11–26.

If the potential function for a conservative one-degree-of-freedom system is $V = (10\cos 2\theta + 25\sin\theta)$ J, where $0° < \theta < 180°$, determine the positions for equilibrium and investigate the stability at each of these positions.

SOLUTION

$$V = 10\cos 2\theta + 25\sin\theta$$

For equilibrium:

$$\frac{dV}{d\theta} = -20\sin 2\theta + 25\cos\theta = 0$$

$$(-40\sin\theta + 25)\cos\theta = 0$$

$$\theta = \sin^{-1}\left(\frac{25}{40}\right) = 38.7° \text{ and } 141° \qquad \textbf{Ans.}$$

and

$$\theta = \cos^{-1} 0 = 90° \qquad \textbf{Ans.}$$

Stability:

$$\frac{d^2V}{d\theta^2} = -40\cos 2\theta - 25\sin\theta$$

$$\theta = 38.7°, \quad \frac{d^2V}{d\theta^2} = -24.4 < 0, \quad \text{Unstable} \qquad \textbf{Ans.}$$

$$\theta = 141°, \quad \frac{d^2V}{d\theta^2} = -24.4 < 0, \quad \text{Unstable} \qquad \textbf{Ans.}$$

$$\theta = 90°, \quad \frac{d^2V}{d\theta^2} = 15 > 0, \quad \text{Stable} \qquad \textbf{Ans.}$$

Ans:
$\theta = 38.7°$ unstable
$\theta = 90°$ stable
$\theta = 141°$ unstable

11–27.

If the potential function for a conservative one-degree-of-freedom system is $V = (8x^3 - 2x^2 - 10)$ J, where x is given in meters, determine the positions for equilibrium and investigate the stability at each of these positions.

SOLUTION

$$V = 8x^3 - 2x^2 - 10$$

$$\frac{dV}{dx} = 24x^2 - 4x = 0$$

$$(24x - 4)x = 0$$

$x = 0$ and $x = 0.167$ m **Ans.**

$$\frac{d^2V}{dx^2} = 48x - 4$$

$x = 0, \quad \frac{d^2V}{dx^2} = -4 < 0$ Unstable **Ans.**

$x = 0.167$ m, $\quad \frac{d^2V}{dx^2} = 4 > 0$ Stable **Ans.**

Ans:
$x = 0.167$ m
$\frac{d^2V}{dx^2} = -4 < 0$ Unstable
$\frac{d^2V}{dx^2} = 4 > 0$ Stable

***11–28.** If the potential energy for a conservative one-degree-of-freedom system is expressed by the relation $V = (24 \sin\theta + 10 \cos 2\theta)$ J, $0° \leq \theta \leq 90°$, determine the equilibrium positions and investigate the stability at each position.

SOLUTION

$V = 24 \sin\theta + 10 \cos 2\theta$

Equilibrium Position:

$$\frac{dV}{d\theta} = 24 \cos\theta - 20 \sin 2\theta = 0$$

$$24 \cos\theta - 40 \sin\theta \cos\theta = 0$$

$$\cos\theta(24 - 40 \sin\theta) = 0$$

$\cos\theta = 0 \qquad \theta = 90°$ **Ans.**

$24 - 40 \sin\theta = 0 \qquad \theta = 36.9°$ **Ans.**

Stability:

$$\frac{d^2V}{d\theta^2} = -40 \cos 2\theta - 24 \sin\theta$$

At $\theta = 90°$ $\quad \dfrac{d^2V}{d\theta^2} = -40 \cos 180° - 24 \sin 90° = 16 > 0$ stable **Ans.**

At $\theta = 36.9°$ $\quad \dfrac{d^2V}{d\theta^2} = -40 \cos 73.7° - 24 \sin 36.9° = -25.6 < 0$ unstable **Ans.**

Ans:
stable at $\theta = 90°$
Unstable at $\theta = 36.9°$

11–29.

If the potential function for a conservative one-degree-of-freedom system is $V = (12 \sin 2\theta + 15 \cos \theta)$ J, where $0° < \theta < 180°$, determine the positions for equilibrium and investigate the stability at each of these positions.

SOLUTION

$$V = 12 \sin 2\theta + 15 \cos \theta$$

$$\frac{dV}{d\theta} = 0; \qquad 24 \cos 2\theta - 15 \sin \theta = 0$$

$$24(1 - 2\sin^2\theta) - 15 \sin \theta = 0$$

$$48 \sin^2\theta + 15 \sin \theta - 24 = 0$$

Choosing the angle $0° < \theta < 180°$

$$\theta = 34.6° \qquad \textbf{Ans.}$$

and

$$\theta = 145° \qquad \textbf{Ans.}$$

$$\frac{d^2V}{d\theta^2} = -48 \sin 2\theta - 15 \cos \theta$$

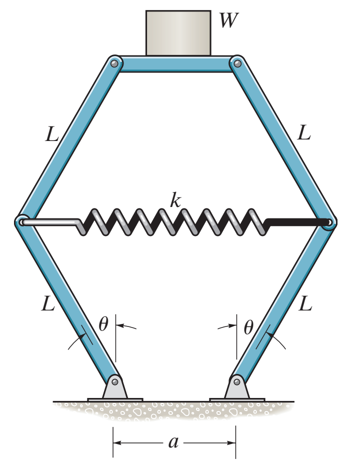
$$\theta = 34.6°, \qquad \frac{d^2V}{d\theta^2} = -57.2 < 0 \qquad \text{Unstable} \qquad \textbf{Ans.}$$

$$\theta = 145°, \qquad \frac{d^2V}{d\theta^2} = 57.2 > 0 \qquad \text{Stable} \qquad \textbf{Ans.}$$

Ans:
Unstable at $\theta = 34.6°$
stable at $\theta = 145°$

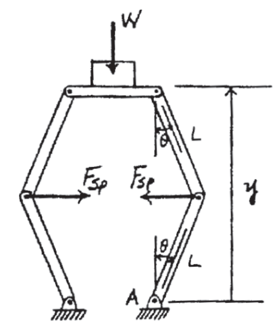
11–30.

The spring of the scale has an unstretched length of a. Determine the angle θ for equilibrium when a weight W is supported on the platform. Neglect the weight of the members. What value W would be required to keep the scale in neutral equilibrium when $\theta = 0°$?



SOLUTION

Potential Function: The datum is established at point A. Since the weight W is above the datum, its potential energy is positive. From the geometry, the spring stretches $x = 2L\sin\theta$ and $y = 2L\cos\theta$.



$$V = V_e + V_g$$

$$= \frac{1}{2}kx^2 + Wy$$

$$= \frac{1}{2}(k)(2L\sin\theta)^2 + W(2L\cos\theta)$$

$$= 2kL^2\sin^2\theta + 2WL\cos\theta$$

Equilibrium Position: The system is in equilibrium if $\dfrac{dV}{d\theta} = 0$.

$$\frac{dV}{d\theta} = 4kL^2\sin\theta\cos\theta - 2WL\sin\theta = 0$$

$$\frac{dV}{d\theta} = 2kL^2\sin 2\theta - 2WL\sin\theta = 0$$

Solving,

$$\theta = 0° \quad \textbf{or} \quad \theta = \cos^{-1}\left(\frac{W}{2kL}\right) \qquad \textbf{Ans.}$$

Stability: To have neutral equilibrium at $\theta = 0°$, $\left.\dfrac{d^2V}{d\theta^2}\right|_{\theta - 0°} = 0$.

$$\frac{d^2V}{d\theta^2} = 4kL^2\cos 2\theta - 2WL\cos\theta$$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta - 0°} = 4kL^2\cos 0° - 2WL\cos 0° = 0$$

$$W = 2kL \qquad \textbf{Ans.}$$

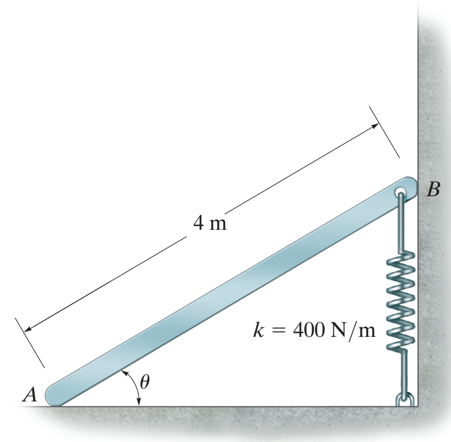
Ans:

$\theta = \cos^{-1}\left(\dfrac{W}{2kL}\right)$

$W = 2kL$

11–31.

The uniform bar has a mass of 80 kg. Determine the angle θ for equilibrium and investigate the stability of the bar when it is in this position. The spring has an unstretched length when $\theta = 90°$.



SOLUTION

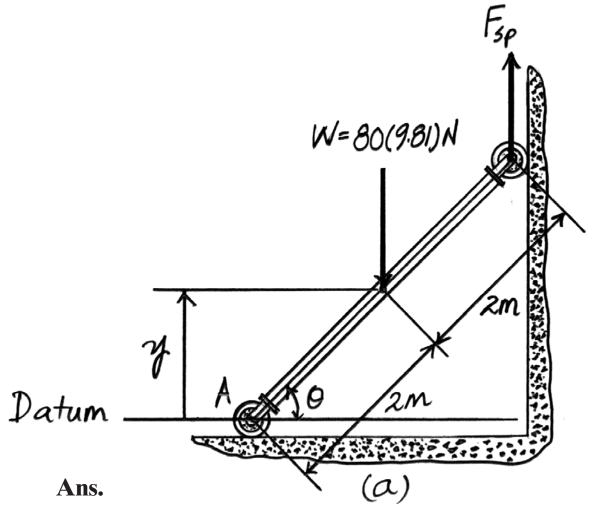
Potential Function. The Datum is established through point A, Fig. a. Since the center of gravity of the bar is above the datum, its potential energy is positive. Here, $y = 2 \sin \theta$ and the spring stretches $x = 4(1 - \sin \theta)$ m. Thus,

$$V = V_e + V_g$$
$$= \frac{1}{2}kx^2 + Wy$$
$$= \frac{1}{2}(400)\left[4(1 - \sin \theta)\right]^2 + 80(9.81)(2 \sin \theta)$$
$$= 3200 \sin^2 \theta - 4830.4 \sin \theta + 3200$$

Equilibrium Position. The bar is in equilibrium of $\frac{dV}{d\theta} = 0$

$$\frac{dV}{d\theta} = 6400 \sin \theta \cos \theta - 4830.4 \cos \theta = 0$$

$$\cos \theta(6400 \sin \theta - 4830.4) = 0$$



Solving,

$$\theta = 90° \text{ or } \theta = 49.00° = 49.0°$$ **Ans.**

Using the trigonometry identify $\sin 2\theta = 2 \sin \theta \cos \theta$,

$$\frac{dV}{d\theta} = 3200 \sin 2\theta - 4830.4 \cos \theta$$

$$\frac{d^2V}{d\theta^2} = 6400 \cos 2\theta + 4830.4 \sin \theta$$

Stability. The equilibrium configuration is stable if $\frac{d^2V}{d\theta^2} > 0$, unstable if $\frac{d^2V}{d\theta^2} < 0$ and neutral if $\frac{d^2V}{d\theta^2} = 0$.

At $\theta = 90°$,

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=90°} = 6400 \cos[2(90°)] + 4830.4 \sin 90° = -1569.6 < 0$$

Thus, the bar is in **unstable equilibrium** at $\theta = 90°$

At $\theta = 49.00°$,

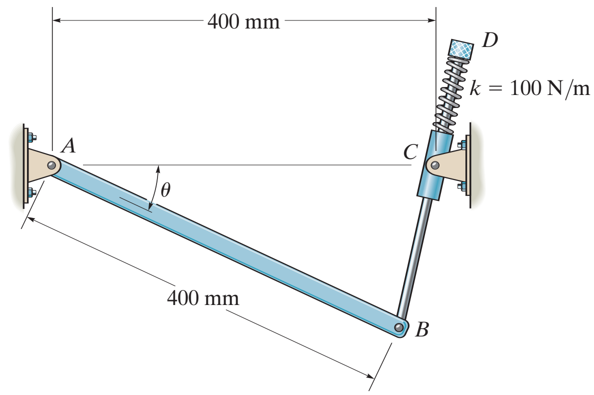
$$\frac{d^2V}{d\theta^2} = 6400 \cos[2(49.00°)] + 4830.4 \sin 49.00° = 2754.26 > 0$$

Thus, the bar is in **stable equilibrium** at $\theta = 49.0°$.

Ans:
Unstable equilibrium at $\theta = 90°$
Stable equilibrium at $\theta = 49.0°$

***11–32.**

The uniform link AB has a mass of 3 kg and is pin connected at both of its ends. The rod BD, having negligible weight, passes through a swivel block at C. If the spring has a stiffness of $k = 100$ N/m and is unstretched when $\theta = 0°$, determine the angle θ for equilibrium and investigate the stability at the equilibrium position. Neglect the size of the swivel block.



SOLUTION

$$s = \sqrt{(0.4)^2 + (0.4)^2 - 2(0.4)^2 \cos\theta}$$

$$= (0.4)\sqrt{2(1 - \cos\theta)}$$

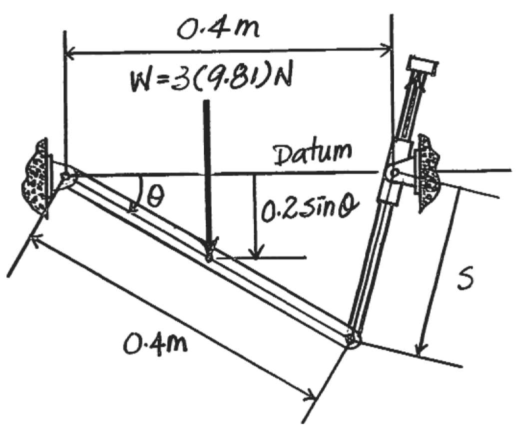
$$V = V_g + V_e$$

$$= -(0.2)(\sin\theta)3(9.81) + \frac{1}{2}(100)\left[(0.4)^2(2)(1 - \cos\theta)\right]$$

$$\frac{dV}{d\theta} = -(5.886)\cos\theta + 16(\sin\theta) = 0 \qquad \textbf{(1)}$$

$$\theta = 20.2° \qquad \textbf{Ans.}$$

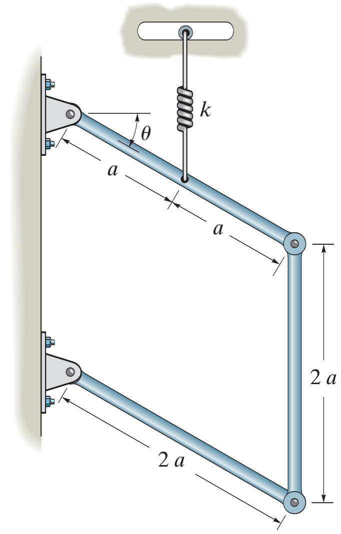
$$\frac{d^2V}{d\theta^2} = 5.886\sin\theta + (16)\cos\theta = 17.0 > 0 \qquad \textbf{stable} \quad \textbf{Ans.}$$



Ans:
$\theta = 20.2°$
stable

11–33.

If each of the three links of the mechanism has a weight W, determine the angle θ for equilibrium. The spring, which always remains vertical, is unstretched when $\theta = 0°$.



SOLUTION

$$y_1 = a\sin\theta \qquad \delta y_1 = a\cos\theta\,\delta\theta$$

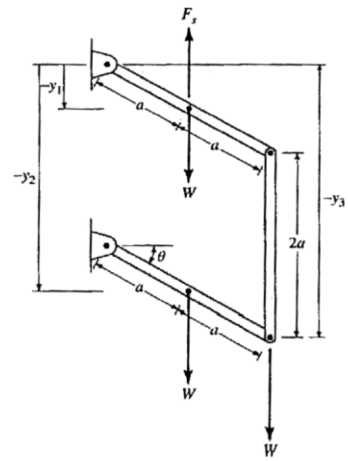
$$y_2 = 2a + a\sin\theta \qquad \delta y_2 = a\cos\theta\,\delta\theta$$

$$y_3 = 2a + 2a\sin\theta \qquad \delta y_3 = 2a\cos\theta\,\delta\theta$$

$$F_s = ka\sin\theta$$

$$\delta U = 0;\ (W - F_s)\delta y_1 + W\delta y_2 + W\delta y_3 = 0$$

$$(W - ka\sin\theta)a\cos\theta\,\delta\theta + Wa\cos\theta\,\delta\theta + W(2a)\cos\theta\,\delta\theta = 0$$



Assume $\theta < 90°$, so $\cos\theta \neq 0$.

$$4W - ka\sin\theta = 0$$

$$\theta = \sin^{-1}\left(\frac{4W}{ka}\right) \qquad \textbf{Ans.}$$

or

$$\theta = 90° \qquad \textbf{Ans.}$$

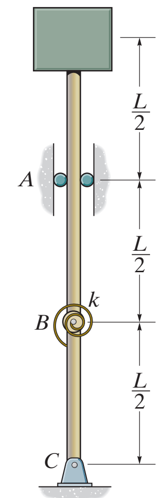
Ans:

$\theta = \sin^{-1}\left(\frac{4W}{ka}\right)$

$\theta = 90°$

11–34.

A spring with a torsional stiffness k is attached to the pin at B. It is unstretched when the rod assembly is in the vertical position. Determine the weight W of the block that results in neutral equilibrium. *Hint*: Establish the potential energy function for a small angle θ. i.e., approximate $\sin\theta \approx 0$, and $\cos\theta \approx 1 - \theta^2/2$.



SOLUTION

Potential Function: With reference to the datum, Fig. a, the gravitational potential energy of the block is positive since its center of gravity is located above the datum. Here, the rods are tilted with a small angle θ. Thus, $y = \frac{L}{2}\cos\theta + L\cos\theta = \frac{3}{2}L\cos\theta$. However, for a small angle θ, $\cos\theta \cong 1 - \frac{\theta^2}{2}$. Thus,

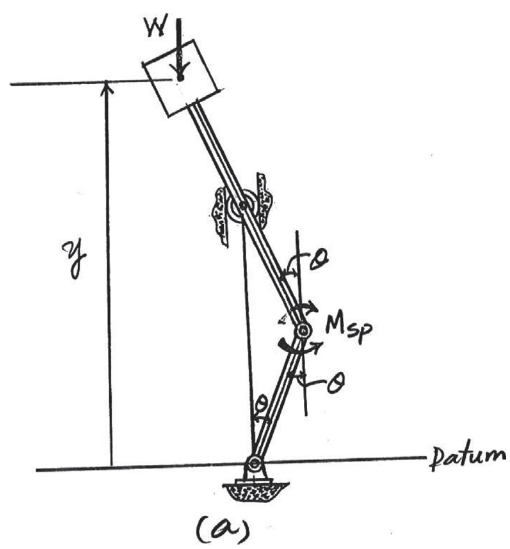
$$V_g = Wy = W\left(\frac{3}{2}L\right)\left(1 - \frac{\theta^2}{2}\right) = \frac{3WL}{2}\left(1 - \frac{\theta^2}{2}\right)$$

The elastic potential energy of the torsional spring can be computed using $V_e = \frac{1}{2}k\beta^2$, where $\beta = 2\theta$. Thus,

$$V_g = \frac{1}{2}k(2\theta)^2 = 2k\theta^2$$

The total potential energy of the system is

$$V = V_g + V_e = \frac{3WL}{2}\left(1 - \frac{\theta^2}{2}\right) + 2k\theta^2$$



Equilibrium Configuration: Taking the first derivative of V, we have

$$\frac{dV}{d\theta} = -\frac{3WL}{2}\theta + 4k\theta = \theta\left(-\frac{3WL}{2} + 4k\right)$$

Equilibrium requires $\frac{dV}{d\theta} = 0$. Thus,

$$\theta\left(-\frac{3WL}{2} + 4k\right) = 0$$

$$\theta = 0°$$

Stability: The second derivative of V is

$$\frac{d^2V}{d\theta^2} = -\frac{3WL}{2} + 4k$$

To have neutral equilibrium at $\theta = 0°$, $\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0°} = 0$. Thus,

$$-\frac{3WL}{2} + 4k = 0$$

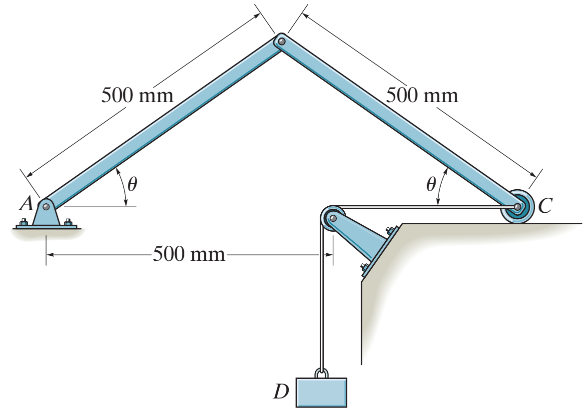
$$W = \frac{8k}{3L}$$ **Ans.**

Note: The equilibrium configuration of the system at $\theta = 0°$ is stable if $W < \frac{8k}{3L}\left(\frac{d^2V}{d\theta^2} > 0\right)$ and is unstable if $W > \frac{8k}{3L}\left(\frac{d^2V}{d\theta^2} < 0\right)$.

Ans:
$W = \frac{8k}{3L}$

11–35.

Determine the angle θ for equilibrium and investigate the stability at this position. The bars each have a mass of 3 kg and the suspended block D has a mass of 7 kg. Cord DC has a total length of 1 m.



SOLUTION

$$l = 500 \text{ mm}$$

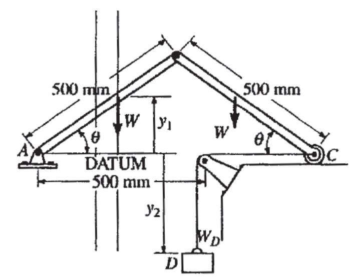
$$y_1 = \frac{l}{2}\sin\theta$$

$$y_2 = l + 2l(1 - \cos\theta) = l(3 - 2\cos\theta)$$

$$V = 2Wy_1 - W_D y_2$$

$$= Wl\sin\theta - W_D l(3 - 2\cos\theta)$$

$$\frac{dV}{d\theta} = l(W\cos\theta - 2W_D\sin\theta) = 0$$



$$\tan\theta = \frac{W}{2W_D} = \frac{3(9.81)}{14(9.81)} = 0.2143$$

$$\theta = 12.1^\circ$$ **Ans.**

$$\frac{d^2V}{d\theta^2} = l(-W\sin\theta - 2W_D\cos\theta)$$

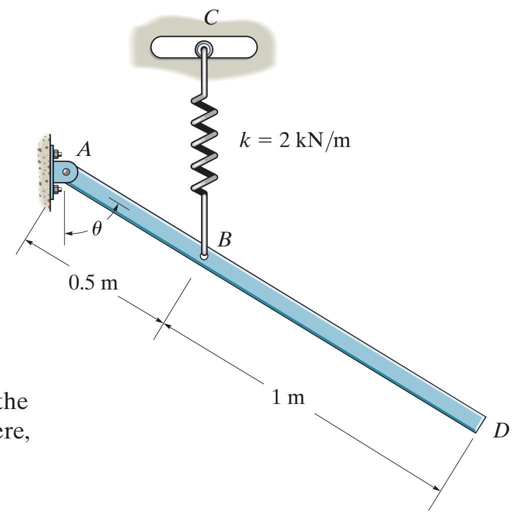
$\theta = 12.1^\circ$, $$\frac{d^2V}{d\theta^2} = 0.5[-3(9.81)\sin 12.1^\circ - 14(9.81)\cos 12.1^\circ]$$

$$= -70.2 < 0$$ Unstable **Ans.**

Ans:
$\theta = 12.1^\circ$
unstable

***11–36.**

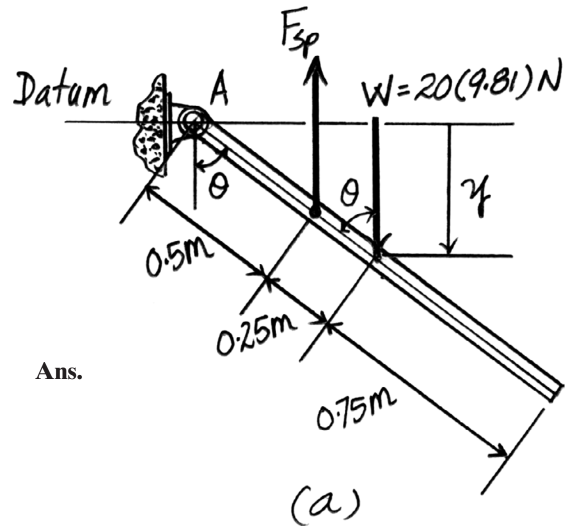
The uniform bar AD has a mass of 20 kg. If the attached spring is unstretched when $\theta = 90°$, determine the angle θ for equilibrium. Note that the spring always remains in the vertical position due to the roller guide. Investigate the stability of the bar when it is in the equilibrium position.



SOLUTION

Potential Function. The Datum is established through point A, Fig. *a*. Since the center of gravity of the bar is below the datum, its potential energy is negative. Here, $y = 0.75 \cos\theta$ and the spring stretches $x = 0.5 \cos\theta$. Thus,

$$V = V_e + V_g$$
$$= \frac{1}{2}kx^2 + \Sigma Wy$$
$$= \frac{1}{2}(2000)(0.5\cos\theta)^2 + \left[-20(9.81)(0.75\cos\theta)\right]$$
$$= 250\cos^2\theta - 147.15\cos\theta$$



Equilibrium Position. The bar is in equilibrium if $\frac{dV}{d\theta} = 0$.

$$\frac{dV}{d\theta} = -500\sin\theta\cos\theta + 147.15\sin\theta = 0$$
$$\sin\theta(147.15 - 500\cos\theta) = 0$$

Solving,

$$\theta = 0° \qquad \theta = 72.88° = 72.9° \qquad \textbf{Ans.}$$

Using the trigonometry identity $\sin 2\theta = 2\sin\theta\cos\theta$,

$$\frac{dV}{d\theta} = -250\sin 2\theta + 147.15\sin\theta$$
$$\frac{d^2V}{d\theta^2} = -500\cos 2\theta + 147.15\cos\theta$$

Stability. The equilibrium configuration is stable if $\frac{d^2V}{d\theta^2} > 0$, unstable if $\frac{d^2V}{d\theta^2} < 0$ and neutral if $\frac{d^2V}{d\theta^2} = 0$.

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0°} = -500\cos 0° + 147.15\cos 0° = -352.85 < 0$$

Thus, the bar is in **unstable equilibrium** at $\theta = 0°$.

At $\theta = 72.88°$,

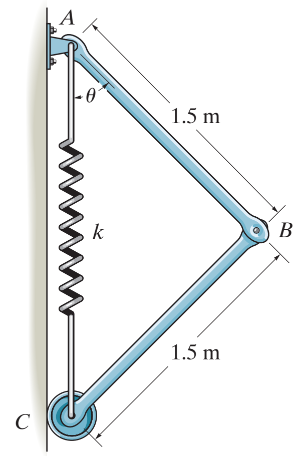
$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=72.88°} = -500\cos[2(72.88°)] + 147.15\cos 72.88° = 456.69 > 0$$

Thus, the bar is in **stable equilibrium** at $\theta = 72.9°$.

Ans:
Unstable equilibrium at $\theta = 0°$
Stable equilibrium at $\theta = 72.9°$

11–37.

The two bars each have a mass of 8 kg. Determine the required stiffness k of the spring so that the two bars are in equilibrium when $\theta = 60°$. The spring has an unstretched length of 1 m. Investigate the stability of the system at the equilibrium position.



SOLUTION

Potential Function. The Datum is established through point A, Fig. a. Since the centers of gravity of the bars are below the datum, their potential energies are negative. Here, $y_1 = 0.75 \cos\theta$, $y_2 = 1.5\cos\theta + 0.75\cos\theta = 2.25\cos\theta$ and the spring stretches $x = 2(1.5\cos\theta) - 1 = (3\cos\theta - 1)$ m. Thus,

$$V = V_e + V_g$$

$$= \frac{1}{2}kx^2 + \Sigma Wy$$

$$= \frac{1}{2}k(3\cos\theta - 1)^2 + [-8(9.81)(0.75\cos\theta)] + [-8(9.81)(2.25\cos\theta)]$$

$$= 4.5\,k\cos^2\theta - 3\,k\cos\theta + 0.5\,k - 235.44\cos\theta$$

$$= 4.5\,k\cos^2\theta - (3\,k + 235.44)\cos\theta + 0.5\,k$$

Equilibrium Position. The system is in equilibrium if $\frac{dV}{d\theta} = 0$

$$\frac{dV}{d\theta} = -9\,k\sin\theta\cos\theta + (3\,k + 235.44)\sin\theta = 0$$

$$\sin\theta(-9\,k\cos\theta + 3\,k + 235.44) = 0$$

Since $\sin\theta \neq 0$, then

$$-9\,k\cos\theta + 3\,k + 235.44 = 0$$

$$k = \frac{235.44}{9\cos\theta - 3}$$

When $\theta = 60°$,

$$k = \frac{235.44}{9\cos 60° - 3} = 156.96\text{ N/m} = 157\text{ N/m} \qquad \textbf{Ans.}$$

Using this result,

$$\frac{dV}{d\theta} = -9(156.96)\sin\theta\cos\theta + [3(156.96) + 235.44]\sin\theta$$

$$= -1412.64\sin\theta\cos\theta + 706.32\sin\theta$$

Using the trigonometry identity $\sin 2\theta = 2\sin\theta\cos\theta$,

$$\frac{dV}{d\theta} = 706.32\sin\theta - 706.32\sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = 706.32\cos\theta - 1412.64\cos 2\theta$$

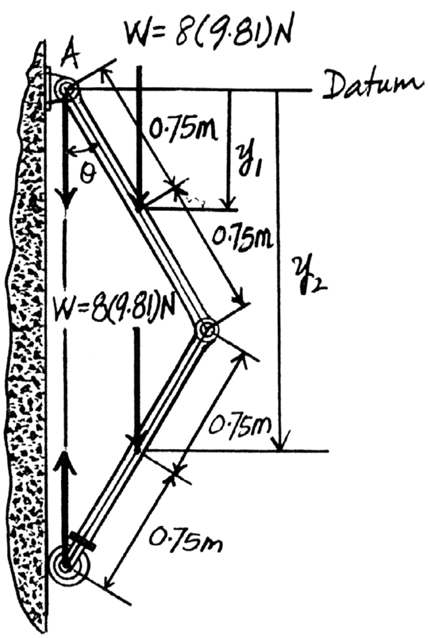
11–37 Continued

Stability. The equilibrium configuration is stable if $\frac{d^2V}{d\theta^2} > 0$, unstable if $\frac{d^2V}{d\theta^2} < 0$, and neutral if $\frac{d^2V}{d\theta^2} = 0$.

At $\theta = 60°$,

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=60°} = 706.32 \cos 60° - 1412.64 \cos [2(60°)] = 1059.16 > 0$$

Thus, the system is in **stable equilibrium** at $\theta = 60°$.

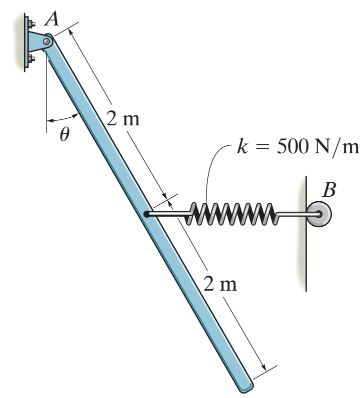


(a)

Ans:
$k = 157$ N/m
Stable equilibrium at $\theta = 60°$

11–38.

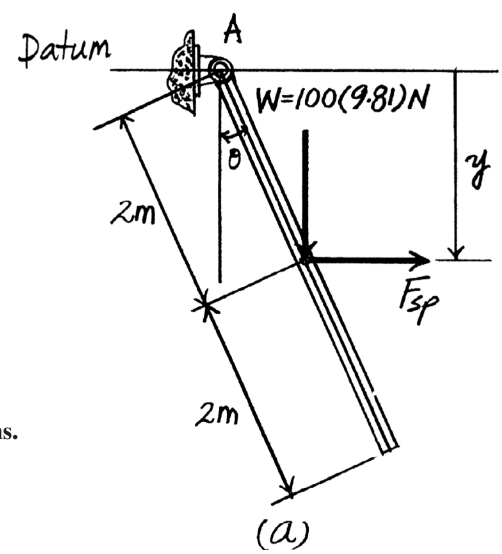
The uniform rod has a mass of 100 kg. If the spring is unstretched when $\theta = 60°$, determine the angle θ for equilibrium and investigate the stability at the equilibrium position. The spring is always in the horizontal position due to the roller guide at B



SOLUTION

Potential Function. The Datum is established through point A, Fig. a. Since the center of gravity of the bar is below the datum, its potential energy is negative. Here $y = 2\cos\theta$, and the spring stretches $x = 2\sin 60° - 2\sin\theta = 2(\sin 60° - \sin\theta)$. Thus

$$\begin{aligned} V &= V_e + V_g \\ &= \frac{1}{2}kx^2 + Wy \\ &= \frac{1}{2}(500)[2(\sin 60° - \sin\theta)]^2 + [-100(9.81)(2\cos\theta)] \\ &= 1000\sin^2\theta - 1000\sqrt{3}\sin\theta - 1962\cos\theta + 750 \end{aligned}$$



Equilibrium Position. The bar is in equilibrium if $\frac{dV}{d\theta} = 0$.

$$\frac{dV}{d\theta} = 2000\sin\theta\cos\theta - 1000\sqrt{3}\cos\theta + 1962\sin\theta$$

Using the trigonometry identity $\sin 2\theta = 2\sin\theta\cos\theta$,

$$\frac{dV}{d\theta} = 1000\sin 2\theta - 1000\sqrt{3}\cos\theta + 1962\sin\theta = 0$$

Solved numerically,

$\theta = 24.62° = 24.6°$ **Ans.**

$$\frac{d^2V}{d\theta^2} = 2000\cos 2\theta + 1000\sqrt{3}\sin\theta + 1962\cos\theta$$

Stability. The equilibrium configuration is stable if $\frac{d^2V}{d\theta^2} > 0$, unstable if $\frac{d^2V}{d\theta^2} < 0$ and neutral if $\frac{d^2V}{d\theta^2} = 0$.

At $\theta = 24.62°$,

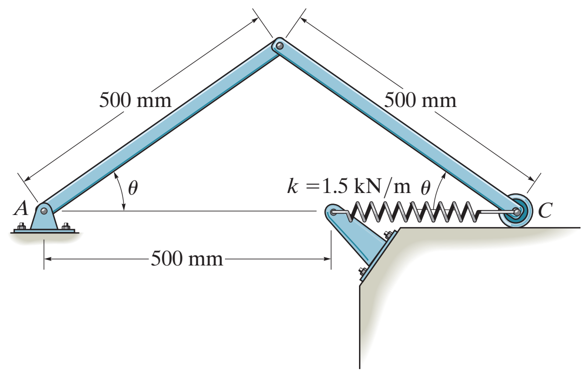
$$\frac{d^2V}{d\theta^2} = 2000\cos[2(24.62°)] + 1000\sqrt{3}\sin 24.62° + 1962\cos 24.62° = 3811.12 > 0$$

Thus, the bar is in **stable equilibrium** at $\theta = 24.6°$.

Ans:
Stable equilibrium at $\theta = 24.6°$

11–39.

Determine the angle θ for equilibrium and investigate the stability at this position. The bars each have a mass of 10 kg and the spring has an unstretched length of 100 mm.



SOLUTION

Potential Function. The Datum is established through point A, Fig. a. Since the centers of gravity of the bars are above the datum, their potential energies are positive. Here $y = 0.25 \sin \theta$ and the spring stretches $x = \left[2(0.5 \cos \theta) - 0.5\right] - 0.1 = (\cos \theta - 0.6)$ m. Thus

$$V = V_e + V_g$$

$$= \frac{1}{2}kx^2 + \Sigma Wy$$

$$= \frac{1}{2}(1500)(\cos \theta - 0.6)^2 + 2[10(9.81)](0.25 \sin \theta)$$

$$= 750 \cos^2 \theta - 900 \cos \theta + 49.05 \sin \theta + 270$$

Equilibrium Position. The system is in equilibrium if $\frac{dV}{d\theta} = 0$.

$$\frac{dV}{d\theta} = -1500 \sin \theta \cos \theta + 900 \sin \theta + 49.05 \cos \theta = 0$$

Solving numerically,

$$\theta = 4.713° = 4.71° \text{ or } \theta = 51.22° = 51.2° \qquad \textbf{Ans.}$$

Using the trigonometry identity $\sin 2\theta = 2 \sin \theta \cos \theta$,

$$\frac{dV}{d\theta} = -750 \sin 2\theta + 900 \sin \theta + 49.05 \cos \theta$$

$$\frac{d^2V}{d\theta^2} = 900 \cos \theta - 1500 \cos 2\theta - 49.05 \sin \theta$$

Stability. The equilibrium configurate is stable if $\frac{d^2V}{d\theta^2} > 0$, unstable if $\frac{d^2V}{d\theta^2} < 0$ and neutral if $\frac{d^2V}{d\theta^2} = 0$.

11–39. Continued

At $\theta = 51.22°$,

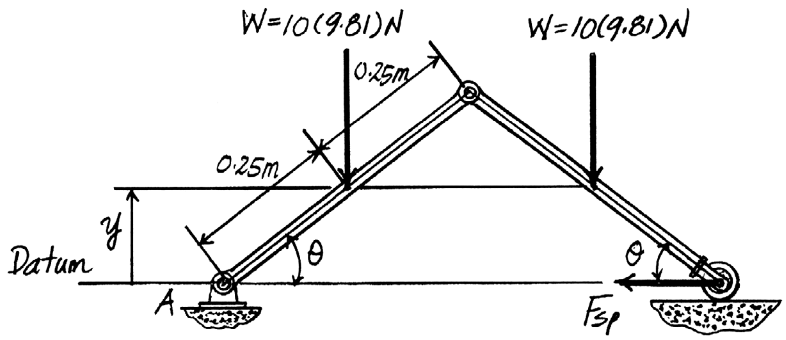
$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=51.22°} = 900 \cos 51.22° - 49.05 \sin 51.22° - 1500 \cos [2(51.22°)] = 848.77 > 0$$

Thus, the system is in **stable equilibrium** at $\theta = 51.2°$.

At $\theta = 4.713°$,

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=4.713°} = 900 \cos 4.713° - 49.05 \sin 4.713° - 1500 \cos [2(4.713°)] = -586.82 < 0$$

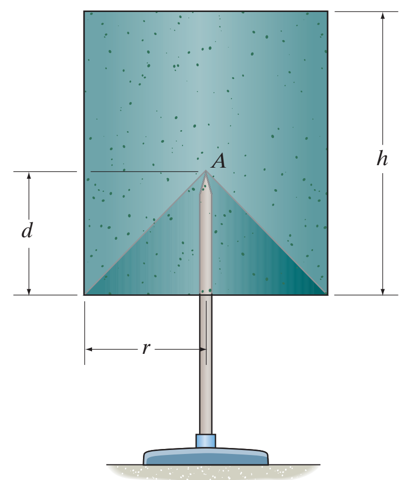
Thus, the system is in **unstable equilibrium** at $\theta = 4.71°$.



(a)

Ans:
Stable equilibrium at $\theta = 51.2°$
Unstable equilibrium at $\theta = 4.71°$

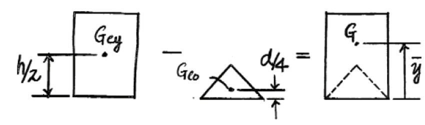
***11–40.** A conical hole is drilled into the bottom of the cylinder, which is supported on the fulcrum at A. Determine the minimum distance d in order for it to remain in stable equilibrium.



SOLUTION

Potential Function: First, we must determine the center of gravity of the cylinder. By referring to Fig. *a*,

$$\bar{y} = \frac{\Sigma y_C m}{\Sigma m} = \frac{\frac{h}{2}(\rho\pi r^2 h) - \frac{d}{4}\left(\frac{1}{3}\rho\pi r^2 d\right)}{\rho\pi r^2 h - \frac{1}{3}\rho\pi r^2 d} = \frac{6h^2 - d^2}{4(3h - d)} \qquad \textbf{(1)}$$



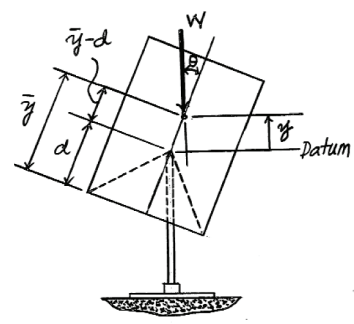
(a)

With reference to the datum, Fig. *a*, the gravitational potential energy of the cylinder is positive since its center of gravity is located above the datum. Here,

$$y = (\bar{y} - d)\cos\theta = \left[\frac{6h^2 - d^2}{4(3h - d)} - d\right]\cos\theta = \left[\frac{6h^2 - 12hd + 3d^2}{4(3h - d)}\right]\cos\theta$$

Thus,

$$V = V_g = Wy = W\left[\frac{6h^2 - 12hd + 3d^2}{4(3h - d)}\right]\cos\theta$$



(b)

Equilibrium Configuration: Taking the first derivative of V,

$$\frac{dV}{d\theta} = -W\left[\frac{6h^2 - 12hd + 3d^2}{4(3h - d)}\right]\sin\theta$$

Equilibrium requires $\frac{dV}{d\theta} = 0$. Thus,

$$-W\left[\frac{6h^2 - 12hd + 3d^2}{4(3h - d)}\right]\sin\theta = 0$$

$$\sin\theta = 0 \qquad \theta = 0^\circ$$

Stability: The second derivative of V is

$$\frac{d^2V}{d^2\theta} = -W\left[\frac{6h^2 - 12hd + 3d^2}{4(3h - d)}\right]\cos\theta$$

To have neutral equilibrium at $\theta = 0^\circ$, $\left.\frac{d^2V}{d^2\theta}\right|_{\theta=0^\circ} = 0$. Thus,

$$-W\left[\frac{6h^2 - 12hd + 3d^2}{4(3h - d)}\right]\cos 0^\circ = 0$$

$$6h^2 - 12hd + 3d^2 = 0$$

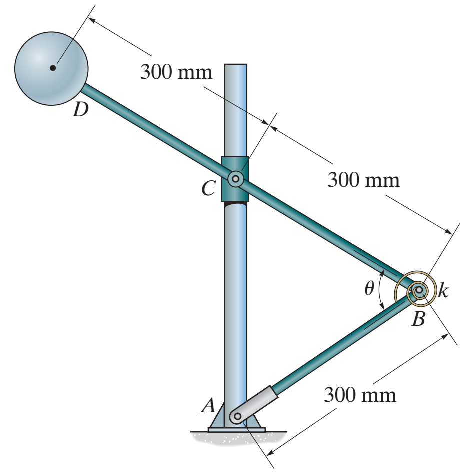
$$d = \frac{12h \pm \sqrt{(-12h)^2 - 4(3)(6h^2)}}{2(3)} = 0.5858h = 0.586h \qquad \textbf{Ans.}$$

Note: If we substitute $d = 0.5858h$ into Eq. (1), we notice that the fulcrum must be at the center of gravity for neutral equilibrium.

Ans:
$d = 0.586h$

11–41.

If the spring has a torsional stiffness $k = 300\ \text{N}\cdot\text{m/rad}$ and is unwound when $\theta = 90°$, determine the angle for equilibrium if the sphere has a mass of 20 kg. Investigate the stability at this position. Collar C can slide freely along the vertical guide. Neglect the weight of the rods and collar C.



SOLUTION

Potential Function: With reference to the datum, Fig. *a,* the gravitational potential energy of the sphere is positive its center of gravity is located above the datum. Here, $y = (0.3\sin(\theta/2) + 0.6\sin(\theta/2))\text{m} = (0.9\sin(\theta/2))$. Thus,

$$V_g = mgy = 20(9.81)(0.9\sin\theta/2) = 176.58\sin(\theta/2)$$

The elastic potential energy of the torsional spring can be computed using $V_e = \frac{1}{2}k\beta^2$, where $\beta = \frac{\pi}{2} - \theta$. Thus,

$$V_e = \frac{1}{2}300\left(\frac{\pi}{2} - \theta\right)^2 = 150\theta^2 - 150\pi\theta + 37.5\pi^2$$

The total potential energy of the system is

$$V = V_g + V_e = 176.58\sin(\theta/2) + 150\theta^2 - 150\pi\theta + 37.5\pi^2$$

Equilibrium Configuration: Taking the first derivative of V, we have

$$\frac{dV}{d\theta} = 88.29\ \cos(\theta/2) + 300\theta - 150\pi$$

Equilibrium requires $\frac{dV}{d\theta} = 0$, Thus,

$$88.29\cos(\theta/2) + 300\theta - 150\pi = 0$$

Solving by trial and error, we obtain

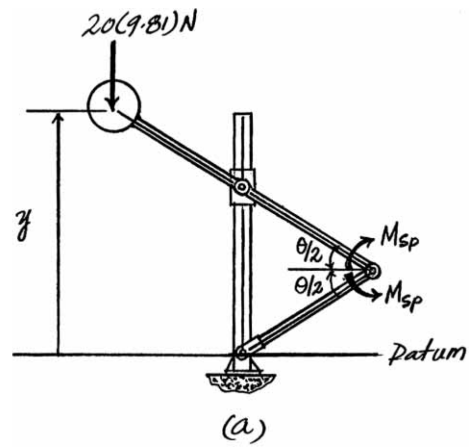
$$\theta = 1.340\text{ rad} = 76.78° = 76.8° \qquad \textbf{Ans.}$$

Stability: The second derivative of V is

$$\frac{d^2V}{d\theta^2} = -44.145\sin(\theta/2) + 300$$

At the equilibrium configuration of $\theta = 76.78°$,

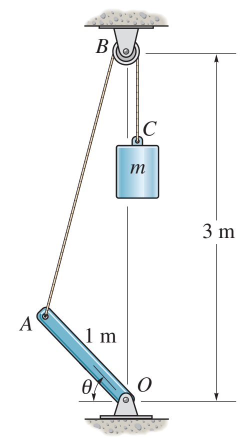
$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=76.78°} = -44.14\sin 38.39° + 300 = 272.58 > 0\ \text{Stable} \qquad \textbf{Ans.}$$



Ans:
$\theta = 76.8°$
Stable

11–42.

If the uniform rod OA has a mass of 12 kg, determine the mass m that will hold the rod in equilibrium when $\theta = 30°$. Point C is coincident with B when OA is horizontal. Neglect the size of the pulley at B.



SOLUTION

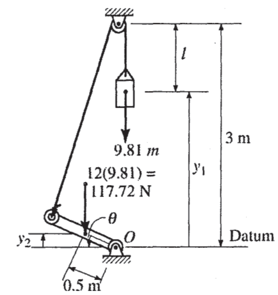
Geometry: Using the law of cosines,

$$l_{A'B} = \sqrt{1^2 + 3^2 - 2(1)(3)\cos(90° - \theta)} = \sqrt{10 - 6\sin\theta}$$

$$l_{AB} = \sqrt{1^2 + 3^2} = \sqrt{10}\text{ m}$$

$$l = l_{AB} - l_{A'B} = \sqrt{10} - \sqrt{10 - 6\sin\theta}$$

Potential Function: The datum is established at point O. Since the center of gravity of the rod and the block are above the datum, their potential energy is positive.



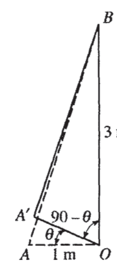
Here, $y_1 = 3 - l = [3 - (\sqrt{10} - \sqrt{10 - 6\sin\theta})]$ m and $y_2 = 0.5\sin\theta$ m.

$$V = V_g = W_1y_1 + W_2y_2$$

$$= 9.81m[3 - (\sqrt{10} - \sqrt{10 - 6\sin\theta})] + 117.72(0.5\sin\theta)$$

$$= 29.43m - 9.81m(\sqrt{10} - \sqrt{10 - 6\sin\theta}) + 58.86\sin\theta$$

Equilibrium Position: The system is in equilibrium if



$$\left.\frac{dV}{d\theta}\right|_{\theta=30°} = 0.$$

$$\frac{dV}{d\theta} = -9.81m\left[-\frac{1}{2}(10 - 6\sin\theta)^{-\frac{1}{2}}(-6\cos\theta)\right] + 58.86\cos\theta$$

$$= -\frac{29.43m\cos\theta}{\sqrt{10 - 6\sin\theta}} + 58.86\cos\theta$$

At $\theta = 30°$,

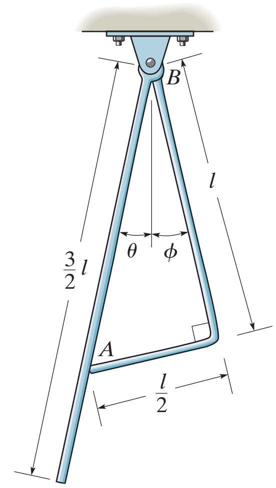
$$\left.\frac{dV}{d\theta}\right|_{\theta=30°} = -\frac{29.43m\cos 30°}{\sqrt{10 - 6\sin 30°}} + 58.86\cos 30° = 0$$

$m = 5.29$ kg **Ans.**

Ans:
$m = 5.29$ kg

11–43.

Each bar has a mass per length of m_0. Determine the angles θ and ϕ at which they are suspended in equilibrium. The contact at A is smooth, and both are pin connected at B.



SOLUTION

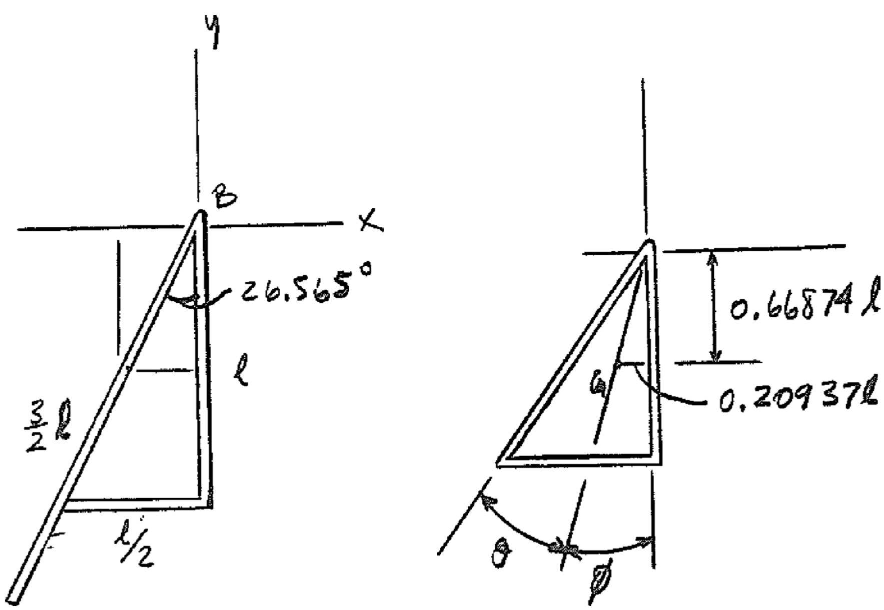
Require G for system to be at its lowest point.

$$\bar{x} = \frac{0 - \frac{l}{4}\left(\frac{l}{2}\right) - (0.75\,l \sin 26.565^\circ)\left(\frac{3}{2}l\right)}{l + \frac{l}{2} + \frac{3}{2}l} = -0.20937\,l$$

$$\bar{y} = \frac{-\left(\frac{1}{2}\right)(l) - l\left(\frac{l_x}{2}\right) - (0.75\,l \cos 26.565^\circ)\left(\frac{3}{2}\,l\right)}{l + \frac{l}{2} + \frac{3}{2}l} = -0.66874\,l$$

$$\phi = \tan^{-1}\left(\frac{0.20937\,l}{0.66874\,l}\right) = 17.38^\circ = 17.4^\circ \qquad \textbf{Ans.}$$

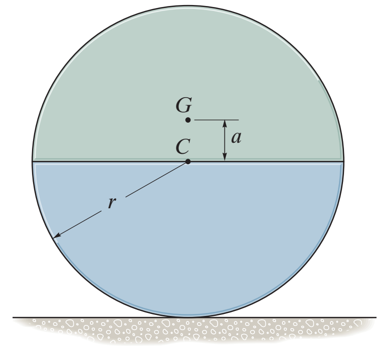
$$\theta = 26.565^\circ - 17.38^\circ = 9.18^\circ \qquad \textbf{Ans.}$$



Ans:
$\phi = 17.4^\circ$
$\theta = 9.18^\circ$

***11–44.**

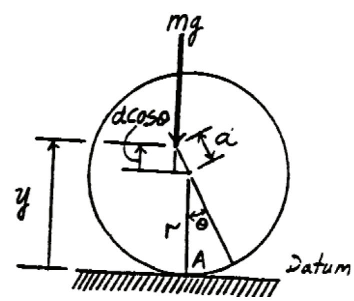
The cylinder is made of two materials such that it has a mass of m and a center of gravity at point G. Show that when G lies above the centroid C of the cylinder, the equilibrium is unstable.



SOLUTION

Potential Function: The datum is established at point A. Since the center of gravity of the cylinder is above the datum, its potential energy is positive. Here, $y = r + d\cos\theta$.

$$V = V_g = Wy = mg(r + d\cos\theta)$$



Equilibrium Position: The system is in equilibrium if $\dfrac{dV}{d\theta} = 0$.

$$\frac{dV}{d\theta} = -mgd\sin\theta = 0$$

$$\sin\theta = 0 \qquad \theta = 0^\circ$$

Stability:

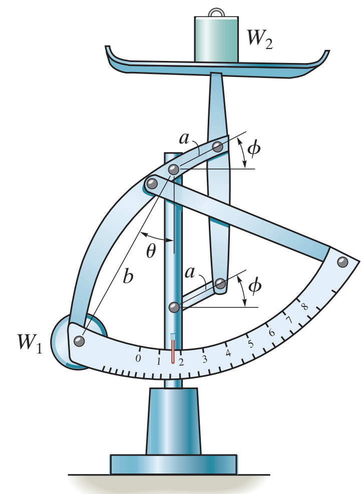
$$\frac{d^2V}{d\theta^2} = -mgd\cos\theta$$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} = -mgd\cos 0^\circ = -mgd < 0$$

Thus, the cylinder is in **unstable equilibrium** at $\theta = 0^\circ$ **(Q.E.D.)**

11–45.

The small postal scale consists of a counterweight W_1, connected to the members having negligible weight. Determine the weight W_2 that is on the pan in terms of the angles θ and ϕ and the dimensions shown. All members are pin connected.



SOLUTION

$$y_1 = b\cos\theta$$

$$y_2 = a\sin\phi = a\sin(90° - \theta - \gamma)$$

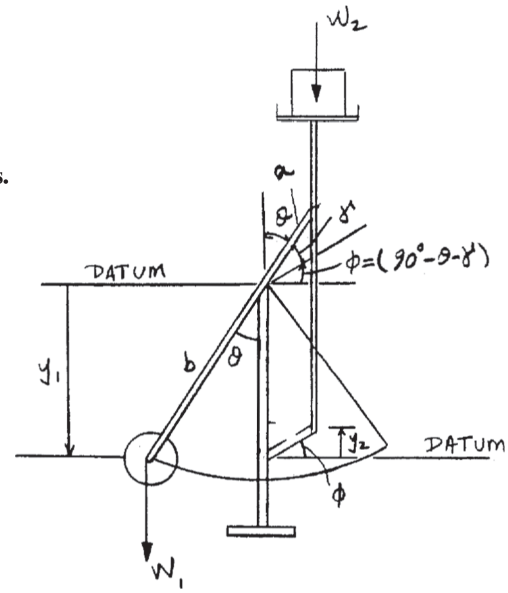
where γ is a constant and $\phi = (90° - \theta - \gamma)$

$$V = -W_1 y_1 + W_2 y_2$$

$$= -W_1 b\cos\theta + W_2 a\sin(90° - \theta - \gamma)$$

$$\frac{dV}{d\theta} = W_1 b\sin\theta - W_2 a\cos(90° - \theta - \gamma)$$

$$W_2 = W_1\left(\frac{b}{a}\right)\frac{\sin\theta}{\cos\phi}$$ **Ans.**

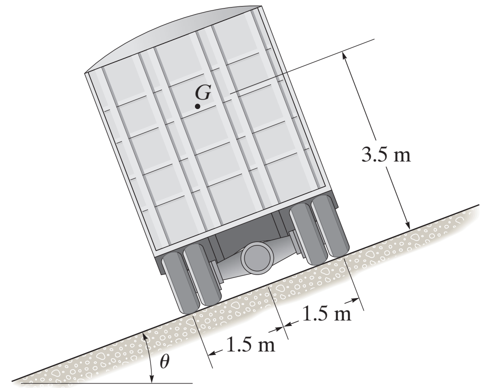


Ans:

$$W_2 = W_1\left(\frac{b}{a}\right)\frac{\sin\theta}{\cos\phi}$$

11–46.

The truck has a mass of 20 Mg and a mass center at G. Determine the steepest grade θ along which it can park without overturning and investigate the stability in this position.

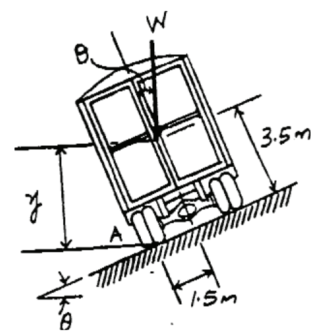


SOLUTION

Potential Function: The datum is established at point A. Since the center of gravity for the truck is above the datum, its potential energy is positive. Here, $y = (1.5 \sin\theta + 3.5\cos\theta)$ m.

$$V = V_g = Wy = W(1.5\sin\theta + 3.5\cos\theta)$$

Equilibrium Position: The system is in equilibrium if $\dfrac{dV}{d\theta} = 0$



$$\frac{dV}{d\theta} = W(1.5\cos\theta - 3.5\sin\theta) = 0$$

Since $W \neq 0$,

$$1.5\cos\theta - 3.5\sin\theta = 0$$

$$\theta = 23.20° = 23.2° \qquad \textbf{Ans.}$$

Stability:

$$\frac{d^2V}{d\theta^2} = W(-1.5\sin\theta - 3.5\cos\theta)$$

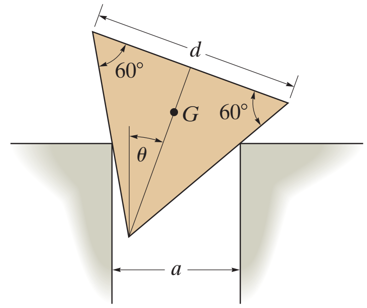
$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=23.20°} = W(-1.5\sin 23.20° - 3.5\cos 23.20°) = -3.81W < 0$$

Thus, the truck is in **unstable equilibrium** at $\theta = 23.2°$ **Ans.**

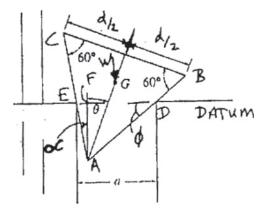
Ans:
Unstable equilibrium at $\theta = 23.2°$

11–47.

The triangular block of weight W rests on the smooth corners which are a distance a apart. If the block has three equal sides of length d, determine the angle θ for equilibrium.



SOLUTION



$$AF = AD\sin\phi = AD\sin(60° - \theta)$$

$$\frac{AD}{\sin\alpha} = \frac{a}{\sin 60°}$$

$$AD = \frac{a}{\sin 60°}(\sin(60° + \theta))$$

$$AF = \frac{a}{\sin 60°}(\sin(60° + \theta))\sin(60° - \theta)$$

$$= \frac{a}{\sin 60°}(0.75\cos^2\theta - 0.25\sin^2\theta)$$

$$y = \frac{d}{\sqrt{3}}\cos\theta - AF$$

$$V = Wy$$

$$\frac{dV}{d\theta} = W\left[(-0.5774\,d)\sin\theta - \frac{a}{\sin 60°}(-1.5\sin\theta\cos\theta - 0.5\sin\theta\cos\theta)\right] = 0$$

Require, $\sin\theta = 0$ $\qquad \theta = 0°$ **Ans.**

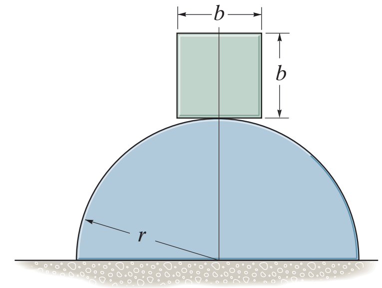
or $\quad -0.5774\,d - \dfrac{a}{\sin 60°}(-2\cos\theta) = 0$

$$\theta = \cos^{-1}\left(\frac{d}{4a}\right)$$ **Ans.**

Ans:
$\theta = 0°$
$\theta = \cos^{-1}\left(\dfrac{d}{4a}\right)$

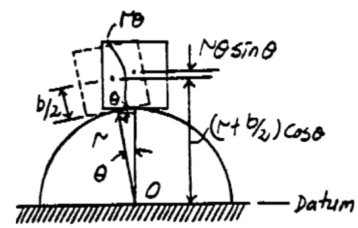
***11–48.**

A homogeneous block rests on top of the cylindrical surface. Derive the relationship between the radius of the cylinder, r, and the dimension of the block, b, for stable equilibrium. *Hint*: Establish the potential energy function for a small angle θ, i.e., approximate $\sin\theta \approx 0$, and $\cos\theta \approx 1 - \theta^2/2$.



SOLUTION

Potential Function: The datum is established at point O. Since the center of gravity for the block is above the datum, its potential energy is positive. Here, $y = \left(r + \frac{b}{2}\right)\cos\theta + r\theta\sin\theta$.



$$V = W_y = W\left[\left(r + \frac{b}{2}\right)\cos\theta + r\theta\sin\theta\right] \qquad \textbf{[1]}$$

For small angle θ, $\sin\theta = \theta$ and $\theta = 1 - \frac{\theta^2}{2}$. Then Eq. [1] becomes

$$V = W\left[\left(r + \frac{b}{2}\right)\left(1 - \frac{\theta^2}{2}\right) + r\theta^2\right]$$

$$= W\left(\frac{r\theta^2}{2} - \frac{b\theta^2}{4} + r + \frac{b}{2}\right)$$

Equilibrium Position: The system is in equilibrium if $\frac{dV}{d\theta} = 0$.

$$\frac{dV}{d\theta} = W\left(r - \frac{b}{2}\right)\theta = 0 \qquad \theta = 0^\circ$$

Stability: To have stable equilibrium, $\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} > 0$.

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} = W\left(r - \frac{b}{2}\right) > 0$$

$$\left(r - \frac{b}{2}\right) > 0$$

$$b < 2r \qquad \textbf{Ans.}$$

Ans:

$b < 2r$