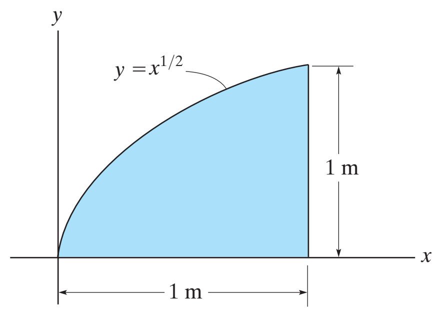
10–1.

Determine the moment of inertia for the shaded area about the x axis.



SOLUTION

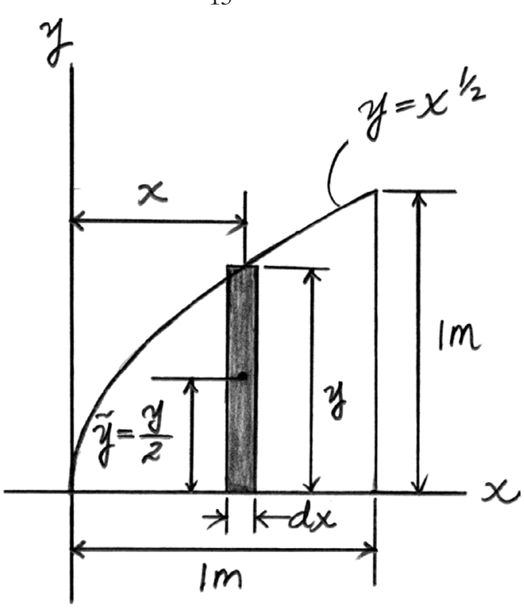
Differential Element. The area of the differential element parallel to the y axis shown shaded in Fig. a is $dA = ydx$. The moment of inertia of this element about the x axis is

$$
\begin{aligned}
dI_x &= d\bar{I}_{x'} + dA\,\tilde{y}^2 \\
&= \frac{1}{12}(dx)y^3 + ydx\left(\frac{y}{2}\right)^2 \\
&= \frac{1}{3}y^3 dx \\
&= \frac{1}{3}(x^{\frac{1}{2}})^3 dx \\
&= \frac{1}{3}x^{\frac{3}{2}} dx
\end{aligned}
$$

Moment of Inertia. Perform the integration.

$$
\begin{aligned}
I_x = \int dI_x &= \int_0^{1\text{ m}} \frac{1}{3}x^{\frac{3}{2}} dx \\
&= \frac{2}{15}x^{\frac{5}{2}}\Big|_0^{1\text{ m}} \\
&= \frac{2}{15}\text{m}^4 = 0.133\,\text{m}^4
\end{aligned}
$$

Ans.

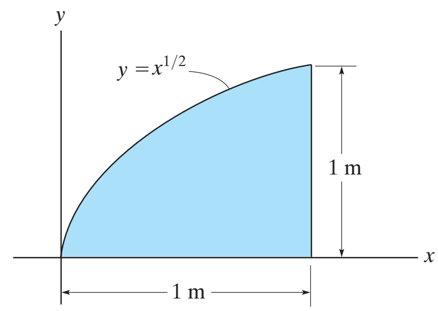


(a)

Ans:
$I_x = 0.133\text{ m}^4$

10–2.

Determine the moment of inertia for the shaded area about the y axis.



SOLUTION

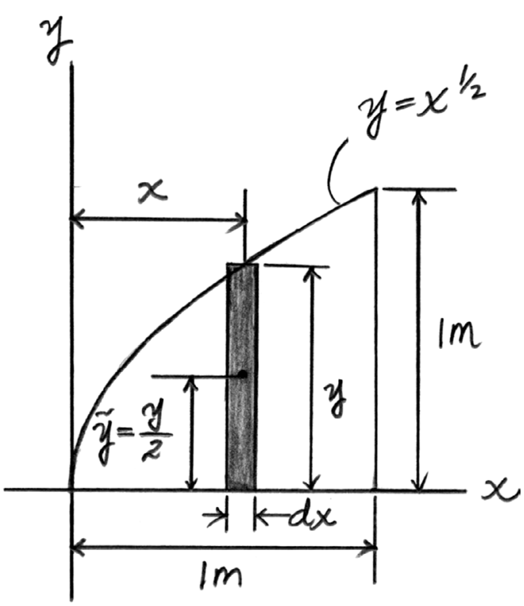
Differential Element. The area of the differential element parallel to the y axis shown shaded in Fig. a is $dA = ydx = x^{\frac{1}{2}}dx$.

Moment of Inertia. Perform the integration,

$$I_y = \int_A x^2 dA = \int_0^{1\text{ m}} x^2\left(x^{\frac{1}{2}}dx\right)$$

$$= \frac{2}{7}x^{\frac{7}{2}}\Big|_0^{1\text{ m}}$$

$$= \frac{2}{7}\text{ m}^4 = 0.286\text{ m}^4 \qquad \textbf{Ans.}$$

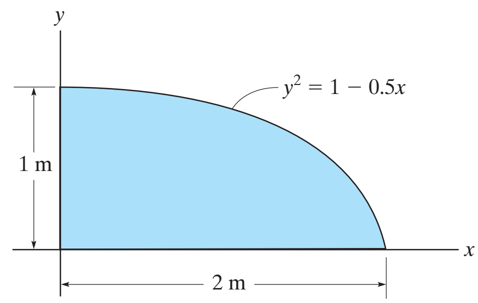


(a)

Ans:
$I_y = 0.286\text{ m}^4$

10–3.

Determine the moment of inertia for the shaded area about the x axis.



SOLUTION

Differential Element. Here $x = 2(1 - y^2)$. The area of the differential element parallel to the x axis shown shaded in Fig. a is $dA = xdy = 2(1 - y^2)dy$.

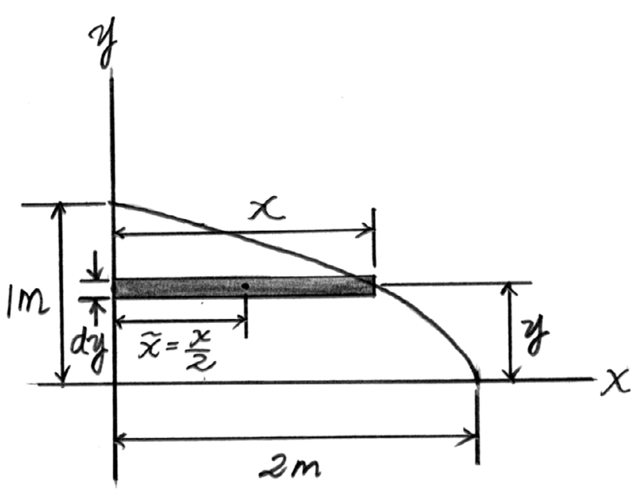
Moment of Inertia. Perform the integration,

$$I_x = \int_A y^2 dA = \int_0^{1\text{ m}} y^2[2(1 - y^2)dy]$$

$$= 2\int_0^{1\text{ m}} (y^2 - y^4)dy$$

$$= 2\left(\frac{y^3}{3} - \frac{y^5}{5}\right)\Bigg|_0^{1\text{ m}}$$

$$= \frac{4}{15}\text{ m}^4 = 0.267\text{ m}^4$$ **Ans.**

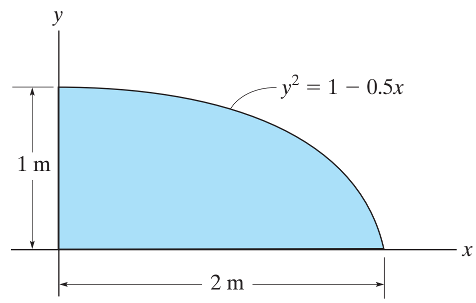


(a)

Ans:
$I_x = 0.267\text{ m}^4$

***10–4.**

Determine the moment of inertia for the shaded area about the y axis.



SOLUTION

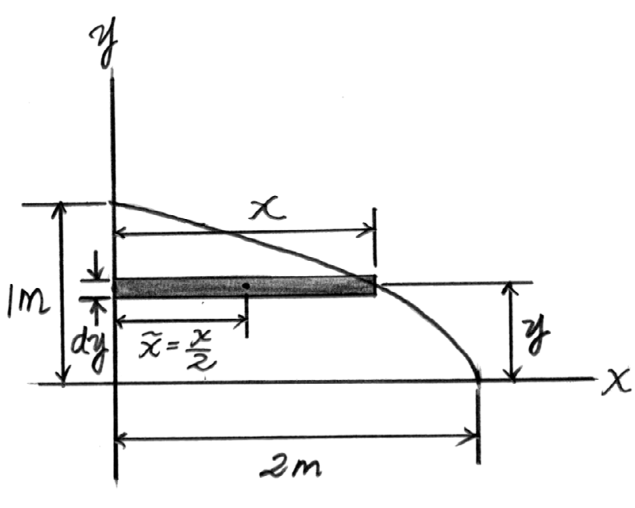
Differential Element. Here $x = 2(1 - y^2)$. The moment of inertia of the differential element parallel to the x axis shown shaded in Fig. *a* about the y axis is

$$
\begin{aligned}
dI_y &= d\bar{I}_{y'} + dA\tilde{x}^2 \\
&= \frac{1}{12}(dy)x^3 + xdy\left(\frac{x}{2}\right)^2 \\
&= \frac{1}{3}x^3 dy \\
&= \frac{1}{3}[2(1 - y^2)]^3\, dy \\
&= \frac{8}{3}(-y^6 + 3y^4 - 3y^2 + 1)dy
\end{aligned}
$$

Moment of Inertia. Perform the integration,

$$
\begin{aligned}
I_y = \int dI_y &= \frac{8}{3}\int_0^{1\text{ m}} (-y^6 + 3y^4 - 3y^2 + 1)dy \\
&= \frac{8}{3}\left(-\frac{y^7}{7} + \frac{3}{5}y^5 - y^3 + y\right)\Bigg|_0^{1\text{ m}} \\
&= \frac{128}{105}\text{ m}^4 = 1.22\text{ m}^4
\end{aligned}
$$

Ans.

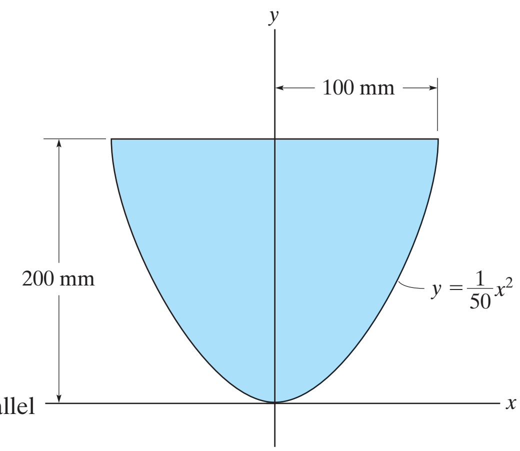


(a)

Ans:
$I_y = 1.22\text{ m}^4$

10–5.

Determine the moment of inertia for the shaded area about the x axis.



SOLUTION

Differential Element. Here $x = \sqrt{50}y^{\frac{1}{2}}$. The area of the differential element parallel to the x axis shown shaded in Fig. a is $dA = 2x\ dy\ = 2\sqrt{50}y^{\frac{1}{2}}dy$.

Moment of Inertia. Perform the integration,

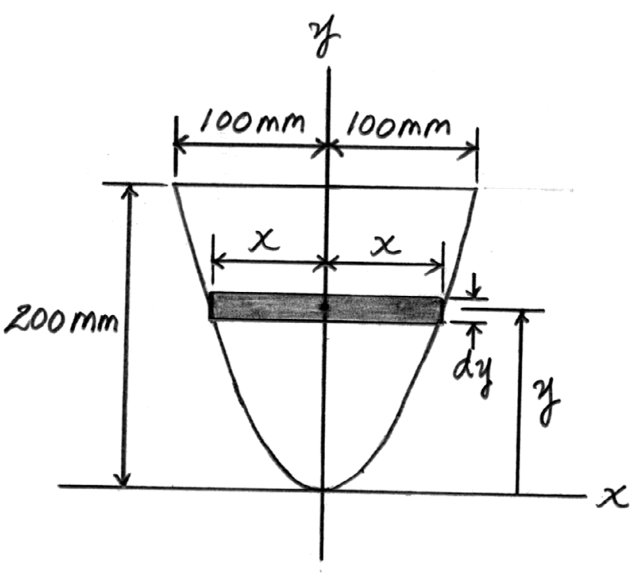
$$I_x = \int_A y^2 dA = \int_0^{200\text{ mm}} y^2\left[2\sqrt{50}y^{\frac{1}{2}}dy\right]$$

$$= 2\sqrt{50}\int_0^{200\text{ mm}} y^{\frac{5}{2}}dy$$

$$= 2\sqrt{50}\left(\frac{2}{7}y^{\frac{7}{2}}\right)\Bigg|_0^{200\text{ mm}}$$

$$= 457.14(10^6)\text{ mm}^4$$

$$= 457(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

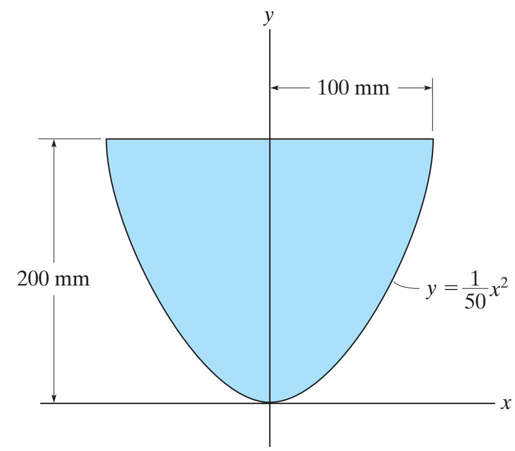


(a)

Ans:
$I_x = 457\left(10^6\right)\text{ mm}^4$

10–6.

Determine the moment of inertia for the shaded area about the y axis.



SOLUTION

Differential Element. Here $x = \sqrt{50}\,y^{\frac{1}{2}}$. The moment of inertia of the differential element parallel to x axis shown in Fig. a about y axis is

$$dI_y = \frac{1}{12}(dy)(2x)^3 = \frac{2}{3}x^3 dy = \frac{2}{3}\left(\sqrt{50}y^{\frac{1}{2}}\right)^3 dy = \frac{100\sqrt{50}}{3}y^{\frac{3}{2}}dy.$$

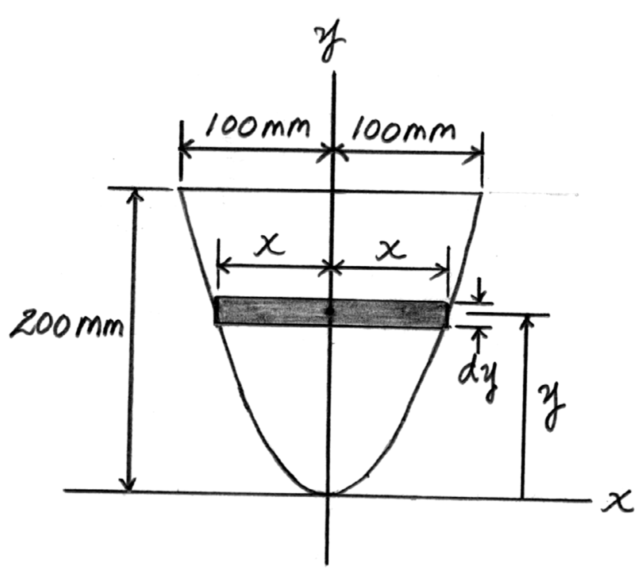
Moment of Inertia. Perform the integration,

$$I_y = \int dI_y = \int_0^{200\text{ mm}} \frac{100\sqrt{50}}{3}\,y^{\frac{3}{2}}dy$$

$$= \frac{100\sqrt{50}}{3}\left(\frac{2}{5}y^{\frac{5}{2}}\right)\Bigg|_0^{200\text{ mm}}$$

$$= 53.33(10^6)\text{ mm}^4$$

$$= 53.3(10^6)\text{ mm}^4$$ **Ans.**

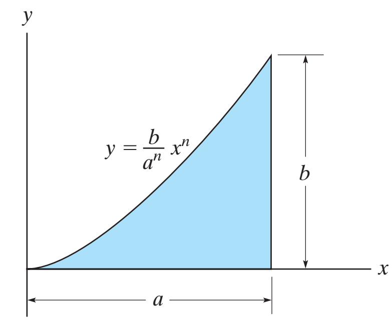


(a)

Ans:
$I_y = 53.3(10^6)\text{ mm}^4$

10–7.

Determine the moment of inertia about the x axis.



SOLUTION

Differential Element. Here $x = \frac{a}{b^{\frac{1}{n}}} y^{\frac{1}{n}}$. The area of the differential element parallel to the x axis shown shaded in Fig. a is $dA = (a - x)dy = \left(a - \frac{a}{b^{\frac{1}{n}}} y^{\frac{1}{n}}\right)dy$.

Moment of Inertia. Perform the integration,

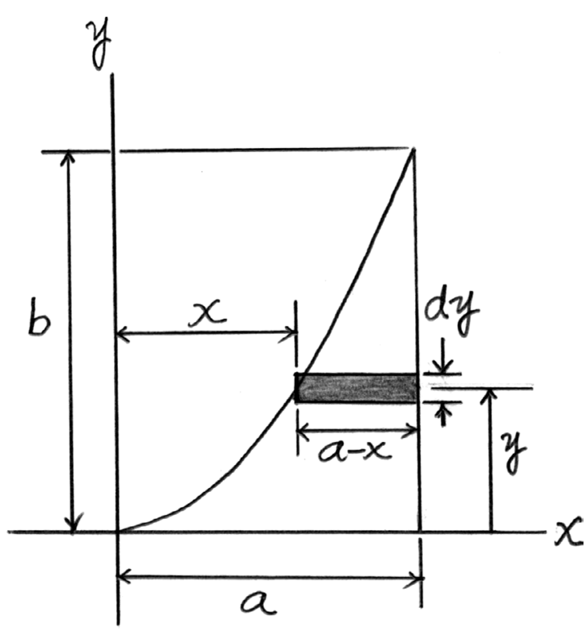
$$I_x = \int_A y^2 dA = \int_0^b y^2\left(a - \frac{a}{b^{\frac{1}{n}}} y^{\frac{1}{n}}\right)dy$$

$$= \int_0^b \left(ay^2 - \frac{a}{b^{\frac{1}{n}}} y^{\frac{1}{n}+2}\right)dy$$

$$= \left[\frac{a}{3}y^3 - \left(\frac{a}{b^{\frac{1}{n}}}\right)\left(\frac{n}{3n+1}\right)y^{\frac{3n+1}{n}}\right]\Bigg|_0^b$$

$$= \frac{1}{3}ab^3 - \left(\frac{n}{3n+1}\right)ab^3$$

$$= \frac{ab^3}{3(3n+1)} \qquad \textbf{Ans.}$$



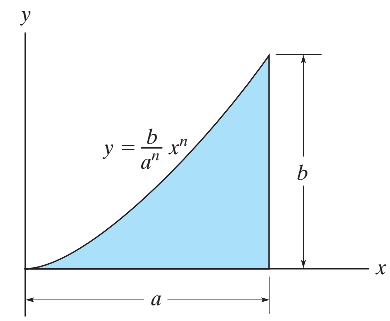
(a)

Ans:

$I_x = \frac{ab^3}{3(3n+1)}$

***10–8.**

Determine the moment of inertia about the y axis.



SOLUTION

Differential Element. The area of the differential element parallel to the y axis shown shaded in Fig. a is $dA = ydx = \frac{b}{a^n}x^n dx$.

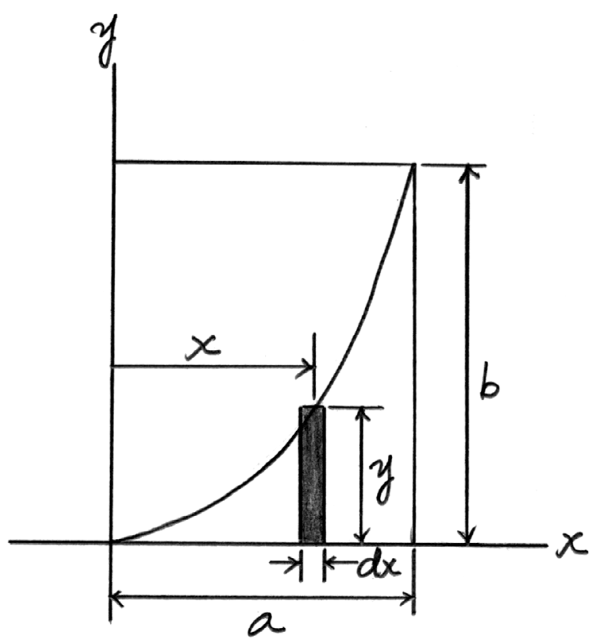
Moment of Inertia. Perform the integration,

$$I_y = \int_A x^2 dA = \int_0^a x^2\left(\frac{b}{a^n}x^n dx\right)$$

$$= \int_0^a \frac{b}{a^n} x^{n+2} dx$$

$$= \frac{b}{a^n}\left(\frac{1}{n+3}\right)\left(x^{n+3}\right)\Big|_0^a$$

$$= \frac{a^3 b}{n+3} \qquad \textbf{Ans.}$$



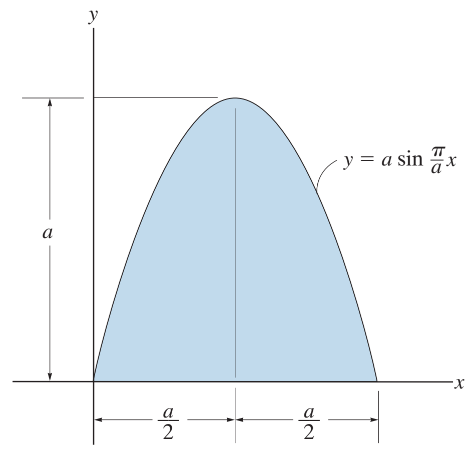
(a)

Ans:

$$I_y = \frac{a^3 b}{n+3}$$

10–9.

Determine the moment of inertia of the shaded area about the x axis.



SOLUTION

Differential Element: The area of the rectangular differential element in Fig. *a* is $dA = y\,dx$. The moment of inertia of this element about the x axis is

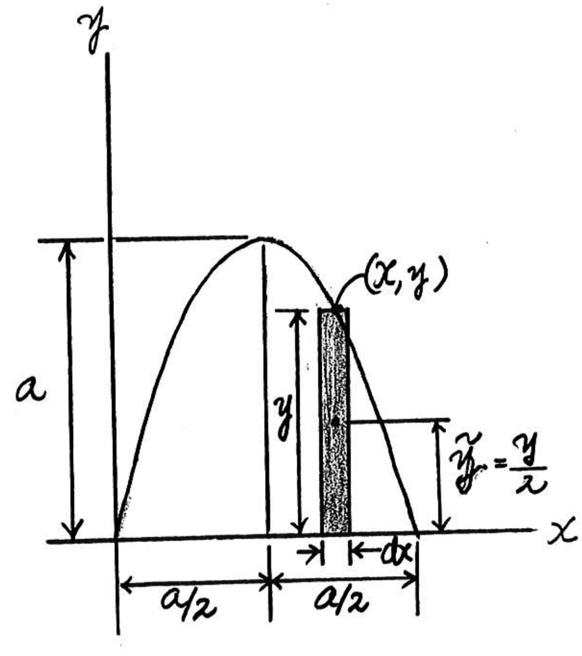
$$dI_x = d\bar{I}_{x'} + dA\tilde{y}^2 = \frac{1}{12}(dx)y^3 + ydx\left(\frac{y}{2}\right)^2 = \frac{1}{3}y^3\,dx = \frac{1}{3}\left(a\sin\frac{\pi}{a}x\right)^3 dx = \frac{a^3}{3}\sin^3\left(\frac{\pi}{a}x\right)dx.$$

Moment of Inertia: Performing the integration, we have

$$I_x = \int dI_x = \int_0^a \frac{a^3}{3}\sin^3\left(\frac{\pi}{a}x\right)dx$$

$$= \frac{a^3}{3}\left\{\left[-\frac{1}{3(\pi/a)}\cos\left(\frac{\pi}{a}x\right)\right]\left[\sin^2\left(\frac{\pi}{a}x\right) + 2\right]\right\}\Bigg|_a^0$$

$$= \frac{4a^4}{9\pi}$$ **Ans.**



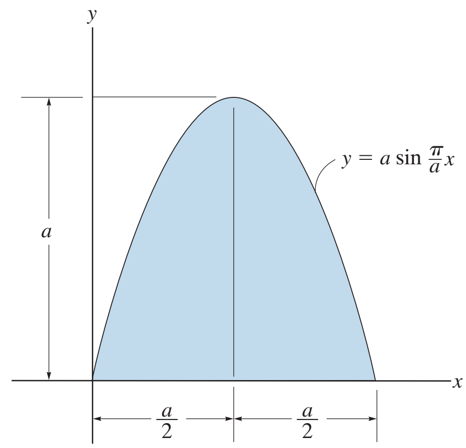
(a)

Ans:

$I_x = \frac{4a^4}{9\pi}$

10–10.

Determine the moment of inertia of the shaded area about the y axis.



SOLUTION

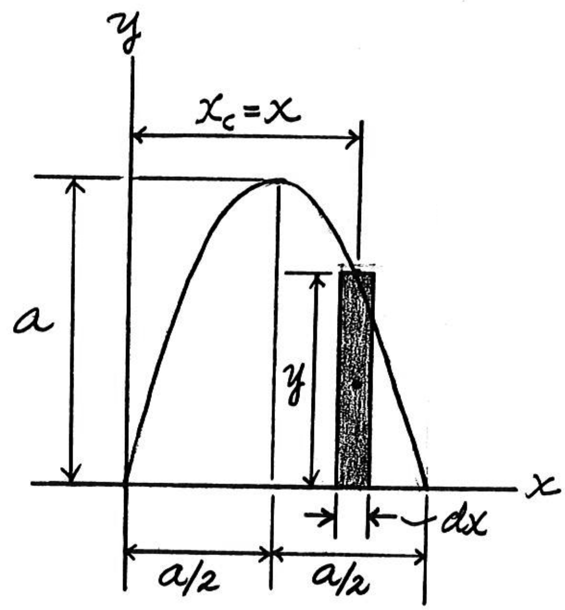
Differential Element: The area of the rectangular differential element in Fig. a is $dA = y\,dx = a \sin\left(\frac{\pi}{a}x\right)dx$.

Moment of Inertia: Applying Eq. 10–1, we have

$$I_y = \int_A x^2 dA = \int_0^a x^2\left(a \sin \frac{\pi}{a}x\right)dx$$

$$= a\left[-\frac{a}{\pi}\left(x^2 \cos \frac{\pi}{a}x\right) + \frac{a^2}{\pi^2}\left(2x \sin \frac{\pi}{a}x\right) + \frac{2a^3}{\pi^3} \cos \frac{\pi}{a}x\right]\Bigg|_0^a$$

$$= \left(\frac{\pi^2 - 4}{\pi^3}\right)a^4$$ **Ans.**

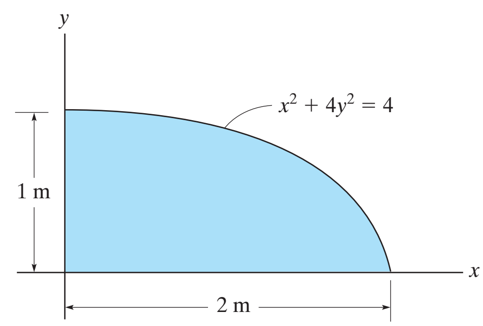


(a)

Ans:
$I_y = \left(\frac{\pi^2 - 4}{\pi^3}\right)a^4$

10–11.

Determine the moment of inertia about the x axis.



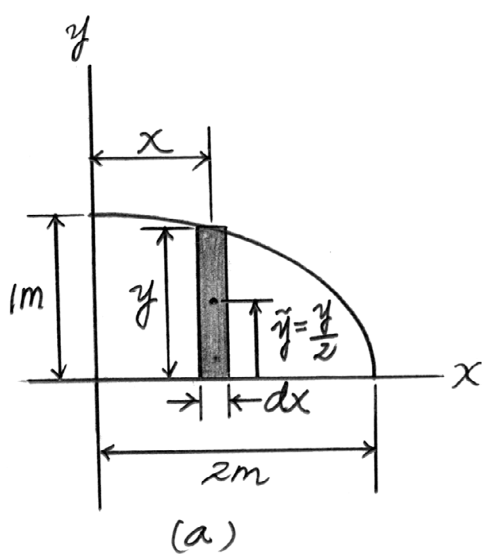
SOLUTION

Differential Element. Here, $y = \frac{1}{2}\sqrt{4 - x^2}$. The moment of inertia of the differential element parallel to the y axis shown shaded in Fig. a about x axis is

$$dI_x = d\bar{I}_{x'} + dA\tilde{y}^2$$
$$= \frac{1}{12}(dx)y^3 + ydx\left(\frac{y}{2}\right)^2$$
$$= \frac{1}{3}y^3 dx$$
$$= \frac{1}{3}\left(\frac{1}{2}\sqrt{4 - x^2}\right)^3 dx$$
$$= \frac{1}{24}\sqrt{(4 - x^2)^3}\, dx$$

Moment of Inertia. Perform the integration.

$$I_x = \int dI_x = \int_0^{2\text{ m}} \frac{1}{24}\sqrt{(4 - x^2)^3}\, dx$$
$$= \frac{1}{96}\left[x\sqrt{(4 - x^2)^3} + 6x\sqrt{4 - x^2} + 24\sin^{-1}\frac{x}{2}\right]\Bigg|_0^{2\text{ m}}$$
$$= \frac{\pi}{8}\text{ m}^4$$ **Ans.**

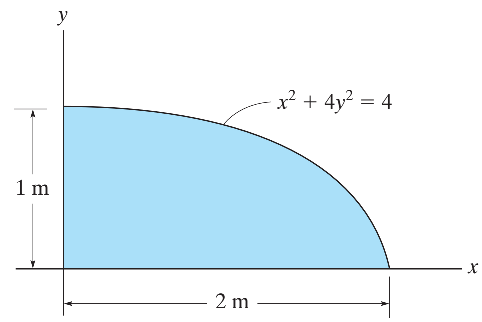


(a)

Ans:
$I_x = \frac{\pi}{8}\text{ m}^4$

***10–12.**

Determine the moment of inertia about the y axis.



SOLUTION

Differential Element. Here, $y = \frac{1}{2}\sqrt{4 - x^2}$. The area of the differential element parallel to the y axis shown shaded in Fig. a is $dA = ydx = \frac{1}{2}\sqrt{4 - x^2}dx$

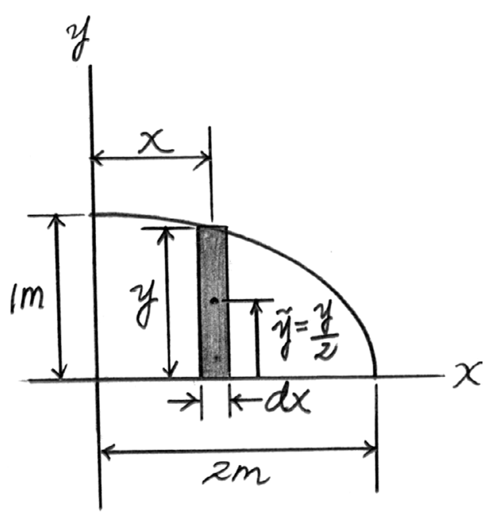
Moment of Inertia. Perform the integration,

$$I_y = \int_A x^2 dA = \int_0^{2\text{ m}} x^2\left[\frac{1}{2}\sqrt{4 - x^2}dx\right]$$

$$= \frac{1}{2}\int_0^{2\text{ m}} x^2\sqrt{4 - x^2}dx$$

$$= \frac{1}{2}\left[-\frac{x}{4}\sqrt{(4 - x^2)^3} + \frac{1}{2}\left(x\sqrt{4 - x^2} + 4\sin^{-1}\frac{x}{2}\right)\right]\Bigg|_0^{2\text{ m}}$$

$$= \frac{\pi}{2}\text{ m}^4 \qquad \textbf{Ans.}$$

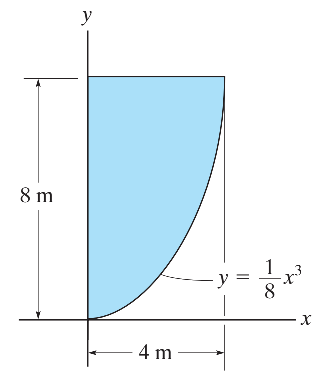


(a)

Ans:
$I_y = \frac{\pi}{2}\text{ m}^4$

10–13.

Determine the moment of inertia for the shaded area about the x axis.



SOLUTION

Differential Element. Here, $x = 2y^{\frac{1}{3}}$. The area of the differential element parallel to the x axis shown shaded in Fig. a is $dA = xdy = 2y^{\frac{1}{3}}\,dy$

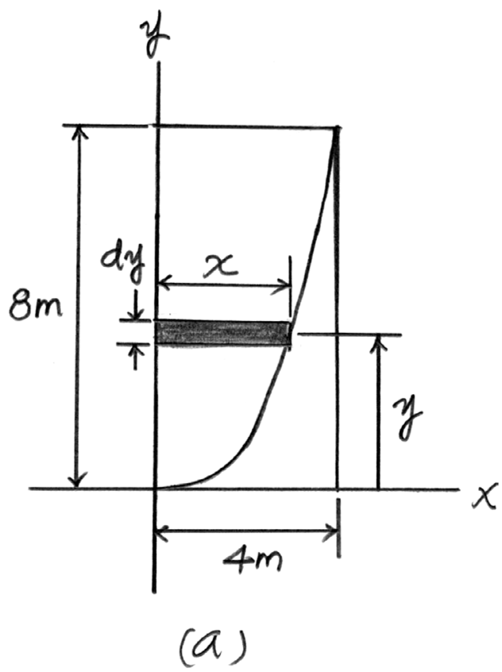
Moment of Inertia. Perform the integration,

$$I_x = \int_A y^2 dA = \int_0^{8\text{ m}} y^2\,(2y^{\frac{1}{3}}\,dy)$$

$$= 2\int_0^{8\text{ m}} y^{\frac{7}{3}}\,dy$$

$$= 2\left(\frac{3}{10}y^{\frac{10}{3}}\right)\Bigg|_0^{8\text{ m}}$$

$$= 614.4\text{ m}^4 = 614\text{ m}^4 \qquad \textbf{Ans.}$$

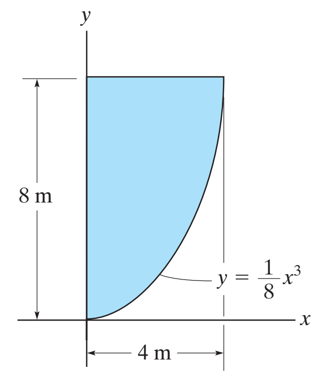


(a)

Ans:
$I_x = 614\text{ m}^4$

10–14.

Determine the moment of inertia for the shaded area about the y axis.



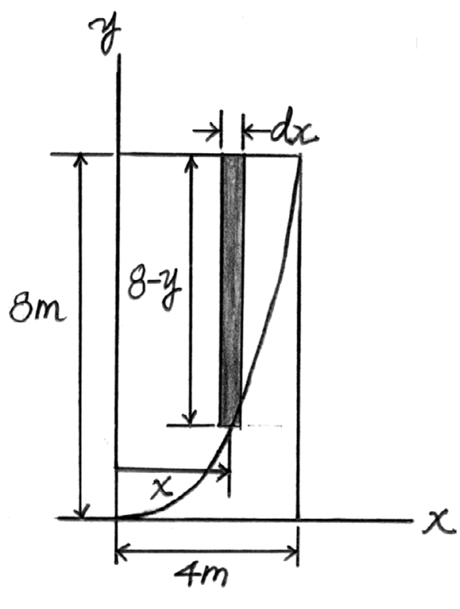
SOLUTION

Differential Element. The area of the differential element parallel to the y axis, shown shaded in Fig. a, is $dA = (8 - y)d_x = \left(8 - \frac{1}{8}x^3\right)dx$

Moment of Inertia. Perform the integration,

$$
\begin{aligned}
I_y = \int_A x^2 dA &= \int_0^{4\text{ m}} x^2\left(8 - \frac{1}{8}x^3\right) dx \\
&= \int_0^{4\text{ m}} \left(8x^2 - \frac{1}{8}x^5\right) dx \\
&= \left(\frac{8}{3}x^3 - \frac{1}{48}x^6\right)\Bigg|_0^{4\text{ m}} \\
&= 85.33\text{ m}^4 = 85.3\text{ m}^4
\end{aligned}
$$

Ans.

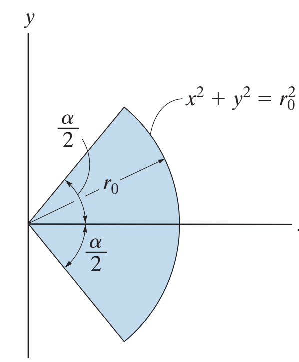


(a)

Ans:
$I_y = 85.3\text{ m}^4$

10–15.

Determine the moment of inertia of the shaded area about the y axis.



SOLUTION

Differential Element: The area of the differential element shown shaded in Fig. *a* is $dA = (rd\theta)dr$.

Moment of Inertia: Applying Eq. 10–1, we have

$$I_y = \int_A x^2 dA = \int_{-\alpha/2}^{\alpha/2}\int_0^{r_0} r^2\cos^2\theta(rd\theta)dr$$

$$= \int_{-\alpha/2}^{\alpha/2}\int_0^{r_0} r^3\cos^2\theta drd\theta$$

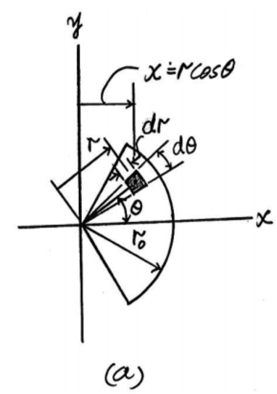
$$= \int_{-\alpha/2}^{\alpha/2}\left(\frac{r^4}{4}\right)\bigg|_0^{r_0}\cos^2\theta d\theta$$

$$= \int_{-\alpha/2}^{\alpha/2}\frac{r_0^{\,4}}{4}\cos^2\theta d\theta$$

However, $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$. Thus,

$$I_y = \int_{-\alpha/2}^{\alpha/2}\frac{r_0^{\,4}}{8}(\cos 2\theta + 1)d\theta$$

$$= \frac{r_0^{\,4}}{8}\left[\frac{1}{2}\sin 2\theta + \theta\right]\bigg|_{-\alpha/2}^{\alpha/2} = \frac{r_0^{\,4}}{8}(\sin\alpha + \alpha) \qquad \textbf{Ans.}$$



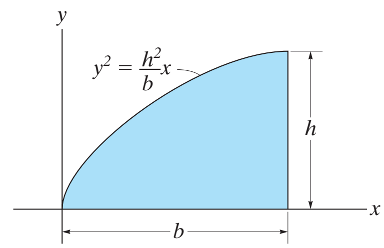
(a)

Ans:

$I_y = \frac{r_0^4}{8}(\sin\alpha + \alpha)$

***10–16.**

Determine the moment of inertia for the shaded area about the x axis.



SOLUTION

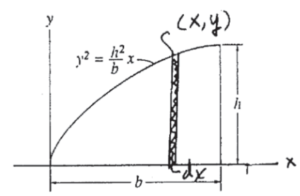
$$d\,I_x = \frac{1}{3}y^3\,dx$$

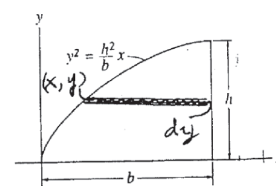
$$I_x = \int d\,I_x$$

$$= \int_0^b \frac{y^3}{3}\,dx = \int_0^b \frac{1}{3}\left(\frac{h^2}{b}\right)^{3/2} x^{3/2}\,dx$$

$$= \frac{1}{3}\left(\frac{h^2}{b}\right)^{3/2}\left(\frac{2}{5}\right)x^{5/2}\Big]_0^b$$

$$= \frac{2}{15}bh^3$$ **Ans.**





Also,

$$dA = (b - x)\,dy = \left(b - \frac{b}{h^2}y^2\right)dy$$

$$I_x = \int y^2\,dA$$

$$= \int_0^h y^2\left(b - \frac{b}{h^2}y^2\right)dy$$

$$= \left[\frac{b}{3}y^3 - \frac{b}{5h^2}y^5\right]_0^h$$

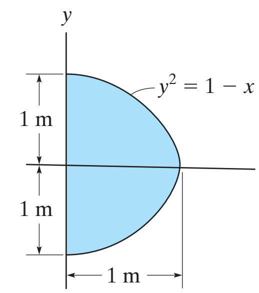
$$= \frac{2}{15}bh^3$$ **Ans.**

Ans:

$$I_x = \frac{2}{15}bh^3$$

10–17.

Determine the moment of inertia for the shaded area about the x axis.



SOLUTION

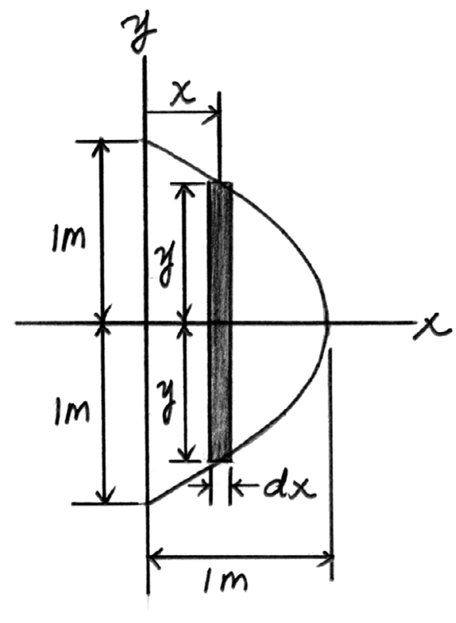
Differential Element. Here $y = (1 - x)^{\frac{1}{2}}$. The moment of inertia of the differential element parallel to the y axis shown shaded in Fig. a about the x axis is $dI_x = \frac{1}{12}(dx)(2y)^3 = \frac{2}{3}y^3 dx = \frac{2}{3}\left[(1 - x)^{\frac{1}{2}}\right]^3 dx = \frac{2}{3}(1 - x)^{\frac{3}{2}} dx.$

Moment of Inertia. Perform the integration,

$$I_x = \int dI_x = \int_0^{1\text{ m}} \frac{2}{3}(1 - x)^{\frac{3}{2}} dx$$

$$= \frac{2}{3}\left[-\frac{2}{5}(1 - x)^{\frac{5}{2}}\right]\Bigg|_0^{1\text{ m}}$$

$$= \frac{4}{15}\text{ m}^4 = 0.267\text{ m}^4$$ **Ans.**

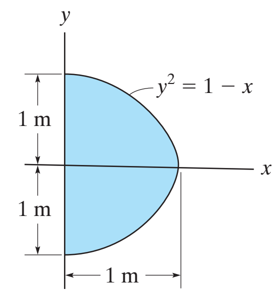


(a)

Ans:
$I_x = 0.267\text{ m}^4$

10–18.

Determine the moment of inertia for the shaded area about the y axis.



SOLUTION

Differential Element. Here $x = 1 - y^2$. The moment of inertia of the differential element parallel to the x axis shown shaded in Fig. a about the y axis is

$$dI_y = d\bar{I}_{y'} + dA\tilde{x}^2$$

$$= \frac{1}{12}(dy)x^3 + xdy\left(\frac{x}{2}\right)^2$$

$$= \frac{1}{3}x^3dy$$

$$= \frac{1}{3}(1 - y^2)^3dy$$

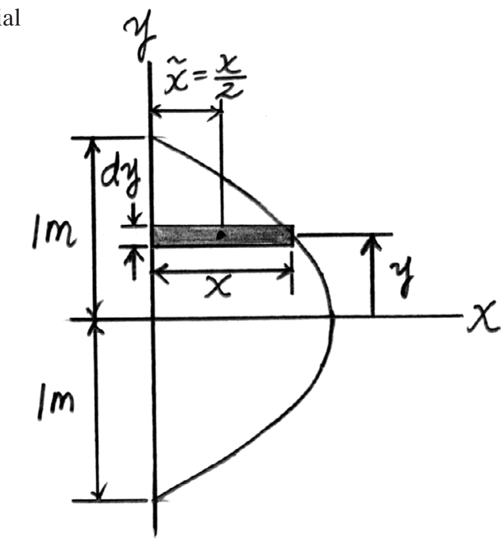
$$= \frac{1}{3}(-y^6 + 3y^4 - 3y^2 + 1)dy$$

Moment of Inertia. Perform the integration,

$$I_y = \int dI_y = \int_{-1\text{ m}}^{1\text{ m}} \frac{1}{3}(-y^6 + 3y^4 - 3y^2 + 1)dy$$

$$= \frac{1}{3}\left(-\frac{y^7}{7} + \frac{3}{5}y^5 - y^3 + y\right)\Bigg|_{-1\text{ m}}^{1\text{ m}}$$

$$= \frac{32}{105}\text{ m}^4 = 0.305\text{ m}^4$$ **Ans.**

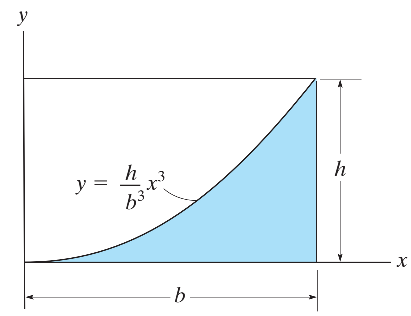


(a)

Ans:
$I_y = 0.305\text{ m}^4$

10–19.

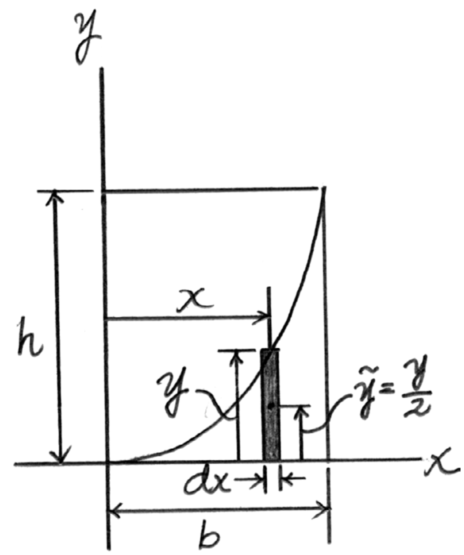
Determine the moment of inertia for the shaded area about the x axis.



SOLUTION

Differential Element. The moment of inertia of the differential element parallel to the y axis shown shaded in Fig. a about the x axis is

$$
\begin{aligned}
dI_x &= d\bar{I}_{x'} + dA\tilde{y}^2 \\
&= \frac{1}{12}(dx)y^3 + ydx\left(\frac{y}{2}\right)^2 \\
&= \frac{1}{3}y^3dx \\
&= \frac{1}{3}\left(\frac{h}{b^3}x^3\right)^3 dx \\
&= \frac{h^3}{3b^9}x^9dx
\end{aligned}
$$



(a)

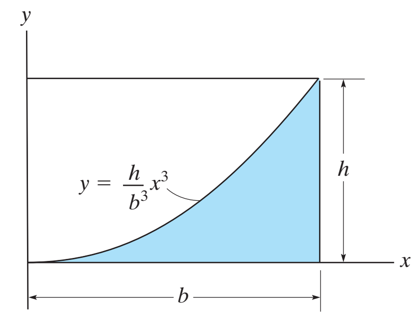
Moment of Inertia. Perform the integration,

$$
\begin{aligned}
I_x = \int dI_x &= \int_0^b \frac{h^3}{3b^9}x^9dx \\
&= \frac{h^3}{3b^9}\left(\frac{x^{10}}{10}\right)\bigg|_0^b \\
&= \frac{1}{30}bh^3 \qquad \textbf{Ans.}
\end{aligned}
$$

Ans:
$I_x = \frac{1}{30}bh^3$

***10–20.**

Determine the moment of inertia for the shaded area about the y axis.



SOLUTION

Differential Element. The area of the differential element parallel to the y axis shown shaded in Fig. a is $dA = ydx = \frac{h}{b^3}x^3dx$

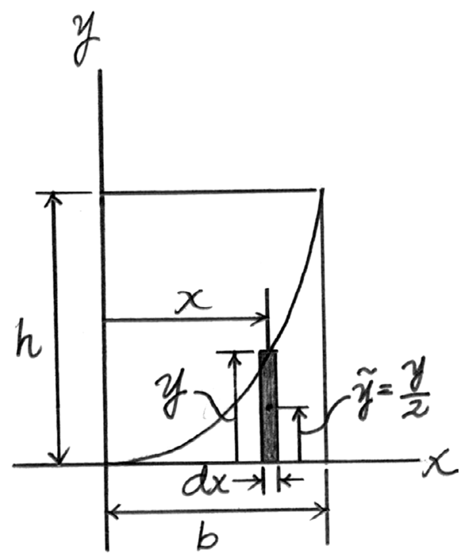
Moment of Inertia. Perform the integration,

$$I_y = \int_A x^2 dA = \int_0^b x^2\left(\frac{h}{b^3}x^3\right)dx$$

$$= \frac{h}{b^3}\int_0^b x^5 dx$$

$$= \frac{h}{b^3}\left(\frac{x^6}{6}\right)\Bigg|_0^6$$

$$= \frac{b^3 h}{6} \qquad \textbf{Ans.}$$



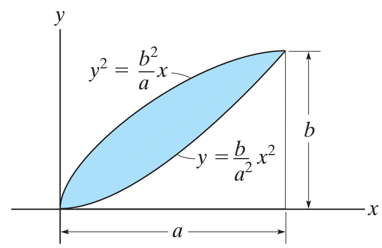
(a)

Ans:

$I_y = \frac{b^3h}{6}$

10–21.

Determine the moment of inertia for the shaded area about the x axis.



SOLUTION

Differential Element. Here $x_2 = \frac{a}{b^{\frac{1}{2}}}y^{\frac{1}{2}}$ and $x_1 = \frac{a}{b^2}y^2$. Thus, the area of the differential element parallel to the x axis shown shaded in Fig. a is $dA = (x_2 - x_1)\,dy = \left(\frac{a}{b^{\frac{1}{2}}}y^{\frac{1}{2}} - \frac{a}{b^2}y^2\right)dy$.

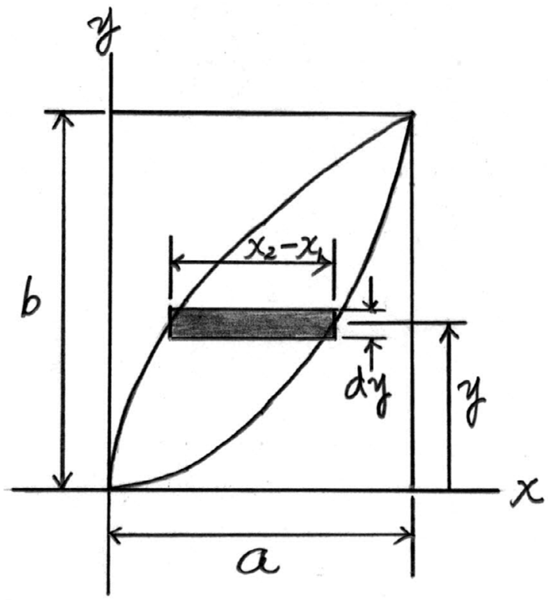
Moment of Inertia. Perform the integration,

$$I_x = \int_A y^2 dA = \int_0^b y^2\left(\frac{a}{b^{\frac{1}{2}}}y^{\frac{1}{2}} - \frac{a}{b^2}y^2\right)dy$$

$$= \int_0^b \left(\frac{a}{b^{\frac{1}{2}}}y^{\frac{5}{2}} - \frac{a}{b^2}y^4\right)dy$$

$$= \left(\frac{2a}{7b^{\frac{1}{2}}}y^{\frac{7}{2}} - \frac{a}{5b^2}y^5\right)\Bigg|_0^b$$

$$= \frac{3ab^3}{35}$$ **Ans.**



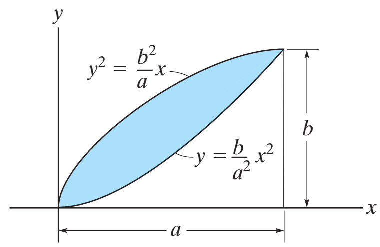
(a)

Ans:

$I_x = \frac{3ab^3}{35}$

10–22.

Determine the moment of inertia for the shaded area about the y axis.



SOLUTION

Differential Element. Here, $y_2 = \frac{b}{a^{\frac{1}{2}}}x^{\frac{1}{2}}$ and $y_1 = \frac{b}{a^2}x^2$. Thus, the area of the differential element parallel to the y axis shown shaded in Fig. a is $dA = (y_2 - y_1)dx = \left(\frac{b}{a^{\frac{1}{2}}}x^{\frac{1}{2}} - \frac{b}{a^2}x^2\right)dx$

Moment of Inertia. Perform the integration,

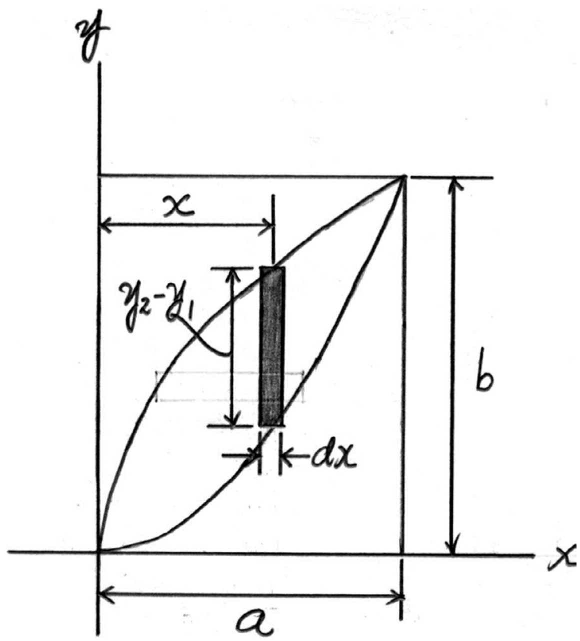
$$I_y = \int_A x^2 dA = \int_0^a x^2\left(\frac{b}{a^{\frac{1}{2}}}x^{\frac{1}{2}} - \frac{b}{a^2}x^2\right)dx$$

$$= \int_0^a \left(\frac{b}{a^{\frac{1}{2}}}x^{\frac{5}{2}} - \frac{b}{a^2}x^4\right)dx$$

$$= \left(\frac{2b}{7a^{\frac{1}{2}}}x^{\frac{7}{2}} - \frac{b}{5a^2}x^5\right)\Bigg|_0^a$$

$$= \frac{2}{7}a^3b - \frac{1}{5}a^3b$$

$$= \frac{3a^3b}{35} \qquad \textbf{Ans.}$$



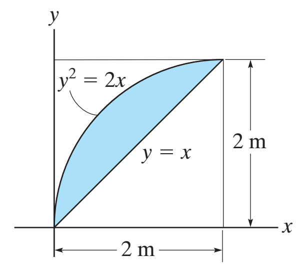
(a)

Ans:

$I_y = \frac{3a^3b}{35}$

10–23.

Determine the moment of inertia for the shaded area about the x axis.



SOLUTION

Differential Element. Here $x_2 = y$ and $x_1 = \frac{1}{2}y^2$. The area of the differential element parallel to the x axis shown shaded in Fig. a is $dA = (x_2 - x_1)dy = \left(y - \frac{1}{2}y^2\right)dy$.

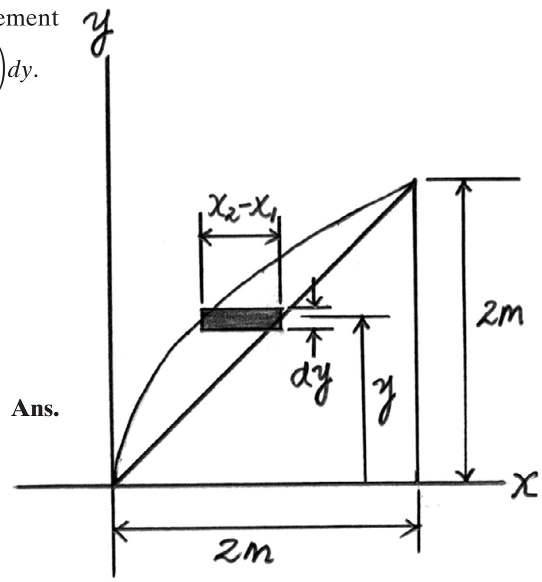
Moment of Inertia. Perform the integration,

$$I_x = \int_A y^2 dA = \int_0^{2\text{ m}} y^2\left(y - \frac{1}{2}y^2\right)dy$$

$$= \int_0^{2\text{ m}} \left(y^3 - \frac{1}{2}y^4\right)dy$$

$$= \left(\frac{y^4}{4} - \frac{y^5}{10}\right)\Bigg|_0^{2\text{ m}}$$

$$= 0.8\text{ m}^4$$ **Ans.**

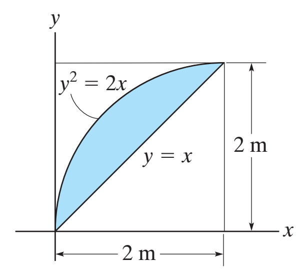


(a)

Ans:
$I_x = 0.8\text{ m}^4$

***10–24.**

Determine the moment of inertia for the shaded area about the y axis.



SOLUTION

Differential Element. Here, $y_2 = \sqrt{2}x^{\frac{1}{2}}$ and $y_1 = x$. The area of the differential element parallel to the y axis shown shaded in Fig. a is $dA = (y_2 - y_1)\,dx = \left(\sqrt{2}x^{\frac{1}{2}} - x\right)dx$.

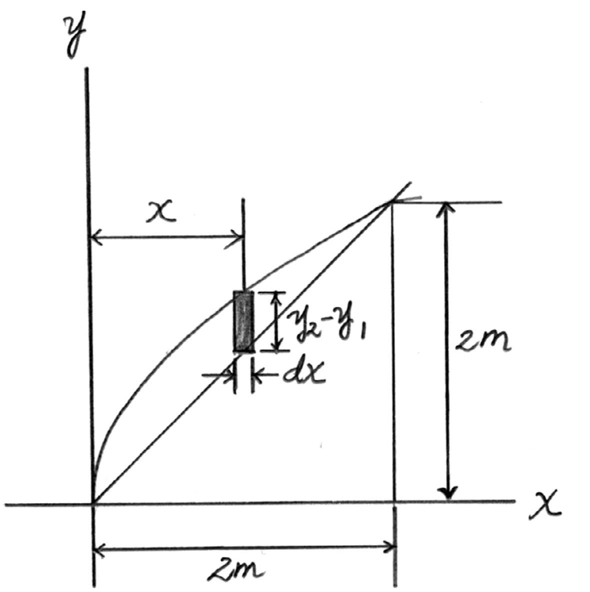
Moment of Inertia. Perform the integration,

$$I_y = \int_A x^2 dA = \int_0^{2\text{ m}} x^2\left(\sqrt{2}x^{\frac{1}{2}} - x\right)dx$$

$$= \int_0^{2\text{ m}} \left(\sqrt{2}x^{\frac{5}{2}} - x^3\right)dx$$

$$= \left(\frac{2\sqrt{2}}{7}x^{\frac{7}{2}} - \frac{x^4}{4}\right)\Bigg|_0^{2\text{ m}}$$

$$= \frac{4}{7}\text{ m}^4 = 0.571\text{ m}^4 \qquad \textbf{Ans.}$$

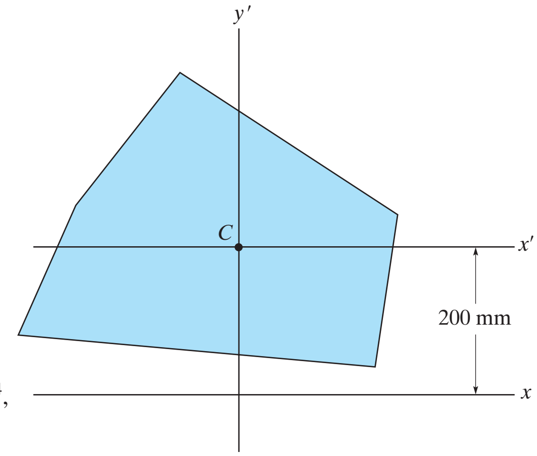


(a)

Ans:
$I_y = 0.571\text{ m}^4$

10–25.

The polar moment of inertia for the area is $J_C = 642\ (10^6)\ \text{mm}^4$, about the z' axis passing through the centroid C. The moment of inertia about the y' axis is $264\ (10^6)\ \text{mm}^4$, and the moment of inertia about the x axis is $938\ (10^6)\ \text{mm}^4$. Determine the area A.



SOLUTION

Applying the parallel-axis theorem with $d_y = 200$ mm and $I_x = 938(10^6)\ \text{mm}^4$,

$$I_x = \bar{I}_{x'} + Ad_y^2$$

$$938(10^6) = \bar{I}_{x'} + A(200^2)$$

$$\bar{I}_{x'} = 938(10^6) - 40(10^3)A$$

with known polar moment of inertia about C,

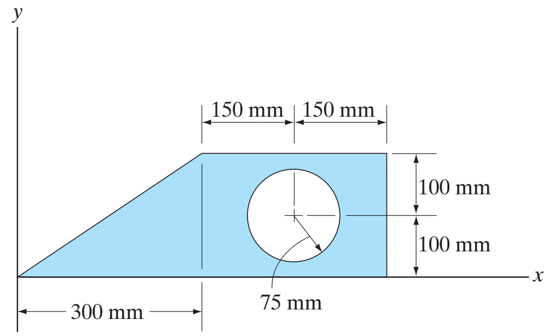
$$\bar{J}_C = \bar{I}_{x'} + \bar{I}_{y'}$$

$$642(10^6) = 938(10^6) - 40(10^3)A + 264(10^6)$$

$$A = 14.0(10^3)\ \text{mm}^2$$ **Ans.**

Ans:
$A = 14.0(10^3)\ \text{mm}^2$

10–26. Determine the moment of inertia of the composite area about the x axis.



SOLUTION

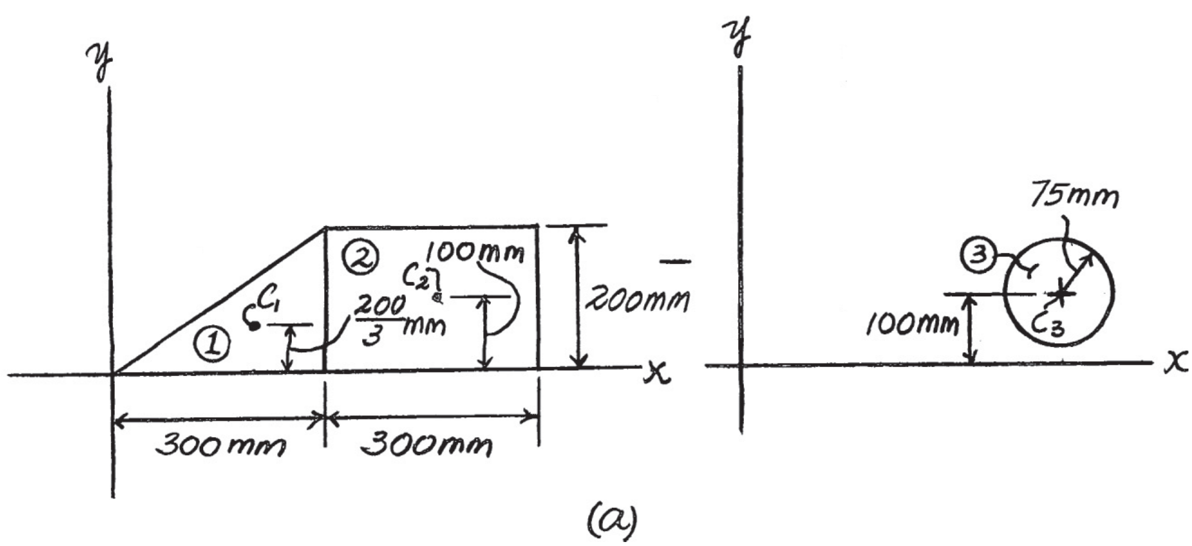
Composite Parts: The composite area can be subdivided into three segments as shown in Fig. *a*. Since segment (3) is a hole, it contributes a negative moment of inertia. The perpendicular distance measured from the centroid of each segment to the x axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the x axis can be determined using the parallel - axis theorem. Thus,

$$I_x = \bar{I}_{x'} + A(d_y)^2$$

$$= \left[\frac{1}{36}(300)(200^3) + \frac{1}{2}(300)(200)\left(\frac{200}{3}\right)^2\right] + \left[\frac{1}{12}(300)(200^3) + 300(200)(100)^2\right] + \left[-\frac{\pi}{4}(75^4) + \left(-\pi(75^2)\right)(100)^2\right]$$

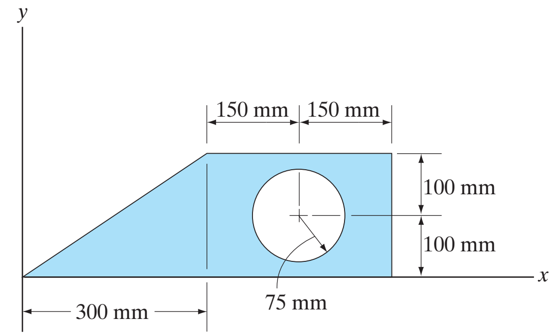
$$= 798(10^6)\,\text{mm}^4$$ **Ans.**



(a)

Ans:
$I_y = 798(10^6)\text{ mm}^4$

10–27. Determine the moment of inertia of the composite area about the y axis.



SOLUTION

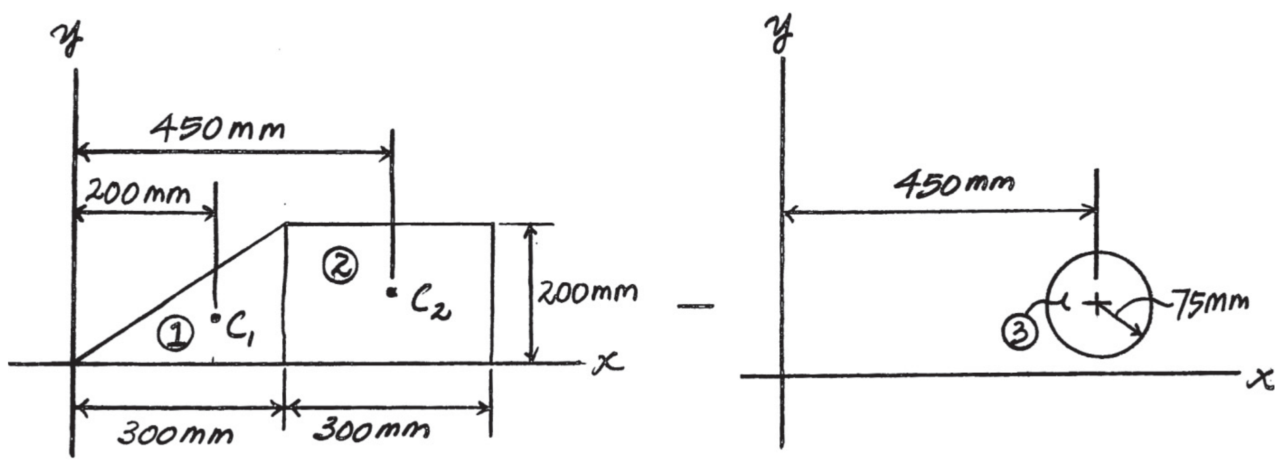
Composite Parts: The composite area can be subdivided into three segments as shown in Fig. *a*. Since segment (3) is a hole, it contributes a negative moment of inertia. The perpendicular distance measured from the centroid of each segment to the y axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem. Thus,

$$I_y = \bar{I}_{y'} + A(d_x)^2$$

$$= \left[\frac{1}{36}(200)(300^3) + \frac{1}{2}(200)(300)(200)^2\right] + \left[\frac{1}{12}(200)(300^3) + 200(300)(450)^2\right] + \left[-\frac{\pi}{4}(75^4) + \left(-\pi(75^2)\right)(450)^2\right]$$

$$= 10.3(10^9)\ \text{mm}^4 \qquad \textbf{Ans.}$$



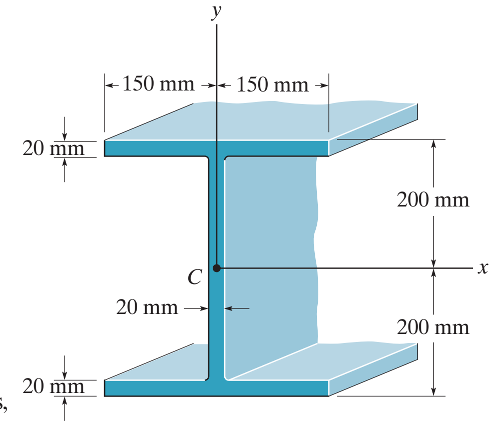
(a)

Ans:

$I_y = 10.3(10^9)\ \text{mm}^4$

***10–28.**

Determine the moment of inertia about the x axis.



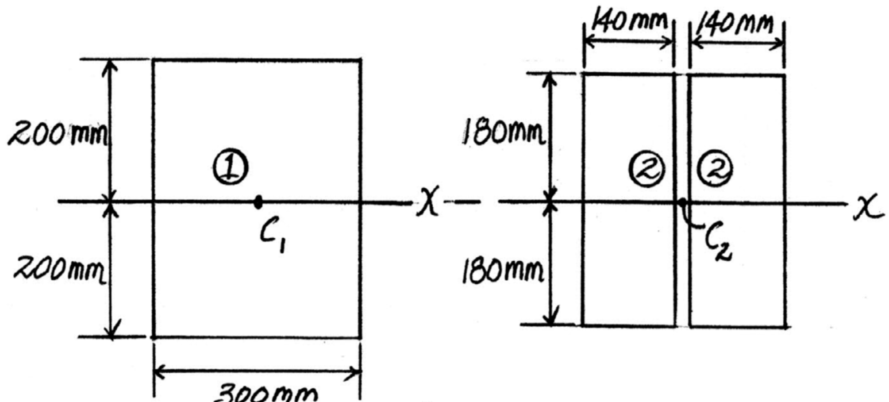
SOLUTION

Moment of Inertia. Since the x axis passes through the centroids of the two segments, Fig. a,

$$I_x = \frac{1}{12}(300)(400^3) - \frac{1}{12}(280)(360^3)$$

$$= 511.36(10^6)\ \text{mm}^4$$

$$= 511(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$

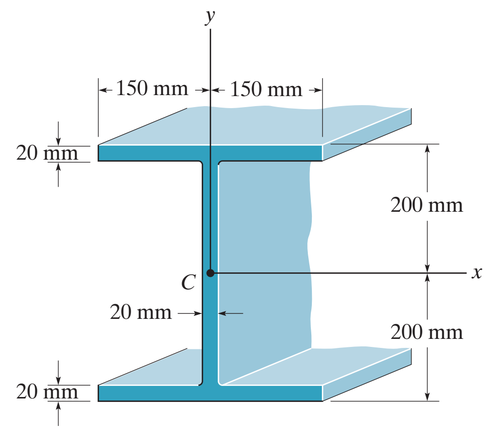


(a)

Ans:
$I_x = 511(10^6)\ \text{mm}^4$

10–29.

Determine the moment of inertia about the y axis.



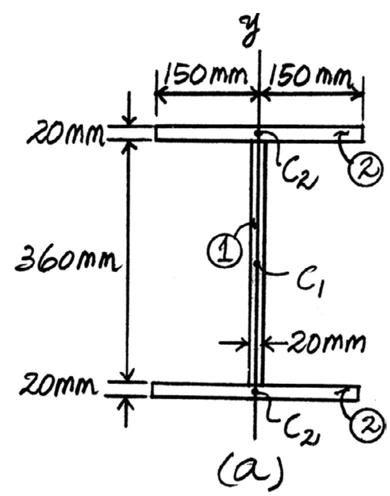
SOLUTION

Moment of Inertia. Since the y axis passes through the centroid of the two segments, Fig. a,

$$I_y = \frac{1}{12}(360)(20^3) + \frac{1}{12}(40)(300^3)$$

$$= 90.24(10^6)\ \text{mm}^4$$

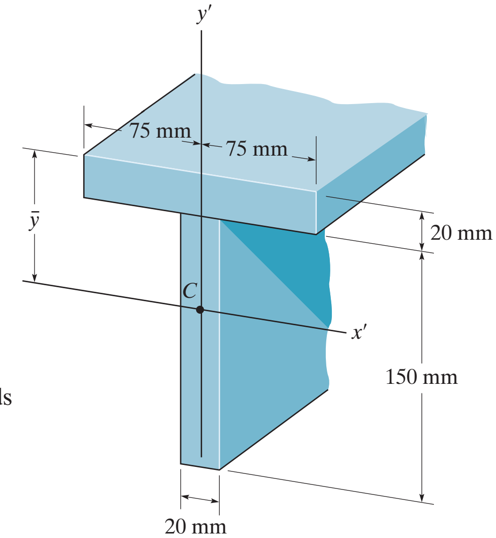
$$= 90.2(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$



Ans:
$I_y = 90.2(10^6)\ \text{mm}^4$

10–30.

Determine $\bar{y}$, which locates the centroidal axis x' for the cross-sectional area of the T-beam, and then find the moments of inertia $I_{x'}$ and $I_{y'}$.



SOLUTION

Centroid. Referring to Fig. *a*, the areas of the segments and their respective centroids are tabulated below.

Segment	$A(\text{mm}^2)$	$\tilde{y}(\text{mm})$	$\tilde{y}A(\text{mm}^3)$
1	150(20)	10	$30(10^3)$
2	20(150)	95	$285(10^3)$
Σ	$6(10^3)$		$315(10^3)$

Thus, $\bar{y} = \dfrac{\Sigma\tilde{y}^2 A}{\Sigma A} = \dfrac{315(10^3)}{6(10^3)} = 52.5\text{ mm}$ **Ans.**

Moment of Inertia. The moment of inertia about the x' axis for each segment can be determined using the parallel axis theorem, $I_{x'} = \bar{I}_{x'} + Ad_y^2$. Referring to Fig. *b*,

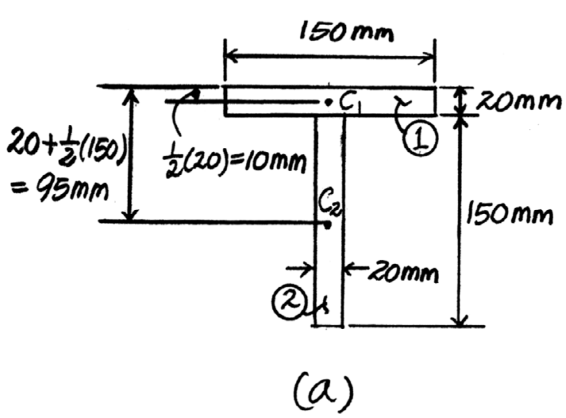
Segment	$A_i(\text{mm}^2)$	$(d_y)_i(\text{mm})$	$(\bar{I}_{x'})_i(\text{mm}^4)$	$(Ad_y^2)_i(\text{mm}^4)$	$(\bar{I}_{x'})_i(\text{mm}^4)$
1	150(20)	42.5	$\frac{1}{12}(150)(20^3)$	$5.41875(10^6)$	$5.51875(10^6)$
2	20(150)	42.5	$\frac{1}{12}(20)(150^3)$	$5.41875(10^6)$	$11.04375(10^6)$

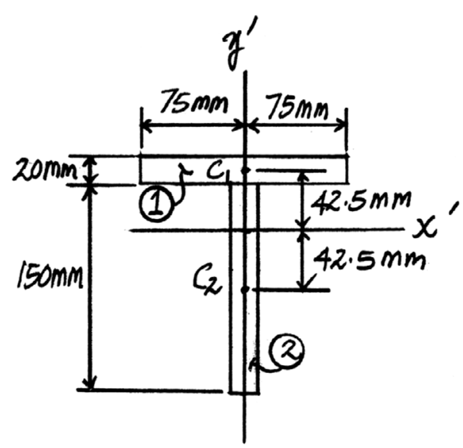
Thus

$$I_{x'} = \Sigma(I_{x'})_i = 16.5625(10^6)\text{ mm}^4 = 16.6(10^6)\text{ mm}^4$$ **Ans.**

Since the y' axis passes through the centroids of segments 1 and 2,

$$I_{y'} = \frac{1}{12}(20)(150^3) + \frac{1}{12}(150)(20^3)$$
$$= 5.725(10^6)\text{ mm}^4$$ **Ans.**

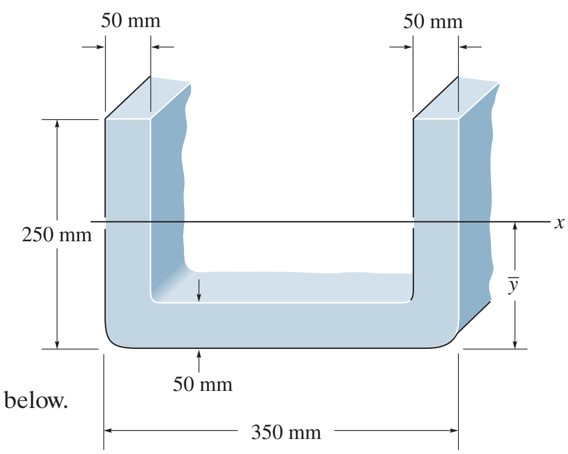




Ans:
$\bar{y} = 52.5\text{ mm}$
$I_{x'} = 16.6(10^6)\text{ mm}^4$
$I_{y'} = 5.725(10^6)\text{ mm}^4$

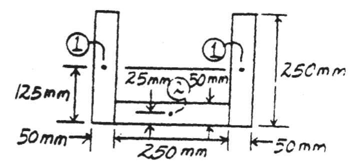
10–31.

Determine the location $\bar{y}$ of the centroid of the channel's cross-sectional area and then calculate the moment of inertia of the area about this axis.



SOLUTION

Centroid: The area of each segment and its respective centroid are tabulated below.

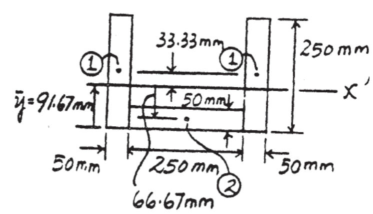


Segment	A (mm^2)	$\tilde{y}$ (mm)	$\tilde{y}A$ (mm^3)
1	100(250)	125	$3.125(10^6)$
2	250(50)	25	$0.3125(10^6)$
Σ	$37.5(10^3)$		$3.4375(10^6)$

Thus,

$$\tilde{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{3.4375(10^6)}{37.5(10^3)} = 91.67 \text{ mm} = 91.7 \text{ mm} \qquad \textbf{Ans.}$$

Moment of Inertia: The moment of inertia about the x' axis for each segment can be determined using the parallel-axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.



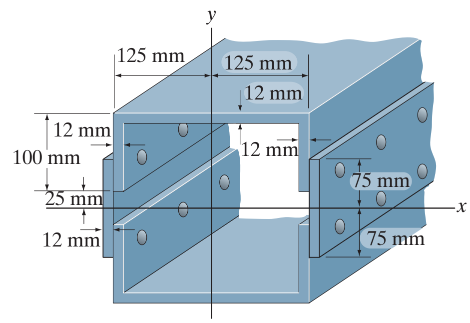
Segment	A_i (mm^2)	$(d_y)_i$ (mm)	$(\bar{I}_{x'})_i$ (mm^4)	$(Ad_y^2)_i$ (mm^4)	$(I_{x'})_i$ (mm^4)
1	100(250)	33.33	$\frac{1}{12}(100)(250^3)$	$27.778(10^6)$	$157.99(10^6)$
2	250(50)	66.67	$\frac{1}{12}(250)(50^3)$	$55.556(10^6)$	$58.16(10^6)$

Thus,

$$I_{x'} = \Sigma(I_{x'})_i = 216.15(10^6) \text{ mm}^4 = 216(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

Ans:
$\tilde{y} = 91.7$ mm
$I_{x'} = 216(10^6)$ mm^4

***10–32.** Determine the moment of inertia of the beam's cross-sectional area about the x axis.



SOLUTION

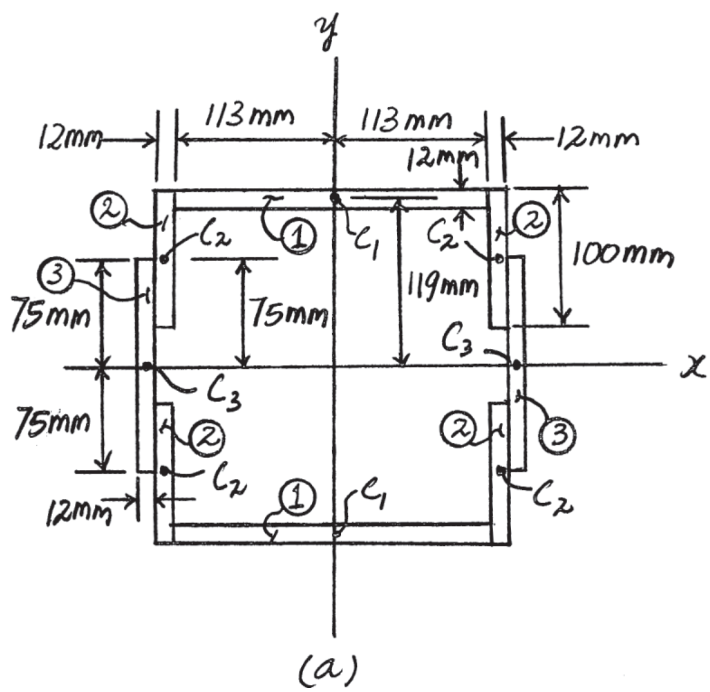
Composite Parts: The composite area can be subdivided into segments as shown in Fig. *a*. The perpendicular distance measured from the centroid of each segment to the x axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the x axis can be determined using the parallel - axis theorem. Thus,

$$I_x = \bar{I}_{x'} + A(d_y)^2$$

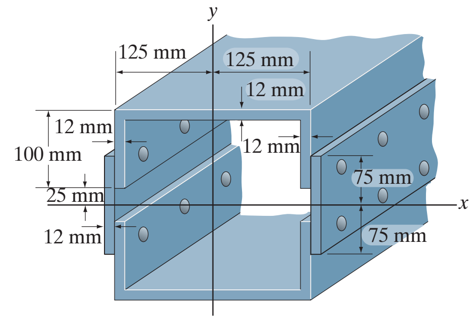
$$= \left[2\left(\frac{1}{12}(226)(12^3)\right) + 2(226)(12)(119)^2\right] + \left[4\left(\frac{1}{12}(12)(100^3)\right) + 4(12)(100)(75)^2\right] + \left[2\left(\frac{1}{12}(12)(150^3)\right) + 2(12)(150)(0)^2\right]$$

$$= 114.62(10^6)\text{ mm}^4 = 115(10^6)\text{ mm}^4$$ **Ans.**



Ans:
$I_x = 115(10^6)\text{ mm}^4$

10–33. Determine the moment of inertia of the beam's cross-sectional area about the y axis.



SOLUTION

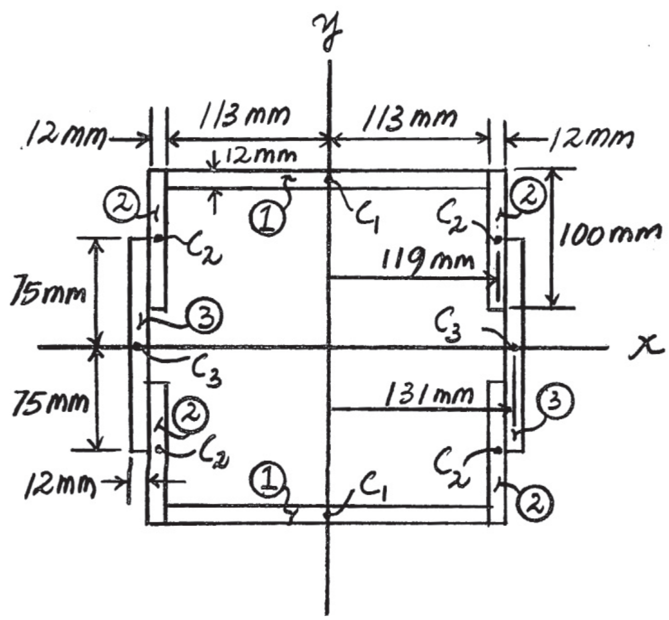
Composite Parts: The composite cross - sectional area of the beam can be subdivided into segments as shown in Fig. *a*. The perpendicular distance measured from the centroid of each segment to the x axis is also indicated.

Moment of Inertia: The moment of inertia of each segment about the y axis can be determined using the parallel - axis theorem. Thus,

$$I_y = \bar{I}_{x'} + A(d_x)^2$$

$$= \left[2\left(\frac{1}{12}(12)(226^3)\right) + 2(226)(12)(0)^2\right] + \left[4\left(\frac{1}{12}(100)(12^3)\right) + 4(100)(12)(119)^2\right] + \left[2\left(\frac{1}{12}(150)(12^3)\right) + 2(150)(12)(131)^2\right]$$

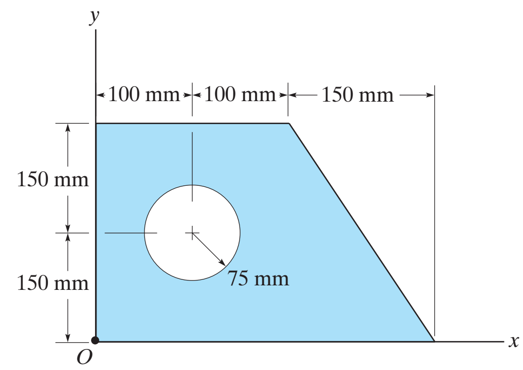
$$= 152.94(10^6)\,\text{mm}^4 = 153(10^6)\,\text{mm}^4$$ **Ans.**



Ans:
$I_y = 153(10^6)\,\text{mm}^4$

10–34.

Determine the moment of inertia I_x of the shaded area about the x axis.



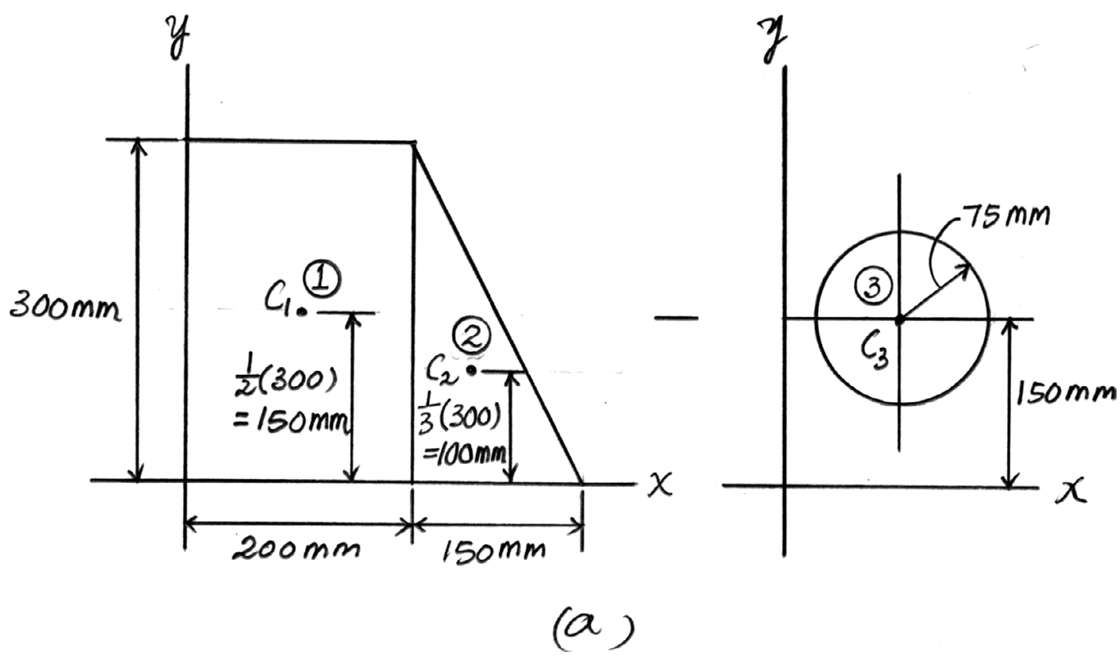
SOLUTION

Moment of Inertia. The moment of inertia about the x axis for each segment can be determined using the parallel axis theorem, $I_x = \bar{I}_{x'} + Ad^2y$. Referring to Fig. a

Segment	$A_i(\text{mm}^2)$	$(d_y)_i\,(\text{mm})$	$(\bar{I}_{x'})_i\,(\text{mm}^4)$	$(Ad_y)_i^2\,(\text{mm}^4)$	$(\bar{I}_x)_i\,(\text{mm}^4)$
1	200(300)	150	$\frac{1}{12}(200)(300^3)$	$1.35(10^9)$	$1.80(10^9)$
2	$\frac{1}{2}(150)(300)$	100	$\frac{1}{36}(150)(300^3)$	$0.225(10^9)$	$0.3375(10^9)$
3	$-\pi(75^2)$	150	$-\frac{\pi(75^4)}{4}$	$-0.3976(10^9)$	$-0.4225(10^9)$

Thus,

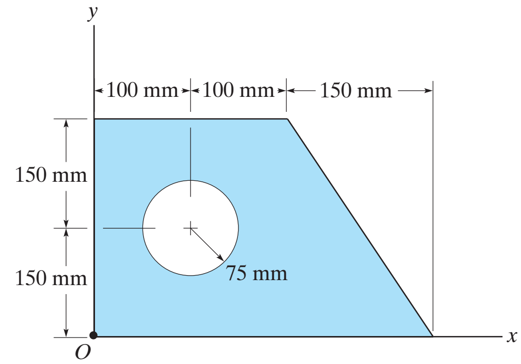
$$I_x = \Sigma(I_x)_i = 1.715(10^9)\ \text{mm}^4 = 1.72(10^9)\ \text{mm}^4 \qquad \textbf{Ans.}$$



(a)

Ans:
$I_x = 1.72(10^9)\ \text{mm}^4$

10–35. Determine the moment of inertia I_x of the shaded area about the y axis.



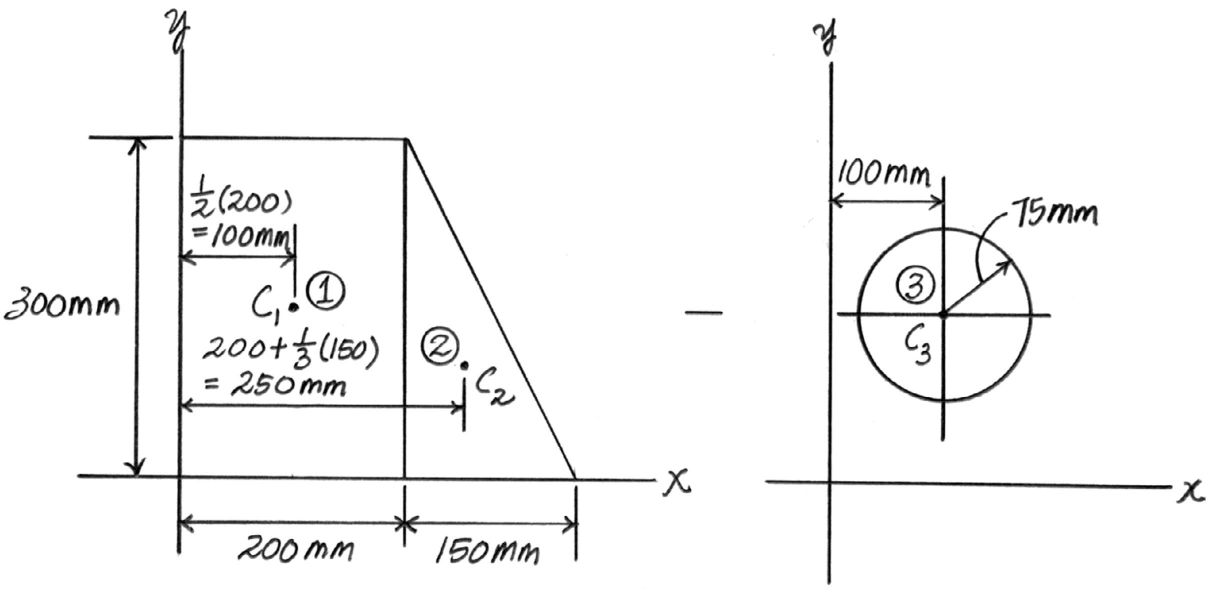
SOLUTION

Moment of Inertia. The moment of inertia about the y axis for each segment can be determined using the parallel-axis theorem, $I_y = \bar{I}_{y'} + Ad_x^2$. Referring to Fig. *a*

Segment	$A_i(\text{mm}^2)$	$(d_x)_i\,(\text{mm})$	$\bar{I}_{y'}(\text{mm}^4)$	$(Ad_x^2)_i\,(\text{mm}^4)$	$(\bar{I}_y)_i\,(\text{mm}^4)$
1	200(300)	100	$\frac{1}{12}(300)(200^3)$	$0.6(10^9)$	$0.800(10^9)$
2	$\frac{1}{2}(150)(300)$	250	$\frac{1}{36}(300)(150^3)$	$1.40625(10^9)$	$1.434375(10^9)$
3	$-\pi(75^2)$	100	$-\frac{\pi(75^4)}{4}$	$-0.1767(10^9)$	$-0.20157(10^9)$

Thus,

$$I_y = \Sigma(I_y)_i = 2.033(10^9)\ \text{mm}^4 = 2.03(10^9)\ \text{mm}^4$$ **Ans.**

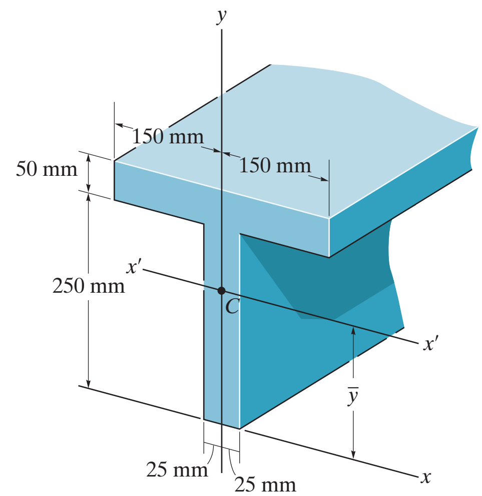


(a)

Ans:
$I_y = 2.03(10^9)\text{mm}^4$

***10–36.**

Determine the moment of inertia of the beam's cross-sectional area about the y axis.

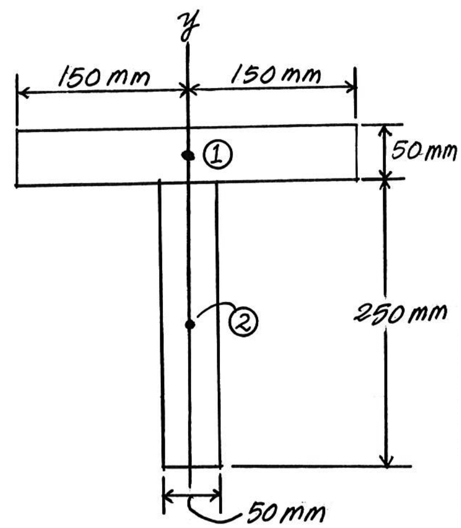


SOLUTION

Moment of Inertia: The dimensions and location of centroid of each segment are shown in Fig. *a*. Since the y axis passes through the centroid of both segments, the moment of inertia about y axis for each segment is simply $(I_y)_i = (I_{y'})_i$.

$$I_y = \Sigma (I_y)_i = \frac{1}{12}(50)(300^3) + \frac{1}{12}(250)(50^3)$$

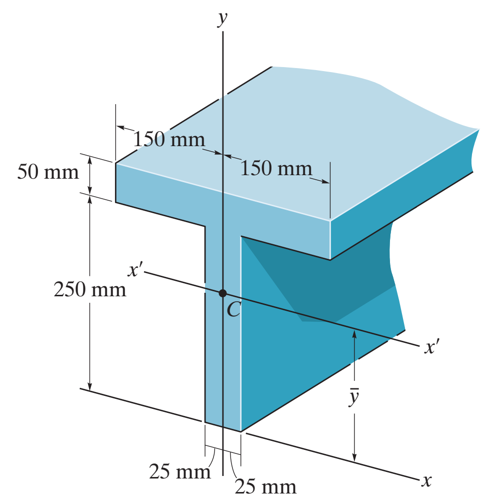
$$= 115.10(10^6)\text{ mm}^4 = 115(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$



(a)

Ans:
$I_y = 115\left(10^6\right)\text{ mm}^4$

10–37. Determine $\bar{y}$, which locates the centroidal axis x' for the cross-sectional area of the T-beam, and then find the moment of inertia about the x' axis.



SOLUTION

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{125(250)(50) + (275)(50)(300)}{250(50) + 50(300)}$$

$$= 206.818 \text{ mm}$$

$$\bar{y} = 207 \text{ mm} \qquad \textbf{Ans.}$$

$$\bar{I}_{x'} = \left[\frac{1}{12}(50)(250)^3 + 50(250)(206.818 - 125)^2\right]$$

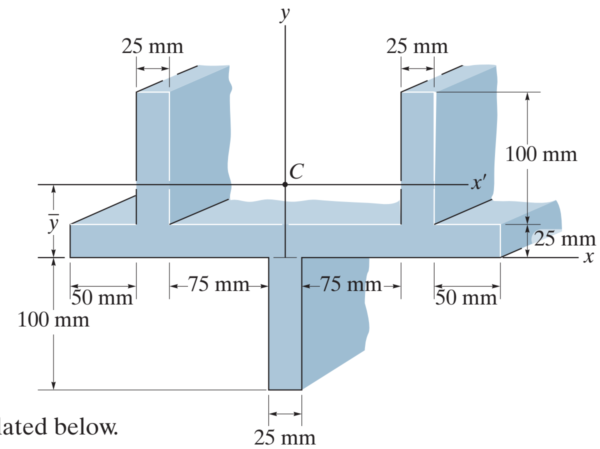
$$+\left[\frac{1}{12}(300)(50)^3 + 50(300)(275 - 206.818)^2\right]$$

$$\bar{I}_{x'} = 222(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

Ans:
$\bar{y} = 207 \text{ mm}$
$\bar{I}_{x'} = 222\left(10^6\right) \text{mm}^4$

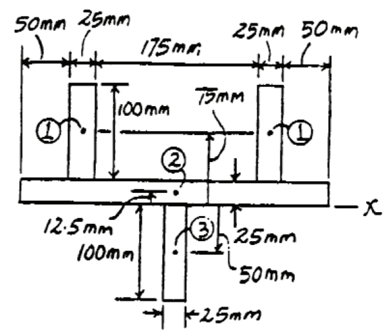
10–38.

Determine the distance $\bar{y}$ to the centroid of the beam's cross-sectional area; then determine the moment of inertia about the x' axis.



SOLUTION

Centroid: The area of each segment and its respective centroid are tabulated below.

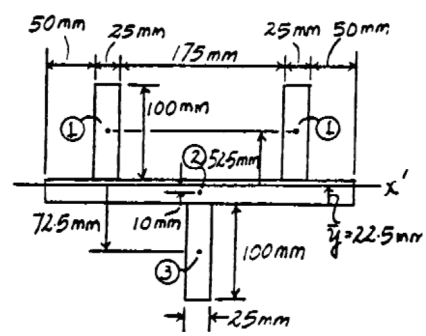


Segment	A (mm^2)	$\bar{y}$ (mm)	$\bar{y}A$ (mm^3)
1	50(100)	75	375(10^3)
2	325(25)	12.5	101.5625(10^3)
3	25(100)	–50	–125(10^3)
Σ	15.625(10^3)		351.5625(10^3)

Thus,

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{351.5625(10^3)}{15.625(10^3)} = 22.5 \text{ mm} \qquad \textbf{Ans.}$$

Moment of Inertia: The moment of inertia about the x' axis for each segment can be determined using the parallel-axis theorem $I_{x'} + \bar{I}_{x'} + Ad_y^2$.



Segment	A_i (mm^2)	$(d_y)_i$ (mm)	$(\bar{I}_{x''})_i$ (mm^4)	$(Ad_y^2)_i$ (mm^4)	$(\bar{I}_{x'})_i$ (mm^4)
1	50(100)	52.5	$\frac{1}{12}(50)(100^3)$	13.781(10^6)	17.948(10^6)
2	325(25)	10	$\frac{1}{12}(325)(25^3)$	0.8125(10^6)	1.236(10^6)
3	25(100)	72.5	$\frac{1}{12}(25)(100^3)$	13.141(10^6)	15.224(10^6)

Thus,

$$I_{x'} = \Sigma (I_{x'})_i = 34.41\left(10^6\right) \text{ mm}^4 = 34.4\left(10^6\right) \text{ mm}^4 \qquad \textbf{Ans.}$$

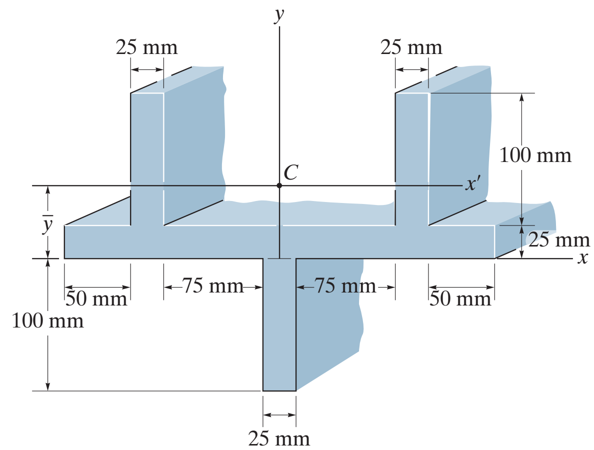
Ans:

$\bar{y} = 22.5$ mm

$I_{x'} = 34.4(10^6)$ mm^4

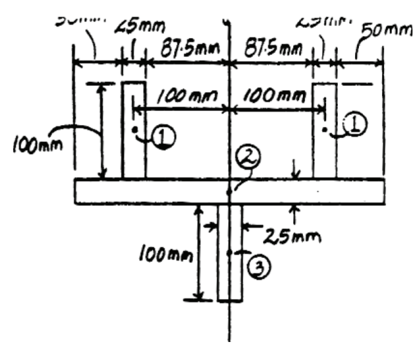
10–39.

Determine the moment of inertia of the beam's cross-sectional area about the y axis.



SOLUTION

Moment of Inertia: The moment of inertia about the y' axis for each segment can be determined using the parallel-axis theorem $I_{y'} = \bar{I}_{y'} + Ad_x^2$.



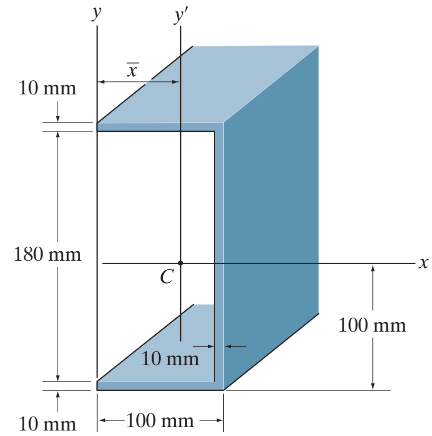
Segment	A_i (mm^2)	$(d_x)_i$ (mm)	$(\bar{I}_{y''})_i$ (mm^4)	$(Ad_x^2)_i$ (mm^4)	$(\bar{I}_{y''})_i$ (mm^4)
1	2[100(25)]	100	$2\left[\frac{1}{12}(100)(25^3)\right]$	$50.0(10^6)$	$50.260(10^6)$
2	25(325)	0	$\frac{1}{12}(25)(325^3)$	0	$71.517(10^6)$
3	100(25)	0	$\frac{1}{12}(100)(25^3)$	0	$0.130(10^6)$

Thus,

$$I_{y'} = \Sigma(I_{y'})_i = 121.91\left(10^6\right)\text{ mm}^4 = 122\left(10^6\right)\text{ mm}^4 \qquad \textbf{Ans.}$$

Ans:
$I_{y'} = 122\left(10^6\right)\text{ mm}^4$

***10–40.** Determine the moment of inertia of the cross-sectional area about the x axis.



SOLUTION

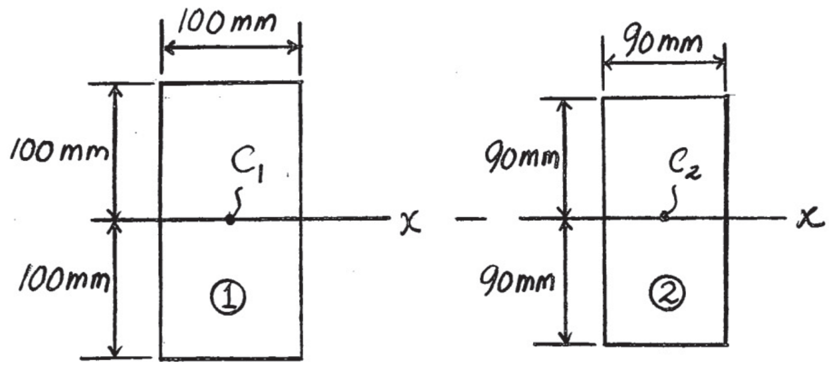
Composite Parts: The composite cross - sectional area of the beam can be subdivided into two segments as shown in Fig. *a*. Here, segment (2) is hole, and so it contributes a negative moment of inertia.

Moment of Inertia: Since the x axis passes through the centroid of both rectangular segments,

$$I_x = (I_x)_1 + (I_x)_2$$

$$= \frac{1}{12}(100)(200^3) - \frac{1}{12}(90)(180^3)$$

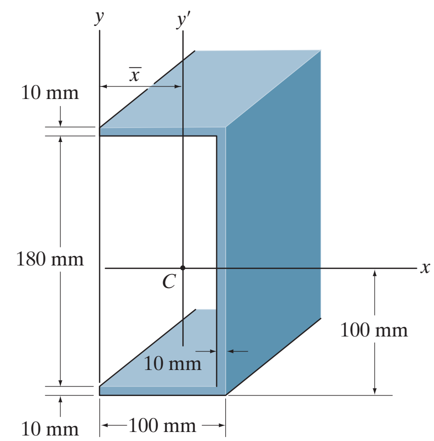
$$= 22.9(10^6)\,\text{mm}^4$$ **Ans.**



(a)

Ans:
$I_x = 22.9(10^6)\,\text{mm}^4$

10–41. Locate the centroid $\bar{x}$ of the beam's cross-sectional area, and then determine the moment of inertia of the area about the centroidal y' axis.



SOLUTION

Composite Parts: The composite cross - sectional area of the beam can be subdivided into segments as shown in Fig. *a*.

Centroid: The perpendicular distance measured from the centroid of each type of segment to the y axis is also indicated in Fig. *a*. Thus,

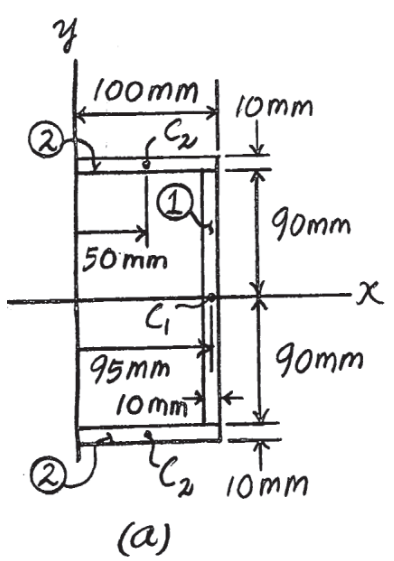
$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{95(10(180)) + 50(2(100)(10))}{10(180) + 2(100)(10)} = \frac{271(10^3)}{3.8(10^3)} = 71.32 \text{ mm} \qquad \textbf{Ans.}$$

Moment of Inertia: The moment of inertia of each segment about the y' axis can be determined using the parallel - axis theorem. The perpendicular distance measured from the centroid of each type of segment to the y' axis is indicated in Fig. *b*.

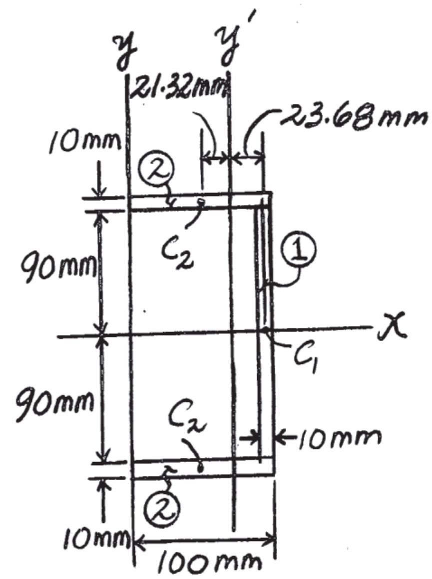
$$I_{y'} = \bar{I}_{y'} + A(d_{x'})^2$$

$$= \left[\frac{1}{12}(180)(10^3) + 180(10)(23.68)^2\right] + \left[2\left(\frac{1}{12}(10)(100^3)\right) + 2(100)(10)(21.32)^2\right]$$

$$= 3.60(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$



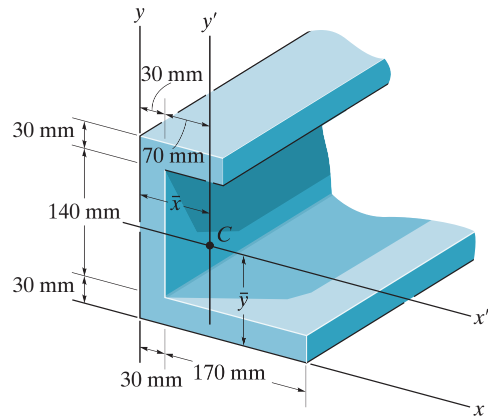
(a)



Ans:
$\bar{x} = 71.32$ mm
$I_{y'} = 3.60(10^6)$ mm^4

10–42.

Determine the moment of inertia of the beam's cross-sectional area about the x axis.



SOLUTION

$$I_x = \frac{1}{12}(170)(30)^3 + 170(30)(15)^2$$

$$+\frac{1}{12}(30)(170)^3 + 30(170)(85)^2$$

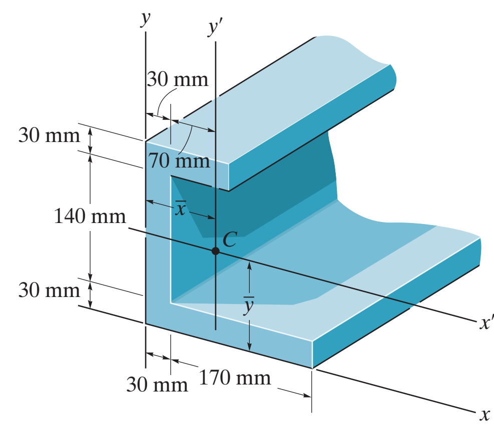
$$+\frac{1}{12}(100)(30)^3 + 100(30)(185)^2$$

$I_x = 154(10^6)\ \text{mm}^4$ **Ans.**

Ans:
$I_x = 154\left(10^6\right)\ \text{mm}^4$

10–43.

Determine the moment of inertia of the beam's cross-sectional area about the y axis.



SOLUTION

$$I_y = \frac{1}{12}(30)(170)^3 + 30(170)(115)^2$$

$$+ \frac{1}{12}(170)(30)^3 + 30(170)(15)^2$$

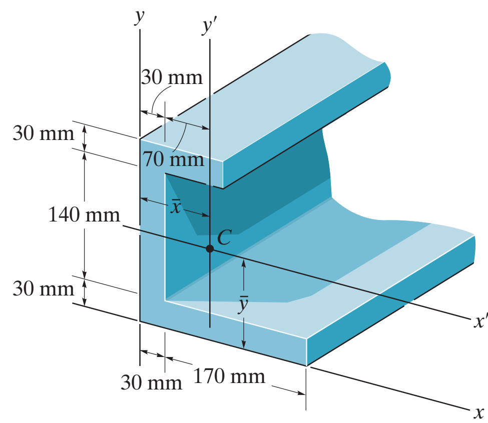
$$+ \frac{1}{12}(30)(100)^3 + 30(100)(50)^2$$

$$I_y = 91.3(10^6)\text{ mm}^4$$ **Ans.**

Ans:
$I_y = 91.3(10^6)\text{ mm}^4$

***10–44.**

Determine the distance $\bar{y}$ to the centroid C of the beam's cross-sectional area and then compute the moment of inertia $\bar{I}_{x'}$ about the x' axis.



SOLUTION

$$\bar{y} = \frac{170(30)(15) + 170(30)(85) + 100(30)(185)}{170(30) + 170(30) + 100(30)}$$

$$= 80.68 = 80.7 \text{ mm}$$ **Ans.**

$$\bar{I}_{x'} = \left[\frac{1}{12}(170)(30)^3 + 170(30)(80.68 - 15)^2\right]$$

$$+\left[\frac{1}{12}(30)(170)^3 + 30(170)(85 - 80.68)^2\right]$$

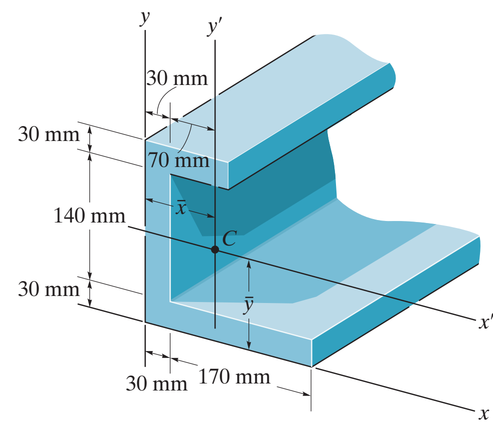
$$+\frac{1}{12}(100)(30)^3 + 100(30)(185 - 80.68)^2$$

$$\bar{I}_{x'} = 67.6(10^6) \text{ mm}^4$$ **Ans.**

Ans:
$\bar{y} = 80.7$ mm
$\bar{I}_{x'} = 67.6(10^6)$ mm^4

10–45.

Determine the distance $\bar{x}$ to the centroid C of the beam's cross-sectional area and then compute the moment of inertia $\bar{I}_{y'}$ about the y' axis.



SOLUTION

$$\bar{x} = \frac{170(30)(115) + 170(30)(15) + 100(30)(50)}{170(30) + 170(30) + 100(30)}$$

$$= 61.59 = 61.6 \text{ mm}$$ **Ans.**

$$\bar{I}_{y'} = \left[\frac{1}{12}(30)(170)^3 + 170(30)(115 - 61.59)^2\right]$$

$$+ \left[\frac{1}{12}(170)(30)^3 + 30(170)(15 - 61.59)^2\right]$$

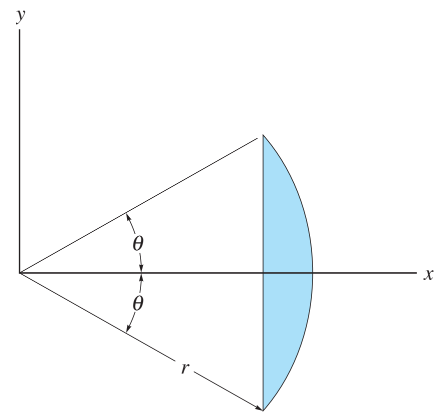
$$+ \frac{1}{12}(30)(100)^3 + 100(30)(50 - 61.59)^2$$

$$\bar{I}_{y'} = 41.2(10^6) \text{ mm}^4$$ **Ans.**

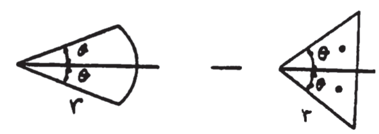
Ans:
$\bar{x} = 61.6$ mm
$\bar{I}_{y'} = 41.2\left(10^6\right)$ mm^4

10–46.

Determine the moment of inertia of the shaded area about the x axis.



SOLUTION



$$I_x = \left[\frac{1}{4}r^4\left(\theta - \frac{1}{2}\sin 2\theta\right)\right]$$

$$- 2\left[\frac{1}{36}(r\cos\theta)(r\sin\theta)^3 + \frac{1}{2}(r\cos\theta)(r\sin\theta)\left(\frac{1}{3}r\sin\theta\right)^2\right]$$

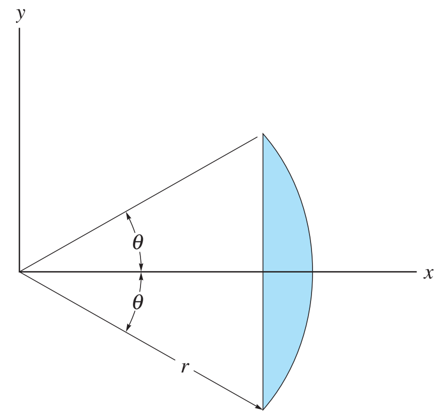
$$= \frac{1}{4}r^4\left(\theta - \frac{1}{2}\sin 2\theta\right) - \frac{1}{18}r^4\cos\theta\sin^3\theta - \frac{1}{9}r^4\cos\theta\sin^3\theta$$

$$= \frac{r^4}{24}(6\theta - 3\sin 2\theta - 4\cos\theta\sin^3\theta)$$ **Ans.**

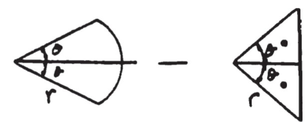
Ans:
$I_x = \dfrac{r^4}{24}(6\theta - 3\sin 2\theta - 4\cos\theta\sin^3\theta)$

10–47.

Determine the moment of inertia of the shaded area about the y axis.



SOLUTION



$$I_y = \left[\frac{1}{4} r^4\left(\theta + \frac{1}{2}\sin 2\theta\right)\right]$$

$$- \left[\frac{1}{36}(2r\sin\theta)(r\cos\theta)^3 + \frac{1}{2}(2r\sin\theta)(r\cos\theta)\left(\frac{2}{3}r\cos\theta\right)^2\right]$$

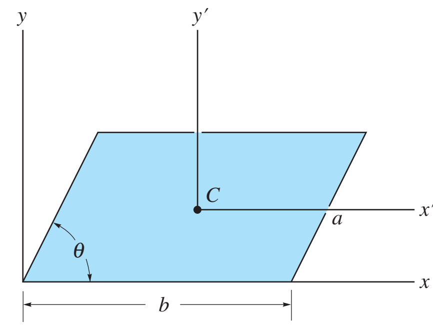
$$= \frac{1}{4}r^4\left(\theta + \frac{1}{2}\sin 2\theta\right) - \left[\frac{1}{18}r^4\sin\theta\cos^3\theta + r^3\sin\theta\cos^3\theta\right]$$

$$= \frac{r^4}{4}\left(\theta + \frac{1}{2}\sin 2\theta - 2\sin\theta\cos^3\theta\right)$$ **Ans.**

Ans:
$$I_y = \frac{r^4}{4}\left(\theta + \frac{1}{2}\sin 2\theta - 2\sin\theta\cos^3\theta\right)$$

***10–48.**

Determine the moment of inertia of the parallelogram about the x' axis, which passes through the centroid C of the area.



SOLUTION

$h = a \sin \theta$

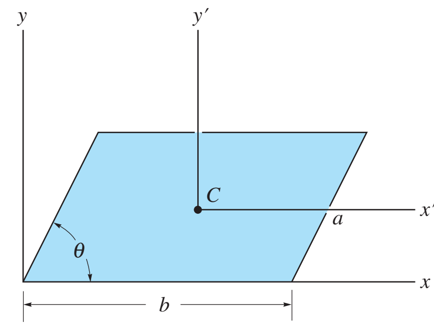
$$\bar{I}_{x'} = \frac{1}{12}bh^3 = \frac{1}{12}(b)(a \sin \theta)^3 = \frac{1}{12}a^3 b \sin^3 \theta \qquad \textbf{Ans.}$$

Ans:

$\bar{I}_{x'} = \dfrac{1}{12} a^3 b \sin^3 \theta$

10–49.

Determine the moment of inertia of the parallelogram about the y' axis, which passes through the centroid C of the area.



SOLUTION

$$\bar{x} = a\cos\theta + \frac{b - a\cos\theta}{2} = \frac{1}{2}(a\cos\theta + b)$$

$$\bar{I}_{y'} = 2\left[\frac{1}{36}(a\sin\theta)(a\cos\theta)^3 + \frac{1}{2}(a\sin\theta)(a\cos\theta)\left(\frac{b}{2} + \frac{a}{2}\cos\theta - \frac{2}{3}a\cos\theta\right)^2\right]$$

$$+ \frac{1}{12}(a\sin\theta)(b - a\cos\theta)^3$$

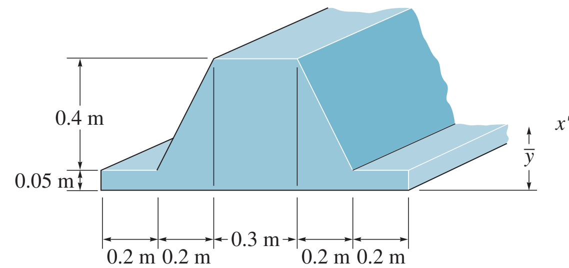
$$= \frac{ab\sin\theta}{12}\left(b^2 + a^2\cos^2\theta\right) \qquad \textbf{Ans.}$$

Ans:

$$I_{y'} = \frac{ab\sin\theta}{12}\left(b^2 + a^2\cos^2\theta\right)$$

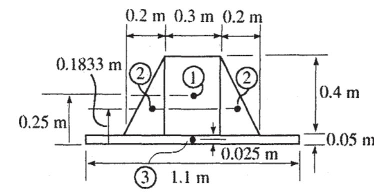
10–50.

Locate the centroid $\bar{y}$ of the cross section and determine the moment of inertia of the section about the x' axis.



SOLUTION

Centroid: The area of each segment and its respective centroid are tabulated below.

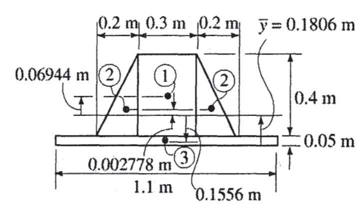


Segment	A (m^2)	$\bar{y}$ (m)	$\bar{y}A$ (m^3)
1	0.3(0.4)	0.25	0.03
2	$\frac{1}{2}(0.4)(0.4)$	0.1833	0.014667
3	1.1(0.05)	0.025	0.001375
Σ	0.255		0.046042

Thus,

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.046042}{0.255} = 0.1806 \text{ m} = 0.181 \text{ m} \qquad \textbf{Ans.}$$

Moment of Inertia: The moment of inertia about the x' axis for each segment can be determined using the parallel-axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.



Segment	A_i (m^2)	$(d_y)_i$ (m)	$(\bar{I}_{x'})_i$ (m^4)	$(Ad_y^2)_i$ (m^4)	$(I_{x'})_i$ (m^4)
1	0.3(0.4)	0.06944	$\frac{1}{12}(0.3)(0.4^3)$	$0.5787(10^{-3})$	$2.1787(10^{-3})$
2	$\frac{1}{2}(0.4)(0.4)$	0.002778	$\frac{1}{36}(0.4)(0.4^3)$	$0.6173(10^{-6})$	$0.7117(10^{-3})$
3	1.1(0.05)	0.1556	$\frac{1}{12}(1.1)(0.05^3)$	$1.3309(10^{-3})$	$1.3423(10^{-3})$

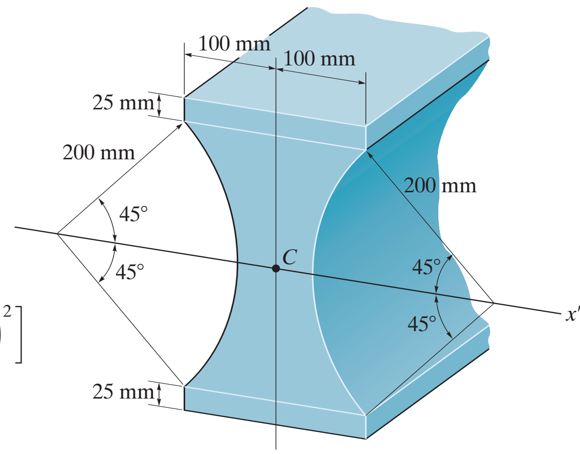
Thus,

$$I_{x'} = \Sigma(I_{x'})_i = 4.233(10^{-3}) \text{ m}^4 = 4.23(10^{-3}) \text{ m}^4 \qquad \textbf{Ans.}$$

Ans:
$\bar{y} = 0.181$ m
$\bar{I}_{x'} = 4.23(10^{-3})$ m^4

10–51.

Determine the moment of inertia for the beam's cross-sectional area about the x' axis passing through the centroid C of the cross section.

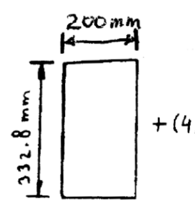


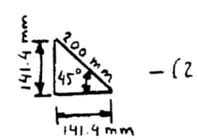
SOLUTION

$$\bar{I}_{x'} = \frac{1}{12}(200)(332.8)^3 + 4\left[\frac{1}{36}(141.4)(141.4)^3 + \left(\frac{1}{2}(141.4)(141.4)\right)\left(\frac{141.4}{3}\right)^2\right]$$

$$- 2\left[\frac{1}{4}(200)^4\left(\frac{\pi}{4} - \frac{1}{2}\sin 90^\circ\right)\right]$$

$$= 520(10^6)\ \text{mm}^4$$ **Ans.**

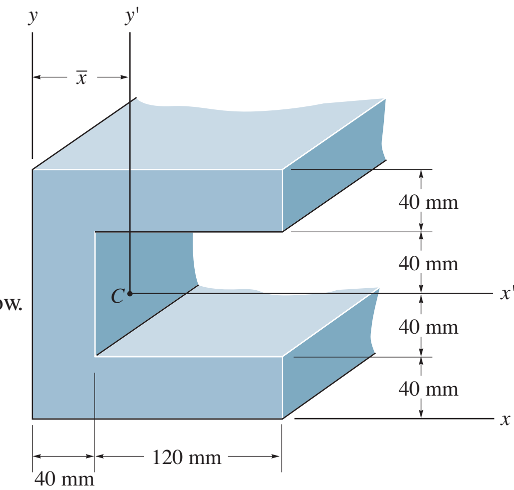




Ans:
$\bar{I}_{x'} = 520(10^6)\ \text{mm}^4$

***10–52.**

Determine the distance $\bar{x}$ to the centroid of the beam's cross-sectional area: then find the moment of inertia about the y' axis.



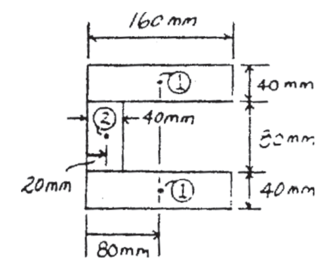
SOLUTION

Centroid: The area of each segment and its respective centroid are tabulated below.

Segment	A (mm^2)	$\bar{x}$ (mm)	$\bar{x}A$ (mm^3)
1	160(80)	80	1.024(10^6)
2	40(80)	20	64.0(10^3)
Σ	16.0(10^3)		1.088(10^6)

Thus,

$$\bar{x} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{1.088(10^6)}{16.0(10^3)} = 68.0 \text{ mm} \qquad \textbf{Ans.}$$

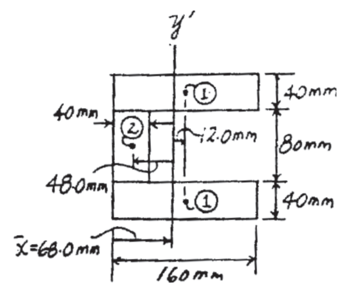


Moment of Inertia: The moment of inertia about the y' axis for each segment can be determined using the parallel–axis theorem $I_{y'} = \bar{I}_{y'} + Ad_x^2$.

Segment	A_i (mm^2)	$(d_x)_i$ (mm)	$(\bar{I}_{y'})_i$ (mm^4)	$(Ad_x^2)_i$ (mm^4)	$(I_{y'})_i$ (mm^4)
1	80(160)	12.0	$\frac{1}{12}(80)(160^3)$	1.8432(10^6)	29.150(10^6)
2	80(40)	48.0	$\frac{1}{12}(80)(40^3)$	7.3728(10^6)	7.799(10^6)

Thus,

$$I_{y'} = \Sigma\left(I_{y'}\right)_i = 36.949\left(10^6\right) \text{ mm}^4 = 36.9\left(10^6\right) \text{ mm}^4 \qquad \textbf{Ans.}$$

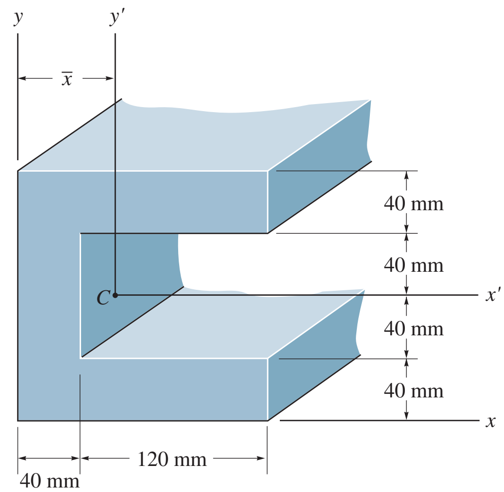


Ans:

$I_{y'} = 36.9\left(10^6\right)$ mm^4

10–53.

Determine the moment of inertia for the beam's cross-sectional area about the x' axis.



SOLUTION

Moment of Inertia: The moment inertia for the rectangle about its centroidal axis can be determined using the formula, $I_{x'} = \frac{1}{12}bh^3$, given on the inside back cover of the textbook.

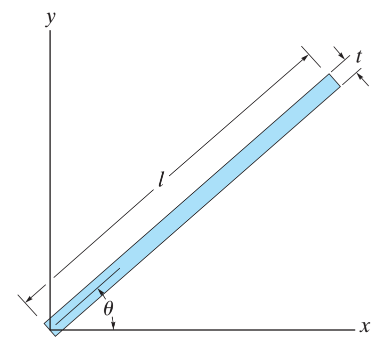
$$I_{x'} = \frac{1}{12}(160)\left(160^3\right) - \frac{1}{12}(120)\left(80^3\right) = 49.5\left(10^6\right)\ \text{mm}^4 \qquad \textbf{Ans.}$$

Ans:

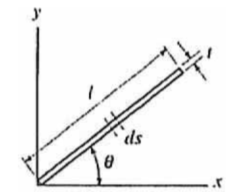
$I_{x'} = 49.5(10^6)\ \text{mm}^4$

10–54.

Determine the product of inertia of the thin strip of area with respect to the x and y axes. The strip is oriented at an angle θ from the x axis. Assume that $t \ll l$.



SOLUTION



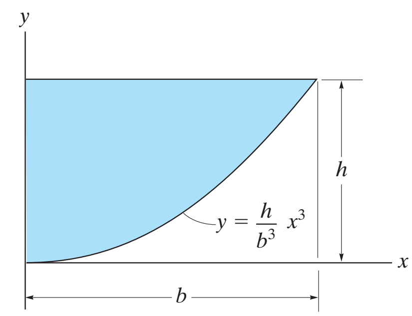
$$l_{xy} = \int_A xy\,dA = \int_0^l (s\cos\theta)(s\sin\theta)t\,ds = \sin\theta\cos\theta t\int_0^1 s^2 ds$$

$$= \frac{1}{6}l^3 t\sin 2\bar{\theta}$$ **Ans.**

Ans:
$I_{xy} = \frac{1}{6}l^3 t\sin 2\bar{\theta}$

10–55.

Determine the product of inertia for the shaded area with respect to the x and y axes.



SOLUTION

$$\tilde{x} = \frac{x}{2}$$

$$\tilde{y} = y$$

$$dA = x\,dy$$

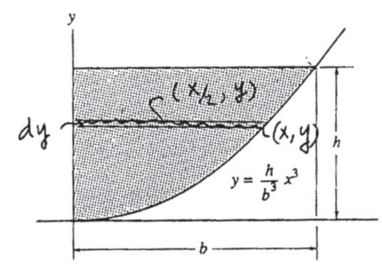
$$d\,I_{xy} = \frac{x^2 y}{2}\,dy$$

$$I_{xy} = \int d\,I_{xy}$$

$$= \int_0^h \frac{1}{2}\left(\frac{b}{h^{1/3}}\right)^2 y^{5/3}\,dy$$

$$= \frac{1}{2}\left[\left(\frac{b^2}{h^{2/3}}\right)\left(\frac{3}{8}\right)y^{8/3}\right]_0^h$$

$$= \frac{3}{16}b^2 h^2$$ **Ans.**

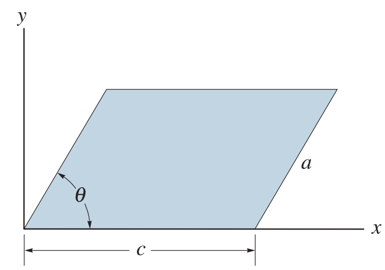


Ans:

$I_{xy} = \frac{3}{16}b^2 h^2$

***10–56.**

Determine the product of inertia of the parallelogram with respect to the x and y axes.



SOLUTION

Product of Inertia of the Triangle: The product of inertia with respect to x and y axes can be determined by integration. The area of the differential element parallel to y axis is $dA = y\,dx = \left(h + \frac{h}{b}x\right)dx$ [Fig. (a)]. The coordinates of the centroid for this element are $\widetilde{x} = x$, $\widetilde{y} = \frac{y}{2} = \frac{1}{2}\left(h + \frac{h}{b}x\right)$. Then the product of inertia for this element is

$$dI_{xy} = d\bar{I}_{x'y'} + dA\,\widetilde{x}\widetilde{y}$$

$$= 0 + \left[\left(h + \frac{h}{b}x\right)dx\right](x)\left[\frac{1}{2}\left(h + \frac{h}{b}x\right)\right]$$

$$= \frac{1}{2}\left(h^2x + \frac{h^2}{b^2}x^3 + \frac{2h^2}{b}x^2\right)dx$$

Performing the integration, we have

$$I_{xy} = \int dI_{xy} = \frac{1}{2}\int_{-b}^{0}\left(h^2x + \frac{h^2}{b^2}x^3 + \frac{2h^2}{b}x^2\right)dx = -\frac{b^2h^2}{24}$$

The product of inertia with respect to centroidal axes, x' and y', can be determined by applying Eq. 10–8 [Fig. (b) or (c)].

$$I_{xy} = \bar{I}_{x'y'} + Ad_xd_y$$

$$-\frac{b^2h^2}{24} = \bar{I}_{x'y'} + \frac{1}{2}bh\left(-\frac{b}{3}\right)\left(\frac{h}{3}\right)$$

$$\bar{I}_{x'y'} = \frac{b^2h^2}{72}$$

Here, $b = a\cos\theta$ and $h = a\sin\theta$. Then, $\bar{I}_{x'y'} = \dfrac{a^4\sin^2\theta\cos^2\theta}{72}$.

Product of inertia of the parallelogram [Fig. (d)] with respect to centroidal x' and y' axes, is

$$\bar{I}_{x'y'} = 2\left[\frac{a^4\cos^2\theta\sin^2\theta}{72} + \frac{1}{2}(a\sin\theta)(a\cos\theta)\left(\frac{3c - a\cos\theta}{6}\right)\left(\frac{a\sin\theta}{6}\right)\right]$$

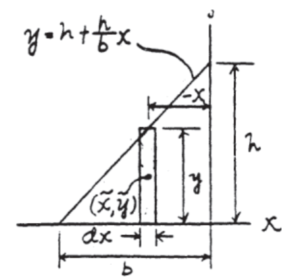
$$= \frac{a^3c\sin^2\theta\cos\theta}{12}$$

The product of inertia of the parallelogram [Fig. (d)] about x and y axes is

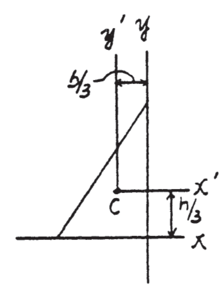
$$I_{xy} = \bar{I}_{x'y'} + Ad_xd_y$$

$$= \frac{a^3c\sin^2\theta\cos\theta}{12} + (a\sin\theta)(c)\left(\frac{c + a\cos\theta}{2}\right)\left(\frac{a\sin\theta}{2}\right)$$

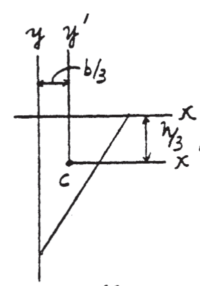
$$= \frac{a^2c\sin^2\theta}{12}(4a\cos\theta + 3c) \qquad \textbf{Ans.}$$



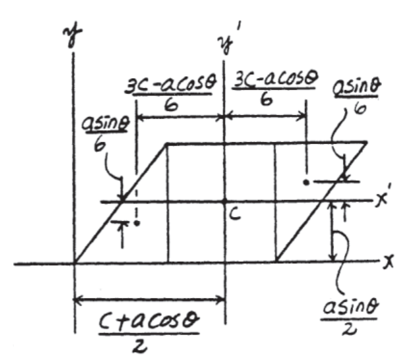
(a)



(b)



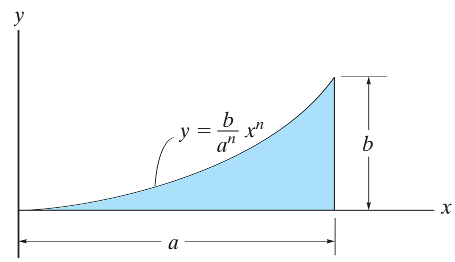
(c)



(d)

10–57.

Determine the product of inertia for the shaded area with respect to the x and y axes.



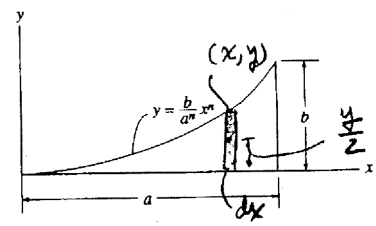
SOLUTION

$$dI_{xy} = d\bar{I}_{xy} + \tilde{x}\tilde{y}\,dA$$

$$I_{xy} = 0 + \int_0^a (x)\left(\frac{y}{2}\right)(y\,dx) = \frac{1}{2}\int_0^a \left(\frac{b^2}{a^{2n}}\right)x^{2n+1}\,dx$$

$$= \left(\frac{b^2}{2a^{2n}}\right)\left(\frac{1}{2n+2}\right)x^{2n+2}\Big|_0^a = \frac{b^2a^{2n+2}}{4(n+1)a^{2n}}$$

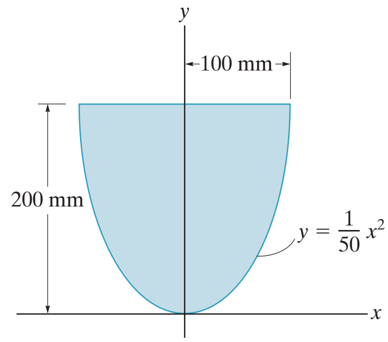
$$= \frac{a^2b^2}{4(n+1)}$$ **Ans.**



Ans:

$$I_{xy} = \frac{a^2b^2}{4(n+1)}$$

10–58. Determine the product of inertia for the shaded portion of the parabola with respect to the x and y axes.



SOLUTION

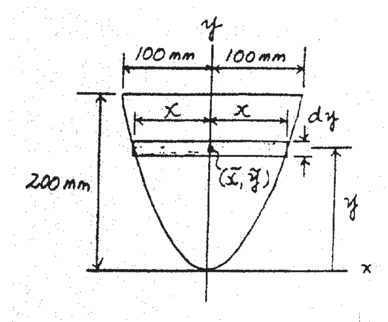
Differential Element: Here, $x = \sqrt{50}y^{\frac{1}{2}}$. The area of the differential element parallel to the x axis is $dA = 2x dy = 2\sqrt{50}y^{\frac{1}{2}}\,dy$. The coordinates of the centroid for this element are $\overline{x} = 0, \overline{y} = y$. Then the product of inertia for this element is

$$\begin{aligned} dI_{xy} &= d\bar{I}_{x'y'} + dA\,\overline{x}\,\overline{y} \\ &= 0 + \left(2\sqrt{50}y^{\frac{1}{2}}\,dy\right)(0)(y) \\ &= 0 \end{aligned}$$

Product of Inertia: Performing the integration, we have

$$I_{xy} = \int dI_{xy} = 0 \qquad \textbf{Ans.}$$

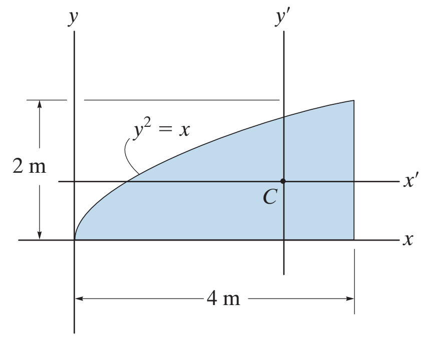
Note: By inspection, $I_{xy} = 0$ since the shaded area is symmetrical about the y axis.



Ans:
$I_{xy} = 0$

10–59.

Determine the product of inertia of the shaded area with respect to the x and y axes, and then use the parallel-axis theorem to find the product of inertia of the area with respect to the centroidal x' and y' axes.



SOLUTION

Differential Element: The area of the differential element parallel to the y axis shown shaded in Fig. a is $dA = y\,dx = x^{1/2}\,dx$. The coordinates of the centroid of this element are $\tilde{x} = x$ and $\tilde{y} = \dfrac{y}{2} = \dfrac{1}{2}x^{1/2}$ Thus, the product of inertia of this element with respect to the x and y axes is

$$dI_{xy} = d\bar{I}_{x'y'} + dA\tilde{x}\tilde{y}$$

$$= 0 + \left(x^{1/2}\,dx\right)(x)\left(\frac{1}{2}x^{1/2}\right)$$

$$= \frac{1}{2}x^2dx$$

Product of Inertia: Performing the integration, we have

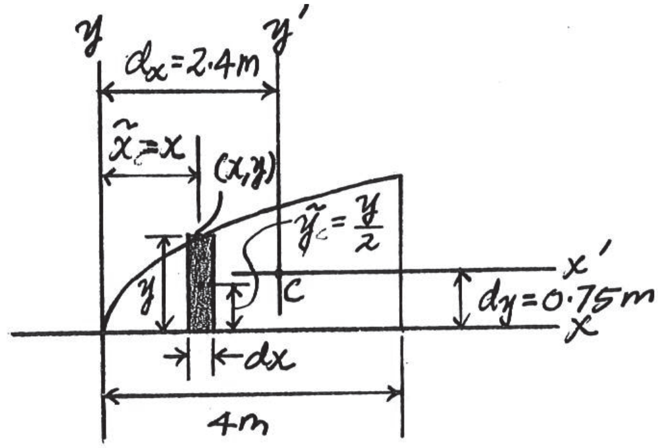
$$I_{xy} = \int dI_{xy} = \int_0^{4\text{ m}} \frac{1}{2}x^2dx = \left(\frac{1}{6}x^3\right)\Bigg|_0^{4\text{ m}} = 10.67\text{ m}^4 = 10.7\text{ m}^4 \quad \textbf{Ans.}$$

Using the information provided on the inside back cover of this book, the location of the centroid of the parabolic area is at $\bar{x} = 4 - \dfrac{2}{5}(4) = 2.4$ m and $\bar{y} = \dfrac{3}{8}(2) = 0.75$ m and its area is given by $A = \dfrac{2}{3}(4)(2) = 5.333\text{ m}^2$. Thus,

$$I_{xy} = \bar{I}_{x'y'} + Ad_xdy$$

$$10.67 = \bar{I}_{x'y'} + 5.333(2.4)(0.75)$$

$$\bar{I}_{x'y'} = 1.07\text{ m}^4 \quad \textbf{Ans.}$$

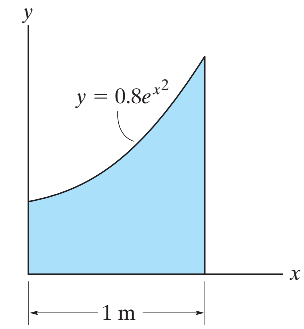


(a)

Ans:
$I_{xy} = 10.7\text{ m}^4$
$\bar{I}_{x'y'} = 1.07\text{ m}^4$

***10–60.**

Determine the product of inertia for the shaded area with respect to the x and y axes. Use Simpson's rule to evaluate the integral.



SOLUTION

$$\bar{x} = x$$

$$\bar{y} = \frac{y}{2}$$

$$d\,A = y\,dx$$

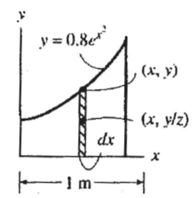
$$dI_{xy} = \frac{xy^2}{2}\,dx$$

$$I_{xy} = \int dI_{xy}$$

$$= \int_0^1 \frac{1}{2}x(0.8\,e^{x^2})^2\,dx$$

$$= 0.32\int_0^1 xe^{2x^2}\,dx$$

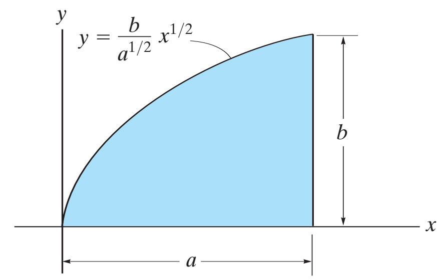
$$= 0.511\text{ m}^4 \qquad \textbf{Ans.}$$



Ans:
$I_{xy} = 0.511\text{ m}^4$

10–61.

Determine the product of inertia for the parabolic area with respect to the x and y axes.



SOLUTION

$$\tilde{x} = x$$

$$\tilde{y} = \frac{y}{2}$$

$$dA = y\,dx$$

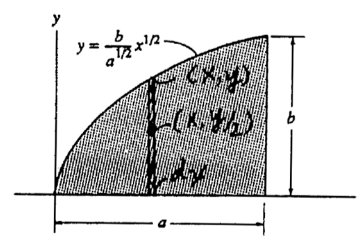
$$dI_{xy} = \frac{xy^2}{2}\,dx$$

$$I_{xy} = \int d\,I_{xy}$$

$$= \int_0^a \frac{1}{2}\left(\frac{b^2}{a}\right)x^2\,dx$$

$$= \frac{1}{6}\left[\left(\frac{b^2}{a}\right)x^3\right]_0^a$$

$$= \frac{1}{6}a^2\,b^2 \qquad \textbf{Ans.}$$

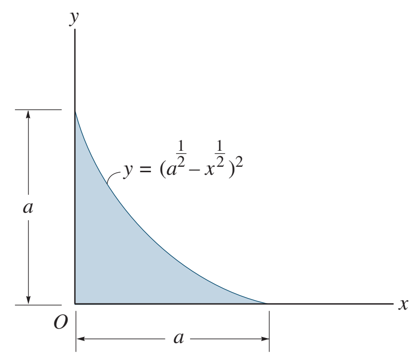


Ans:

$$I_{xy} = \frac{1}{6}a^2b^2$$

10–62.

Determine the product of inertia of the shaded area with respect to the x and y axes.



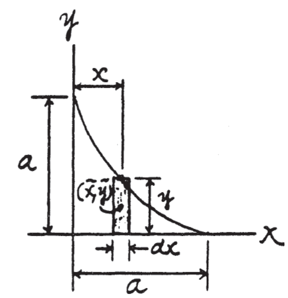
SOLUTION

Differential Element: The area of the differential element parallel to the y axis is $dA = ydx = \left(a^{\frac{1}{2}} - x^{\frac{1}{2}}\right)^2 dx$. The coordinates of the centroid for this element are $\widetilde{x} = x, \widetilde{y} = \dfrac{y}{2} = \dfrac{1}{2}\left(a^{\frac{1}{2}} - x^{\frac{1}{2}}\right)^2$. Then the product of inertia for this element is

$$dI_{xy} = d\bar{I}_{x'y'} + dA\widetilde{x}\widetilde{y}$$

$$= 0 + \left[\left(a^{\frac{1}{2}} - x^{\frac{1}{2}}\right)^2 dx\right](x)\left[\frac{1}{2}\left(a^{\frac{1}{2}} - x^{\frac{1}{2}}\right)^2\right]$$

$$= \frac{1}{2}\left(x^3 + a^2x + 6ax^2 - 4a^{\frac{3}{2}}x^{\frac{3}{2}} - 4a^{\frac{1}{2}}x^{\frac{5}{2}}\right)dx$$



Product of Inertia: Performing the integration, we have

$$I_{xy} = \int dI_{xy} = \frac{1}{2}\int_0^a \left(x^3 + a^2x + 6ax^2 - 4a^{\frac{3}{2}}x^{\frac{3}{2}} - 4a^{\frac{1}{2}}x^{\frac{5}{2}}\right)dx$$

$$= \frac{1}{2}\left(\frac{x^4}{4} + \frac{a^2}{2}x^2 + 2ax^3 - \frac{8}{5}a^{\frac{3}{2}}x^{\frac{5}{2}} - \frac{8}{7}a^{\frac{1}{2}}x^{\frac{7}{2}}\right)\Bigg|_0^a$$

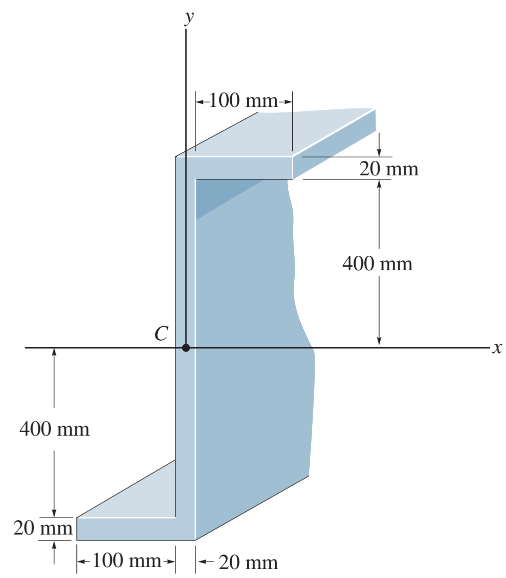
$$= \frac{a^4}{280}$$ **Ans.**

Ans:

$I_{xy} = \dfrac{a^4}{280}$

10–63.

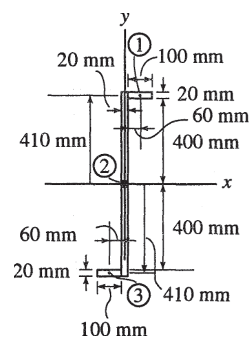
Determine the product of inertia of the cross-sectional area with respect to the x and y axes.



SOLUTION

Product of Inertia: The area for each segment, its centroid and product of inertia with respect to x and y axes are tabulated below.

Segment	A_i **(mm²)**	$(d_x)_i$ **(mm)**	$(d_y)_i$ **(mm)**	$(I_{xy})_i$ **(mm⁴)**
1	100(20)	60	410	$49.2(10^6)$
2	840(20)	0	0	0
3	100(20)	−60	−410	$49.2(10^6)$

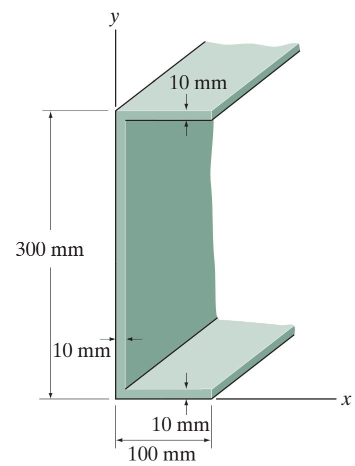


Thus,

$$I_{xy} = \Sigma(I_{xy})_i = 98.4(10^6)\text{mm}^4$$ **Ans.**

Ans:
$I_{xy} = 98.4(10^6)$ mm^4

***10–64.** Determine the product of inertia of the beam's cross-sectional area with respect to the x and y axes.



SOLUTION

Composite Parts: The composite cross - sectional area of the beam can be subdivided into three segments. The perpendicular distances measured from the centroid of each element to the x and y axes are also indicated.

Product of Inertia: Since the centroidal axes are the axes of all the segments are the axes of symmetry, then $\bar{I}_{x'y'} = 0$. Thus, the product of inertia of each segment with respect to the x and y axes can be determined using the parallel - axis theorem.

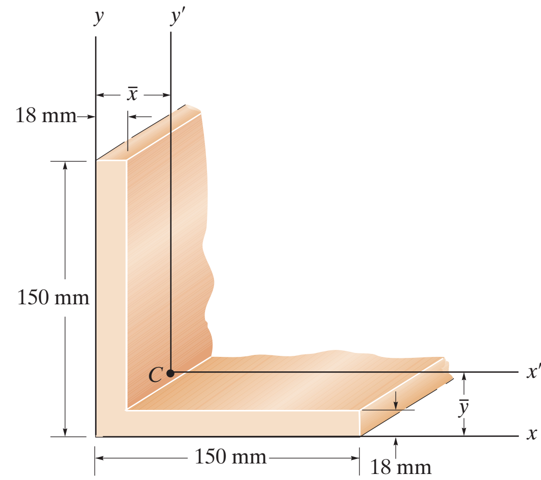
$$I_{xy} = \bar{I}_{x'y'} + Ad_xd_y = Ad_xd_y$$

$$= 90(10)(55)(295) + 300(10)(5)(150) + 90(10)(55)(5) = \Sigma I_{xy} = 17.1(10^6)\text{mm}^4$$ **Ans.**

Ans:
$I_{xy} = 17.1(10^6)\text{mm}^4$

10–65.

Determine the location $(\bar{x}, \bar{y})$ of the centroid C of the angle's cross-sectional area, and then compute the product of inertia with respect to the x' and y' axes.

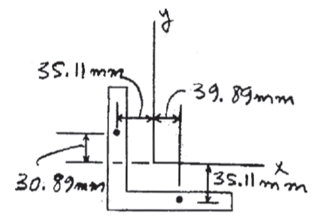


SOLUTION

Centroid:

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{9(18)(150) + 84(18)(132)}{18(150) + 18(132)} = 44.11 \text{ mm} = 44.1 \text{ mm} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{75(18)(150) + 9(18)(132)}{18(150) + 18(132)} = 44.11 \text{ mm} = 44.1 \text{ mm} \qquad \textbf{Ans.}$$



Product of inertia about x' and y' axes:

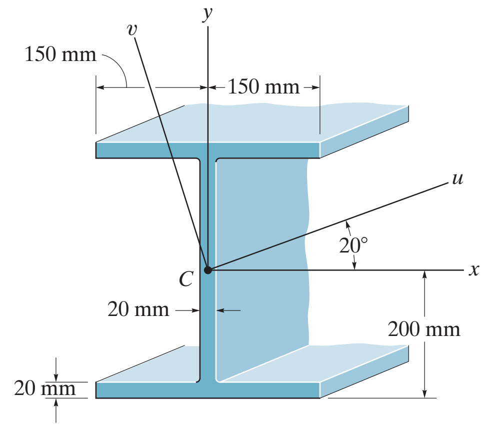
$$I_{x'y'} = 18(150)(-35.11)(30.89) + 18(132)(39.89)(-35.11)$$

$$= -6.26(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

Ans:
$\bar{x} = \bar{y} = 44.1$ mm
$I_{x'y'} = -6.26(10^6)$ mm^4

10–66.

Determine the product of inertia for the beam's cross-sectional area with respect to the u and v axes.



SOLUTION

Moments of inertia I_x and I_y

$$I_x = \frac{1}{12}(300)(400)^3 - \frac{1}{12}(280)(360)^3 = 511.36(10)^6 \text{ mm}^4$$

$$I_y = 2\left[\frac{1}{12}(20)(300)^3\right] + \frac{1}{12}(360)(20)^3 = 90.24(10)^6 \text{ mm}^4$$

The section is symmetric about both x and y axes; therefore $I_{xy} = 0$.

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

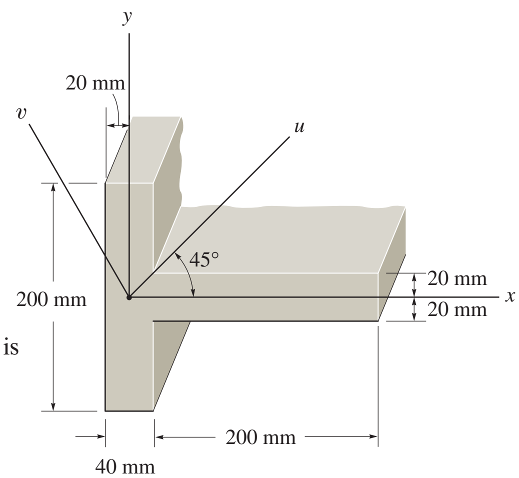
$$= \left(\frac{511.36 - 90.24}{2}\sin 40° + 0\cos 40°\right)10^6$$

$$= 135(10)^6 \text{ mm}^4$$ **Ans.**

Ans:
$I_{uv} = 135(10)^6 \text{ mm}^4$

10–67.

Determine the moments of inertia I_u and I_v of the shaded area.



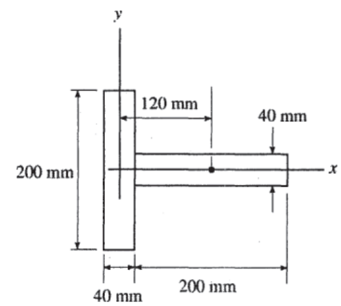
SOLUTION

Moment and Product of Inertia about x and y Axes: Since the shaded area is symmetrical about the x axis, $I_{xy} = 0$.

$$I_x = \frac{1}{12}(200)(40^3) + \frac{1}{12}(40)(200^3) = 27.73(10^6)\text{ mm}^4$$

$$I_y = \frac{1}{12}(40)(200^3) + 40(200)(120^2) + \frac{1}{12}(200)(40^3)$$

$$= 142.93(10^6)\text{ mm}^4$$



Moment of Inertia about the Inclined u and v Axes: Applying Eq. 10–9 with $\theta = 45°$, we have

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \left(\frac{27.73 + 142.93}{2} + \frac{27.73 - 142.93}{2}\cos 90° - 0(\sin 90°)\right)(10^6)$$

$$= 85.3(10^6)\text{ mm}^4$$ **Ans.**

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

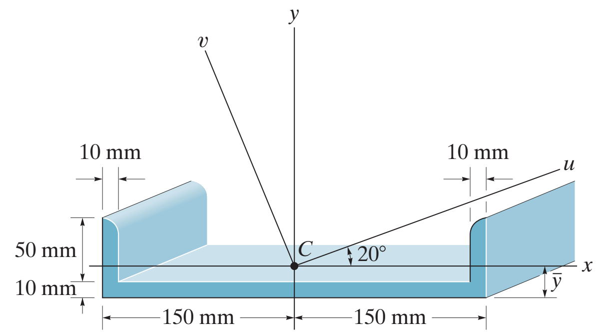
$$= \left(\frac{27.73 + 142.93}{2} - \frac{27.73 - 142.93}{2}\cos 90° - 0(\sin 90°)\right)(10^6)$$

$$= 85.3(10^6)\text{ mm}^4$$ **Ans.**

Ans:
$I_u = 85.3(10^6)\text{ mm}^4$
$I_v = 85.3(10^6)\text{ mm}^4$

***10–68.**

Determine the distance $\bar{y}$ to the centroid of the area and then calculate the moments of inertia I_u and I_v for the channel's cross-sectional area. The u and v axes have their origin at the centroid C. For the calculation, assume all corners to be square.



SOLUTION

$$\bar{y} = \frac{300(10)(5) + 2[(50)(10)(35)]}{300(10) + 2(50)(10)} = 12.5 \text{ mm}$$ **Ans.**

$$I_x = \left[\frac{1}{12}(300)(10)^3 + 300(10)(12.5 - 5)^2\right]$$

$$+ 2\left[\frac{1}{12}(10)(50)^3 + 10(50)(35 - 12.5)^2\right]$$

$$= 0.9083(10^6) \text{ mm}^4$$

$$I_y = \frac{1}{12}(10)(300)^3 + 2\left[\frac{1}{12}(50)(10)^3 + 50(10)(150 - 5)^2\right]$$

$$= 43.53(10^6) \text{ mm}^4$$

$$I_{xy} = 0 \qquad \text{(By symmetry)}$$

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{0.9083(10^6) + 43.53(10^6)}{2} + \frac{0.9083(10^6) - 43.53(10^6)}{2}\cos 40° - 0$$

$$= 5.89(10^6) \text{ mm}^4$$ **Ans.**

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

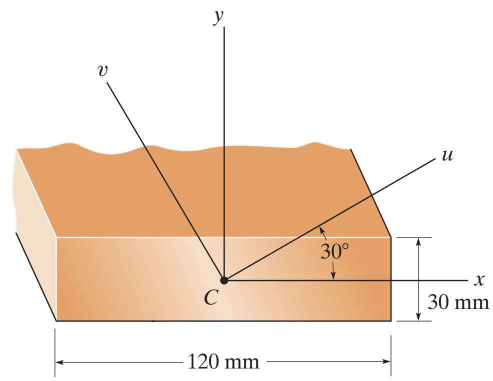
$$= \frac{0.9083(10^6) + 43.53(10^6)}{2} - \frac{0.9083(10^6) - 43.53(10^6)}{2}\cos 40° + 0$$

$$= 38.5(10^6) \text{ mm}^4$$ **Ans.**

Ans:
$I_u = 5.89(10^6)$ mm^4
$I_v = 38.5(10^6)$ mm^4

10–69.

Determine the moments of inertia I_u, I_v and the product of inertia I_{uv} for the rectangular area. The u and v axes pass through the centroid C.



SOLUTION

Moment And Product of Inertia About *x* and *y* Axes. Since the rectangular area is symmetrical about the x and y axes, $I_{xy} = 0$.

$$I_x = \frac{1}{12}(120)(30^3) = 0.270(10^6)\ \text{mm}^4 \quad I_y = \frac{1}{12}(30)(120^3) = 4.32(10^6)\ \text{mm}^4$$

Moment And Product of Inertia About The Inclined *u* and *v* Axes. With $\theta = 30°$,

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \left[\frac{0.270 + 4.32}{2} + \frac{0.27 - 4.32}{2}\cos 60° - 0 \sin 60°\right](10^6)$$

$$= 1.2825(10^6)\ \text{mm}^4 = 1.28(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= \left[\frac{0.27 + 4.32}{2} - \frac{0.27 - 4.32}{2}\cos 60° + 0 \sin 60°\right](10^6)$$

$$= 3.3075(10^6)\ \text{mm}^4 = 3.31(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

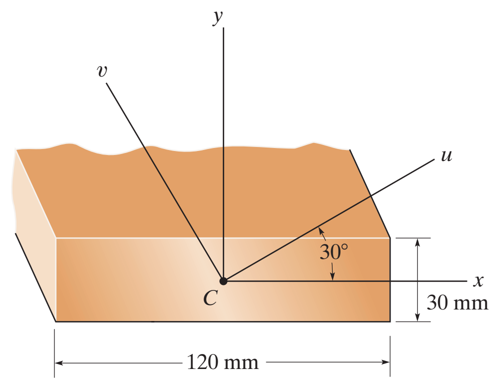
$$= \left[\frac{0.270 - 4.32}{2}\sin 60° + 0 \cos 60°\right](10^6)$$

$$= -1.7537(10^6)\ \text{mm}^4 = -1.75(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$

Ans:
$I_u = 1.28(10^6)\ \text{mm}^4$
$I_v = 3.31(10^6)\ \text{mm}^4$
$I_{uv} = -1.75(10^6)\ \text{mm}^4$

10–70.

Solve Prob. 10–69 using Mohr's circle. *Hint*: To solve, find the coordinates of the point $P(I_u, I_{uv})$ on the circle, measured counterclockwise from the radial line OA. (See Fig. 10–19.) The point $Q(I_v, -I_{uv})$ is on the opposite side of the circle.



SOLUTION

Moment And Product of Inertia About *x* And *y* Axes. Since the rectangular Area is symmetrical about the x and y axes, $I_{xy} = 0$.

$$I_x = \frac{1}{12}(120)(30^3) = 0.270(10^6)\ \text{mm}^4 \qquad I_y = \frac{1}{12}(30)(120^3) = 4.32(10^6)\ \text{mm}^4$$

Construction of The Circle. The Coordinates of center O of the circle are

$$\left(\frac{I_x + I_y}{2}, 0\right) = \left(\frac{0.270 + 4.32}{2}, 0\right)(10^6) = (2.295, 0)(10^6)$$

And the reference point A is

$$(I_x, I_{xy}) = (0.270, 0)(10^6)$$

Thus, the radius of the circle is

$$R = OA = \left(\sqrt{(2.295 - 0.27)^2 + 0^2}\right)(10^6) = 2.025(10^6)\ \text{mm}^4$$

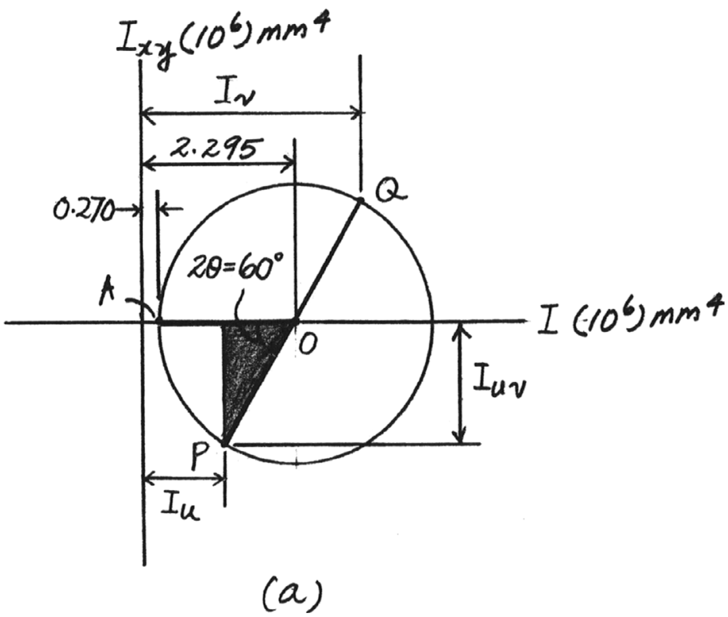
Using these Results, the circle shown in Fig. *a* can be constructed. Rotate radial line OA counterclockwise $2\theta = 60°$ to coincide with radial line OP where coordinate of point P is (I_u, I_{uv}). Then

$$I_u = (2.295 - 2.025\cos 60°)(10^6) = 1.2825(10^6)\ \text{mm}^4 = 1.28(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$

$$I_{uv} = -2.025(10^6)\sin 60° = -1.7537(10^6)\ \text{mm}^4 = -1.75(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$

I_v is represented by the coordinate of point Q. Thus,

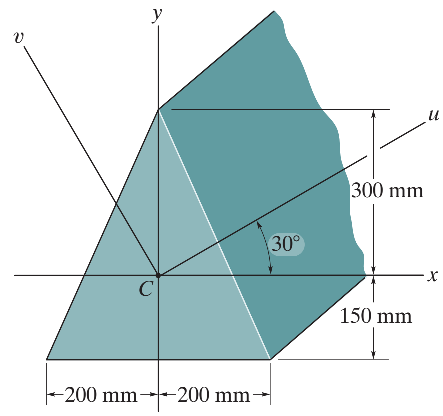
$$I_v = (2.295 + 2.025\cos 60°)(10^6) = 3.3075(10^6)\ \text{mm}^4 = 3.31(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$



(a)

Ans:
$I_u = 1.28(10^6)\ \text{mm}^4$
$I_{uv} = -1.75(10^6)\ \text{mm}^4$
$I_v = 3.31(10^6)\ \text{mm}^4$

10–71.



Determine the moments of inertia and the product of inertia of the beam's cross sectional area with respect to the u and v axes.

SOLUTION

Moments and product of Inertia with Respect to the x and y Axes: The perpendicular distances measured from the centroid of the triangular segment to the y axis are indicated in Fig. *a*.

$$I_x = \frac{1}{36}(400)(450^3) = 1012.5(10^6)\ \text{mm}^4$$

$$I_y = 2\left[\frac{1}{36}(450)(200^3) + \frac{1}{2}(450)(200)(66.67^2)\right] = 600(10^6)\ \text{mm}^4$$

Since the cross-sectional area is symmetrical about the y axis, $I_{xy} = 0$.

Moment and product of Inertia with Respect to the u and v Axes: Applying Eq. 10.8 with $\theta = 30°$, we have

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \left[\frac{1012.5 + 600}{2} + \left(\frac{1012.5 - 600}{2}\right)\cos 60° - 0\sin 60°\right](10^6)$$

$$= 909.375(10^6)\ \text{mm}^4 = 909(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

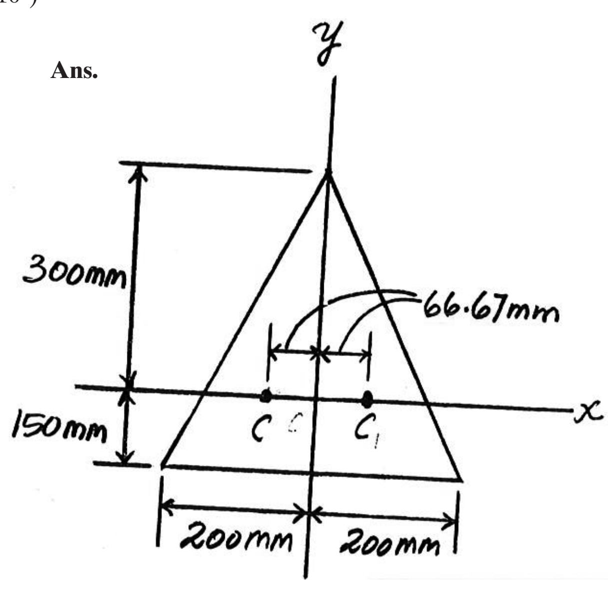
$$= \left[\frac{1012.5 + 600}{2} - \left(\frac{1012.5 - 600}{2}\right)\cos 60° + 0\sin 60\right](10^6)$$

$$= 703.125(10^6)\ \text{mm}^4 = 703(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= \left[\left(\frac{1012.5 - 600}{2}\right)\sin 60° + 0\cos 60°\right](10^6)$$

$$= 178.62(10^6)\ \text{mm}^4 = 179(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$

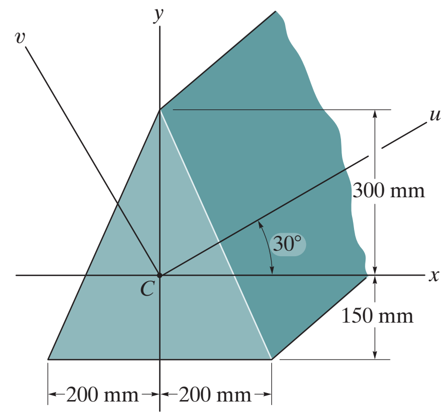


(a)

Ans:
$I_u = 909(10^6)\ \text{mm}^4$
$I_v = 703(10^6)\ \text{mm}^4$
$I_{uv} = 179(10^6)\ \text{mm}^4$

***10–72.**

Solve Prob. 10–71 using Mohr's circle. *Hint:* Once the circle is established, rotate $2\theta = 60°$ counterclockwise from the reference OA, then find the coordinates of the points that define the diameter of the circle.



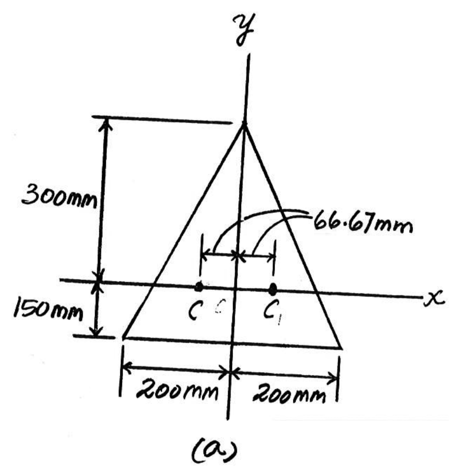
SOLUTION

Moments and product of Inertia with Respect to the x and y Axes: The perpendicular distances measured from the centroid of the triangular segment to the y axis are indicated in Fig. *a*.

$$I_x = \frac{1}{36}(400)(450^3) = 1012.5(10^6)\text{ mm}^4$$

$$I_y = 2\left[\frac{1}{36}(450)(200^3) + \frac{1}{2}(450)(200)(66.67^2)\right] = 600(10^6)\text{ mm}^4$$

Since the cross-sectional area is symmetrical about the y axis, $I_{xy} = 0$.



Construction of Mohr's Circle: The center of C of the circle lies along the I axis at a distance

$$I_{avg} = \frac{I_x + I_y}{2} = \left(\frac{1012.5 + 600}{2}\right)(10^6)\text{mm}^4 = 806.25(10^6)\text{ mm}^4$$

The coordinates of the reference point A are $[1012.5, 0](10^6)\text{ mm}^4$. The circle can be constructed as shown in Fig. *b*. The radius of the circle is

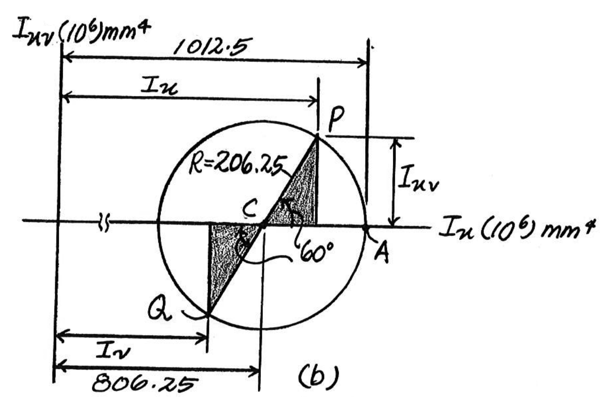
$$R = CA = (1012.5 - 806.25)(10^6) = 206.25(10^6)\text{ mm}^4$$

Moment and Product of Inertia with Respect to the u and v Axes: By referring to the geometry of the circle, we obtain

$$I_u = (806.25 + 206.25\cos 60°)(10^6) = 909(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

$$I_v = (806.25 - 206.25\cos 60°)(10^6) = 703(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

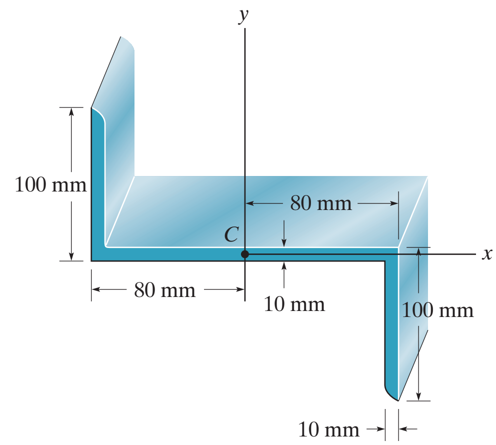
$$I_{uv} = 206.25\sin 60° = 179(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$



Ans:
$I_u = 909(10^6)\text{ mm}^4$
$I_v = 703(10^6)\text{ mm}^4$
$I_{uv} = 179(10^6)\text{ mm}^4$

10–73.

Determine the orientation of the principal axes having an origin at point C, and the principal moments of inertia of the cross section about these axes.



SOLUTION

Moment And Product of Inertia About *x* and *y* Axes. Using the parallel-axis theorem by referring to Fig. *a*

$$I_x = \Sigma\left(\bar{I}_{x'} + Ad_y^2\right); \qquad I_x = \frac{1}{12}(140)(10^3) + 2\left[\frac{1}{12}(10)(100^3) + 10(100)(45^2)\right]$$
$$= 5.7283\,(10^6)\text{ mm}^4$$

$$I_y = \Sigma\left(\bar{I}_{y'} + Ad_x^2\right); \qquad I_y = \frac{1}{12}(10)(140^3) + 2\left[\frac{1}{12}(100)(10^3) + 100(10)(75^2)\right]$$
$$= 13.5533\,(10^6)\text{ mm}^4$$

$$I_{xy} = \Sigma(\bar{I}_{x'y'} + Ad_x d_y); \qquad I_{xy} = 0 + \left[0 + 10(100)(-75)(45)\right]$$
$$+ [0 + 10(100)(75)(-45)] = -6.75(10^6)\text{ mm}^4$$

Principal Moments of Inertia.

$$I_{\substack{\max\\ \min}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x + I_y}{2}\right)^2 + I_{xy}^2}$$

$$= \left[\frac{5.7283 + 13.5533}{2} \pm \sqrt{\left(\frac{5.7283 - 13.5533}{2}\right)^2 + (-6.75)^2}\right](10^6)$$

$$= (9.6408 \pm 7.8019)(10^6)$$

$$I_{\max} = 17.44(10^6)\text{ mm}^4 = 17.4(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

$$I_{\min} = 1.8389(10^6)\text{ mm}^4 = 1.84(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

10–73. Continued

The orientation of the principal axes can now be determined

$$\tan 2\theta_p = \frac{-I_{xy}}{(I_x - I_y)/2} = \frac{-(-6.75)}{(5.7283 - 13.5533)/2} = -1.7252$$

$$2\theta_p = -59.90^\circ \quad \text{and} \quad 120.10^\circ$$

$$\theta_p = -29.95^\circ \quad \text{and} \quad 60.04^\circ$$

Substitute $\theta_p = 60.05^\circ$ into the equation for I_u,

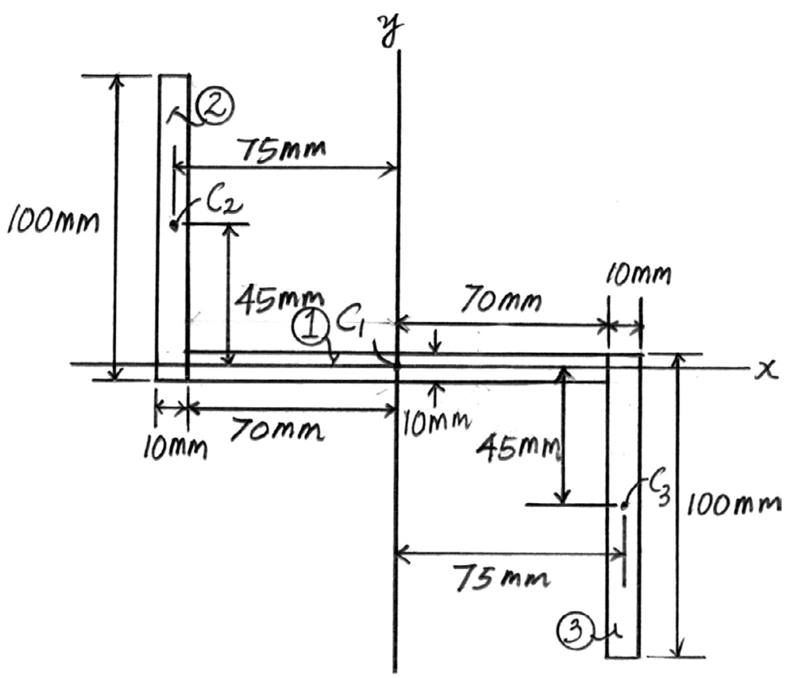
$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \left[\frac{5.7283 + 13.5533}{2} + \frac{5.7283 - 13.5533}{2}\cos 120.10^\circ - (-6.75)\sin 120.10^\circ\right](10^6)$$

$$= 17.44(10^6)\text{ mm}^4$$

Thus,

$(\theta_p)_1 = 60.0^\circ \qquad (\theta_p)_2 = -30.0^\circ$ **Ans.**

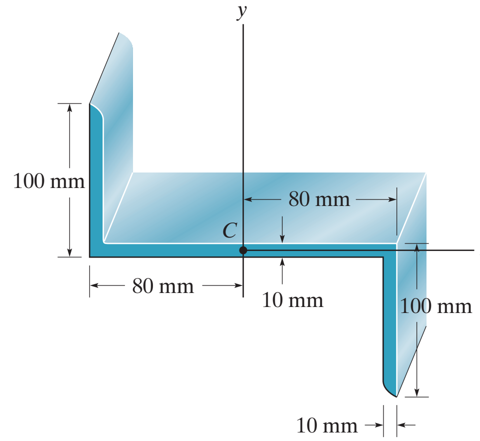


(a)

Ans:
$I_{max} = 17.4(10^6)\text{ mm}^4$
$I_{min} = 1.84(10^6)\text{ mm}^4$
$(\theta_p)_1 = 60.0^\circ$
$(\theta_p)_2 = -30.0^\circ$

10–74.

Solve Prob. 10–73 using Mohr's circle.



SOLUTION

Moment And Product of Inertia About *x* and *y* Axes. Using the parallel-axis theorem by referring to Fig. *a*

$$I_x = \Sigma\left(\bar{I}_{x'} + Ad_y^2\right); \qquad I_x = \frac{1}{12}(140)(10^3) + 2\left[\frac{1}{12}(10)(100^3) + 10(100)(45^2)\right]$$
$$= 5.7283\,(10^6)\text{ mm}^4$$

$$I_y = \Sigma\left(\bar{I}_{y'} + Ad_x^2\right); \qquad I_y = \frac{1}{12}(10)(140^3) + 2\left[\frac{1}{12}(100)(10^3) + 100(10)(75^2)\right]$$
$$= 13.5533\,(10^6)\text{ mm}^4$$

$$I_{xy} = \Sigma\left(\bar{I}_{x'y'} + Ad_x d_y\right); \qquad I_{xy} = 0 + \left[0 + 10(100)(-75)(45)\right]$$
$$+ [0 + 10(100)(75)(-45)] = -6.75(10^6)\text{ mm}^4$$

Construction of the circle. The coordinates of center O of the circle are

$$\left(\frac{I_x + I_y}{2}, 0\right) = \left(\frac{5.7283 + 13.5533}{2}, 0\right)(10^6) = (9.6408, 0)(10^6)$$

And the reference point A is

$$(I_x, I_{xy}) = (5.7283, -6.75)(10^6)$$

Thus, the radius of the circle is

$$R = OA = \left(\sqrt{(9.6408 - 5.7283)^2 + (-6.75)^2}\right)(10^6) = 7.8019(10^6)$$

Using these results, the circle shown in Fig. *b* can be constructed. Here, the coordinates of points B and C represent I_{min} and I_{max} respectively. Thus

$$I_{\text{max}} = (9.6408 + 7.8019)(10^6) = 17.44(10^6)\text{mm}^4 = 17.4(10^6)\text{mm}^4 \qquad \textbf{Ans.}$$

$$I_{\text{min}} = (9.6408 - 7.8019)(10^6) = 1.8389(10^6)\text{mm}^4 = 1.84(10^6)\text{mm}^4 \qquad \textbf{Ans.}$$

10–74. Continued

The orientation of the principal axes can be determined from the geometry of the shaded triangle on the circle

$$\tan 2(\theta_p)_2 = \frac{6.75}{9.6408 - 5.7283}$$

$$2(\theta_p)_2 = 59.90°$$

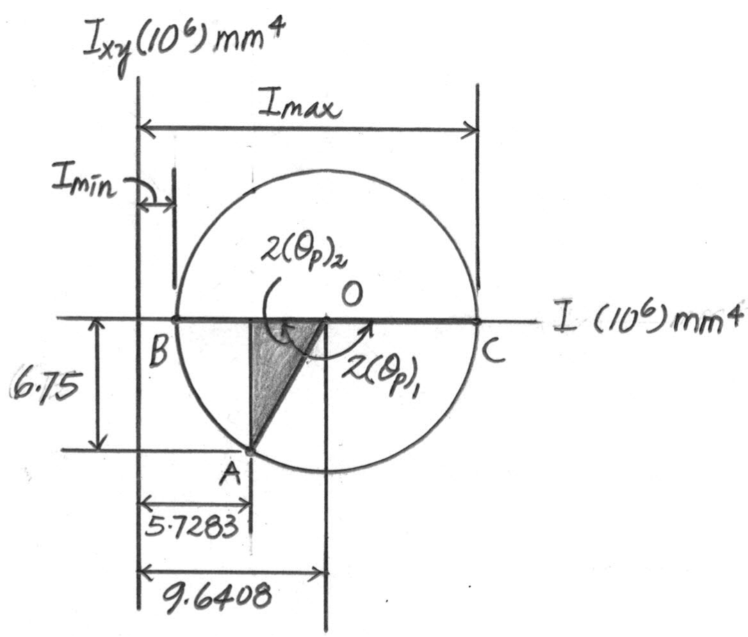
$$(\theta_p)_2 = 29.95° = 30.0° \curvearrowright \qquad \textbf{Ans.}$$

And

$$2(\theta_p)_1 = 180° - 2(\theta_p)_1$$

$$2(\theta_p)_1 = 180° - 59.90° = 120.10°$$

$$(\theta_p)_1 = 60.04° = 60.0° \curvearrowleft \qquad \textbf{Ans.}$$



(b)

Ans:

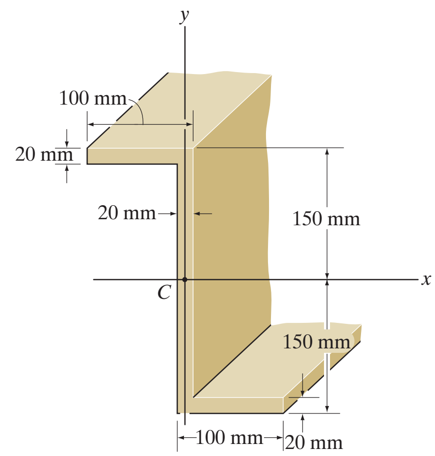
$I_{\max} = 17.4\left(10^6\right)\,\text{mm}^4$

$I_{\min} = 1.84\left(10^6\right)\,\text{mm}^4$

$(\theta_p)_2 = 30.0° \curvearrowright$

$(\theta_p)_1 = 60.0° \curvearrowleft$

10–75. Determine the orientation of the principal axes, which have their origin at centroid C of the beam's cross-sectional area. Also, find the principal moments of inertia.



SOLUTION

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from each subdivided segment to the x and y axes are indicated in Fig. *a*. Applying the parallel - axis theorem,

$$I_x = 2\left[\frac{1}{12}(80)(20^3) + 80(20)(140^2)\right] + \frac{1}{12}(20)(300^3) = 107.83(10^6)\text{mm}^4$$

$$I_y = 2\left[\frac{1}{12}(20)(80^3) + 20(80)(50^2)\right] + \frac{1}{12}(300)(20^3) = 9.907(10^6)\text{mm}^4$$

$$I_{xy} = 80(20)(-50)(140) + 80(20)(50)(-140) = -22.4(10^6)\text{mm}^4$$

Principal Moment of Inertia:

$$I_{\substack{\max\\\min}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy^2}}$$

$$= \left[\frac{107.83 + 9.907}{2} \pm \sqrt{\left(\frac{107.83 - 9.907}{2}\right)^2 + (-22.4)^2}\right](10^6)$$

$$= 58.867 \pm 53.841$$

$$I_{\max} = 112.71(10^6) = 113(10^6)\text{mm}^4$$ **Ans.**

$$I_{\min} = 5.026(10^6) = 5.03(10^6)\text{mm}^4$$

Orientation of Principal Axes:

$$\tan 2\theta_p = \frac{-I_{xy}}{(I_x - I_y)/2} = \frac{-(-22.4)(10^6)}{(107.83 - 9.907)(10^6)/2} = 0.4575$$

$$2\theta_p = 24.58^\circ \text{ and } -155.42^\circ$$

$$\theta_p = 12.29^\circ \text{ and } -77.71^\circ$$

Substituting $\theta = \theta_p = 12.29^\circ$

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{107.83 + 9.907}{2} + \left(\frac{107.83 - 9.907}{2}\right)\cos 24.58^\circ - (-22.4)\sin 24.58^\circ$$

$$= 112.71(10^6)\text{mm}^4 = I_{\max}$$

10–75. Continued

This shows that I_{max}

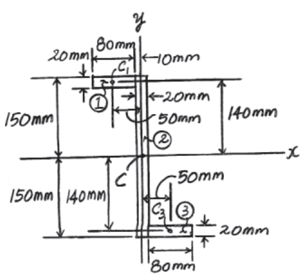
corresponds to the principal axis orientated at

$I_{max} = 113(10^6)\,\text{mm}^4 \qquad (\theta_p)_1 = 12.3°$

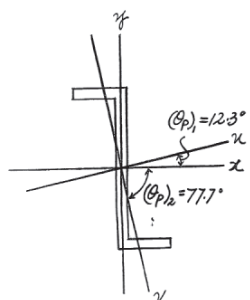
and I_{min} corresponds to the principal axis orientated at

$I_{min} = 5.03(10^6)\,\text{mm}^4 \qquad (\theta_p)_2 = -77.7°$

The orientation of the principal axes are shown in Fig. *b*.



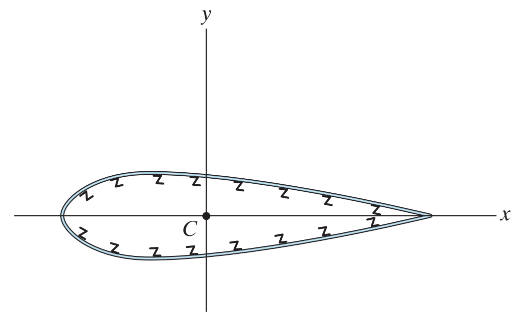
(a)



(b)

Ans:
$I_{max} = 113(10^6)$ mm^4
$I_{min} = 5.03(10^6)$ mm^4
$(\theta_p)_1 = 12.3°$
$(\theta_p)_2 = -77.0°$

***10–76.** The area of the cross section of an airplane wing has the following properties about the x and y axes passing through the centroid C: $\bar{I}_x = 180\,(10^{-6})\text{ m}^4$, $\bar{I}_y = 720\,(10^{-6})\text{ m}^4$, $\bar{I}_{xy} = 60\,(10^{-6})\text{ m}^4$. Determine the orientation of the principal axes and the principal moments of inertia.



SOLUTION

Principal Moment of Inertia. Applying Eq. 10-11,

$$I_{\substack{\max\\\min}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right) + I_{xy^2}}$$

$$= \left[\frac{180 + 720}{2} \pm \sqrt{\left(\frac{180 - 720}{2}\right)^2 + 60^2}\right](10^{-6})$$

$$= (450 \pm 276.59)(10^{-6})\text{ m}^4$$

$$I_{\max} = 726.59(10^{-6})\text{ m}^4 = 727(10^{-6})\text{ m}^4 \qquad \textbf{Ans.}$$

$$I_{\min} = 173.41(10^{-6})\text{ m}^4 = 173(10^{-6})\text{ m}^4 \qquad \textbf{Ans.}$$

Orientation of the Principal Axes. Applying Eq. 10-10,

$$\tan 2\theta_p = \frac{-I_{xy}}{(I_x - I_y)/2} = \frac{-60}{(180 - 720)/2} = 0.2222$$

$$2\theta_p = 12.53° \quad \text{and} \quad 192.53$$

$$\theta_p = 6.264° \quad \text{and} \quad 96.26°$$

Substitute $\theta_p = 96.26°$ into first of Eq. 10-9 (I_u),

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \left[\frac{180 + 720}{2} + \frac{180 - 720}{2}\cos 192.53° - 60\sin 192.53°\right](10^{-6})\text{ m}^4$$

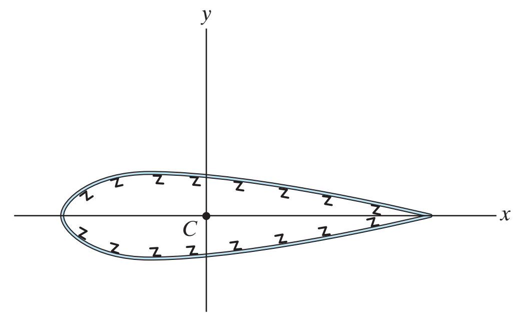
$$= 726.59(10^{-6})\text{ m}^4$$

Therefore

$$(\theta_p)_1 = 96.26° = 96.3° \quad (\theta_p)_2 = 6.264° = 6.26° \qquad \textbf{Ans.}$$

Ans:
$I_{\max} = 726.59(10^{-6})\text{ m}^4 = 727(10^{-6})\text{ m}^4$
$I_{\min} = 173.41(10^{-6})\text{ m}^4 = 173(10^{-6})\text{ m}^4$
$(\theta_p)_1 = 96.3°$
$(\theta_p)_2 = 6.26°$

10–77. Solve Prob. 10–76 using Mohr's circle.



SOLUTION

Construction of the Circle: The coordinates of center O of the circle is

$$O\left(\frac{I_x + I_y}{2}, 0\right) = \left(\frac{180 + 720}{2}, 0\right)(10^{-6}) = (450{,}0)(10^{-6})$$

And the coordinates of reference point A is

$$A(I_x, I_{xy}) = A(180, 60)(10^{-6})$$

Thus, the radius of the circle is

$$R = OA = \left(\sqrt{(450 - 180)^2 + (0 - 60)^2}\right)(10^{-6}) = 276.59(10^{-6})$$

Using these results, the circle shown in Fig. *a* can be constructed. Here, the coordinates of points C and B represent $I_{\max}$ and $I_{\min}$ respectively. Thus

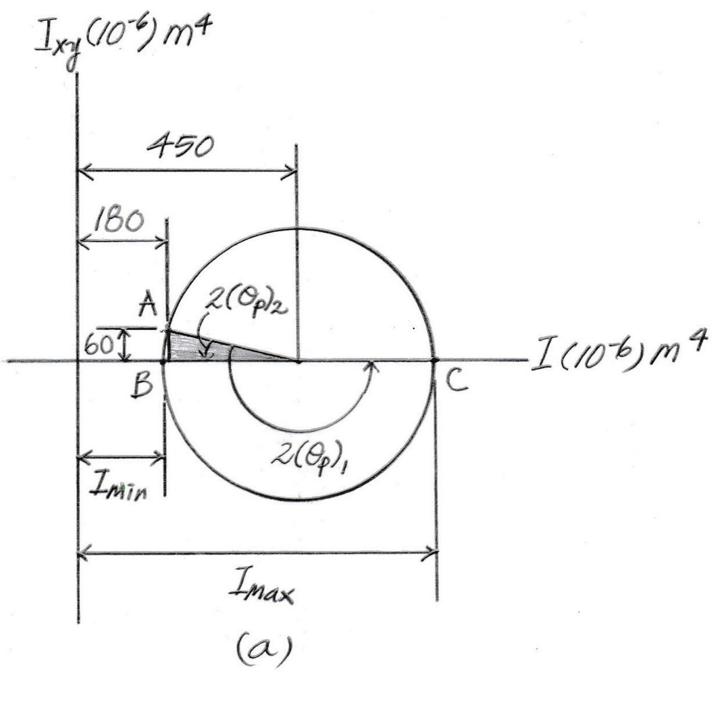
$$I_{\max} = (450 + 276.59)(10^{-6}) = 726.59(10^{-6})\text{ m}^4 = 727(10^{-6})\text{ m}^4 \qquad \textbf{Ans.}$$

$$I_{\min} = (450 - 276.59)(10^{-6}) = 173.41(10^{-6})\text{ m}^4 = 173(10^{-6})\text{ m}^4 \qquad \textbf{Ans.}$$

The orientation of the principal axes can be determined from the geometry of the shaded triangle on the circle.

$$\tan 2(\theta_p)_2 = \frac{60}{450 - 180}; \quad 2(\theta_p)_2 = 12.53^\circ \quad (\theta_p)_2 = 6.264^\circ = 6.26^\circ \circlearrowright \qquad \textbf{Ans.}$$

$$2(\theta_p)_1 = 2(\theta_p)_2 + 180^\circ = 12.53^\circ + 180^\circ = 192.53^\circ \quad (\theta_p)_1 = 96.26^\circ = 96.3^\circ \circlearrowright \qquad \textbf{Ans.}$$



(a)

Ans:

$I_{max} = (450 + 276.59)(10^{-6}) = 727(10^{-6})\text{ m}^4$

$I_{min} = (450 - 276.59)(10^{-6}) = 173(10^{-6})\text{ m}^4$

$\tan 2(\theta_p)_2 = \frac{60}{450 - 180}$; $2(\theta_p)_2 = 12.53^\circ$

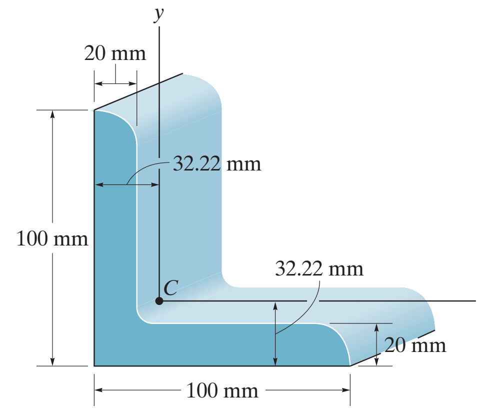
$(\theta_p)_2 = 6.26^\circ \circlearrowright$

$2(\theta_p)_1 = 2(\theta_p)_2 + 180^\circ = 12.53^\circ + 180^\circ = 192.53^\circ$

$(\theta_p)_1 = 96.26^\circ = 96.3^\circ \circlearrowright$

10–78.

Determine the principal moments of inertia for the angle's cross-sectional area with respect to a set of principal axes that have their origin located at the centroid C. Use the equation developed in Section 10.7. For the calculation, assume all corners to be square.



SOLUTION

$$I_x = \left[\frac{1}{12}(20)(100)^3 + 100(20)(50 - 32.22)^2\right]$$

$$+ \left[\frac{1}{12}(80)(20)^3 + 80(20)(32.22 - 10)^2\right]$$

$$= 3.142(10^6)\text{ mm}^4$$

$$I_y = \left[\frac{1}{12}(100)(20)^3 + 100(20)(32.22 - 10)^2\right]$$

$$+ \left[\frac{1}{12}(20)(80)^3 + 80(20)(60 - 32.22)^2\right]$$

$$= 3.142(10^6)\text{ mm}^4$$

$$I_{xy} = \Sigma \bar{x}\bar{y}A$$

$$= -(32.22 - 10)(50-32.22)(100)(20) - (60-32.22)(32.22-10)(80)(20)$$

$$= -1.778(10^6)\text{ mm}^4$$

$$I_{max/min} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$= 3.142(10^6) \pm \sqrt{0 + \{(-1.778)(10^6)\}^2}$$

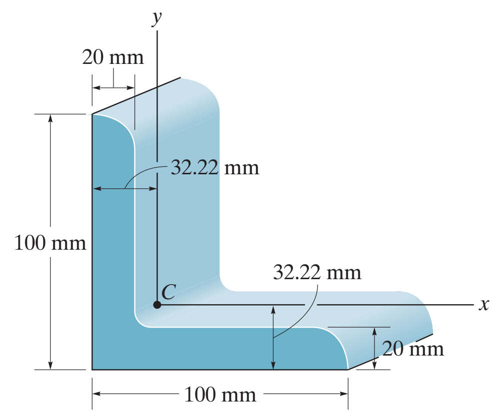
$I_{max} = 4.92(10^6)\text{ mm}^4$ **Ans.**

$I_{min} = 1.36(10^6)\text{ mm}^4$ **Ans.**

Ans:
$I_{max} = 4.92(10^6)\text{ mm}^4$
$I_{min} = 1.36(10^6)\text{ mm}^4$

10–79.

Solve Prob. 10–78 using Mohr's circle.



SOLUTION

Solve Prob. 10–78.

$I_x = 3.142(10^6)\text{ mm}^4$

$I_y = 3.142(10^6)\text{ mm}^4$

$I_{xy} = -1.778(10^6)\text{ mm}^4$

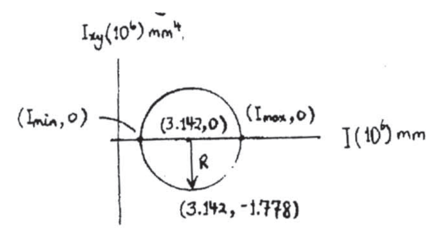
Center of circle:

$$\frac{I_x + I_y}{2} = 3.142(10^6)\text{ mm}^4$$

$$R = \sqrt{(3.142 - 3.142)^2 + (-1.778)^2}(10^6) = 1.778(10^6)\text{ mm}^4$$

$I_{max} = 3.142(10^6) + 1.778(10^6) = 4.92(10^6)\text{ mm}^4$ **Ans.**

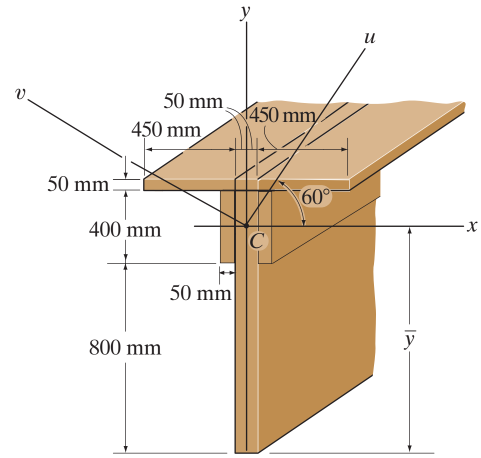
$I_{min} = 3.142(10^6) - 1.778(10^6) = 1.36(10^6)\text{ mm}^4$ **Ans.**



Ans:

$I_{max} = 4.92(10^6)\text{ mm}^4$
$I_{min} = 1.36(10^6)\text{ mm}^4$

***10–80.** Locate the centroid $\bar{y}$ of the beam's cross-sectional area and then determine the moments of inertia and the product of inertia of this area with respect to the u and v axes.



SOLUTION

Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the bottom of the beam's cross – sectional area are indicated in Fig. *a*. Thus,

$$\bar{y} = \frac{\Sigma y_C A}{\Sigma A} = \frac{1225(1000)(50) + 2[1000(400)(50)] + 600(1200)(100)}{1000(50) + 2(400)(50) + 1200(100)} = 825 \text{ mm} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from the centroid of each segment to the x and y axes are indicated in Fig. *b*. Using the parallel – axis theorem,

$$I_x = \left[\frac{1}{12}(1000)(50^3) + 1000(50)(400)^2\right] + 2\left[\frac{1}{12}(50)(400^3) + 50(400)(175)^2\right] + \left[\frac{1}{12}(100)(1200^3) + 100(1200)(225)^2\right]$$
$$= 302.44\,(10^8)\text{ mm}^4$$

$$I_y = \frac{1}{12}(50)(1000^3) + 2\left[\frac{1}{12}(400)(50^3) + 400(50)(75)^2\right] + \frac{1}{12}(1200)(100^3)$$
$$= 45\,(10^8)\text{ mm}^4$$

Since the cross – sectional area is symmetrical about the y axis, $I_{xy} = 0$.

Moment and Product of Inertia with Respect to the *u* and *v* Axes: With $\theta = 60$,

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$
$$= \left[\frac{302.44 + 45}{2} + \frac{302.44 - 45}{2}\cos 120° - 0\sin 120°\right](10^8)$$
$$= 109.36\,(10^8)\text{ mm}^4 = 109\,(10^8)\text{ mm}^4 \qquad \textbf{Ans.}$$

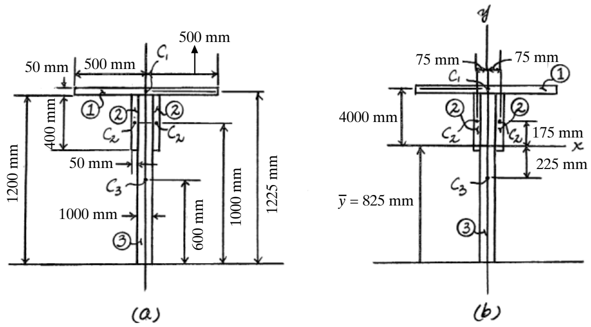
$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$
$$= \left[\frac{302.44 + 45}{2} - \frac{302.44 - 45}{2}\cos 120° + 0\sin 120°\right](10^8)$$
$$= 238.08\,(10^8)\text{ mm}^4 = 238\,(10^8)\text{ mm}^4 \qquad \textbf{Ans.}$$

***10–80. Continued**

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

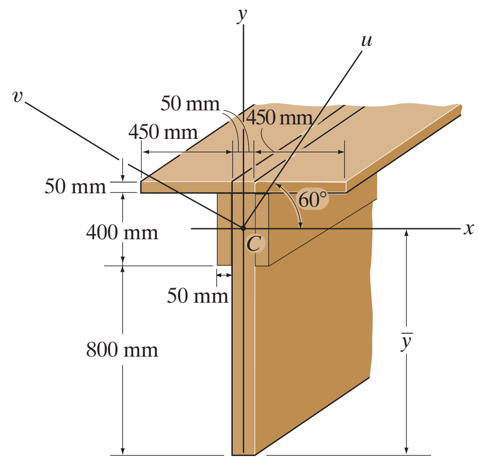
$$= \left[\frac{302.44 - 45}{2}\sin 120° + 0\cos 120°\right](10^8)$$

$$= 111.47\,(10^8)\text{ mm}^4 = 111\,(10^8)\text{ mm}^4$$ **Ans.**



Ans:
$\bar{y} = 825$ mm
$I_u = 109\,(10^8)$ mm^4
$I_v = 238\,(10^8)$ mm^4
$I_{uv} = 111\,(10^8)$ mm^4

10–81. Solve Prob. 10–80 using Mohr's circle.



SOLUTION

Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the bottom of the beam's cross – sectional area are indicated in Fig. *a*. Thus,

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{1225(1000)(50) + 2[1000(400)(50)] + 600(1200)(100)}{1000(50) + 2(400)(50) + 1200(100)} = 825 \text{ mm} \qquad \textbf{Ans.}$$

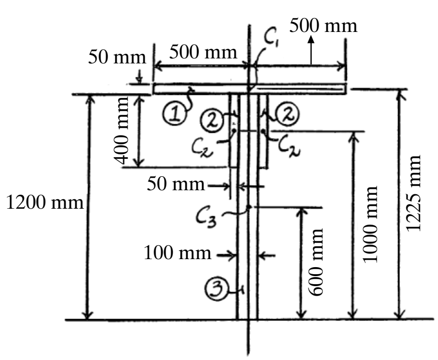
Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from the centroid of each segment to the *x* and *y* axes are indicated in Fig. *b*. Using the parallel – axis theorem,

$$I_x = \left[\frac{1}{12}(1000)(50^3) + 1000(50)(400)^2\right] + 2\left[\frac{1}{12}(50)(400^3) + 50(400)(175)^2\right] + \left[\frac{1}{12}(100)(1200^3) + 100(1200)(225)^2\right]$$
$$= 302.44\ (10^8)\ \text{mm}^4$$

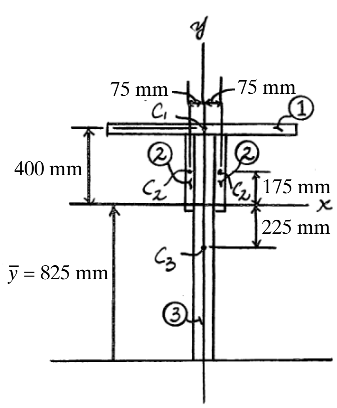
$$I_y = \frac{1}{12}(50)(1000^3) + 2\left[\frac{1}{12}(400)(50^3) + 400(50)(75)^2\right] + \frac{1}{12}(1200)(100^3)$$
$$= 45\ (10^8)\ \text{mm}^4$$

$$I_{xy} = 60(5)(-14.35)(13.15) + 55(15)(15.65)(-14.35)$$
$$= -11.837\ (10^4)\ \text{mm}^4$$

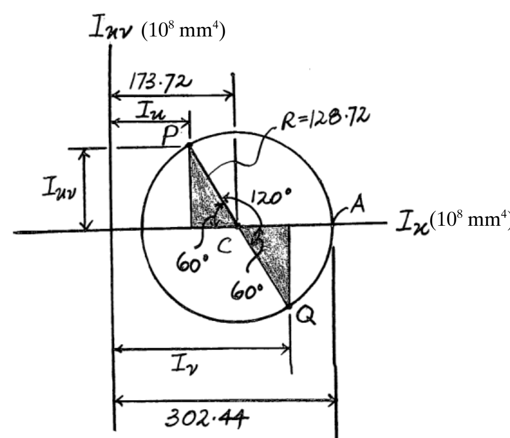
Since the cross – sectional area is symmetrical about the *y* axis, $I_{xy} = 0$.



(a)



(b)



(c)

10–81. Continued

Construction of Mohr's Circle: The center C of the circle lies along the u axis at a distance

$$I_{\text{avg}} = \frac{I_x + I_y}{2} = \left(\frac{302.44 + 45}{2}\right)(10^8) = 173.72\,(10^8)\text{ mm}^4$$

The coordinates of the reference point A are $(302.44, 0)\,(10^8)\text{ mm}^4$. The circle can be constructed as shown in Fig. *c*. The radius of the circle is

$$R = CA = (302.44 - 173.72)\,(10^8) = 128.72\,(10^8)\text{ mm}^4$$

Moment and Product of Inertia with Respect to the *u* and *v* Axes: By referring to the geometry of the circle,

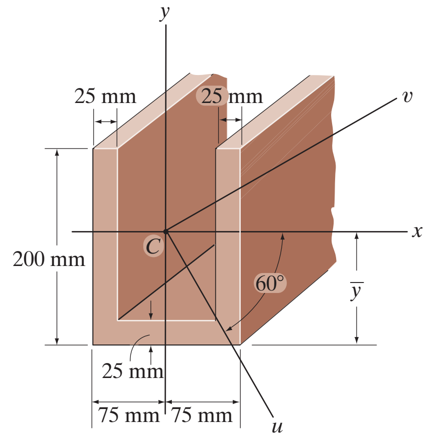
$$I_u = (173.72 - 128.72\cos 60°)\,(10^8) = 109\,(10^8)\text{ mm}^4$$ **Ans.**

$$I_v = (173.72 + 128.72\cos 60°)\,(10^8) = 238\,(10^8)\text{ mm}^4$$ **Ans.**

$$I_{uv} = (128.72\sin 60°)\,(10^8) = 111\,(10^8)\text{ mm}^4$$ **Ans.**

Ans:
$\bar{y} = 825$ mm
$I_u = 109\,(10^8)\text{ mm}^4$
$I_v = 238\,(10^8)\text{ mm}^4$
$I_{uv} = 111\,(10^8)\text{ mm}^4$

10–82. Locate the centroid $\bar{y}$ of the beam's cross-sectional area and then determine the moments of inertia of this area and the product of inertia with respect to the u and v axes. The axes have their origin at the centroid C.



SOLUTION

Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the bottom of the beam's cross - sectional area are indicated in Fig. *a*. Thus,

$$\bar{y} = \frac{\Sigma y_c A}{\Sigma A} = \frac{2[100(200)(25)] + 12.5(2.5)(100)}{2(200)(25) + 25(100)} = 82.5 \text{ mm} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from the centroid of each segment to the x and y axes are indicated in Fig. *b*. Using the parallel - axis theorem

$$I_x = 2\left[\frac{1}{12}(25)(200^3) + 25(200)(17.5)^2\right] + \left[\frac{1}{12}(100)(25^3) + 100(25)(70)^2\right]$$

$$= 48.78(10^6) \text{ mm}^4$$

$$I_y = 2\left[\frac{1}{12}(200)(25^3) + 200(25)(62.5)^2\right] + \frac{1}{12}(25)(100^3)$$

$$= 41.67(10^6) \text{ mm}^4$$

Since the cross - sectional area is symmetrical about the y axis, $I_{xy} = 0$.

Moment and Product of Inertia with Respect to the *u* and *v* Axes: With $\theta = -60°$,

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \left[\frac{48.78 + 41.67}{2} + \left(\frac{48.78 - 41.67}{2}\right)\cos(-120°) - 0\sin(-120°)\right](10^6)$$

$$= 43.4(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

$$I_v = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \left[\frac{48.78 + 41.67}{2} + \left(\frac{48.78 - 41.67}{2}\right)\cos(-120°) + 0\sin(-120°)\right](10^6)$$

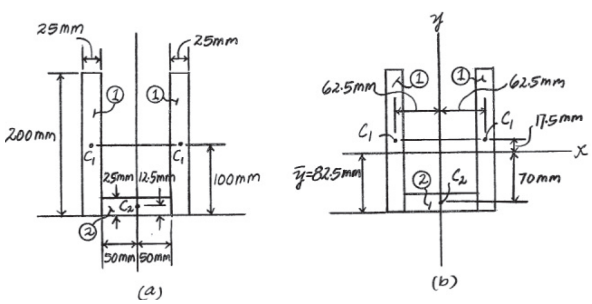
$$= 47.0(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

10–82. Continued

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

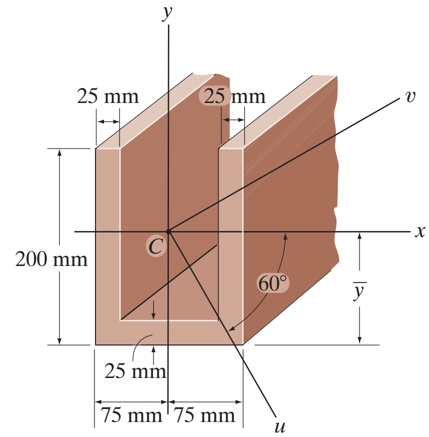
$$= \left(\frac{48.78 - 41.67}{2}\right)\sin(-120^\circ) + 0\cos(-120^\circ)$$

$$= -3.08(10^6)\,\text{mm}^4$$ **Ans.**



Ans:
$\bar{y} = 82.5$ mm
$I_u = 43.4\,(10^6)$ mm^4
$I_v = 47.0\,(10^6)$ mm^4
$I_{uv} = -3.08\,(10^6)$ mm^4

10–83. Solve Prob. 10–82 using Mohr's circle.



SOLUTION

Centroid: The perpendicular distances measured from the centroid of each subdivided segment to the bottom of the beam's cross - sectional area are indicated in Fig. *a*. Thus,

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{2[100(200)(25)] + 12.5(2.5)(100)}{2(200)(25) + 25(100)} = 82.5 \text{ mm} \qquad \textbf{Ans.}$$

Moment and Product of Inertia with Respect to the *x* and *y* Axes: The perpendicular distances measured from the centroid of each segment to the *x* and *y* axes are indicated in Fig. *b*. Using the parallel - axis theorem

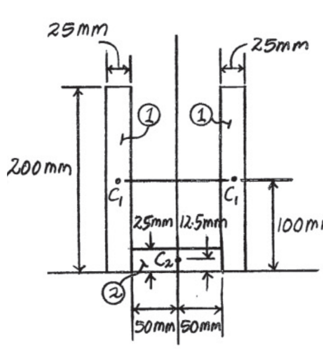
$$I_x = 2\left[\frac{1}{12}(25)(200^3) + 25(200)(17.5)^2\right] + \left[\frac{1}{12}(100)(25^3) + 100(25)(70)^2\right]$$

$$= 48.78(10^6) \text{ mm}^4$$

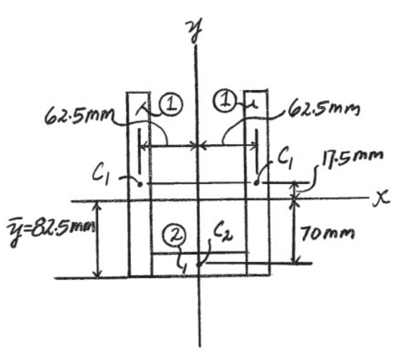
$$I_y = 2\left[\frac{1}{12}(200)(25^3) + 200(25)(62.5)^2\right] + \frac{1}{12}(25)(100^3)$$

$$= 44.67(10^6) \text{ mm}^4$$

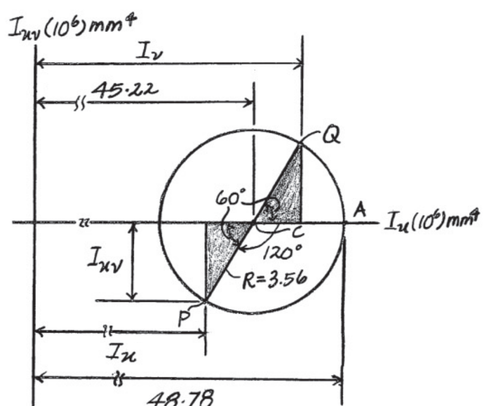
Since the cross - sectional area is symmetrical about the *y* axis, $I_{xy} = 0$.



(a)



(b)



(c)

10–83. Continued

Construction of Mohr's Circle: The center C of the circle lies along the u axis at a distance

$$I_{avg} = \frac{I_x + I_y}{2} = \left(\frac{48.78 + 41.67}{2}\right)(10^6)\text{ mm}^4 = 45.22(10^6)\text{ mm}^4$$

The coordinates of the reference point A are $[48.78, 0](10^6)\text{ mm}^4$. The circle can be constructed as shown in Fig. *a*. The radius of the circle is

$$R = CA = (48.78 - 45.22)(10^6) = 3.56(10^6)\text{ mm}^4$$

Moment and Product of Inertia with Respect to the *u* and *v* Axes: By referring to the geometry of the circle,

$$I_u = (45.22 - 3.56\cos 60°)(10^6) = 43.4(10^6)\text{ mm}^4$$ **Ans.**

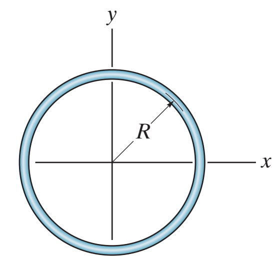
$$I_v = (45.22 + 3.56\cos 60°)(10^6) = 47.0(10^6)\text{ mm}^4$$ **Ans.**

$$I_{uv} = -3.56\sin 60° = -3.08(10^6)\text{ mm}^4$$ **Ans.**

Ans:
$\bar{y} = 82.5$ mm
$I_u = 43.4\ (10^6)\text{ mm}^4$
$I_v = 47.0\ (10^6)\text{ mm}^4$
$I_{uv} = -3.08\ (10^6)\text{ mm}^4$

***10–84.**

Determine the moment of inertia of the thin ring about the z axis. The ring has a mass m.

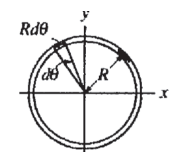


SOLUTION

$$I_z = \int_0^{2\pi} \rho\, A(R\, d\theta)R^2 = 2\pi\, \rho\, A\, R^3$$

$$m = \int_0^{2\pi} \rho\, A\, R\, d\theta = 2\pi\, \rho\, A\, R$$

Thus,

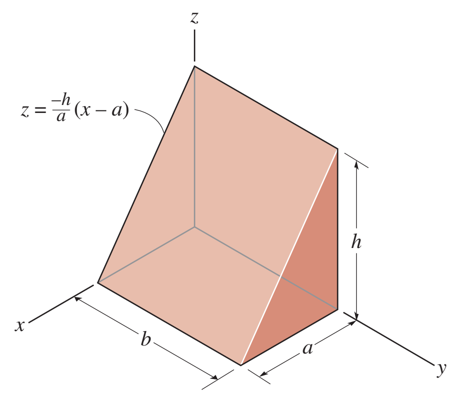


$$I_z = m R^2$$ **Ans.**

Ans:
$I_z = m\, R^2$

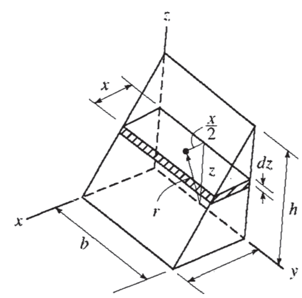
10–85.

Determine the moment of inertia of the homogenous triangular prism with respect to the y axis. Express the result in terms of the mass m of the prism. *Hint:* For integration, use thin plate elements parallel to the x-y plane having a thickness of dz.



SOLUTION

Differential Thin Plate Element: Here, $x = a\left(1 - \frac{z}{h}\right)$. The mass of the differential thin plate element is $dm = \rho dV = \rho bx dz = \rho ab\left(1 - \frac{z}{h}\right) dz$. The mass moment of inertia of this element about y axis is



$$dI_y = dI_G + dmr^2$$

$$= \frac{1}{12}dmx^2 + dm\left(\frac{x^2}{4} + z^2\right)$$

$$= \frac{1}{3}x^2\,dm + z^2\,dm$$

$$= \left[\frac{a^2}{3}\left(1 - \frac{z}{h}\right)^2 + z^2\right]\left[\rho ab\left(1 - \frac{z}{h}\right) dz\right]$$

$$= \frac{\rho ab}{3}\left(a^2 + \frac{3a^2}{h^2}z^2 - \frac{3a^2}{h}z - \frac{a^2}{h^3}z^3 + 3z^2 - \frac{3z^3}{h}\right) dz$$

Total Mass: Performing the integration, we have

$$m = \int_m dm = \int_0^h \rho ab\left(1 - \frac{z}{h}\right) dz = \rho ab\left(z - \frac{z^2}{2h}\right)\Bigg|_0^h = \frac{1}{2}\rho abh$$

Mass Moment of Inertia: Performing the integration, we have

$$I_y = \int dI_y = \int_0^h \frac{\rho ab}{3}\left(a^2 + \frac{3a^2}{h^2}z^2 - \frac{3a^2}{h}z - \frac{a^2}{h^3}z^3 + 3z^2 - \frac{3z^3}{h}\right) dz$$

$$= \frac{\rho ab}{3}\left(a^2 z + \frac{a^2}{h^2}z^3 - \frac{3a^2}{2h}z^2 - \frac{a^2}{4h^3}z^4 + z^3 - \frac{3z^4}{4h}\right)\Bigg|_0^h$$

$$= \frac{\rho abh}{12}(a^2 + h^2)$$

The mass moment of inertia expressed in terms of the total mass is

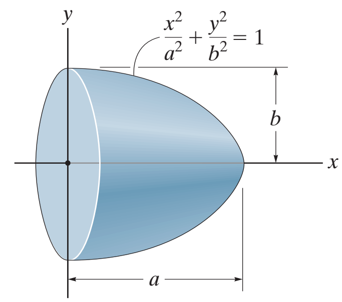
$$I_y = \frac{1}{6}\left(\frac{\rho abh}{2}\right)(a^2 + h^2) = \frac{m}{6}(a^2 + h^2) \qquad \textbf{Ans.}$$

Ans:

$$I_y = \frac{m}{6}(a^2 + h^2)$$

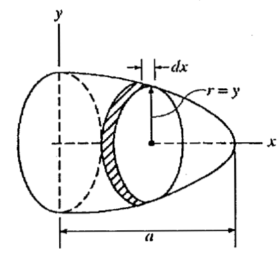
10–86.

Determine the moment of inertia of the semi-ellipsoid with respect to the x axis and express the result in terms of the mass m of the semiellipsoid. The material has a constant density ρ.



SOLUTION

Differential Disk Element: Here, $y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$. The mass of the differential disk element is $dm = \rho dV = \rho\pi\, y^2\, dx = \rho\pi\, b^2\left(1 - \frac{x^2}{a^2}\right) dx$. The mass moment of inertia of this element is $dI_x = \frac{1}{2}dmy^2 = \frac{1}{2}\left[\rho\pi\, b^2\left(1 - \frac{x^2}{a^2}\right) dx\right]\left[b^2\left(1 - \frac{x^2}{a^2}\right)\right] = \frac{\rho\pi\, b^4}{2}\left(\frac{x^4}{a^4} - \frac{2x^2}{a^2} + 1\right) dx$.



Total Mass: Performing the integration, we have

$$m = \int_m dm = \int_0^a \rho\pi\, b^2\left(1 - \frac{x^2}{a^2}\right)dx = \rho\pi\, b^2\left(x - \frac{x^3}{3a^2}\right)\Big|_0^a$$

$$= \frac{2}{3}\rho\pi ab^2$$

Mass Moment of Inertia: Performing the integration, we have

$$I_x = \int dI_x = \int_0^a \frac{\rho\pi\, b^4}{2}\left(\frac{x^4}{a^4} - \frac{2x^2}{a^2} + 1\right) dx$$

$$= \frac{\rho\pi\, b^4}{2}\left(\frac{x^5}{5a^4} - \frac{2x^3}{3a^2} + x\right)\Big|_0^a$$

$$= \frac{4}{15}\rho\pi\, ab^4$$

The mass moment of inertia expressed in terms of the total mass is.

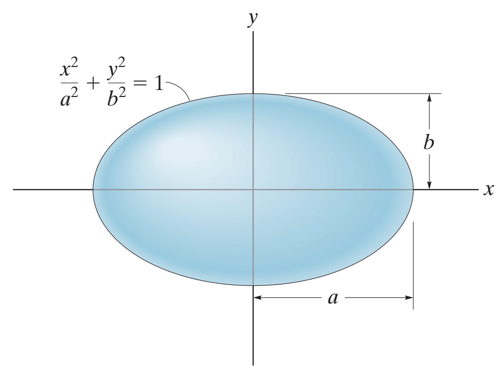
$$I_x = \frac{2}{5}\left(\frac{2}{3}\rho\pi\, ab^2\right) b^2 = \frac{2}{5}mb^2$$ **Ans.**

Ans:

$I_x = \frac{2}{5}mb^2$

10–87.

Determine the moment of inertia of the ellipsoid with respect to the x axis and express the result in terms of the mass m of the ellipsoid. The material has a constant density ρ.



SOLUTION

$$dm = \int \pi y^2 dx$$

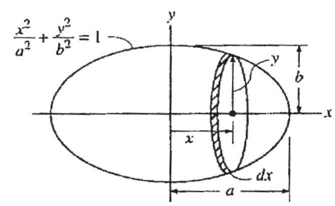
$$dI_x = \frac{y^2 dm}{2}$$

$$m = \int_V \rho\, dV = \int_{-a}^{a} \rho \pi b^2 \left(1 - \frac{x^2}{a^2}\right) dx = \frac{4}{3}\pi \rho a b^2$$

$$I_x = \int_{-a}^{a} \frac{1}{2}\rho \pi b^4 \left(1 - \frac{x^2}{a^2}\right)^2 dx = \frac{8}{15}\pi \rho a b^4$$

Thus,

$$I_x = \frac{2}{5} m b^2$$ **Ans.**

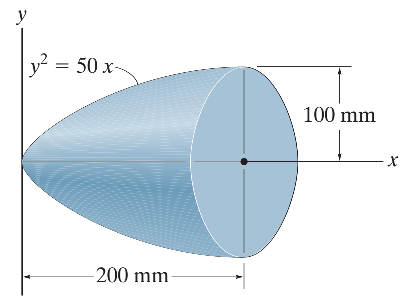


Ans:

$I_x = \frac{2}{5} m b^2$

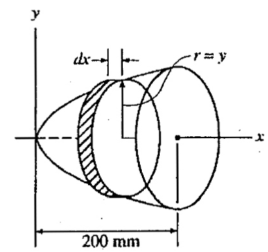
***10–88.**

Determine the radius of gyration k_x of the paraboloid. The density of the material is $\rho = 5\ \text{Mg/m}^3$.



SOLUTION

Differential Disk Element: The mass of the differential disk element is $dm = \rho dV = \rho\pi y^2\,dx = \rho\pi(50x)\,dx$. The mass moment of inertia of this element is $dI_x = \frac{1}{2}dmy^2 = \frac{1}{2}[\rho\pi(50x)\,dx](50x) = \frac{\rho\pi}{2}(2500x^2)\,dx$.



Total Mass: Performing the integration, we have

$$m = \int_m dm = \int_0^{200\text{ mm}} \rho\pi(50x)dx = \rho\pi(25x^2)\Big|_0^{200\text{ mm}} = 1(10^6)\rho\pi$$

Mass Moment of Inertia: Performing the integration, we have

$$I_x = \int dI_x = \int_0^{200\text{ mm}} \frac{\rho\pi}{2}(2500x^2)\,dx$$

$$= \frac{\rho\pi}{2}\left(\frac{2500x^3}{3}\right)\Bigg|_0^{200\text{ mm}}$$

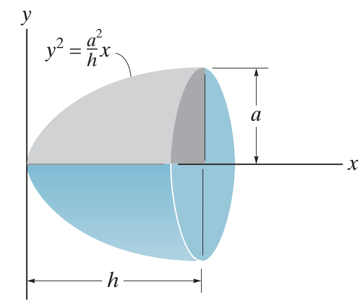
$$= 3.333(10^9)\rho\pi$$

The radius of gyration is

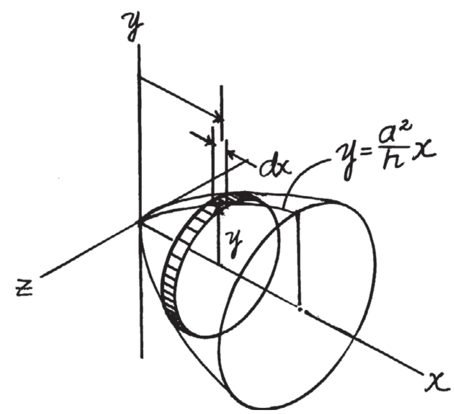
$$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{3.333(10^9)\rho\pi}{1(10^6)\rho\pi}} = 57.7\text{ mm}$$ **Ans.**

Ans:
$k_x = 57.7$ mm

10–89. The paraboloid is formed by revolving the shaded area around the x axis. Determine the moment of inertia about the x axis and express the result in terms of the total mass m of the paraboloid. The material has a constant density ρ.



SOLUTION



$$dm = \rho\, dV = \rho\,(\pi\, y^2\, dx)$$

$$d\,I_x = \frac{1}{2}\,dm\, y^2 = \frac{1}{2}\rho\, \pi\, y^4\, dx$$

$$I_x = \int_0^h \frac{1}{2}\rho\, \pi\left(\frac{a^4}{h^2}\right)x^2\, dx$$

$$= \frac{1}{6}\,\pi\, \rho a^4\, h$$

$$m = \int_0^h \frac{1}{2}\rho\, \pi\left(\frac{a^2}{h}\right)x\, dx$$

$$= \frac{1}{2}\,\rho\, \pi\, a^2\, h$$

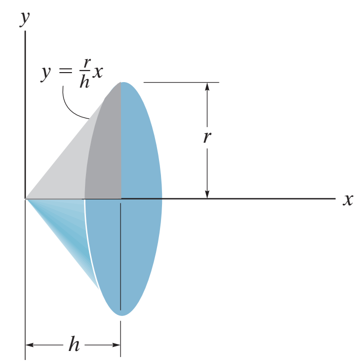
$$I_x = \frac{1}{3}\,ma^2$$ **Ans.**

Ans:

$$I_x = \frac{1}{3}\,ma^2$$

10–90.

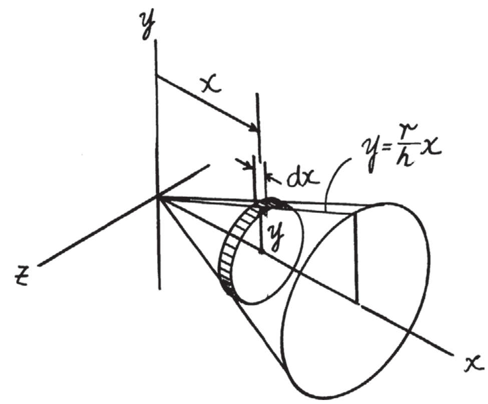
The right circular cone is formed by revolving the shaded area around the x axis. Determine the moment of inertia I_x and express the result in terms of the total mass m of the cone. The cone has a constant density ρ.



SOLUTION

$dm = \rho\, dV = \rho(\pi\, y^2\, dx)$

$$m = \int_0^h \rho(\pi)\left(\frac{r^2}{h^2}\right)x^2\,dx = \rho\pi\left(\frac{r^2}{h^2}\right)\left(\frac{1}{3}\right)h^3 = \frac{1}{3}\rho\pi\, r^2 h$$



$$dI_x = \frac{1}{2}\,y^2\,dm$$

$$= \frac{1}{2}\,y^2\,(\rho\pi\, y^2\, dx)$$

$$= \frac{1}{2}\,\rho(\pi)\left(\frac{r^4}{h^4}\right)x^4\,dx$$

$$I_x = \int_0^h \frac{1}{2}\rho(\pi)\left(\frac{r^4}{h^4}\right)x^4\,dx = \frac{1}{10}\rho\pi\, r^4\, h$$

Thus,

$$I_x = \frac{3}{10}\,m\,r^2$$ **Ans.**

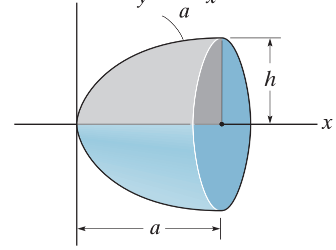
Ans:

$I_x = \frac{3}{10}mr^2$

10–91.

Determine the radius of gyration k_x of the solid formed by revolving the shaded area about x axis. The density of the material is ρ.

$y^n = \frac{h^n}{a} x$



SOLUTION

Differential Disk Element. The mass of the differential disk element shown shaded in Fig. a is $dm = \rho dv = \rho\pi y^2 dx$. Here $y = \dfrac{h}{a^{\frac{1}{n}}} x^{\frac{1}{n}}$. Thus, $dm = \rho\pi\left(\dfrac{h}{a^{\frac{1}{n}}} x^{\frac{1}{n}}\right)^2 dx = \dfrac{\rho\pi h^2}{a^{\frac{2}{n}}} x^{\frac{2}{n}} dx$. The mass moment of Inertia of this element about the x axis is

$$dI_x = \frac{1}{2}(dm)y^2 = \frac{1}{2}\left(\frac{\rho\pi h^2}{a^{2/n}} x^{\frac{2}{n}}\, dx\right)\left(\frac{h}{a^{\frac{2}{n}}} x^{\frac{1}{n}}\right)^2 = \left(\frac{\rho\chi h^4}{2a^{\frac{4}{n}}} x^{\frac{4}{n}}\, dx\right)$$

Total Mass. Perform the integration,

$$m = \int_m dm = \int_0^a \frac{\rho\pi h^2}{a^{\frac{2}{n}}}\left(x^{\frac{2}{n}} dx\right)$$

$$= \left(\frac{\rho\pi h^2}{a^{\frac{2}{n}}}\right)\left(\frac{n}{n+2}\right)\left(x^{\frac{n+2}{n}}\right)\Bigg|_0^a$$

$$= \left(\frac{n}{n+2}\right)\rho\pi a h^2$$

Mass Moment of Inertia. Perform the integration,

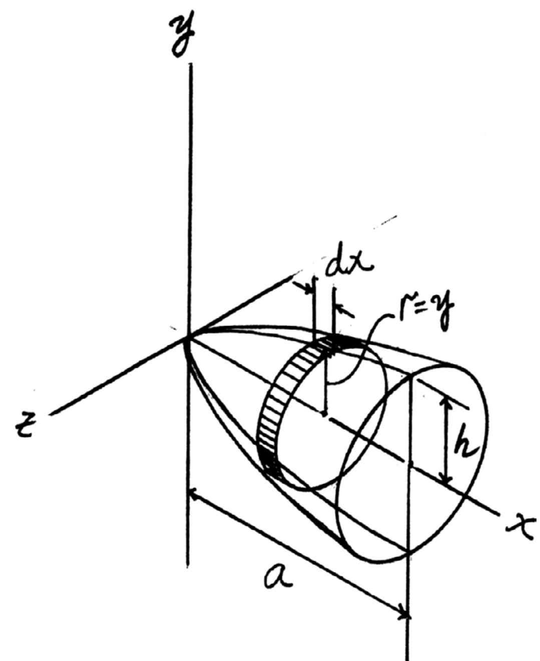
$$I_x = \int dI_x = \int_0^a \frac{\rho\pi h^4}{2a^{\frac{4}{n}}}\left(x^{\frac{4}{n}}\, dx\right)$$

$$= \left(\frac{\rho\pi h^4}{2a^{4/n}}\right)\left(\frac{n}{n+4}\right)\left(x^{\frac{n+4}{n}}\right)\Bigg|_0^a$$

$$= \left[\frac{n}{2(n+4)}\right]\rho\pi a h^4$$

The radius of gyration is

$$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{\left[\dfrac{n}{2(n+4)}\right]\rho\pi\, a h^4}{\left(\dfrac{n}{n+4}\right)\rho\pi\, a h^2}} = \sqrt{\frac{n+2}{2(n+4)}}\, h$$ **Ans.**

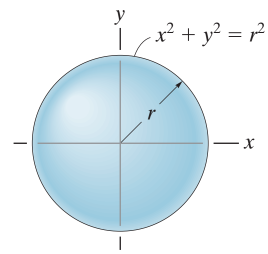


(a)

Ans:
$k_x = \sqrt{\dfrac{n+2}{2(n+4)}}\, h$

***10–92.**

Determine the moment of inertia I_x of the sphere and express the result in terms of the total mass m of the sphere. The sphere has a constant density ρ.

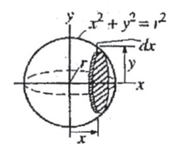


SOLUTION

$$d\,I_x = \frac{y^2\,dm}{2}$$

$$dm = \rho\,dV = \rho(\pi y^2\,dx) = \rho\pi(r^2 - x^2)dx$$

$$d\,I_x = \frac{1}{2}\rho\pi(r^2 - x^2)^2\,dx$$



$$I_x = \int_{-r}^{r} \frac{1}{2}\rho\pi(r^2 - x^2)^2\,dx$$

$$= \frac{8}{15}\pi\rho r^5$$

$$m = \int_{-r}^{r} \rho\pi(r^2 - x^2)\,dx$$

$$= \frac{4}{3}\rho\pi r^3$$

Thus,

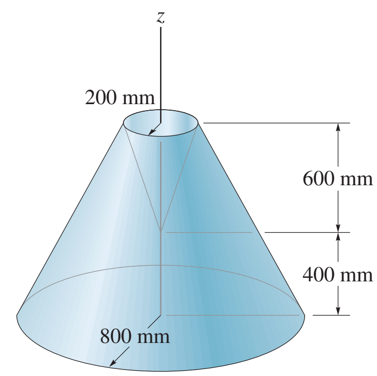
$$I_x = \frac{2}{5}mr^2 \qquad \textbf{Ans.}$$

Ans:

$$I_x = \frac{2}{5}mr^2$$

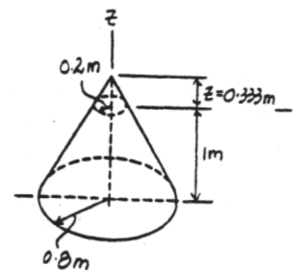
10–93.

Determine the moment of inertia I_z of the frustrum of the cone which has a conical depression. The material has a density of $2000\ \text{kg/m}^3$.



SOLUTION

Mass Moment of Inertia About z Axis: From similar triangles, $\dfrac{z}{0.2} = \dfrac{z+1}{0.8}$, $z = 0.333\ \text{m}$. The mass moment of inertia of each cone about z axis can be determined using $I_z = \dfrac{3}{10}mr^2$.

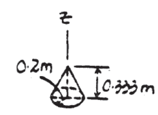


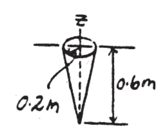
$$I_z = \Sigma(I_z)_i = \frac{3}{10}\left[\frac{\pi}{3}(0.8^2)(1.333)(2000)\right](0.8^2)$$

$$-\frac{3}{10}\left[\frac{\pi}{3}(0.2^2)(0.333)(2000)\right](0.2^2)$$

$$-\frac{3}{10}\left[\frac{\pi}{3}(0.2^2)(0.6)(2000)\right](0.2^2)$$

$$= 342\ \text{kg}\cdot\text{m}^2$$ **Ans.**

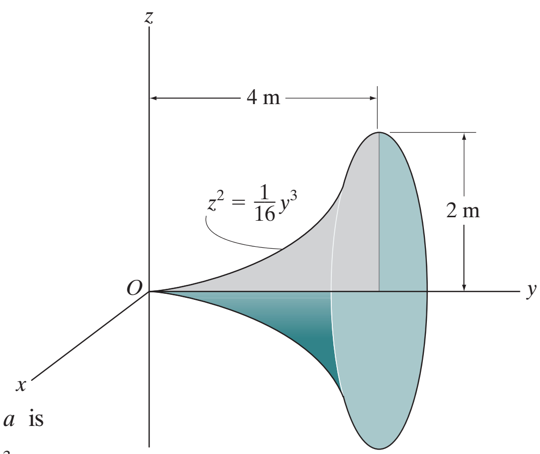




Ans:
$I_z = 342\ \text{kg}\cdot\text{m}^2$

10–94.

Determine the mass moment of inertia I_y of the solid formed by revolving the shaded area around the y axis. The total mass of the solid is 1500 kg.



SOLUTION

Differential Element: The mass of the disk element shown shaded in Fig. *a* is $dm = \rho dV = \rho\pi r^2 dy$. Here, $r = z = \frac{1}{4}y^{3/2}$. Thus, $dm = \rho\pi\left(\frac{1}{4}y^{3/2}\right)^2 dy = \frac{\rho\pi}{16}y^3 dy$. The mass moment of inertia of this element about the y axis is $dI_y = \frac{1}{2}dmr^2 = \frac{1}{2}\left(\rho\pi r^2 dy\right)r^2 = \frac{\rho\pi}{2}r^4 dy = \frac{\rho\pi}{2}\left(\frac{1}{4}y^{3/2}\right)^4 dy = \frac{\rho\pi}{512}y^6 dy$.

Mass: The mass of the solid can be determined by integrating dm. Thus,

$$m = \int dm = \int_0^{4\text{ m}} \frac{\rho\pi}{16}y^3 dy = \frac{\rho\pi}{16}\left(\frac{y^4}{4}\right)\Bigg|_0^{4\text{ m}} = 4\pi\rho$$

The mass of the solid is $m = 1500$ kg. Thus,

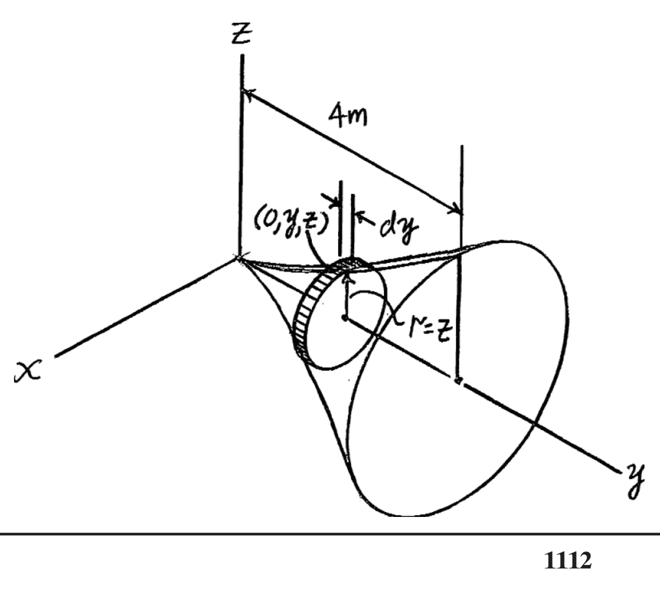
$$1500 = 4\pi\rho \qquad \rho = \frac{375}{\pi}\text{ kg/m}^3$$

Mass Moment of Inertia: Integrating dI_y,

$$I_y = \int dI_y = \int_0^{4\text{ m}} \frac{\rho\pi}{512}y^6 dy = \frac{\rho\pi}{512}\left(\frac{y^7}{7}\right)\Bigg|_0^{4\text{ m}} = \frac{32\pi}{7}\rho$$

Substituting $\rho = \frac{375}{\pi}$ kg/m^3 into I_y,

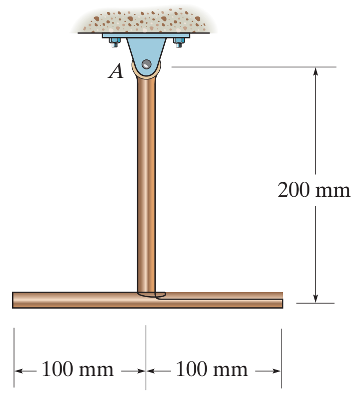
$$I_y = \frac{32\pi}{7}\left(\frac{375}{\pi}\right) = 1.71(10^3)\text{ kg}\cdot\text{m}^2$$ **Ans.**



Ans:
$I_y = 1.71\left(10^3\right)\text{ kg}\cdot\text{m}^2$

10–95.

The slender rods have a mass of 4 kg/m. Determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through point A.



SOLUTION

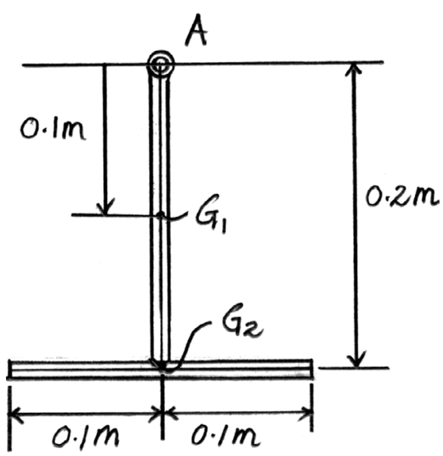
Mass Moment of Inertia About An Axis Through A. The mass of each segment is $m_i = (4\text{ kg/m})(0.2\text{ m}) = 0.8\text{ kg}$. The mass moment inertia of each segment shown in Fig. a about an axis through their center of mass can be determined using $(I_G)_i = \frac{1}{12} m_i l_i^2$.

$$I_A = \Sigma\left[(I_G)_i + m_i d_i^2\right]$$

$$= \left[\frac{1}{12}(0.8)(0.2^2) + 0.8(0.1^2)\right] + \left[\frac{1}{12}(0.8)(0.2^2) + 0.8(0.2^2)\right]$$

$$= 0.04533\text{ kg}\cdot\text{m}^2$$

$$= 0.0453\text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

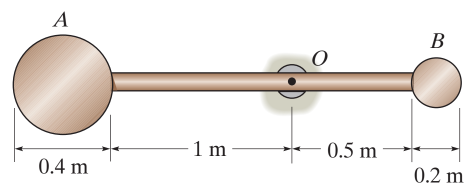


(a)

Ans:
$I_A = 0.0453\text{ kg}\cdot\text{m}^2$

***10–96.**

The pendulum consists of a 8-kg circular disk A, a 2-kg circular disk B, and a 4-kg slender rod. Determine the radius of gyration of the pendulum about an axis perpendicular to the page and passing through point O.



SOLUTION

Mass Moment of Inertia About An Axis Through *O*. The mass moment of inertia of each rod segment and disk segment shown in Fig. *a* about an axis passes through their center of mass can be determined using $(I_G)_i = \frac{1}{12} m_i l_i^2$ and $(I_G)_i = \frac{1}{2} m_i r_i^2$.

$$I_O = \Sigma\left[(I_G)_i + m_i d_i^2\right]$$

$$= \left[\frac{1}{12}(4)(1.5^2) + 4(0.25^2)\right] + \left[\frac{1}{2}(2)(0.1^2) + 2(0.6^2)\right]$$

$$+ \left[\frac{1}{2}(8)(0.2^2) + 8(1.2^2)\right]$$

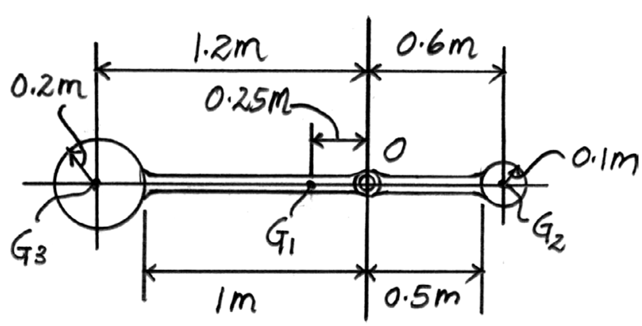
$$= 13.41 \text{ kg} \cdot \text{m}^2$$

The total mass is

$$8 \text{ kg} + 2 \text{ kg} + 4 \text{ kg} = 14 \text{ kg}$$

The radius of gyration is

$$k_O = \sqrt{\frac{I_O}{m}} = \sqrt{\frac{13.41 \text{ kg} \cdot \text{m}^2}{14 \text{ kg}}} = 0.9787 \text{ m} = 0.979 \text{ m} \qquad \textbf{Ans.}$$

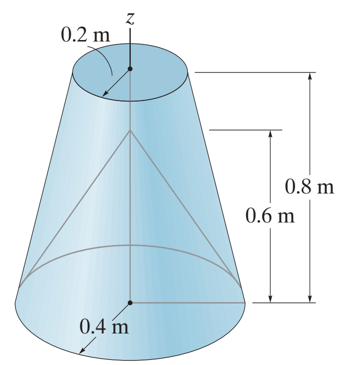


(a)

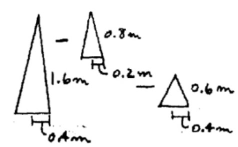
Ans:
$k_O = 0.979$ m

10–97.

Determine the moment of inertia I_z of the frustum of the cone which has a conical depression. The material has a density of 200 kg/m^3.



SOLUTION



$$I_z = \frac{3}{10}[\frac{1}{3}\pi\,(0.4)^2(1.6)(200)](0.4)^2$$

$$-\frac{3}{10}[\frac{1}{3}\pi\,(0.2)^2(0.8)(200)](0.2)^2$$

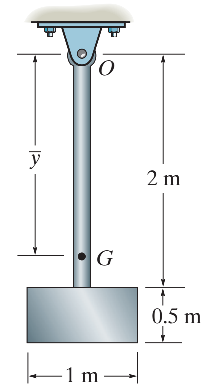
$$-\frac{3}{10}[\frac{1}{3}\pi\,(0.4)^2(0.6)(200)](0.4)^2$$

$I_z = 1.53\ \text{kg}\cdot\text{m}^2$ **Ans.**

Ans:
$I_z = 1.53\ \text{kg}\cdot\text{m}^2$

10–98.

The pendulum consists of the 3-kg slender rod and the 5-kg thin plate. Determine the location $\bar{y}$ of the center of mass G of the pendulum; then find the mass moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.



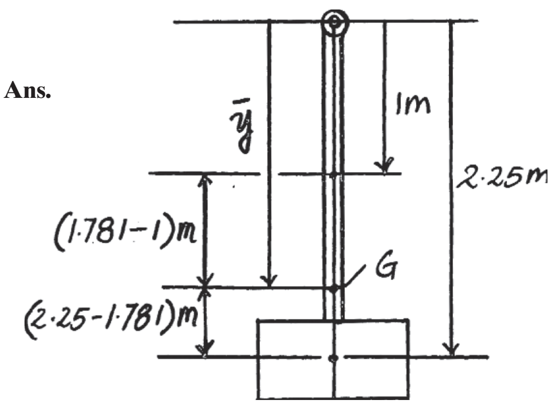
SOLUTION

$$\bar{y} = \frac{\Sigma \bar{y}m}{\Sigma m} = \frac{1(3) + 2.25(5)}{3 + 5} = 1.781 \text{ m} = 1.78 \text{ m} \qquad \textbf{Ans.}$$

$$I_G = \Sigma \bar{I}_{G'} + md^2$$

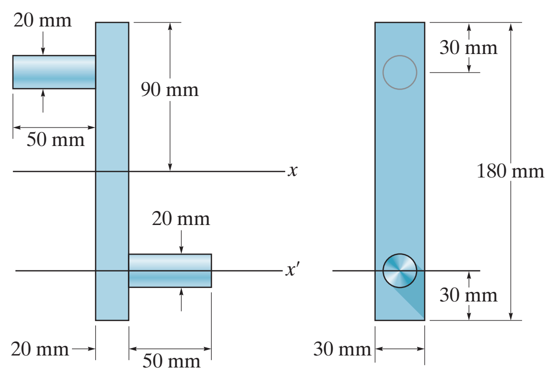
$$= \frac{1}{12}(3)(2)^2 + 3(1.781 - 1)^2 + \frac{1}{12}(5)(0.5^2 + 1^2) + 5(2.25 - 1.781)^2$$

$$= 4.45 \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans.}$$



Ans:
$\bar{y} = 1.78$ m
$I_G = 4.45$ kg · m^2

10–99. Determine the mass moment of inertia of the overhung crank about the x axis. The material is steel having a density of $\rho = 7.85\ \text{Mg/m}^3$.



SOLUTION

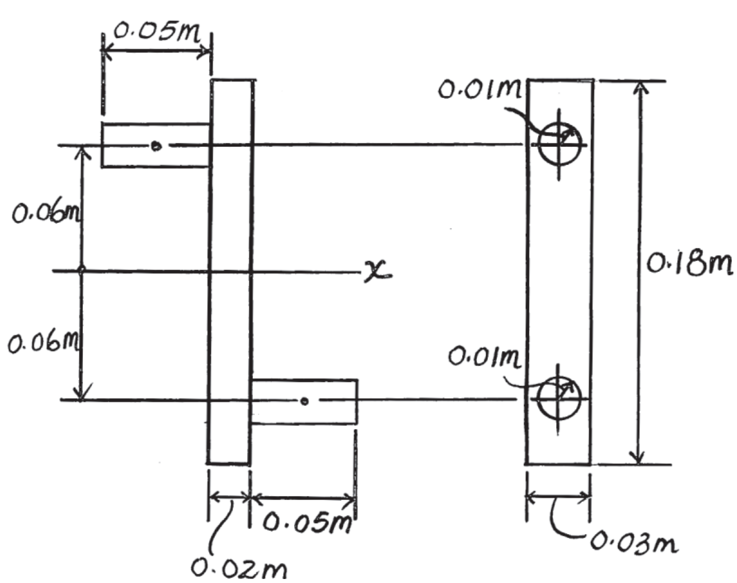
$$m_e = 7.85(10^3)\big((0.05)\pi(0.01)^2\big) = 0.1233\ \text{kg}$$

$$m_p = 7.85(10^3)((0.03)(0.180)(0.02)) = 0.8478\ \text{kg}$$

$$I_x = 2\left[\frac{1}{2}(0.1233)(0.01)^2 + (0.1233)(0.06)^2\right]$$

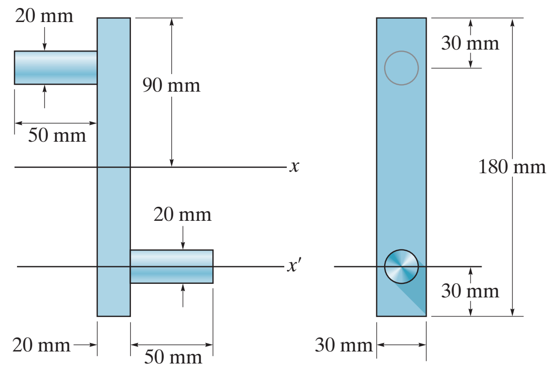
$$+\left[\frac{1}{12}(0.8478)((0.03)^2 + (0.180)^2)\right]$$

$$= 0.00325\ \text{kg}\cdot\text{m}^2 = 3.25\ \text{g}\cdot\text{m}^2$$ **Ans.**



Ans:
$I_x = 3.25\ \text{g}\cdot\text{m}^2$

***10–100.** Determine the mass moment of inertia of the overhung crank about the x' axis. The material is steel having a density of $\rho = 7.85\ \text{Mg/m}^3$.



SOLUTION

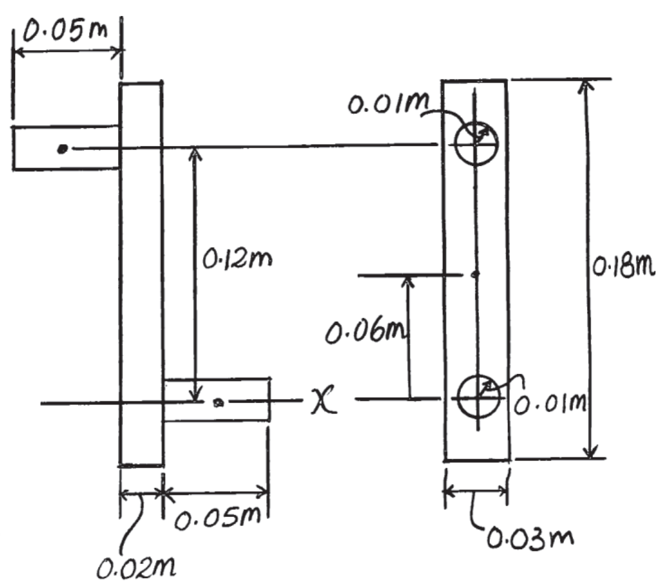
$$m_e = 7.85(10^3)\left((0.05)\pi(0.01)^2\right) = 0.1233\ \text{kg}$$

$$m_p = 7.85(10^3)((0.03)(0.180)(0.02)) = 0.8478\ \text{kg}$$

$$I_{x'} = \left[\frac{1}{2}(0.1233)(0.01)^2\right] + \left[\frac{1}{2}(0.1233)(0.02)^2 + (0.1233)(0.120)^2\right]$$

$$+ \left[\frac{1}{12}(0.8478)\left((0.03)^2 + (0.180)^2\right) + (0.8478)(0.06)^2\right]$$

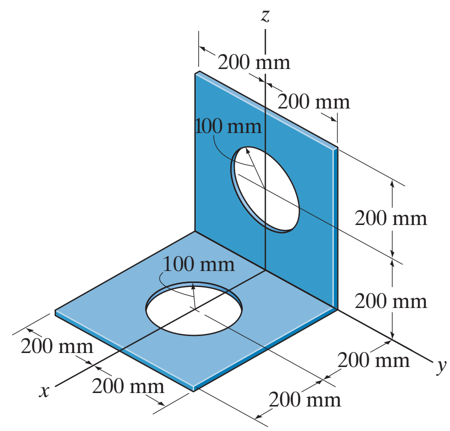
$$= 0.00719\ \text{kg}\cdot\text{m}^2 = 7.19\ \text{g}\cdot\text{m}^2$$ **Ans.**



Ans:
$I_{x'} = 7.19\ \text{g}\cdot\text{m}^2$

10–101.

The thin plate has a mass per unit area of $10\ \text{kg/m}^2$. Determine its mass moment of inertia about the y axis.



SOLUTION

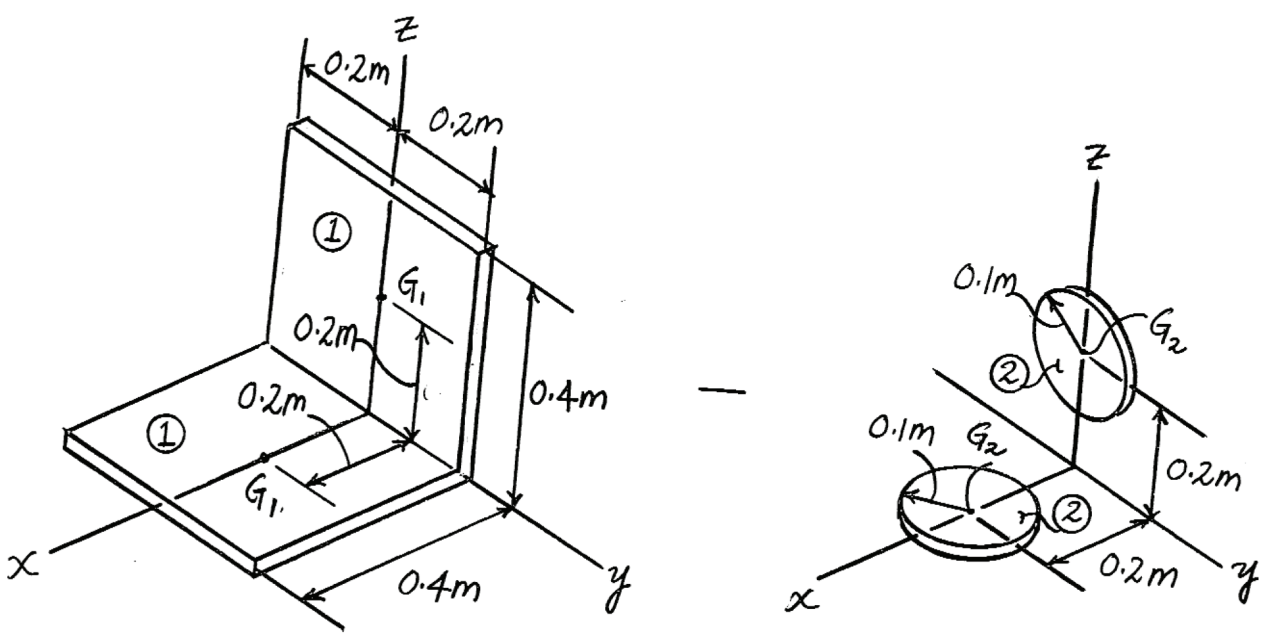
Composite Parts: The thin plate can be subdivided into segments as shown in Fig. *a*. Since the segments labeled (2) are both holes, the y should be considered as negative parts.

Mass moment of Inertia: The mass of segments (1) and (2) are $m_1 = 0.4(0.4)(10) = 1.6$ kg and $m_2 = \pi(0.1^2)(10) = 0.1\pi$ kg. The perpendicular distances measured from the centroid of each segment to the y axis are indicated in Fig. *a*. The mass moment of inertia of each segment about the y axis can be determined using the parallel-axis theorem.

$$I_y = \Sigma\left(I_y\right)_G + md^2$$

$$= 2\left[\frac{1}{12}(1.6)(0.4^2) + 1.6(0.2^2)\right] - 2\left[\frac{1}{4}(0.1\pi)(0.1^2) + 0.1\pi(0.2^2)\right]$$

$$= 0.144\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

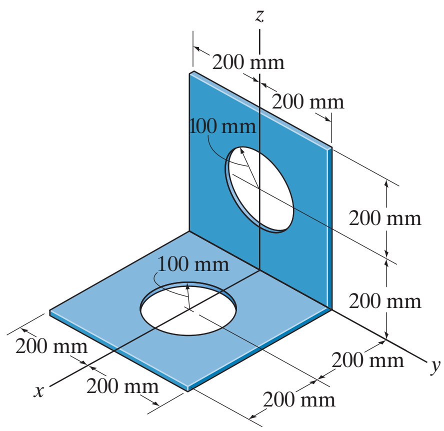


(a)

Ans:
$I_y = 0.144\ \text{kg}\cdot\text{m}^2$

10–102.

The thin plate has a mass per unit area of $10\ \text{kg/m}^2$. Determine its mass moment of inertia about the z axis.



SOLUTION

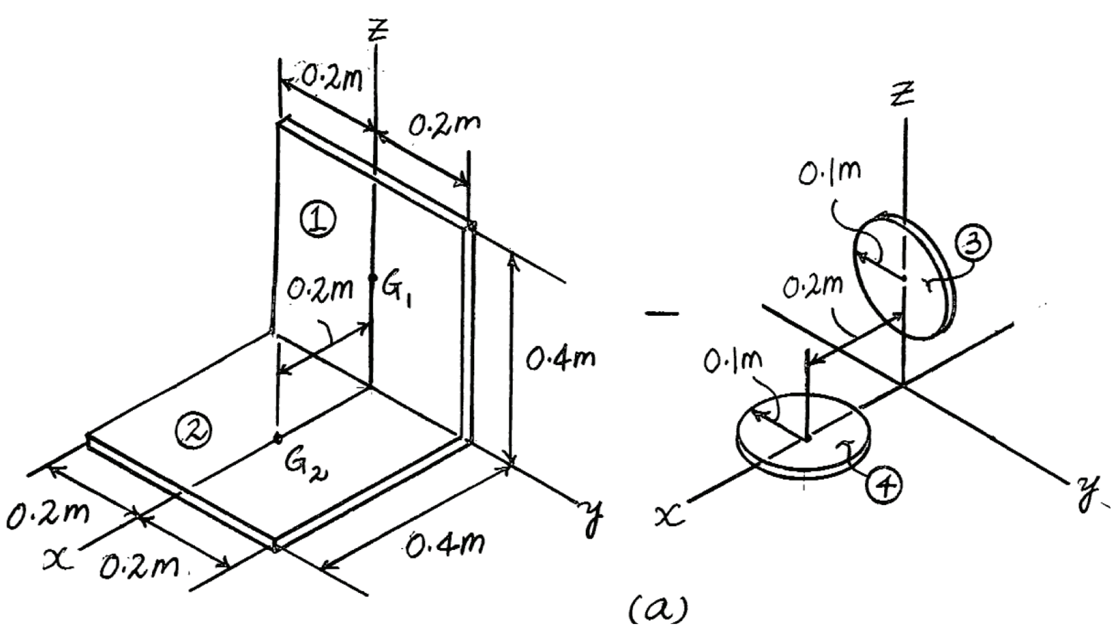
Composite Parts: The thin plate can be subdivided into four segments as shown in Fig. *a*. Since segments (3) and (4) are both holes, the y should be considered as negative parts.

Mass moment of Inertia: Here, the mass for segments (1), (2), (3), and (4) are $m_1 = m_2 = 0.4(0.4)(10) = 1.6$ kg and $m_3 = m_4 = \pi(0.1^2)(10) = 0.1\pi$ kg. The mass moment of inertia of each segment about the z axis can be determined using the parallel-axis theorem.

$$I_z = \Sigma\left(I_z\right)_G + md^2$$

$$= \frac{1}{12}(1.6)(0.4^2) + \left[\frac{1}{12}(1.6)(0.4^2 + 0.4^2) + 1.6(0.2^2)\right] - \frac{1}{4}(0.1\pi)(0.1^2) - \left[\frac{1}{2}(0.1\pi)(0.1^2) + 0.1\pi(0.2^2)\right]$$

$$= 0.113\ \text{kg}\cdot\text{m}^2$$ **Ans.**

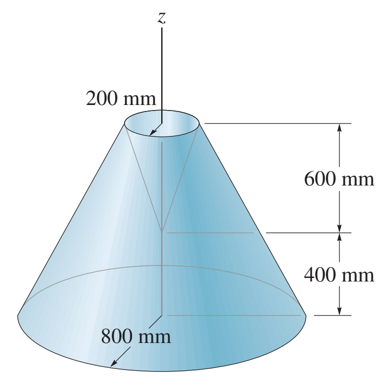


(a)

Ans:
$I_z = 0.113\ \text{kg}\cdot\text{m}^2$

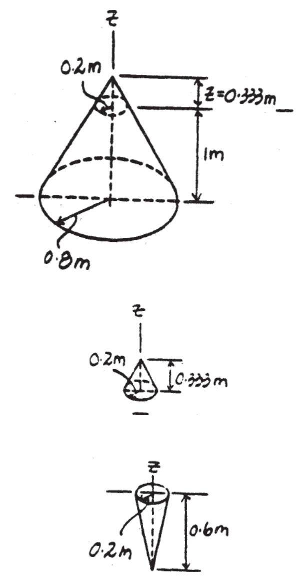
10–103.

Determine the moment of inertia I_z of the frustrum of the cone which has a conical depression. The material has a density of 200 kg/m^3.



SOLUTION

Mass Moment of Inertia About z Axis: From similar triangles, $\dfrac{z}{0.2} = \dfrac{z+1}{0.8}$, $z = 0.333$ m. The mass moment of inertia of each cone about z axis can be determined using $I_z = \dfrac{3}{10}mr^2$.



$$I_z = \Sigma(I_z)_i = \frac{3}{10}\left[\frac{\pi}{3}(0.8^2)(1.333)(200)\right](0.8^2)$$

$$-\frac{3}{10}\left[\frac{\pi}{3}(0.2^2)(0.333)(200)\right](0.2^2)$$

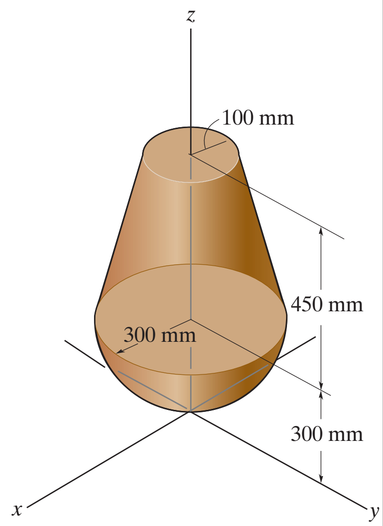
$$-\frac{3}{10}\left[\frac{\pi}{3}(0.2^2)(0.6)(200)\right](0.2^2)$$

$$= 34.2 \text{ kg} \cdot \text{m}^2$$ **Ans.**

Ans:
$I_z = 34.2 \text{ kg} \cdot \text{m}^2$

***10–104.**

Determine the mass moment of inertia of the assembly about the z axis. The density of the material is 7.85 Mg/m^3.



SOLUTION

Composite Parts: The assembly can be subdivided into two circular cone segments (1) and (3) and a hemispherical segment (2) as shown in Fig. *a.* Since segment (3) is a hole, it should be considered as a negative part. From the similar triangles, we obtain

$$\frac{z}{0.45 + z} = \frac{0.1}{0.3} \qquad z = 0.225m$$

Mass: The mass of each segment is calculated as

$$m_1 = \rho V_1 = \rho\left(\frac{1}{3}\pi r^2 h\right) = 7.85(10^3)\left[\frac{1}{3}\pi(0.3^2)(0.675)\right] = 158.9625\pi\text{ kg}$$

$$m_2 = \rho V_2 = \rho\left(\frac{2}{3}\pi r^3\right) = 7.85(10^3)\left[\frac{2}{3}\pi(0.3^3)\right] = 141.3\pi\text{ kg}$$

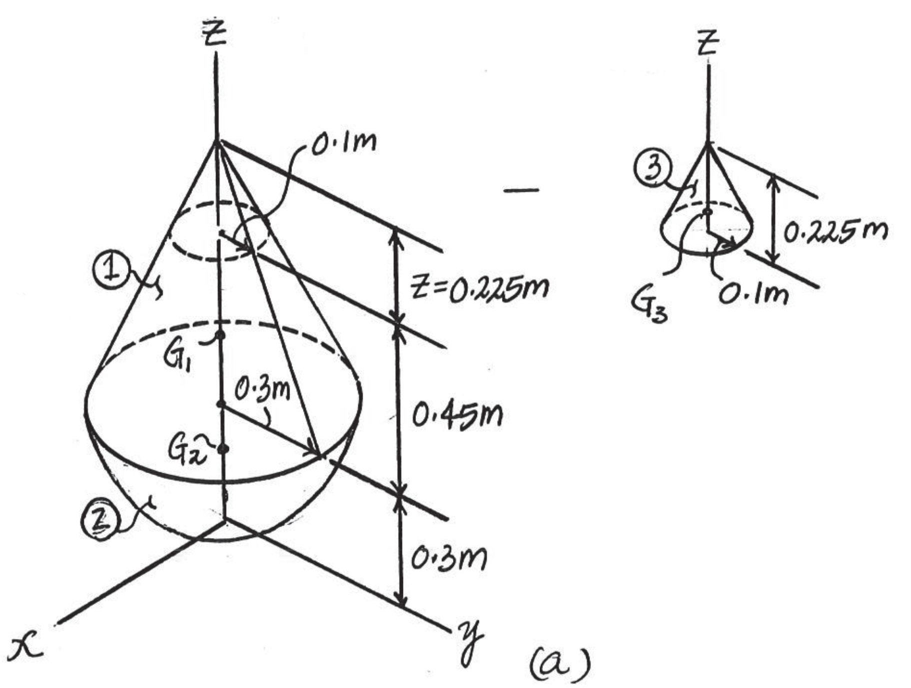
$$m_3 = \rho V_3 = \rho\left(\frac{1}{3}\pi r^2 h\right) = 7.85(10^3)\left[\frac{1}{3}\pi(0.1^2)(0.225)\right] = 5.8875\pi\text{ kg}$$

Mass Moment of Inertia: Since the z axis is parallel to the axis of the cone and the hemisphere and passes through their center of mass, the mass moment of inertia can be computed from $(I_z)_1 = \frac{3}{10}m_1r_1^2$, $(I_z)_2 = \frac{2}{5}m_2r_2^2$, and $\frac{3}{10}m_3r_3^2$. Thus,

$$I_z = \Sigma(I_z)_i$$

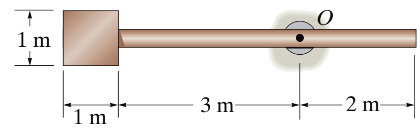
$$= \frac{3}{10}(158.9625\pi)(0.3^2) + \frac{2}{5}(141.3\pi)(0.3^2) - \frac{3}{10}(5.8875\pi)(0.1^2)$$

$$= 29.4\text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$



Ans:
$I_z = 29.4\text{ kg}\cdot\text{m}^2$

10–105. The pendulum consists of a plate having a weight of 60 kg and a slender rod having a weight of 20 kg. Determine the radius of gyration of the pendulum about an axis perpendicular to the page and passing through point O.



SOLUTION

Given:

$W_p = 60\text{ kg}$ $a = 1\text{ m}$
$W_r = 20\text{ kg}$ $b = 1\text{ m}$
$c = 3\text{ m}$
$d = 2\text{ m}$

$$I_0 = \frac{1}{12} \cdot W_r(c + d)^2 + W_r \cdot \left(\frac{c + d}{2} - c\right)^2 + \frac{1}{12} \cdot W_p(a^2 + b^2) + W_p \cdot \left(c + \frac{b}{2}\right)^2$$

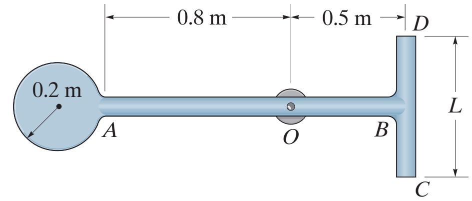
$$k_0 = \sqrt{\frac{I_0}{W_p + W_r}} \qquad k_0 = 3.15\text{ m}$$

Ans.

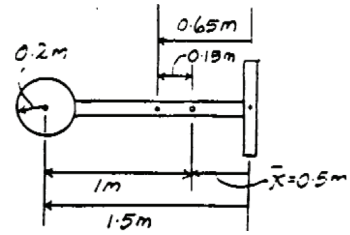
Ans:
$k_0 = 3.15\text{ m}$

10–106.

The pendulum consists of a disk having a mass of 6 kg and slender rods AB and DC which have a mass of $2\ \text{kg/m}$. Determine the length L of DC so that the center of mass is at the bearing O. What is the moment of inertia of the assembly about an axis perpendicular to the page and passing through point O?



SOLUTION



Location of Centroid: This problem requires $\bar{x} = 0.5$ m.

$$\bar{x} = \frac{\Sigma \bar{x} m}{\Sigma m}$$

$$0.5 = \frac{1.5(6) + 0.65[1.3(2)] + 0[L(2)]}{6 + 1.3(2) + L(2)}$$

$$L = 6.39 \text{ m} \qquad \textbf{Ans.}$$

Mass Moment of Inertia About an Axis Through Point O: The mass moment of inertia of each rod segment and disk about an axis passing through the center of mass can be determine using $(I_G)_i = \frac{1}{12}ml^2$ and $(I_G)_i = \frac{1}{2}mr^2$. Applying Eq. 10–15, we have

$$I_O = \Sigma (I_G)_i + m_i d_i^2$$

$$= \frac{1}{12}[1.3(2)]\left(1.3^2\right) + [1.3(2)]\left(0.15^2\right)$$

$$+ \frac{1}{12}[6.39(2)]\left(6.39^2\right) + [6.39(2)]\left(0.5^2\right)$$

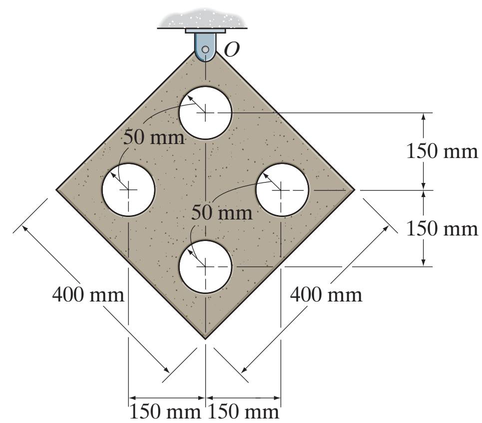
$$+ \frac{1}{2}(6)\left(0.2^2\right) + 6\left(1^2\right)$$

$$= 53.2 \text{ kg} \cdot \text{m}^2 \qquad \textbf{Ans.}$$

Ans:
$L = 6.39$ m
$I_o = 53.2\ \text{kg} \cdot \text{m}^2$

10–107.

Determine the mass moment of inertia of the thin plate about an axis perpendicular to the page and passing through point O. The material has a mass per unit area of $20\ \text{kg/m}^2$.



SOLUTION

Composite Parts: The plate can be subdivided into the segments shown in Fig. *a*. Here, the four similar holes of which the perpendicular distances measured from their centers of mass to point C are the same and can be grouped as segment (2). This segment should be considered as a negative part.

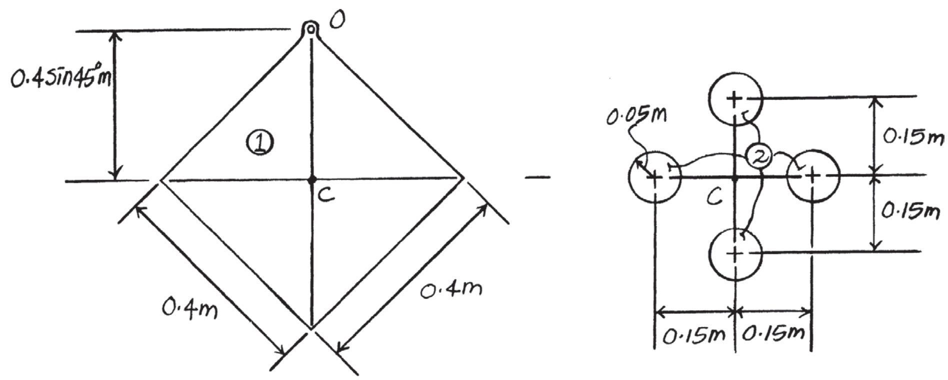
Mass Moment of Inertia: The mass of segments (1) and (2) are $m_1 = (0.4)(0.4)(20) = 3.2\ \text{kg}$ and $m_2 = \pi(0.05^2)(20) = 0.05\pi\ \text{kg}$, respectively. The mass moment of inertia of the plate about an axis perpendicular to the page and passing through point C is

$$I_C = \frac{1}{12}(3.2)(0.4^2 + 0.4^2) - 4\left[\frac{1}{2}(0.05\pi)(0.05^2) + 0.05\pi(0.15^2)\right]$$

$$= 0.07041\ \text{kg}\cdot\text{m}^2$$

The mass moment of inertia of the wheel about an axis perpendicular to the page and passing through point O can be determined using the parallel-axis theorem $I_O = I_C + md^2$, where $m = m_1 - m_2 = 3.2 - 4(0.05\pi) = 2.5717\ \text{kg}$ and $d = 0.4 \sin 45° m$. Thus,

$$I_O = 0.07041 + 2.5717(0.4 \sin 45°)^2 = 0.276\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

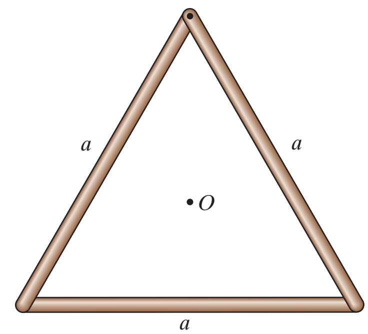


(a)

Ans:
$I_O = 0.276\ \text{kg}\cdot\text{m}^2$

***10–108.**

Each of the three slender rods has a mass m. Determine the moment of inertia of the assembly about an axis that is perpendicular to the page and passes through the center point O.



SOLUTION

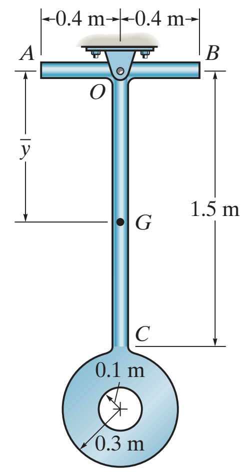
$$I_O = 3\left[\frac{1}{12}ma^2 + m\left(\frac{a\sin 60^\circ}{3}\right)^2\right] = \frac{1}{2}ma^2$$ **Ans.**

Ans:

$I_O = \dfrac{1}{2}ma^2$

10–109.

The pendulum consists of two slender rods AB and OC which have a mass of 3 kg/m. The thin plate has a mass of 12 kg/m^2. Determine the location $\overline{y}$ of the center of mass G of the pendulum, then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.



SOLUTION

$$\overline{y} = \frac{1.5(3)(0.75) + \pi(0.3)^2(12)(1.8) - \pi(0.1)^2(12)(1.8)}{1.5(3) + \pi(0.3)^2(12) - \pi(0.1)^2(12) + 0.8(3)}$$

$$= 0.8878 \text{ m} = 0.888 \text{ m} \qquad \textbf{Ans.}$$

$$I_G = \frac{1}{12}(0.8)(3)(0.8)^2 + 0.8(3)(0.8878)^2$$

$$+\frac{1}{12}(1.5)(3)(1.5)^2 + 1.5(3)(0.75 - 0.8878)^2$$

$$+\frac{1}{2}[\pi(0.3)^2(12)(0.3)^2 + [\pi(0.3)^2(12)](1.8 - 0.8878)^2$$

$$-\frac{1}{2}[\pi(0.1)^2(12)(0.1)^2 - [\pi(0.1)^2(12)](1.8 - 0.8878)^2$$

$$I_G = 5.61 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans.}$$

Ans:
$\overline{y} = 0.888$ m
$I_G = 5.61$ kg·m^2