

TEMPERATURE AND KINETIC THEORY

Responses to Questions

- 1. Because the atomic mass of copper is smaller than that of lead, an atom of copper has less mass than an atom of lead. Thus, 1 kg of copper will have more atoms than 1 kg of lead.
- 2. Properties of materials that can be exploited for the making of a thermometer include:
 - (i) Volume of a liquid (mercury or alcohol thermometer)
 - (ii) Electrical resistance (covered in a later chapter)
 - (iii) Color (frequency) of emitted light from a heated object (covered in a later chapter)
 - (iv) Volume of a gas
 - (v) Expansion of a metal (bimetallic strip)
- 3. 1 C° is larger than 1 F°. There are 100 C° between the freezing and boiling temperatures of water, while there are 180 F° between the same two temperatures.
- 4. To be precise, ℓ_0 is to be the initial length of the object. In practice, however, since the value of the coefficient of expansion is so small, there will be little difference in the calculation of $\Delta \ell$ caused by using either the initial or final length, unless the temperature change is quite large.
- 5. When heated, the aluminum expands more than the iron, because the expansion coefficient of aluminum is larger than that of iron. Thus the aluminum will be on the outside of the curve.
- 6. The steam pipe can have a large temperature change as the steam enters or leaves the pipe. If the pipe is fixed at both ends and the temperature changes significantly, then there will be large thermal stresses that might break joints. The "U" in the pipe allows for expansion and contraction, which is not possible at the fixed ends. This is similar to the joints placed in concrete roadway surfaces to allow expansion and contraction.
- 7. The bimetallic strip is made of two types of metal joined together. The metal of the outside strip has a higher coefficient of linear expansion than that of the inside strip, so it will expand and contract more dramatically. As the temperature cools, the strip will coil more tightly. That will tilt the liquid mercury vessel so that the mercury goes over the two contacts, making a complete circuit and turning on the heater. As the temperature rises, the strip will "uncoil," moving the liquid mercury off of the contacts and shutting off the heater. Moving the temperature setting lever changes the initial position of the

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13-2 Chapter 13

glass vessel and thus changes the temperature at which the coil has expanded or shrunk enough to move the liquid mercury.

- 8. If one part is heated or cooled more than another part, there will be more expansion or contraction of one part of the glass compared to an adjacent part. This causes internal stress forces that may exceed the maximum strength of the glass.
- 9. If water is added quickly to an overheated engine, it comes into contact with the very hot metal parts of the engine. Some areas of the metal parts will cool off very rapidly; others will not. Some of the water will quickly turn to steam and will expand rapidly. The net result can be a cracked engine block or radiator, due to the thermal stress and/or the emission of high-temperature steam from the radiator. Water should always be added slowly, with the engine running. The water will mix with the hotter water already in the system and will circulate through the engine, gradually cooling all parts at about the same rate.
- 10. The coefficient of expansion is derived from a ratio of lengths: $\alpha = \frac{\Delta \ell}{\ell_0} \frac{1}{\Delta T}$. The length units cancel,

so the coefficient does not depend on the specific length unit used in its determination, as long as the same units are used for both $\Delta \ell$ and ℓ_0 .

- 11. The glass is the first to warm due to the hot water, so the glass will initially expand a small amount. As the glass initially expands, the alcohol level will decrease. As thermal equilibrium is reached, the alcohol will expand more than the glass expands, since alcohol has a larger coefficient of expansion than water, and the alcohol level will rise to indicate the higher temperature.
- 12. Since Pyrex glass has a smaller coefficient of linear expansion than ordinary glass, it will expand less than ordinary glass when heated, making it less likely to crack from internal stresses. Pyrex glass is therefore more suitable for applications involving heating and cooling. An ordinary glass mug may expand to the point of cracking if boiling water is poured in it, whereas a Pyrex mug will not.
- 13. On a hot day, the pendulum will be slightly longer than at 20°C, due to thermal expansion of the brass rod. Since the period of a pendulum is proportional to the square root of its length, the period will be slightly longer on the hot day, meaning that the pendulum takes more time for one oscillation. Thus, the clock will run slow.
- 14. The soda is mostly water. As water cools below 4°C it expands. There is more expansion of the soda as it cools below 4°C and freezes than there is available room in the can (the can has actually shrunk a small amount, making the mismatch more pronounced), so the freezing soda pushes against the can surfaces hard enough to push them outward. Evidently the top and bottom of the can are the weakest parts.
- 15. The buoyant force on the aluminum sphere is the weight of the water displaced by the sphere, which is the volume of the sphere times the density of water times *g*. As the substances are heated, the volume of the sphere increases and the density of the water decreases (because of its increased volume). Since the volume expansion coefficient of the water is almost three times larger than that of the aluminum, the fractional decrease in the water density is larger than the fractional increase in the aluminum volume. Thus, the product of the volume of the sphere and the density of water decreases, and the buoyant force gets smaller.
- 16. For an absolute vacuum, no. But for most "vacuums," there are still a few molecules in the containers, and the temperature can be determined from the (very low) pressure.

- 17. (a) Because the escape velocity for the Moon is 1/5 that of the Earth, heavy molecules with lower speeds will be able to escape. The Moon may have started with an atmosphere, but over time almost all of the molecules of gas have escaped.
 - (b) Hydrogen is the lightest gas. For a given kinetic energy (temperature) it has the highest speed and will be most likely to escape.
- 18. Boiling occurs when the saturated vapor pressure equals the external pressure. When we say the oxygen "boils" at -183°C, we mean that the saturated vapor pressure for oxygen will be 1 atm (the same as atmospheric pressure) at a temperature of -183°C. At this temperature and pressure, liquid oxygen will vaporize.
- 19. The freezing point of water decreases slightly with higher pressure. The wire exerts a large pressure on the ice (due to the weights hung at each end). The ice under the wire will melt, allowing the wire to move lower into the block. Once the wire has passed a given position, the water now above the wire will have only atmospheric pressure on it and will refreeze. This process allows the wire to pass all the way through the block and yet leave a solid block of ice behind.
- 20. (a) A pressure cooker is sealed, so as the temperature of its contents increases, the number of particles inside and the volume are kept constant. Thus, the pressure increases according to the ideal gas law. Assuming there is water inside the pressure cooker, an increased pressure yields a higher boiling point for the water. The water in which the food is prepared will boil at a higher temperature than normal, thereby cooking the food faster.
 - (b) At high altitudes, the atmospheric pressure is less than it is at sea level. If atmospheric pressure decreases, the boiling point of water will decrease, so boiling occurs at a lower temperature than at sea level. Food being cooked in an open pot (including pasta and rice) will need to cook longer at this lower temperature to be properly prepared.
 - (c) It is actually easier to boil water at higher altitude, because the water boils at a lower temperature. Thus, it will take less time to add enough heat to the water to bring it to the boiling temperature.
- 21. Liquids boil when their saturated vapor pressure equals the external pressure. For water, from Table 13–3, the saturated vapor pressure of water at 20°C is about 0.023 atm. So if the external pressure is lowered to that level (about 2.3% of normal air pressure), the water will boil at that low temperature.
- 22. Exhaled air contains a large amount of water vapor and is initially at a temperature equal to body temperature. When the exhaled air comes into contact with the external air on a cold day, it cools rapidly and reaches the dew point. At the dew point temperature, the air can no longer hold all the water vapor and water condenses into little droplets, forming a cloud. The white cloud seen is due to the condensed water vapor.
- 24. The water in the radiator of an overheated automobile engine is under pressure. Similar to a pressure cooker, that high pressure keeps the water in the liquid state even though the water is quite hot—hotter than 100°C. When the cap is opened, the pressure is suddenly lowered, and the superheated water boils quickly and violently. That hot steam can cause severe burns if it contacts the skin. Also, the violent bursting forth of steam propels some of the overheated water out of the radiator as well, which can spray onto the person opening the cap and again cause serious burns.

Responses to MisConceptual Questions

- 1. (c) Students may confuse thermal expansion with elasticity and surmise that the narrower rod would expand more for the same temperature change. However, in thermal expansion the change in length is independent of the rod's diameter or cross-sectional area.
- 2. (d) Equation 13–1a for thermal expansion shows that the change in length depends upon the initial length, the change in temperature, and the coefficient of thermal expansion (which depends upon the type of material).
- 3. (c) Many students have the misconception that as the plate expands, the hole will get smaller. To understand what actually happens, imagine that the hole is filled with a steel disk. As the plate and the steel disk are heated, both will expand. After the plate and disk are heated, the disk is removed from the plate. Since the disk expanded, the hole that is left must also have expanded. As the steel ring is heated its circumference will expand, causing its interior to also expand.
- 4. (b) A common error is to treat the temperature as doubling. If the temperature doubled, the pressure would also double. However, the temperature is given in degrees Celsius, not kelvins. When the temperature is converted to kelvins, it is easy to see that the temperature only increases by about 25%, from 373 K to 437 K. The pressure then also increases by about 25%. (The actual amount is 26.8%.)
- 5. (b) The absolute (Kelvin) scale must be used in the ideal gas law. If the Celsius scale is used, it appears that the temperature has doubled. However, 0°C is an arbitrarily chosen point on the temperature scale and cannot be used to determine temperature ratios. When the temperatures are converted to kelvins (293 K and 313 K) it can be seen that their ratio is about 1.07.
- 6. (c) By the ideal gas law, when the temperature is held constant, the pressure is proportional to the number of moles and inversely proportional to the volume. If the second bottle has twice the volume with only half the number of moles, it would only experience one-fourth the pressure.
- 7. (e) A common misconception is that if the temperature increases, both the pressure and volume will increase. However, by the ideal gas law, when the temperature increases, the product of the pressure and volume must increase. This increase can occur by increasing the pressure, increasing the volume, or both.
- 8. (c) The temperature of the gas is a measure of the average kinetic energy of the gas molecules.
- 9. (e) Some students might erroneously relate the temperature of the gas to the velocity of the gas molecules and surmise that the rms speeds would be equal. However, when the two gases are at the same temperature, the molecules will have the same average kinetic energy. The kinetic energy is proportional to the mass of the molecule and the square of the rms speed. Since mass B is half the mass of A, the speed of molecules of mass B must be $\sqrt{2} \approx 1.4 \times$ greater than the speed of molecules of mass A.
- 10. (*a*) In the mixture, the oxygen molecules and helium atoms will be at the same temperature, which means that their average molecular kinetic energies will be the same. Since a helium atom has less mass than an oxygen molecule, the helium atoms will be moving faster than the oxygen molecules on average.
- 11. (d) The temperature of an ideal gas is a measure of the average kinetic energy of the gas, so increasing the temperature will increase the average kinetic energy—(a) is true. Even though the temperature is proportional to the average kinetic energy of the molecules, each molecule can

have a random kinetic energy (according to the Maxwell distribution of speeds). The speeds of individual molecules will vary about this average, so (d) is false. Per the ideal gas law, the product of the pressure and volume is proportional to the temperature. Therefore, if pressure or volume is held constant as the temperature increases, the other parameter (volume or pressure) must increase, so (b) and (c) are true. For a gas to be ideal, it is assumed that the space occupied by the gas is mostly empty. For this to be true, the molecules are assumed to be far apart compared with their size. Thus, (e) is also true.

- 12. (*a*) Students frequently do not understand that gauge pressure and temperature in Celsius are a comparison of pressure and temperature to an arbitrary zero point. For the ideal gas law to hold, the temperature, pressure, and volume must be measured relative to the true zero points: absolute zero pressure, absolute zero temperature, and zero volume. The volume units are not critical, since changing them only affects the value of the ideal gas constant. Gauge pressure and Celsius temperature have additive terms that change the functional form of the ideal gas law.
- 13. (c) The rms speed of a gas is an average molecular speed found by taking the square root of the average of the square of the molecule speeds. Consider a gas in which 2/3 of the molecules are at rest and 1/3 move at a constant speed. The most probable speed is 0, but the rms speed is greater than zero. Therefore, the most probable speed is not necessarily the rms speed, so (a) is false. The rms speed is an average speed; therefore, it will always be equal to or smaller than the maximum speed, so (b) is false. The temperature of a gas is determined by the average molecular kinetic energy. As the temperature increases, the molecular kinetic energy increases; therefore, the rms speed must also increase, so (c) is true.

Solutions to Problems

In solving these problems, the authors did not always follow the rules of significant figures rigidly. We tended to take quoted temperatures as correct to the number of digits shown, especially where other values might indicate that.

1. The number of atoms in a pure substance can be found by dividing the mass of the substance by the mass of a single atom. Take the atomic masses of gold and silver from the periodic table.

$$\frac{\frac{N_{\text{Au}}}{N_{\text{Ag}}}}{\frac{196.96655 \text{ u/atom}(1.66 \times 10^{-27} \text{ kg/u})}{\frac{2.75 \times 10^{-2} \text{ kg}}{(107.8682 \text{ u/atom})(1.66 \times 10^{-27} \text{ kg/u})}} = \frac{107.8682}{196.96655} = 0.548 \rightarrow N_{\text{Au}} = 0.548 N_{\text{Ag}}$$

Because a gold atom is heavier than a silver atom, there are fewer gold atoms in the given mass.

2. The number of atoms is found by dividing the mass of the substance by the mass of a single atom. Take the atomic mass of copper from the periodic table.

$$N_{\rm Cu} = \frac{3.4 \times 10^{-3} \text{ kg}}{(63.546 \text{ u/atom})(1.66 \times 10^{-27} \text{ kg/u})} = 3.2 \times 10^{22} \text{ atoms of Cu}$$

3. (a)
$$T(^{\circ}C) = \frac{5}{9}[T(^{\circ}F) - 32] = \frac{5}{9}[68 - 32] = 20^{\circ}C$$

(b) $T(^{\circ}F) = \frac{9}{5}T(^{\circ}C) + 32 = \frac{9}{5}(1900) + 32 = 3452^{\circ}F \approx 3500^{\circ}F$

- 4. High: $T(^{\circ}C) = \frac{5}{9}[T(^{\circ}F) 32] = \frac{5}{9}[136 32] = \boxed{57.8^{\circ}C}$ Low: $T(^{\circ}C) = \frac{5}{9}[T(^{\circ}F) - 32] = \frac{5}{9}[-129 - 32] = \boxed{-89.4^{\circ}C}$
- 5. $T(^{\circ}F) = \frac{9}{5}T(^{\circ}C) + 32 = \frac{9}{5}(38.9^{\circ}C) + 32 = 102.0^{\circ}F$
- 6. (a) $T(^{\circ}F) = \frac{9}{5}T(^{\circ}C) + 32 = \frac{9}{5}(-18) + 32 = -0.4^{\circ}F \approx \boxed{0^{\circ}F}$
 - (b) $T(^{\circ}C) = \frac{5}{9}[T(^{\circ}F) 32] = \frac{5}{9}[-18 32] = -27.78^{\circ}C \approx \boxed{-28^{\circ}C}$
- 7. The two temperatures, as given by the conversions, are to be the same.

$$T(^{\circ}F) = \frac{9}{5}T(^{\circ}C) + 32 \rightarrow x = \frac{9}{5}x + 32 \rightarrow -32 = \frac{4}{5}x \rightarrow x = (-32)\frac{5}{4} = -40$$

Thus, $-40^{\circ}\text{C} = -40^{\circ}\text{F}$.

8. Assume that the temperature and the length are linearly related. The change in temperature per unit length change is as follows:

$$\frac{\Delta T}{\Delta \ell} = \frac{100.0^{\circ}\text{C} - 0.0^{\circ}\text{C}}{22.79 \text{ cm} - 12.61 \text{ cm}} = 9.823 \text{C}^{\circ}/\text{cm}$$

Then the temperature corresponding to length ℓ is $T(\ell) = 0.0^{\circ}\text{C} + (\ell - 12.61 \text{ cm})(9.823 \text{C}^{\circ}/\text{cm})$.

- (a) $T(18.70 \text{ cm}) = 0.0^{\circ}\text{C} + (18.70 \text{ cm} 12.61 \text{ cm})(9.823 \text{C}^{\circ}/\text{cm}) = 59.8^{\circ}\text{C}$
- (b) $T(14.60 \text{ cm}) = 0.0^{\circ}\text{C} + (14.60 \text{ cm} 12.61 \text{ cm})(9.823 \text{C}^{\circ}/\text{cm}) = 19.5^{\circ}\text{C}$
- 9. Take the 300-m height to be the height in January. Then the increase in the height of the tower as the temperature rises is given by Eq. 13–1a.

$$\Delta \ell = \alpha \ell_0 \Delta T = (12 \times 10^{-6} / \text{C}^\circ)(300 \text{ m})(25^\circ \text{C} - 2^\circ \text{C}) = 0.08 \text{ m}$$

10. When the concrete cools in the winter, it will contract, and there will be no danger of buckling. Thus, the low temperature in the winter is not a factor in the design of the highway. But when the concrete warms in the summer, it will expand. A crack must be left between the slabs equal to the increase in length of the concrete as it heats from 15°C to 50°C.

$$\Delta \ell = \alpha \ell_0 \Delta T = (12 \times 10^{-6} / \text{C}^\circ)(12 \text{ m})(50^\circ \text{C} - 15^\circ \text{C}) = 5.0 \times 10^{-3} \text{ m}$$

11. The increase in length of the table is given by Eq. 13–1a.

$$\Delta \ell = \alpha \ell_0 \Delta T = (0.20 \times 10^{-6} / \text{C}^\circ)(1.8 \text{ m})(6.0 \text{ C}^\circ) = 2.2 \times 10^{-6} \text{ m}$$

For steel, $\Delta \ell = \alpha \ell_0 \Delta T = (12 \times 10^{-6} / \text{C}^\circ)(1.8 \text{ m})(8.0 \text{ C}^\circ) = 1.7 \times 10^{-4} \text{ m}$

The change for Super Invar is only $\frac{1}{60}$ of the change for steel.

12. The increase in length of the rod is given by Eq. 13–1a.

$$\Delta \ell = \alpha \ell_0 \Delta T \quad \rightarrow \quad \Delta T = \frac{\Delta \ell}{\alpha \ell_0} \quad \rightarrow \quad T = T_0 + \frac{\Delta \ell}{\alpha \ell_0} = 25^\circ \text{C} + \frac{0.015}{19 \times 10^{-6}/\text{C}^\circ} = 814.5^\circ \text{C} \approx \boxed{810^\circ \text{C}}$$

13. The rivet must be cooled so that its diameter becomes the same as the diameter of the hole.

$$\Delta \ell = \alpha \ell_0 \Delta T \quad \to \quad \ell - \ell_0 = \alpha \ell_0 (T - T_0)$$

$$T = T_0 + \frac{\ell - \ell_0}{\alpha \ell_0} = 22^{\circ} \text{C} + \frac{1.870 \text{ cm} - 1.872 \text{ cm}}{(12 \times 10^{-6} / \text{C}^{\circ})(1.872 \text{ cm})} = -67^{\circ} \text{C} \approx \boxed{-70^{\circ} \text{C}}$$

The temperature of "dry ice" is about -80°C, so this process will be successful.

14. The change in volume of the aluminum is given by the volume expansion formula, Eq. 13–2. The percent change is found by taking the change, dividing by the original volume, and then multiplying by 100.

$$\frac{\Delta V}{V_0}(100) = \frac{\beta V_0 \Delta T}{V_0}(100) = \beta \Delta T(100) = (75 \times 10^{-6}/\text{C}^\circ)(160^\circ\text{C} - 30^\circ\text{C})(100) = 0.9\ 75 = \boxed{0.98\%}$$

15. (a) The amount of water lost is the final volume of the water minus the final volume of the container. Also note that the original volumes of the water and the container are the same.

$$V_{\text{lost}} = (V_0 + \Delta V)_{\text{H}_2\text{O}} - (V_0 + \Delta V)_{\text{container}} = \Delta V_{\text{H}_2\text{O}} - \Delta V_{\text{container}} = \beta_{\text{H}_2\text{O}} V_0 \Delta T - \beta_{\text{container}} V_0 \Delta T$$
$$\beta_{\text{container}} = \beta_{\text{H}_2\text{O}} - \frac{V_{\text{lost}}}{V_0 \Delta T} = 210 \times 10^{-6} / \text{C}^\circ - \frac{(0.35 \text{ g}) \left(\frac{1 \text{ mL}}{0.98324 \text{ g}}\right)}{(55.50 \text{ mL})(60^\circ \text{C} - 20^\circ \text{C})} = 5.0 \times 10^{-5} / \text{C}^\circ}$$

(b) From Table 13–1, the most likely material is copper.

16. The sum of the original diameter plus the expansion must be the same for both the plug and the ring.

$$(\ell_0 + \Delta \ell)_{\text{iron}} = (\ell_0 + \Delta \ell)_{\text{brass}} \rightarrow \ell_{\text{iron}} + \alpha_{\text{iron}} \ell_{\text{iron}} \Delta T = \ell_{\text{brass}} + \alpha_{\text{brass}} \ell_{\text{brass}} \Delta T$$

$$\Delta T = \frac{\ell_{\text{brass}} - \ell_{\text{iron}}}{\alpha_{\text{iron}} \ell_{\text{iron}} - \alpha_{\text{brass}} \ell_{\text{brass}}} = \frac{8.755 \text{ cm} - 8.741 \text{ cm}}{(12 \times 10^{-6} / \text{C}^\circ)(8.741 \text{ cm}) - (19 \times 10^{-6} / \text{C}^\circ)(8.755 \text{ cm})}$$

$$= -228 \text{C}^\circ = T_{\text{final}} - T_{\text{initial}} = T_{\text{final}} - 15^\circ \text{C} \rightarrow T_{\text{final}} = -213^\circ \text{C} \approx \boxed{-210^\circ \text{C}}$$

17. Since the coefficient of volume expansion is much larger for the coolant than for the aluminum and the steel, the coolant will expand more than the aluminum and steel, so coolant will overflow the cooling system. Use Eq. 13–2.

$$\Delta V = \Delta V_{\text{coolant}} - \Delta V_{\text{aluminum}} - \Delta V_{\text{steel}} = \beta_{\text{coolant}} V_{\text{coolant}} \Delta T - \beta_{\text{alumin um}} V_{\text{aluminum}} \Delta T - \beta_{\text{steel}} V_{\text{steel}} \Delta T$$

= $(\beta_{\text{coolant}} V_{\text{coolant}} - \beta_{\text{aluminum}} V_{\text{aluminum}} - \beta_{\text{steel}} V_{\text{steel}}) \Delta T$
= $[(410 \times 10^{-6} / \text{C}^{\circ})(14.0 \text{ L}) - (75 \times 10^{-6} / \text{C}^{\circ})(3.5 \text{ L}) - (35 \times 10^{-6} / \text{C}^{\circ})(10.5 \text{ L})](12 \text{ C}^{\circ})$
= $0.06132 \text{ L} \approx \overline{[61.3 \text{ mL}]}$

18. The thermal stress must compensate for the thermal expansion. *E* is Young's modulus for the aluminum.

Stress =
$$F/A = \alpha E \Delta T = (25 \times 10^{-6}/\text{C}^{\circ})(70 \times 10^{9} \text{N/m}^{2})(35^{\circ}\text{C} - 12^{\circ}\text{C}) = 4.0 \times 10^{7} \text{ N/m}^{2}$$

19. Notation: we will use *T* for the period, and "Temp" for the temperature. The pendulum has a period of $T_0 = 2\pi \sqrt{\ell_0/g}$ at 17°C and a period of $T = 2\pi \sqrt{\ell/g}$ at 28°C. Notice that $T > T_0$ since $\ell > \ell_0$. With every swing of the clock, the heated clock will indicate that a time T_0 has passed, but the actual amount of time that has passed is *T*. Thus, the heated clock is "losing time" by an amount of ΔT

 $\Delta T = T - T_0$ every swing. The fractional loss is given by $\frac{\Delta T}{T_0}$ and is found as follows:

$$\frac{\Delta T}{T_0} = \frac{T - T_0}{T_0} = \frac{2\pi\sqrt{\ell/g} - 2\pi\sqrt{\ell_0/g}}{2\pi\sqrt{\ell_0/g}} = \frac{\sqrt{\ell} - \sqrt{\ell_0}}{\sqrt{\ell_0}} = \frac{\sqrt{\ell_0 + \Delta\ell} - \sqrt{\ell_0}}{\sqrt{\ell_0}} = \frac{\sqrt{\ell_0 + \alpha\ell_0(\Delta \text{Temp})} - \sqrt{\ell_0}}{\sqrt{\ell_0}}$$
$$= \sqrt{1 + \alpha(\Delta \text{Temp})} - 1 = \sqrt{1 + (19 \times 10^{-6}/\text{C}^\circ)(12 \text{ C}^\circ)} - 1 = 1.1399 \times 10^{-4}$$

Thus, the amount of time lost in any time period T_0 is $\Delta T = (1.1399 \times 10^{-4})T_0$. For one year, we have the following:

$$\Delta T = (1.1399 \times 10^{-4})(3.156 \times 10^7 \text{ s}) = 3598 \text{ s} \approx 60 \text{ min}$$

20. Use the relationship that $T(K) = \frac{5}{9}[T(^{\circ}F) - 32] + 273.15$.

$$T(K) = \frac{5}{9}[T(^{\circ}F) - 32] + 273.15 \rightarrow$$

$$T(^{\circ}F) = \frac{9}{5}[T(K) - 273.15] + 32 = \frac{9}{5}[0 \ K - 273.15] + 32 = -459.67^{\circ}F$$

21. Use the relationship that $T(K) = T(^{\circ}C) + 273.15$.

(a)
$$T(K) = T(^{\circ}C) + 273.15 = 4270 \text{ K} \approx 4300 \text{ K}$$
; $T(K) = T(^{\circ}C) + 273.15 = 15 \times 10^{6} \text{ K}$

(b) % error
$$= \frac{\Delta T}{T(K)} \times 100 = \frac{273.15}{T(K)} \times 100$$

4000°C: $\frac{273.15}{4270} \times 100 \approx 6.4\%$ 15×10⁶°C: $\frac{273.15}{15 \times 10^6} \times 100 \approx 1.8 \times 10^{-3}\%$

22. Assume the gas is ideal. Since the amount of gas is constant, the value of PV/T is constant.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \rightarrow V_2 = V_1 \frac{P_1}{P_2} \frac{T_2}{T_1} = (3.50 \text{ m}^3) \left(\frac{1.00 \text{ atm}}{3.20 \text{ atm}}\right) \frac{(273 + 38.0) \text{ K}}{273 \text{ K}} = \boxed{1.25 \text{ m}^3}$$

23. Assume the air is an ideal gas. Since the amount of air is constant, the value of PV/T is constant.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \to \quad T_2 = T_1 \frac{P_2}{P_1} \frac{V_2}{V_1} = (293 \text{ K}) \left(\frac{40 \text{ atm}}{1 \text{ atm}}\right) \left(\frac{1}{9}\right) = 1302 \text{ K} = 1029^\circ \text{C} \approx \boxed{1000^\circ \text{C}}$$

24. (a) Assume that the helium is an ideal gas and then use the ideal gas law to calculate the volume. Absolute pressure must be used, even though gauge pressure is given.

$$PV = nRT \rightarrow V = \frac{nRT}{P} = \frac{(16.00 \text{ mol})(8.314 \text{J/mol} \cdot \text{K})(283.15 \text{ K})}{(1.350 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})} = \boxed{0.2754 \text{ m}^3}$$

(b) Since the amount of gas is not changed, the value of PV/T is constant.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \to \quad T_2 = T_1 \frac{P_2 V_2}{P_1 V_1} = (283.15 \text{ K}) \left(\frac{2.00 \text{ atm}}{1.350 \text{ atm}}\right) \left(\frac{1}{2}\right) = 210 \text{ K} = \boxed{-63^\circ \text{C}}$$

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25. (a) Assume the nitrogen is an ideal gas. The number of moles of nitrogen is found from the atomic weight, and then the ideal gas law is used to calculate the volume of the gas.

$$n = (28.5 \text{ kg}) \frac{1 \text{ mole N}_2}{28.01 \times 10^{-3} \text{ kg}} = 1017 \text{ mol}$$

$$PV = nRT \quad \rightarrow \quad V = \frac{nRT}{P} = \frac{(1017 \text{ mol})(8.314 \text{J/mol} \cdot \text{K})(273 \text{ K})}{1.013 \times 10^5 \text{ Pa}} = 22.79 \text{ m}^3 \approx \boxed{22.8 \text{ m}^3}$$

(b) Hold the volume and temperature constant and again use the ideal gas law.

.

$$n = (28.5 \text{ kg} + 32.2 \text{ kg}) \frac{1 \text{ mole N}_2}{28.01 \times 10^{-3} \text{ kg}} = 2167 \text{ mol}$$

$$PV = nRT \rightarrow$$

$$P = \frac{nRT}{V} = \frac{(2167 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(273 \text{ K})}{22.79 \text{ m}^3} = \boxed{2.16 \times 10^5 \text{ Pa} = 2.13 \text{ atm}}$$

26. We assume that the mass of air is unchanged and the volume of air is unchanged (since the tank is rigid). Use the ideal gas law.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \rightarrow T_2 = T_1 \frac{P_2}{P_1} = [(273 + 29)\text{K}] \left(\frac{191 \text{ atm}}{204 \text{ atm}}\right) = 283 \text{ K} = 10^{\circ}\text{C}$$

The answer has 2 significant figures.

27. Assume the argon is an ideal gas. The number of moles of argon is found from the atomic weight, and then the ideal gas law is used to find the pressure.

$$n = (105.0 \text{ kg}) \frac{1 \text{ mole Ar}}{39.95 \times 10^{-3} \text{ kg}} = 2628 \text{ mol}$$

$$PV = nRT \quad \rightarrow \quad P = \frac{nRT}{V} = \frac{(2628 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(273.15 \text{ K} + 21.6 \text{ K})}{(38.0 \text{ L})(1.00 \times 10^{-3} \text{ m}^3/\text{L})} = \boxed{1.69 \times 10^8 \text{ Pa}}$$

This is 1670 atm.

28. We assume that the gas is ideal, that the amount of gas is constant, and that the volume of the gas is constant.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \rightarrow T_2 = T_1 \frac{P_2}{P_1} = [(273.15 + 20.0)\text{K}] \left(\frac{2.00 \text{ atm}}{1.00 \text{ atm}}\right) = 586.3 \text{ K} = 313.15^{\circ}\text{C} \approx \boxed{313^{\circ}\text{C}}$$

Assume that the air is an ideal gas and that the volume of the tire remains constant. We consider two 29. states: the original state 1, with temperature T_1 , pressure P_1 , and amount of gas n_1 ; and the final state 2, with temperature T_2 , pressure P_1 , and amount of gas n_2 . The ideal gas law is used.

$$P_1V = n_1RT_1; \ P_1V = n_2RT_2 \rightarrow \frac{n_2}{n_1} = \frac{T_1}{T_2} = \frac{(273+15)K}{(273+38)K} = 0.926$$

Thus, 1 - 0.926 = 0.074 = 7.4% must be removed.

Assume the oxygen is an ideal gas. Since the amount of gas is constant, the value of PV/T is constant. 30.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \rightarrow P_2 = P_1\frac{V_1}{V_2}\frac{T_2}{T_1} = (2.45 \text{ atm})\left(\frac{61.5 \text{ L}}{38.8 \text{ L}}\right)\frac{(273.15 + 56.0)\text{K}}{(273.15 + 18.0)\text{K}} = \boxed{4.39 \text{ atm}}$$

31. Assume the helium is an ideal gas. Since the amount of gas is constant, the value of PV/T is constant. We assume that since the outside air pressure decreases by 30%, the air pressure inside the balloon will also decrease by 30%.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \rightarrow \frac{V_2}{V_1} = \frac{P_1}{P_2}\frac{T_2}{T_1} = \left(\frac{1.0}{0.70}\right)\frac{(273+5.0)\text{K}}{(273+20.0)\text{K}} = 1.3954 \approx 1.4 \text{ times the original volume}$$

32. We calculate the density of water vapor, with a molecular mass of 18.0 grams per mole, from the ideal gas law.

$$PV = nRT \quad \rightarrow \quad \frac{n}{V} = \frac{P}{RT} \quad \rightarrow$$
$$\rho = \frac{m}{V} = \frac{Mn}{V} = \frac{MP}{RT} = \frac{(0.0180 \text{ kg/mol})(1.013 \times 10^5 \text{ Pa})}{(8.314 \text{ J/mol} \cdot \text{K})(373 \text{ K})} = \boxed{0.588 \text{ kg/m}^3}$$

The density from Table 10–1 is 0.598 kg/m^3 . Because this gas is very "near" a phase change state (water can also exist as a liquid at this temperature and pressure), we would not expect it to act like an ideal gas. It is reasonable to expect that the molecules will have other interactions besides purely elastic collisions. That is evidenced by the fact that steam can form droplets, indicating an attractive force between the molecules.

33. We ignore the weight of the stopper. Initially there is a net force (due to air pressure) on the stopper of 0, because the pressure is the same both above and below the stopper. With the increase in temperature, the pressure inside the tube will increase, so there will be a net upward force given by $F_{\text{net}} = (P_{\text{in}} - P_{\text{out}})A$, where A is the cross-sectional area of the stopper. The inside pressure can be expressed in terms of the inside temperature by means of the ideal gas law for a constant volume and constant mass of gas.

$$\frac{P_{\rm in}V_{\rm tube}}{T_{\rm in}} = \frac{P_0V_{\rm tube}}{T_0} \rightarrow P_{\rm in} = P_0 \frac{T_{\rm in}}{T_0}$$

$$F_{\rm net} = (P_{\rm in} - P_{\rm out})A = \left(P_0 \frac{T_{\rm in}}{T_0} - P_0\right)A = P_0 \left(\frac{T_{\rm in}}{T_0} - 1\right)A \rightarrow$$

$$T_{\rm in} = \left(\frac{F_{\rm net}}{P_0A} + 1\right)T_0 = \left[\frac{(10.0 \text{ N})}{(1.013 \times 10^5 \text{ Pa})\pi (0.0075 \text{ m})^2} + 1\right](273 \text{ K} + 18 \text{ K}) = 454 \text{ K} = 181^{\circ}\text{C}$$

34. The ideal gas law can be used to relate the volume at the surface to the submerged volume of the bubble. We assume the amount of gas in the bubble doesn't change as it rises. The pressure at the submerged location is found from Eq. 10–3c.

$$PV = nRT \rightarrow \frac{PV}{T} = nR = \text{constant} \rightarrow \frac{P_{\text{surface}}V_{\text{surface}}}{T_{\text{surface}}} = \frac{P_{\text{submerged}}V_{\text{submerged}}}{T_{\text{submerged}}} \rightarrow \frac{PV}{T}$$

$$V_{\text{surface}} = V_{\text{submerged}} \frac{P_{\text{submerged}}}{P_{\text{surface}}} \frac{T_{\text{surface}}}{T_{\text{submerged}}} = V_{\text{submerged}} \frac{P_{\text{atm}} + \rho gh}{P_{\text{atm}}} \frac{T_{\text{surface}}}{T_{\text{submerged}}}$$
$$= (1.00 \text{ cm}^3) \frac{[1.013 \times 10^5 + (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(41.0 \text{ m})]}{(1.013 \times 10^5) \text{Pa}} \frac{(273.15 + 18.5)\text{K}}{(273.15 + 5.5)\text{K}}$$
$$= 5.198 \text{ cm}^3 = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad r = \left[\frac{3(5.198 \text{ cm}^3)}{4\pi}\right]^{1/3} = \boxed{1.07 \text{ cm}}$$

35. At STP, 1 mole of ideal gas occupies 22.4 L.

(b)

$$\frac{1 \text{ mole}}{22.4 \text{ L}} \left(\frac{6.02 \times 10^{23} \text{ molecules}}{\text{ mole}} \right) \left(\frac{1 \text{ L}}{10^{-3} \text{ m}^3} \right) = \boxed{2.69 \times 10^{25} \text{ molecules/m}^3}$$

36. We assume that the water is at 4° C, so its density is 1000 kg/m^3 .

$$1.000 L\left(\frac{10^{-3} m^{3}}{1 L}\right)\left(\frac{1000 kg}{1 m^{3}}\right)\left(\frac{1 mol}{(15.9994 + 2 \times 1.00794) \times 10^{-3} kg}\right) = 55.51 mol$$

$$55.51 mol\left(\frac{6.022 \times 10^{23} molecules}{1 mol}\right) = 3.343 \times 10^{25} molecules$$

37. (a) Since the average depth of the oceans is very small compared to the radius of the Earth, the ocean's volume can be calculated as that of a spherical shell with surface area $4\pi R_{\text{Earth}}^2$ and a thickness Δy . Then use the density of sea water to find the mass and the molecular weight of water to find the number of moles.

$$Volume = 0.75(4\pi R_{Earth}^2)\Delta y = 0.75(4\pi)(6.38 \times 10^6 \text{ m})^2(3 \times 10^3 \text{ m}) = 1.15 \times 10^8 \text{ m}^3$$
$$1.15 \times 10^{18} \text{ m}^3 \left(\frac{1025 \text{ kg}}{\text{m}^3}\right) \left(\frac{1 \text{ mol}}{18 \times 10^{-3} \text{ kg}}\right) = 6.55 \times 10^{22} \text{ moles} \approx 7 \times 10^{22} \text{ moles}$$
$$6.55 \times 10^{22} \text{ moles}(6.02 \times 10^{23} \text{ molecules/1 mol}) \approx 4 \times 10^{46} \text{ mol ecules}$$

38. Assume the gas is ideal at those low pressures and use the ideal gas law.

$$PV = NkT \rightarrow \frac{N}{V} = \frac{P}{kT} = \frac{1 \times 10^{-12} \text{ N/m}^2}{(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K})} = \left(3 \times 10^8 \frac{\text{molecules}}{\text{m}^3}\right) \left(\frac{10^{-6} \text{ m}^3}{1 \text{ cm}^3}\right)$$
$$= \boxed{300 \text{ molecules/cm}^3}$$

39. We assume an ideal gas at STP. Example 13–9 shows that the molar volume of this gas is 22.4 L. We calculate the actual volume of one mole of gas particles, assuming a volume of ℓ_0^3 , and then find the ratio of the actual volume of the particles to the volume of the gas.

$$\frac{V_{\text{molecules}}}{V_{\text{gas}}} = \frac{(6.02 \times 10^{23} \text{ molecules})((3.0 \times 10^{-10} \text{ m})^3/\text{molecule})}{(22.4 \text{ L})(1 \times 10^{-3} \text{ m}^3/1 \text{ L})} = \boxed{7.3 \times 10^{-4}}$$

The molecules take up less than 0.1% of the volume of the gas.

40. (a) The average translational kinetic energy of a gas molecule is $\frac{3}{2}kT$.

$$KE_{avg} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K}) = 5.65 \times 10^{-21} \text{ J}$$

(b) The total translational kinetic energy is the average kinetic energy per molecule times the number of molecules.

$$KE_{total} = N(KE_{avg}) = (1.0 \text{ mol}) \left(\frac{6.02 \times 10^{23} \text{ molecules}}{1 \text{ mol}} \right) \frac{3}{2} (1.38 \times 10^{-23} \text{ J/K})(298 \text{ K})$$
$$= \boxed{3700 \text{ J}}$$

41. The rms speed is given by Eq. 13–9, $v_{\rm rms} = \sqrt{3kT/m}$. Helium has an atomic mass of 4.0.

$$\nu_{\rm rms} = \sqrt{3kT/m} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(6000 \text{ K})}{4.0(1.66 \times 10^{-27} \text{ kg})}} = 6116 \text{ m/s} \approx 6 \times 10^3 \text{ m/s}}$$

42. The rms speed is given by Eq. 13–9, $v_{\rm rms} = \sqrt{3kT/m}$. Since the rms speed is proportional to the square root of the absolute temperature, to triple the rms speed without changing the mass, the absolute temperature must be multiplied by a factor of 9.

$$T_{\text{fast}} = 4T_{\text{slow}} = 9(273 + 20)\text{K} = 2637 \text{ K} = 2364^{\circ}\text{C} \approx 2360^{\circ}\text{C}$$

43. The rms speed is given by Eq. 13–9, $v_{\rm rms} = \sqrt{3kT/m}$.

$$\frac{(\upsilon_{\rm rms})_2}{(\upsilon_{\rm rms})_1} = 1.040 = \frac{\sqrt{3kT_2/m}}{\sqrt{3kT_1/m}} = \sqrt{\frac{T_2}{T_1}} \rightarrow T_2 = T_1(1.040)^2 = (293.15 \text{ K})(1.040)^2 = 317.07 \text{ K} = \boxed{43.9^{\circ}\text{C}}$$

- 44. From the ideal gas law, PV = nRT, if the volume and amount of gas are held constant, the temperature is proportional to the pressure, $PV = nRT \rightarrow P = \frac{nR}{V}T = (\text{constant})T$. Thus, the temperature will be tripled. Since the rms speed is proportional to the square root of the temperature, $v_{\text{rms}} = \sqrt{3kT/m} = (\text{constant})\sqrt{T}$, v_{rms} will be multiplied by a factor of $\sqrt{3} \approx 1.73$.
- 45. The rms speed is given by Eq. 13–9, $v_{\rm rms} = \sqrt{3kT/m}$. The temperature can be found from the ideal gas law, $PV = NkT \rightarrow kT = PV/N$. The mass of the gas is the mass of a molecule times the number of molecules: M = Nm, and the density of the gas is the mass per unit volume, $\rho = \frac{M}{V}$. Combining these relationships gives the following:

$$\upsilon_{\rm rms} = \sqrt{3kT/m} = \sqrt{\frac{3PV}{Nm}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3P}{\rho}}$$

46. The rms speed is given by Eq. 13–9, $v_{\rm rms} = \sqrt{3kT/m}$, where *m* is the mass of one particle.

$$\frac{(\upsilon_{\rm rms})_2}{(\upsilon_{\rm rms})_1} = \frac{\sqrt{3kT/m_2}}{\sqrt{3kT/m_1}} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{m_1N_{\rm A}}{m_2N_{\rm A}}} = \sqrt{\frac{M_1}{M_2}} \rightarrow \frac{(\upsilon_{\rm rms})_2}{(\upsilon_{\rm rms})_1} = \sqrt{\frac{M_1}{M_2}}$$

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47. The rms speed is given by Eq. 13–9, $v_{\rm rms} = \sqrt{3kT/m}$.

$$\frac{(\upsilon_{\rm rms})_{(^{235}{\rm UF}_6)}}{(\upsilon_{\rm rms})_{(^{238}{\rm UF}_6)}} = \frac{\sqrt{3kT/m_{(^{235}{\rm UF}_6)}}}{\sqrt{3kT/m_{(^{238}{\rm UF}_6)}}} = \sqrt{\frac{m_{(^{238}{\rm UF}_6)}}{m_{(^{235}{\rm UF}_6)}}} = \sqrt{\frac{238.050788 + 6(18.998403)}{235.043930 + 6(18.998403)}} = 1.004298 \approx \boxed{1.004}$$

48. (a) The rms speed is given by Eq. 13–9, $v_{\rm rms} = \sqrt{3kT/m}$.

$$v_{\rm rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K})}{2(15.9994)32(1.66 \times 10^{-27} \text{ kg})}} = 461 \text{ m/s}$$

(b) Assuming that the particle has no preferred direction, we have the following:

$$v_{\text{rms}}^2 = v_x^2 + v_y^2 + v_z^2 = 3v_x^2 \rightarrow v_x = v_{\text{rms}}/\sqrt{3}$$

The time for one crossing of the room is then given by $t = d/v_x = \sqrt{3}d/v_{\rm rms}$, so the time for a round trip is $2\sqrt{3}d/v_{\rm rms}$. Thus, the number of back and forth round trips per second is the reciprocal of this time, $\frac{v_{\rm rms}}{2\sqrt{3}d}$.

round trips per second =
$$\frac{v_{\text{rms}}}{2\sqrt{3}d} = \frac{461 \text{ m/s}}{2\sqrt{3}(5.0 \text{ m})} = 26.62 \approx 27 \text{ round trip s per second}$$

- 49. From Fig. 13–23, we see that CO_2 is a vapor at 35 atm and 35°C.
- 50. (a) From Fig. 13–23, at atmospheric pressure, CO₂ can exist as solid or vapor
 - (b) From Fig. 13–23, for CO₂ to exist as a liquid, $5.11 \text{ atm} \le P \le 73 \text{ atm}$ and $-56.6^{\circ}\text{C} \le T \le 31^{\circ}\text{C}$.
- 51. (a) From Fig. 13–22, water is vapor when the pressure is 0.01 atm and the temperature is 90° C.
 - (b) From Fig. 13–22, water is solid when the pressure is 0.01 atm and the temperature is -20° C.
- 52. (a) At the initial conditions, the water is a liquid. As the pressure is lowered, it becomes a vapor at some pressure between 1.0 atm and 0.006 atm. It would still be a vapor at 0.004 atm.
 - (b) At the initial conditions, the water is a liquid. As the pressure is lowered, it becomes a solid at a pressure of 1.0 atm, and then becomes a vapor at some pressure lower than 0.006 atm. It would be a vapor at 0.004 atm.
- 53. From Table 13–3, the saturated vapor pressure at 30°C is 4240 Pa. Since the relative humidity is 75%, the partial pressure of water is as follows:

 $P_{\text{water}} = 0.75 P_{\text{saturated}} = 0.75(4240 \text{ Pa}) = 3180 \text{ Pa} \approx 3200 \text{ Pa}$

- 54. At the boiling temperature, the external air pressure equals the saturated vapor pressure. Thus, from Table 13–3, for 80°C the saturated air pressure is 355 torr or $4.73 \times 10^4 \text{ Pa}$ or 0.467 atm.
- 55. From Table 13–3, if the temperature is 25°C, the saturated vapor pressure is 23.8 torr. If the relative humidity is 65%, then the partial pressure of water is 65% of the saturated vapor pressure, or 15.47 torr. The dew point is the temperature at which the saturated vapor pressure is 15.47 torr, and

from Table 13–3 that is between 15°C and 20°C. Since there is no entry for 15.47 torr, the temperature can be estimated by a linear interpolation. Between 15°C and 20°C, the temperature change per torr is as follows:

$$\frac{(20-15) \text{ C}^{\circ}}{(17.5-12.8) \text{ torr}} = 1.064 \text{ C}^{\circ}/\text{torr}$$

Thus, the temperature corresponding to 15.47 torr is as follows:

$$15^{\circ}C + [(15.47 - 12.8) \text{ torr}](1.064 \text{ C}^{\circ}/\text{torr}) = 17.8^{\circ}C \approx |18^{\circ}C|$$

56. At the boiling temperature, the air pressure equals the saturated vapor pressure. The pressure of 0.80 atm is equal to 8.10×10^4 Pa. From Table 13–3, the temperature is between 90°C and 100°C. Since there is no entry for 8.10×10^4 Pa, the temperature can be estimated by a linear interpolation. Between 90°C and 100°C, the temperature change per Pa is as follows:

$$\frac{(100-90) \text{ C}^{\circ}}{(10.1-7.01)\times 10^4 \text{ Pa}} = 3.236 \times 10^{-4} \text{ C}^{\circ}/\text{Pa}$$

Thus, the temperature corresponding to 7.27×10^4 Pa is

$$90^{\circ}\text{C} + [(8.10 - 7.01) \times 10^{4} \text{ Pa}](3.236 \times 10^{-4} \text{ C}^{\circ}/\text{Pa}) = 93.53^{\circ}\text{C} \approx 94^{\circ}\text{C}.$$

57. The volume, temperature, and pressure of the water vapor are known. We use the ideal gas law to calculate the mass. The pressure is found from Table 13–3 to be 3170 Pa.

$$PV = nRT \rightarrow n = \frac{PV}{RT} = \frac{(0.55)(3170 \text{ Pa})(5.0 \text{ m})(6.0 \text{ m})(2.4 \text{ m})}{(8.314 \text{ J/mol} \cdot \text{K})(273.15 \text{ K} + 25 \text{ K})} = 50.48 \text{ mol}$$
$$m_{\text{H}_2\text{O}} = (50.48 \text{ mol})(0.018 \text{ kg/mol}) = \boxed{0.91 \text{ kg}}$$

58. For boiling to occur at 120°C, the pressure inside the cooker must be the saturated vapor pressure of water at that temperature. That value can be found in Table 13–3. For the mass to stay in place and contain the steam inside the cooker, the weight of the mass must be greater than the force exerted by the gauge pressure from the gas inside the cooker. The limiting case, to hold the temperature right at 120°C, would be with the mass equal to that force.

$$mg = F_{\text{gauge}} = (P_{\text{inside}} - P_{\text{atm}})A = (P_{\text{inside}} - P_{\text{atm}})\pi r^2 \rightarrow$$
$$m = \frac{(P_{\text{inside}} - P_{\text{atm}})\pi r^2}{g} = \frac{(1.99 \times 10^5 \text{ Pa} - 1.01 \times 10^5 \text{ Pa})\pi (1.5 \times 10^{-3} \text{ m})^2}{9.80 \text{ m/s}^2} = 0.07068 \text{ kg} \approx \boxed{71 \text{ g}}$$

59. The outside air is saturated at 5°C, so the vapor pressure of water is 872 Pa, as read from Table 13–3. The ideal gas law gives the following result for the change in volume of the given mass of air:

$$PV = nRT \rightarrow \frac{P}{nR} = \frac{T}{V} = \text{constant} \rightarrow \frac{T_1}{V_1} = \frac{T_2}{V_2}$$

Thus the vapor pressure of a given mass of air that moves from outside to inside is as follows:

$$P_{\rm in} = \frac{nRT_{\rm in}}{V_{\rm in}} = \frac{nRT_{\rm out}}{V_{\rm out}} = P_{\rm out} = 872 \text{ Pa}$$

The saturated vapor pressure at 22°C is estimated to be 2666 Pa by using linear interpolation.

$$\frac{P_{25^{\circ}C} - P_{20^{\circ}C}}{25^{\circ}C - 20^{\circ}C} = \frac{P_{22^{\circ}C} - P_{20^{\circ}C}}{22^{\circ}C - 20^{\circ}C} \rightarrow \frac{3170 \text{ Pa} - 2330 \text{ Pa}}{5 \text{ C}^{\circ}} = \frac{P_{22^{\circ}C} - 2330 \text{ Pa}}{2\text{ C}^{\circ}} \rightarrow P_{22^{\circ}C} = 2666 \text{ Pa}$$

Thus the relative humidity is as follows:

relative humidity =
$$\frac{872 \text{ Pa}}{2666 \text{ Pa}} = 0.327 \approx 33\%$$

- 60. (a) The true atmospheric pressure will be greater than the reading from the barometer. In Fig. 10–8, if there is a vapor pressure at the top of the tube, then $P_{\text{atm}} \rho gh = P_{\text{vapor}}$. The reading from the barometer will be $\rho gh = P_{\text{atm}} P_{\text{vapor}} < P_{\text{atm}}$.
 - (b) The percent error is found from the atmospheric pressure and the vapor pressure.

% error =
$$\left(\frac{\rho g h - P_{\text{atm}}}{P_{\text{atm}}}\right) \times 100 = \left(-\frac{P_{\text{vapor}}}{P_{\text{atm}}}\right) \times 100 = \left(-\frac{0.0015 \text{ mm} - \text{Hg}}{760 \text{ mm} - \text{Hg}}\right) \times 100$$

= $\left[\left(-2.0 \times 10^{-4}\right)\%\right]$

(c) From Table 13–3, the saturated water vapor pressure at STP is 611 Pa.

% error =
$$\left(\frac{P_{\text{vapor}}}{P_{\text{atm}}}\right) \times 100 = \left(\frac{611 \text{ Pa}}{1.013 \times 10^5 \text{ Pa}}\right) \times 100 = \boxed{0.603 \%}$$

61. From Example 13–19, we have an expression for the time to diffuse a given distance. Divide the distance by the time to get the average speed.

$$t = \frac{\overline{C}}{\Delta C} \frac{(\Delta x)^2}{D} = \frac{\frac{1}{2}(1.00 + 0.50) \text{ mol/m}^3}{(1.00 - 0.50) \text{ mol/m}^3} \frac{(25 \times 10^{-6} \text{ m})^2}{(95 \times 10^{-11} \text{ m}^2/\text{s})} = 0.9868 \text{ s} \approx \boxed{0.99 \text{ s}}$$
$$\upsilon_{\text{diffuse}} = \frac{\Delta x}{t} = \frac{25 \times 10^{-6} \text{ m}}{0.9868 \text{ s}} = \boxed{2.5 \times 10^{-5} \text{ m/s}}$$

The rms thermal speed is given by Eq. 13–9, $v_{\rm rms} = \sqrt{3kT/m}$.

$$\upsilon_{\rm rms} = \sqrt{3kT/m} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})}{75(1.66 \times 10^{2} \, {}^{27} \text{ kg})}} = 312.1 \text{ m/s} \approx \overline{310 \text{ m/s}}$$
$$\frac{\upsilon_{\rm diffuse}}{\upsilon_{\rm rms}} = \frac{2.5 \times 10^{-5} \text{ m/s}}{312.1 \text{ m/s}} = 8.0 \times 10^{-8} \text{ (about 7 orders of magnitude difference)}$$

62. (a) Use the ideal gas law to find the concentration of the oxygen. We assume that the air pressure is 1.00 atm, so the pressure caused by the oxygen is 0.21 atm.

$$PV = nRT$$
 →
 $\frac{n}{V} = \frac{P}{RT} = \frac{(0.21 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})}{(8.315 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 8.732 \text{ mol/m}^3 \approx \boxed{8.7 \text{ mol/m}^3}$

Half of this is 4.366 mol/m^3 .

(b) Use Eq. 13–10 to calculate the diffusion rate.

$$J = DA \frac{C_1 - C_2}{\Delta x} = (1 \times 10^{-5} \text{ m}^2/\text{s})(2 \times 10^{-9} \text{ m}^2) \left(\frac{8.732 \text{ mol/m}^3 - 4.366 \text{ mol/m}^3}{2 \times 10^{-3} \text{ m}}\right)$$
$$= 4.366 \times 10^{-11} \text{ mol/s} \approx \boxed{4 \times 10^{-11} \text{ mol/s}}$$

From Example 13–19, we have an expression for the time to diffuse a given distance. (*c*)

$$t = \frac{\overline{C}}{\Delta C} \frac{(\Delta x)^2}{D} = \frac{\frac{1}{2} (8.732 \text{ mol/m}^3 + 4.366 \text{ mol/m}^3)}{(8.732 \text{ mol/m}^3 - 4.366 \text{ mol/m}^3)} \frac{(2 \times 10^{-3} \text{ m})^2}{1 \times 10^{-5} \text{ m}^2/\text{s}} = \boxed{0.6 \text{ s}}$$

63. (a) At 37°C, the tape will expand from its calibration, so it will read low.

(b)
$$\frac{\Delta \ell}{\ell_0} = \alpha \Delta T = (12 \times 10^{-6} / ^{\circ}\text{C})(37^{\circ}\text{C} - 14^{\circ}\text{C}) = 2.76 \times 10^{-4} \approx (2.8 \times 10^{-2})\%$$

64. The net force on each side of the box will be the pressure difference between the inside and outside of the box times the area of a side of the box. The outside pressure is 1 atmosphere. The ideal gas law is used to find the pressure inside the box, assuming that the mass of gas and the volume are constant.

$$\frac{P}{T} = \frac{nR}{V} = \text{constant} \quad \rightarrow \quad \frac{P_2}{T_2} = \frac{P_1}{T_1} \quad \rightarrow \quad P_2 = P_1 \frac{T_2}{T_1} = (1.00 \text{ atm}) \frac{(273 + 165) \text{ K}}{(273 + 15) \text{ K}} = 1.521 \text{ atm}$$

The area of a side of the box is given by the following.

Area =
$$\ell^2 = [(\text{Volume of box})^{1/3}]^2 = (6.15 \times 10^{-2} \text{ m}^2)^{2/3} = 1.5581 \times 10^{-1} \text{ m}^2$$

The net force on a side of the box is the pressure difference times the area.

$$F = (\Delta \text{ Pressure})(\text{Area}) = (0.521 \text{ atm})(1.013 \times 10^5 \text{ Pa})(1.5581 \times 10^{-1} \text{ m}^2) = 8221 \text{ N} \approx 8200 \text{ N}$$

65. Assume the helium is an ideal gas. The volume of the cylinder is constant, and we assume that the temperature of the gas is also constant in the cylinder. From the ideal gas law, PV = nRT, under these conditions the amount of gas is proportional to the absolute pressure.

$$PV = nRT \rightarrow \frac{P}{n} = \frac{RT}{V} = \text{constant} \rightarrow \frac{P_1}{n_1} = \frac{P_2}{n_2} \rightarrow \frac{n_2}{n_1} = \frac{P_2}{P_1} = \frac{5 \text{ atm} + 1 \text{ atm}}{32 \text{ atm} + 1 \text{ atm}} = \frac{6}{33}$$

Thus, $6/33 = 0.182 \approx 18\%$ of the original gas remains in the cylinder.

66.

Assume that the air in the lungs is an ideal gas, that the amount of gas is constant, and that the temperature is constant. The ideal gas law then says that the value of PV is constant. The pressure a distance h below the surface of the water is discussed in Chapter 10 and is given by $P = P_0 + \rho gh$, where P_0 is atmospheric pressure and ρ is the density of the water.

$$(PV)_{\text{surface}} = (PV)_{\text{submerged}} \rightarrow V_{\text{surface}} = V_{\text{submerged}} \frac{P_{\text{submerged}}}{P_{\text{surface}}} = V_{\text{submerged}} \frac{P_{\text{atm}} + \rho g h}{P_{\text{atm}}}$$
$$= (5.5 \text{ L}) \frac{1.013 \times 10^5 \text{ Pa} + (1.0 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(9 \text{ m})}{1.013 \times 10^5 \text{ Pa}} = 10.29 \text{ L} \approx 10 \text{ L}$$

It is obviously very dangerous to have the lungs attempt to inflate to almost twice their volume. Thus, it is not advisable to quickly rise to the surface.

- 67. To do this problem, the "molecular weight" of air is needed. If we approximate air as 70% N₂ (molecular weight 28) and 30% O₂ (molecular weight 32), then the average molecular weight is 0.70(28) + 0.30(32) = 29.
 - (a) Treat the air as an ideal gas. Assume that the pressure is 1.00 atm.

$$PV = nRT \rightarrow n = \frac{PV}{RT} = \frac{(1.013 \times 10^{5} \text{ Pa})(1200 \text{ m}^{3})}{(8.315 \text{ J/mol} \cdot \text{K})(288 \text{ K})} = 5.076 \times 10^{4} \text{ moles}$$
$$m = (5.076 \times 10^{4} \text{ moles})(29 \times 10^{-3} \text{ kg/mol}) = 1472 \text{ kg} \approx 1500 \text{ kg}$$

(b) Find the mass of air at the lower temperature, and then subtract the mass at the higher temperature.

$$n = \frac{PV}{RT} = \frac{(1.013 \times 10^5 \text{ Pa})(1200 \text{ m}^3)}{(8.315 \text{ J/mol} \cdot \text{K})(258 \text{ K})} = 5.666 \times 10^4 \text{ moles}$$
$$m = (5.666 \times 10^4 \text{ moles})(29 \times 10^{-3} \text{ kg/mol}) = 1643 \text{ kg}$$

The mass entering the house is $1643 \text{ kg} - 1472 \text{ kg} = 171 \text{ kg} \approx 200 \text{ kg}$.

68. Assume the air is an ideal gas, and that the pressure is 1.0 atm.

$$PV = NkT \rightarrow N = \frac{PV}{kT} = \frac{(1.013 \times 10^5 \text{ Pa})(6.0 \times 3.0 \times 2.5) \text{ m}^3}{(1.38 \times 10^{-23} \text{ J/K})(273 + 22) \text{ K}} = 1.1197 \times 10^{27} \text{ molecules} \approx 1.1197 \times 10^{27} \text{ molecules} \left(\frac{1 \text{ mole}}{6.02 \times 10^{23} \text{ molecules}}\right) = 1860 \text{ moles} \approx 1900 \text{ moles}$$

- 69. (a) The iron floats in the mercury because ρ_{Hg} > ρ_{Fe}, as seen in Table 10–1. As the substances are heated, the density of both substances will decrease due to volume expansion. The density of the mercury decreases more upon heating than the density of the iron, because β_{Hg} > β_{Fe}, as seen in Table 13–1. The net effect is that the densities get closer together, so relatively more mercury will have to be displaced to hold up the iron, and the iron will float lower in the mercury.
 - (b) The fraction of the volume submerged is V_{Hg} / V_{Fe} . Both volumes expand as heated. The displaced "is dropped for convenience."

fractional change =
$$\frac{V_{\text{Hg}}/V_{\text{Fe}} - V_{0 \text{Hg}}/V_{0 \text{Fe}}}{V_{0 \text{Hg}}/V_{0 \text{Fe}}} = \frac{\frac{V_{0 \text{Hg}}(1 + \beta_{\text{Hg}}\Delta T)}{V_{0 \text{Fe}}(1 + \beta_{\text{Fe}}\Delta T)} - V_{0 \text{Hg}}/V_{0 \text{Fe}}}{V_{0 \text{Hg}}/V_{0 \text{Fe}}} = \frac{(1 + \beta_{\text{Hg}}\Delta T)}{(1 + \beta_{\text{Fe}}\Delta T)} - 1$$
$$= \frac{1 + (180 \times 10^{-6}/\text{C}^{\circ})(25\text{C}^{\circ})}{1 + (35 \times 10^{-6})(25\text{C}^{\circ})} - 1 = \frac{1.0045}{1.000875} - 1 = 3.6 \times 10^{-3}$$
% change = $(3.6 \times 10^{-3})(100) = \boxed{0.36\%}$

70. We find the number of moles of helium in the balloon from the ideal gas law.

$$PV = nRT \rightarrow n = \frac{PV}{RT} = \frac{(1.08)(1.013 \times 10^5 \text{ Pa})\frac{4}{3}\pi(0.240 \text{ m})^3}{(8.314 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 2.6006 \text{ mol} \approx 2.6006 \text{ mol}$$

2.6006 mol $\left(\frac{4.00 \text{ g}}{1 \text{ mol}}\right) = 10.4 \text{ g}$

71. We assume the temperature is constant. As the oxygen pressure drops to atmospheric pressure, we can find the volume that it occupies at atmospheric pressure. We assume the final pressure inside the cylinder is atmospheric pressure. The gas would quit flowing at that pressure.

$$P_1V_1 = P_2V_2 \rightarrow V_2 = V_1 \frac{P_1}{P_2} = (14 \text{ L}) \frac{(1.38 \times 10^7 \text{ Pa} + 1.013 \times 10^5 \text{ Pa})}{1.013 \times 10^5 \text{ Pa}} = 1921 \text{ L}$$

14 L of that gas is not available—it is left in the container. So there is a total of 1907 L available.

$$(1907 \text{ L}) \frac{1 \text{ min}}{2.1 \text{ L}} = 908.1 \text{ min} \approx 910 \text{ min} \approx 15 \text{ h}$$

72. The gap will be the radius of the lid minus the radius of the jar. Also note that the original radii of the lid and the jar are the same.

$$r_{\text{gap}} = (r_0 + \Delta r)_{\text{lid}} - (r_0 + \Delta r)_{\text{jar}} = \Delta r_{\text{lid}} - \Delta r_{\text{jar}} = (\alpha_{\text{brass}} - \alpha_{\text{glass}})r_0\Delta T$$
$$= (19 \times 10^{-6}/\text{C}^\circ - 9 \times 10^{-6}/\text{C}^\circ)(4.0 \text{ cm})(40 \text{ C}^\circ) = \boxed{1.6 \times 10^{-3} \text{ cm}}$$

73. (a) Assume that a mass M of gasoline with volume V_0 at 0°C is under consideration, so its density is $\rho_0 = M/V_0$. At a temperature of 33°C, the same mass has a volume $V = V_0(1 + \beta \Delta T)$.

$$\rho = \frac{M}{V} = \frac{M}{V_0 (1 + \beta \Delta T)} = \frac{\rho_0}{1 + \beta \Delta T} = \frac{0.68 \times 10^3 \text{ kg/m}^3}{1 + (950 \times 10^{-6}/\text{C}^\circ)(33 \text{ C}^\circ)} = 0.6593 \times 10^3 \text{ kg/m}^3$$
$$\approx \boxed{0.66 \times 10^3 \text{ kg/m}^3}$$

(b) Calculate the percentage change in the density.

% change =
$$\frac{(0.6593 - 0.68) \times 10^3 \text{ kg/m}^3}{0.68 \times 10^3 \text{ kg/m}^3} \times 100 = \boxed{-3.0\%}$$

74. The original length of the steel band is $\ell_0 = 2\pi R_{\text{Earth}}$. At the higher temperature, the length of the band is $\ell = \ell_0 + \Delta \ell = 2\pi R = 2\pi (R_{\text{Earth}} + \Delta R)$. The change in radius, ΔR , would be the height above the Earth.

$$\Delta \ell = \alpha \ell_0 \Delta T = 2\pi \Delta R \quad \rightarrow$$

$$\Delta R = \frac{\alpha \ell_0 \Delta T}{2\pi} = \alpha R_{\text{Earth}} \Delta T = (12 \times 10^{-6} / \text{C}^\circ)(6.38 \times 10^6 \text{ m})(55^\circ \text{C} - 25^\circ \text{C}) = 2297 \text{ m} \approx 2300 \text{ m}$$

75. The rms speed is given by Eq. 13–9, $v_{\rm rms} = \sqrt{3kT/m}$. Hydrogen atoms have a mass of 1 atomic mass unit.

$$v_{\rm rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(2.7 \text{ K})}{1(1.66 \times 10^{-27} \text{ kg})}} = 260 \text{ m/s}$$

The pressure is found from the ideal gas law, PV = NkT.

$$PV = NkT \rightarrow P = \frac{NkT}{V} = \frac{(1)(1.38 \times 10^{-23} \text{ J/K})(2.7 \text{ K})}{1 \text{ cm}^3 \left(\frac{1 \times 10^{-6} \text{ m}^3}{1 \text{ cm}^3}\right)} = 3.726 \times 10^{-17} \text{ Pa}\left(\frac{1 \text{ atm}}{1.01 \times 10^5 \text{ Pa}}\right)$$
$$= 3.689 \times 10^{-22} \text{ atm} \approx \boxed{3.7 \times 10^{-22} \text{ atm}}$$

76. It is stated in the text that the relationship $v_{\rm rms} = \sqrt{3kT/m}$ is applicable to molecules within living cells at body temperature (37°C). The rms speed is given by Eq. 13–9.

(a)
$$v_{\rm rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \,\text{J/K})(310 \,\text{K})}{(89 \,\text{u})(1.66 \times 10^{-27} \,\text{kg/u})}} = 294.7 \,\text{m/s} \approx 290 \,\text{m/s}$$

(b) $v_{\rm rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \,\text{J/K})(310 \,\text{K})}{(8.5 \times 10^4 \,\text{u})(1.66 \times 10^{-27} \,\text{kg/u})}} = 9.537 \,\text{m/s} \approx 9.537 \,\text{m/s}$

77. The rms speed is given by Eq. 13–9, $v_{\rm rms} = \sqrt{3kT/m}$. Set the escape velocity to $v_{\rm rms}$ solve for *T*.

(*a*) For oxygen molecules:

$$T = \frac{m\nu_{\rm rms}^2}{3k} = \frac{2(15.9994)(1.66 \times 10^{-27} \text{ kg})(1.12 \times 10^4 \text{ m/s})^2}{3(1.38 \times 10^{-23} \text{ J/K})} = \boxed{1.61 \times 10^5 \text{ K}}$$

(b) For helium atoms:

$$T = \frac{mv_{\rm rms}^2}{3k} = \frac{(4.002602)(1.66 \times 10^{-27} \text{ kg})(1.12 \times 10^4 \text{ m/s})^2}{3(1.38 \times 10^{-23} \text{ J/K})} = \boxed{2.01 \times 10^4 \text{ K}}$$

- (c) Because the "escape temperature" is so high for oxygen, very few oxygen molecules ever escape the atmosphere. But helium, with one-eighth the mass, can escape at a much lower temperature. While the temperature of the Earth is not close to 2.0×10^4 K today, during the Earth's formation its temperature was possibly much hotter—presumably hot enough that helium was able to escape the atmosphere.
- 78. The temperature can be found from the rms speed by Eq. 13–9, $v_{\rm rms} = \sqrt{3kT/m}$. The molecular mass of nitrogen molecules is 28.

$$\nu_{\rm rms} = \sqrt{3kT/m} \rightarrow T = \frac{m\nu_{\rm rms}^2}{3k} = \frac{(28)(1.66 \times 10^{-27} \text{ kg}) \left[(4.2 \times 10^4 \text{ km/h}) \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) \right]^2}{3(1.38 \times 10^{-23} \text{ J/K})} = 1.5 \times 10^5 \text{ K}$$

79. Following the development of the kinetic molecular theory in Section 13–10 of the textbook, the tennis balls hitting the trash can lid are similar to the particles colliding with the walls of a container causing pressure. Quoting from the text, "the average force averaged over many collisions will be equal to the force exerted during one collision divided by the time between collisions." That average force must be the weight of the trash can lid in order to suspend it. The fact that the collisions are elastic means that the change in momentum of one of the balls is twice its original momentum.

$$F_{\text{avg}} = M_{\text{lid}}g; \ F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{m_{\text{ball}}\upsilon_{\text{ball}} - (-m_{\text{ball}}\upsilon_{\text{ball}})}{\Delta t} = \frac{2m_{\text{ball}}\upsilon_{\text{ball}}}{\Delta t} \rightarrow \Delta t = \frac{2m_{\text{ball}}\upsilon_{\text{ball}}}{M_{\text{lid}}g}$$

The above expression is "seconds per ball," so its reciprocal will be "balls per second."

balls/s =
$$\frac{1}{\Delta t} = \frac{M_{\text{lid}}g}{2m_{\text{ball}}v_{\text{ball}}} = \frac{(0.50 \text{ kg})(9.80 \text{ m/s}^2)}{2(0.060 \text{ kg})(15 \text{ m/s})} = \frac{2.7 \text{ balls/s}}{2.7 \text{ balls/s}}$$

80. Assume that the water vapor behaves like an ideal gas. At 20°C, the saturated vapor pressure is 2.33×10^3 Pa. Using the ideal gas law, find the number of moles of water in the air at both 95% and 40%. Subtract those mole amounts to find the amount of water that must be removed.

$$PV = nRT \rightarrow n = \frac{PV}{RT} \rightarrow$$

$$n_1 - n_2 = \frac{V}{RT}(P_1 - P_2) = \frac{(105 \text{ m}^2)(2.4 \text{ m})}{(8.314 \text{ J/mol} \cdot \text{K})(293 \text{ K})}(2.33 \times 10^3 \text{ Pa})(0.95 - 0.40) = 134.8 \text{ mol}$$

$$134.8 \text{ mol}\left(\frac{18 \times 10^{-3} \text{ kg}}{1 \text{ mol}}\right) = 2.426 \text{ kg} \approx \boxed{2.4 \text{ kg}}$$

Solutions to Search and Learn Problems

1. The first method for calculating the thermal expansion is given in Eq. 13–2, with the coefficient of expansion given in Table 13–1. Using this method, we calculate the volume change for each temperature difference as follows:

$$\Delta V_{\rm A} = \beta V_0 \Delta T_{\rm A} = (3400 \times 10^{-6} / \text{C}^\circ)(1000 \text{ L})(0^\circ \text{C} - -100^\circ \text{C}) = \boxed{340 \text{ L}}$$
$$\Delta V_{\rm B} = \beta V_0 \Delta T_{\rm B} = (3400 \times 10^{-6} / \text{C}^\circ)(1000 \text{ L})(100^\circ \text{C} - 0^\circ \text{C}) = \boxed{340 \text{ L}}$$

Note that the textbook states that this method is accurate only if the change in volume is small compared to the actual volume. This is obviously not the case here, so the values above are not particularly accurate.

The second method is to use the ideal gas law of Eq. 13–3.

$$pV = nRT \rightarrow \frac{pV_1}{pV_2} = \frac{nRT_1}{nRT_2} \rightarrow V_2 = V_1 \frac{T_2}{T_1} \rightarrow \Delta V = V_2 - V_1 = V_1 \left(\frac{T_2}{T_1} \right)$$

The temperature must be converted to kelvins for this method.

$$\Delta V_{\rm A} = V_1 \left(\frac{T_{2\rm A}}{T_{1\rm A}} - 1 \right) = (1000 \text{ L}) \left(\frac{273 \text{ K}}{173 \text{ K}} - 1 \right) = \boxed{580 \text{ L}}$$
$$\Delta V_{\rm B} = V_1 \left(\frac{T_{2\rm B}}{T_{1\rm B}} - 1 \right) = (1000 \text{ L}) \left(\frac{373 \text{ K}}{273 \text{ K}} - 1 \right) = \boxed{370 \text{ L}}$$

The answers are different because the first method assumed small temperature changes, such that the relationship between the temperature and volume could be assumed linear. The second method accounts for a nonlinear relationship over large changes in volume and temperature.

2. (a) We treat the air as an ideal gas. Since the amount of air and temperature of the air are the same in both cases, the ideal gas law says PV = nRT is a constant.

$$P_2V_2 = P_1V_1 \rightarrow V_2 = V_1 \frac{P_1}{P_2} = (11.3 \text{ L}) \frac{180 \text{ atm}}{1.00 \text{ atm}} = 2034 \text{ L} \approx 2030 \text{ L}$$

(b) Before entering the water, the air coming out of the tank will be at 1.00 atm pressure, so the person will be able to breathe 2034 L of air.

$$t = 2034 \text{ L}\left(\frac{1 \text{ breath}}{2.0 \text{ L}}\right) \left(\frac{1 \text{ min}}{12 \text{ breaths}}\right) = 84.75 \text{ min} \approx \overline{85 \text{ min}}$$

(c) When the person is underwater, the temperature and pressure will be different. Use the ideal gas law to relate the original tank conditions to the underwater breathing conditions. The amount of gas will be constant, so PV/T = nR will be constant. The pressure a distance *h* below the surface of the water is given in Eq. 10–3c, $P = P_0 + \rho gh$, where P_0 is atmospheric pressure and ρ is the density of the sea water.

$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1} \rightarrow V_2 = V_1 \frac{P_1}{P_2} \frac{T_2}{T_1}$$

$$V_2 = (11.3 \text{ L}) \left[\frac{(180 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})}{1.013 \times 10^5 \text{ Pa} + (1.025 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(23.0 \text{ m})} \right] \left(\frac{283 \text{ K}}{291 \text{ K}} \right)$$

$$= 602.95 \text{ L} \qquad t = (602.95 \text{ L}) \left(\frac{1 \text{ breath}}{2.0 \text{ L}} \right) \left(\frac{1 \text{ min}}{12 \text{ breaths}} \right) = 25.12 \text{ min} \approx 25 \text{ min}$$

3. There are three forces to consider: the buoyant force upward (which is the weight of the cold air displaced by the volume of the balloon), the downward weight of the hot air inside the balloon, and the downward weight of the passengers and equipment. For the balloon to rise at constant speed, the buoyant force must equal the sum of the two weights.

 $F_{\text{buovant}} = m_{\text{hot}}g + 3300 \text{ N} \rightarrow V \rho_{\text{cold}}g = V \rho_{\text{hot}}g + 3300 \text{ N}$

The ideal gas law can be written in terms of the gas density ρ and the molecular mass M as follows:

$$PV = nRT = \frac{m}{M}RT \rightarrow \frac{PM}{R} = \frac{m}{V}T = \rho I$$

The gas inside and outside the balloon is air, so *M* is the same for inside and outside. Also, since the balloon is open to the atmosphere, the pressure inside the balloon is the same as the pressure outside the balloon. Thus, the ideal gas law reduces to $\rho T = \text{constant} = (\rho T)_{\text{cold}} = (\rho T)_{\text{hot}}$. This equation can be used to eliminate the density of the hot gas from the force equation, and the resulting equation can be solved for the temperature of the gas inside the balloon.

$$V \rho_{\text{cold}} g = V \rho_{\text{hot}} g + 3300 \text{ N} = V \rho_{\text{cold}} \frac{T_{\text{cold}}}{T_{\text{hot}}} g + 3300 \text{ N} \rightarrow$$

$$T_{\text{hot}} = \frac{V \rho_{\text{cold}} T_{\text{cold}} g}{(V \rho_{\text{cold}} g - 3300 \text{ N})} = \frac{(1800 \text{ m}^3)(1.29 \text{ kg/m}^3)(273 \text{ K})(9.80 \text{ m/s}^2)}{[(1800 \text{ m}^3)(1.29 \text{ kg/m}^3)(9.80 \text{ m/s}^2) - 3300 \text{ N}]}$$

$$= 319.3 \text{ K} \quad (2 \text{ significant figures}) \rightarrow 319.3 \text{ K} - 273.15 \text{ K} = 46.15^{\circ} \text{ C} \approx \boxed{50^{\circ} \text{ C}}$$

One factor limiting the maximum altitude would be that as the balloon rises, the density of the air decreases, and thus the temperature required gets higher. Eventually the air would be too hot and the balloon fabric might be damaged.

4. We assume that the last breath Galileo took has been spread uniformly throughout the atmosphere since his death and that each molecule remains in the atmosphere. Calculate the number of molecules in Galileo's last breath and divide it by the volume of the atmosphere to get "Galileo molecules/m³." Multiply that factor times the size of a breath to find the number of Galileo molecules in one of our breaths.

$$PV = NkT \rightarrow N = \frac{PV}{kT} = \frac{(1.01 \times 10^{5} \text{ Pa})(2.0 \times 10^{-3} \text{ m}^{3})}{(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})} = 4.9 \times 10^{22} \text{ molecules}$$

Atmospheric volume $= 4 \pi R_{\text{Earth}}^{2} h = 4 \pi (6.38 \times 10^{6} \text{ m})^{2} (1.0 \times 10^{4} \text{ m}) = 5.1 \times 10^{18} \text{ m}^{3}$
 $\frac{\text{Galileo molecules}}{\text{m}^{3}} = \frac{4.9 \times 10^{22} \text{ molecules}}{5.8 \times 10^{18} \text{ m}^{3}} = 9.6 \times 10^{3} \text{ molecules/m}^{3}$
 $\frac{\text{Galileo molecules}}{\text{breath}} = 9.6 \times 10^{3} \frac{\text{molecules}}{\text{m}^{3}} \left(\frac{2.0 \times 10^{-3} \text{ m}^{3}}{1 \text{ breath}}\right) = 19 \frac{\text{molecule s}}{\text{breath}} \approx 20 \frac{\text{molecules}}{\text{breath}}$

5. (a) We calculate the volume per molecule from the ideal gas law and assume the molecular volume is spherical.

$$PV = NkT \rightarrow \frac{V}{N} = \frac{kT}{P} = \frac{4}{3}\pi r^{3} \rightarrow$$

$$r_{\text{inter-molecular}} = \left(\frac{3kT}{4\pi P}\right)^{1/3} = \left(\frac{3(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K})}{4\pi (1.01 \times 10^{5} \text{ Pa})}\right)^{1/3} = \boxed{2.07 \times 10^{-9} \text{ m}}$$

The intermolecular distance would be twice this "radius," so about 4×10^{-9} m. This is about 14 times larger than the molecular diameter.

$$\frac{d_{\text{inter-}}}{d_{\text{actual}}}_{\text{molecule}} \approx \frac{4.14 \times 10^{-9} \text{ m}}{3 \times 10^{-10} \text{ m}} = 13.8 \quad \rightarrow \quad \left| \frac{d_{\text{inter-}}}{d_{\text{actual}}}_{\text{molecule}} \approx 14 \right|$$

- (b) Now we scale the molecular diameter up to 4 cm. The intermolecular distance would be 13.8 times that, which is about 55 cm.
- (c) We replace the pressure in part (a) with 3 atm.

$$r_{\text{inter-molecular}} = \left(\frac{3kT}{4\pi P}\right)^{1/3} = \left(\frac{3(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K})}{4\pi (3 \text{ atm})(1.01 \times 10^5 \text{ Pa/atm})}\right)^{1/3} = \boxed{1.44 \times 10^{-9} \text{ m}}$$
$$\frac{d_{\text{inter-molecular}}}{d_{\text{actual molecular}}} \approx \frac{2(1.44) \times 10^{-9} \text{ m}}{3 \times 10^{-10} \text{ m}} = 9.58 \rightarrow \boxed{\frac{d_{\text{inter-molecular}}}{d_{\text{actual molecular}}}} \approx 9.6$$

We see that the intermolecular distance has decreased with the increased pressure, from about 14 times larger to only about 10 times larger.

1/2

(d) We are to calculate the volume occupied by the molecules themselves, compared to the total volume of the gas. The total volume is the volume of the molecules plus the volume of the intermolecular distance. Since we don't know the actual number of gas particles, we use the volume of one gas particle and the volume associated with the intermolecular distance.

