8

ROTATIONAL MOTION

Responses to Questions

1. The reading on an odometer designed for 27-inch wheels increases by the circumference of a 27-inch wheel $(27\pi'')$ for every revolution of the wheel. If a 24-inch wheel is used, the odometer will still register $(27\pi'')$ for every revolution, but only $24\pi''$ of linear distance will have been traveled. Thus the odometer will read a distance that is farther than you actually traveled, by a factor of $27/24 = 1.125$. The odometer will read 12.5% too high.

2. (*a*) A point on the rim of a disk rotating with constant angular velocity has no tangential acceleration since the tangential speed is constant. It does have radial acceleration. Although the point's speed is not changing, its velocity is, since the velocity vector is changing direction. The point has a centripetal acceleration, which is directed radially inward.

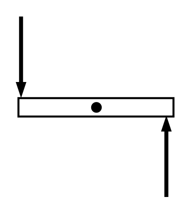
   (*b*) If the disk's angular velocity increases uniformly, the point on the rim will have both radial and tangential acceleration, since it is both moving in a circle and speeding up.

   (*c*) The magnitude of the radial component of acceleration will increase in case (*b*), but the tangential component will be constant. In case (*a*), neither component of linear acceleration will change.

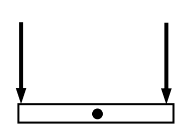
3. Since the torque involves the product of a force times its lever arm, a small force can exert a greater torque than a larger force if the small force has a large enough lever arm.

4. When you do a sit-up from a laying-down position, torque from your abdominal muscles must rotate the upper half of the body. The larger the moment of inertia of the upper half of the body, the more torque is needed, and thus the harder the sit-up is to do. With the hands behind the head, the moment of inertia of the upper half of the body is larger than with the hands out in front.

5. If the net force on a system is zero, the net torque need not be zero. Consider a uniform object with two equal forces on it, as shown in the first diagram. The net force on the object is zero (it would not start to translate under the action of these forces), but there is a net counterclockwise torque about the center of the rod (it would start to rotate under the action of these forces).

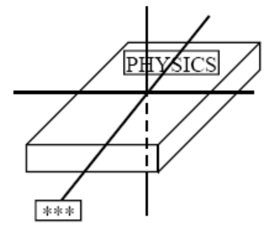
   

   If the net torque on a system is zero, the net force need not be zero. Consider an object with two equal forces on it, as shown in the second diagram. The net torque on the object is zero (it would not start to rotate under the action of these forces), but there is a net downward force on the rod (it would start to translate under the action of these forces).

6. Running involves rotating the leg about the point where it is attached to the rest of the body. Therefore, running fast requires the ability to change the leg's rotation easily. The smaller the moment of inertia of an object, the smaller the resistance to a change in its rotational motion. The closer the mass is to the axis of rotation, the smaller the moment of inertia. Concentrating flesh and muscle high and close to the body minimizes the moment of inertia and increases the angular acceleration possible for a given torque, improving the ability to run fast.

7. Refer to the diagram of the book laying on a table. The moment of inertia about the "starred" axis (the axis parallel to the longest dimension of the book) will be the smallest. Relative to this axis, more of the mass is concentrated close to the axis.



8. No, the mass cannot be considered as concentrated at the CM when considering rotational motion. If all of the mass were at the CM, then the object would have a rotational inertia of 0. That means it could not have any rotational kinetic energy or angular momentum, for example. The distribution of the mass is fundamental when describing rotational motion.

9. The moment of inertia will be larger when considering an axis through a point on the edge of the disk, because most the mass of the disk will be farther from the axis of rotation than it was with the original axis position.

10. Applying conservation of energy at the top and bottom of the incline, assuming that there is no work done by friction, gives $E_{\text{top}} = E_{\text{bottom}} \quad \rightarrow \quad Mgh = \frac{1}{2}M\upsilon^2 + \frac{1}{2}I\omega^2$. For a solid ball, $I = \frac{2}{5}MR^2$. If the ball rolls without slipping (no work done by friction) then $\omega = \upsilon/R$, so

$$Mgh = \frac{1}{2}M\upsilon^2 + \frac{1}{2}\frac{2}{5}MR^2\upsilon^2/R^2 \quad \rightarrow \quad \upsilon = \sqrt{10gh/7}$$

This speed is independent of the angle of the incline, so both balls will have the same speed at the bottom. The ball on the incline with the smaller angle will take more time to reach the bottom than the ball on the incline with the larger angle.

11. The two spheres have different rotational inertias. The sphere that is hollow will have a larger rotational inertia than the solid sphere. If the two spheres are allowed to roll down an incline without slipping, the sphere with the smaller moment of inertia (the solid one) will reach the bottom of the ramp first. See Question 12 below for a detailed explanation of why this happens.

$\boxed{12.}$ (*a*) The sphere will reach the bottom first because it has a smaller rotational inertial. A detailed analysis of that is given below.

(*b*) The sphere will have the greater speed at the bottom, so it will have more translational kinetic energy than the cylinder.

(*c*) Both will have the same energy at the bottom, because they both started with the same potential energy at the top of the incline.

(*d*) The cylinder will have the greater rotational kinetic energy at the bottom, because it has less translational kinetic energy than the sphere.

Here is a detailed analysis of the motion:

Applying conservation of energy at the top and bottom of the incline, assuming that there is no work done by friction, gives $E_{\text{top}} = E_{\text{bottom}} \quad \rightarrow \quad Mgh = \frac{1}{2}M\upsilon^2 + \frac{1}{2}I\omega^2$. If the objects roll without slipping, then $\omega = \upsilon/R$, so $Mgh = \frac{1}{2}M\upsilon^2 + \frac{1}{2}I(\upsilon/R)^2 \quad \rightarrow \quad \upsilon = \sqrt{\dfrac{2Mgh}{M + I/R^2}}$. For a solid ball, $I = \frac{2}{5}MR^2$, and for a cylinder, $I = \frac{1}{2}MR^2$. Thus $\upsilon_{\text{sphere}} = \sqrt{10gh/7}$ and $\upsilon_{\text{cyl}} = \sqrt{4gh/3}$. Since $\upsilon_{\text{sphere}} > \upsilon_{\text{cyl}}$, the sphere has the greater speed at the bottom. That is true for any amount of height change, so the sphere is always moving faster than the cylinder after they start to move. Thus the sphere will reach the bottom first. Since both objects started with the same potential energy, both have the same total kinetic energy at the bottom. But since both objects have the same mass and the cylinder is moving slower, the cylinder has the smaller translational KE and thus the greater rotational KE. Since rotational kinetic energy is $\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2$, then $\underset{\text{sphere}}{\text{KE}_{\text{rot}}} = \frac{2}{7}mgh$ and $\underset{\text{cylinder}}{\text{KE}_{\text{rot}}} = \frac{1}{3}mgh$.

13. The long rod increases the rotational inertia of the walkers. If a walker gets off-center from the tightrope, gravity will exert a torque on the walker, causing the walker to rotate with their feet as a pivot point. With a larger rotational inertia, the angular acceleration caused by that gravitational torque will be smaller, and the walker will therefore have more time to compensate.

The long rod also allows the walkers to make small shifts in their center of mass to bring themselves back to being centered on the tightrope. It is much easier for a walker to move a long, narrow object with the precision needed for small adjustments than a short, heavy object like a barbell.

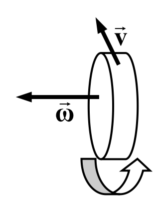
14. Momentum and angular momentum are conserved for closed systems—systems in which there are no external forces or torques applied to the system. Probably no macroscopic systems on Earth are truly closed, so external forces and torques (like those applied by air friction, for example) affect the systems over time.

15. In order to do a somersault, the diver needs some initial angular momentum when she leaves the diving board, because angular momentum will be conserved during the free-fall motion of the dive. She cannot exert a torque about her CM on herself in isolation, so if there is no angular momentum initially, there will be no rotation during the rest of the dive.

16. Once the motorcycle leaves the ground, there is no net torque on it and angular momentum must be conserved. If the throttle is on, the rear wheel will spin faster as it leaves the ground because there is no torque from the ground acting on it. The front of the motorcycle must rise up, or rotate in the direction opposite the rear wheel, in order to conserve angular momentum.

17. While in mid-air, the shortstop cannot exert a torque on himself, so his angular momentum will be conserved. If the upper half of his body (including his hips) rotates in a certain direction during the throwing motion, then to conserve angular momentum, the lower half of his body (including his legs) will rotate in the opposite direction.

18. See the diagram. To the left is west, the direction of the angular velocity. The direction of the linear velocity of a point on the top of the wheel would be north, into the page. If the angular acceleration is east, which is opposite the angular velocity, the wheel is slowing down—its angular speed is decreasing. The tangential linear acceleration of the point on top will be opposite to its linear velocity—it will point south.



19. Using the right-hand rule, point the fingers in the direction of the Earth's rotation, from west to east. Then the thumb points north. Thus the Earth's angular velocity points along its axis of rotation, toward the North Star.

20. Consider a helicopter in the air with the rotor spinning. To change the rotor's angular speed, a torque must be applied to the rotor. That torque has to come from the helicopter. By Newton's third law, an equal and opposite torque will be applied by the rotor to the helicopter. Any change in rotor speed would therefore cause the body of the helicopter to spin in a direction opposite to the change in the rotor's angular velocity.

Some large helicopters have two rotor systems, spinning in opposite directions. That makes any change in the speed of the rotor pair require a net torque of zero, so the helicopter body would not tend to spin. Smaller helicopters have a tail rotor that rotates in a vertical plane, causing a sideways force (thrust) on the tail of the helicopter in the opposite direction of the tendency of the tail to spin.

Responses to MisConceptual Questions

1. (*c*) A common misconception is that if the riders complete the revolution at the same time, they must have the same linear velocities. The time for a rotation is the same for both riders, but Bonnie, at the outer edge, travels in a larger circle than Jill. Bonnie therefore has a greater linear velocity.

2. (*b*) Students may think that the rider would travel half the distance in half the time. This would be true if the object had constant angular speed. However, it is accelerating, so it will travel a shorter distance, $\frac{1}{4}\theta$, in the first half of the time.

3. (*b*) A common error is to think that increasing the radius of the tires would increase the speed measured by the speedometer. This is actually backward. Increasing the size of the tires will cause the car to travel faster than it would with smaller tires, when the wheels have the same angular speed. Therefore, the speed of the car will be greater than the speed measured by the speedometer.

4. (*c*) Torque is the product of the lever arm and the component of the force perpendicular to the arm. Although the 1000-N force has the greatest magnitude, it acts at the pivot. Thus, the lever arm is zero, and the torque is also zero. The 800-N force is parallel to the lever arm and also exerts no torque. Of the three 500-N forces, (*c*) is both perpendicular to the lever arm and farthest from the pivot.

5. (*c, e, f*) Equations 8–10 show that there are three ways in which the torque can be written. It can be the product of the force, the lever arm, and the sine of the angle between them as in answer (*c*). It can be the product of the force and the component of the lever arm perpendicular to the force, as in answer (*e*). It can also be written as the product of the lever arm and the force perpendicular to the lever arm, as in answer (*f*). Doing the calculations shows that all three torques are equal.

6. (*b*) The location of the mass is very important. Imagine taking the material from the solid sphere and compressing it outward to turn the solid sphere into a hollow sphere of the same mass and radius. As you do this, you would be moving mass farther from the axis of rotation, which would increase the moment of inertia. Therefore, the hollow sphere has a greater moment of inertia than the solid sphere.

7. (*b*) If you don't consider how the location of the mass affects the moment of inertia, you might think that the two kinetic energies are nearly the same. However, a hollow cylinder has twice the moment of inertia as a solid cylinder of the same mass and radius. The kinetic energy is proportional to the moment of inertia, so at the same angular speed the wheel with the spokes will have nearly double the kinetic energy of the solid cylinder. It is only "nearly double" because some of the mass is in the spokes, so the moment of inertia is not exactly double.

8. (*b*) It takes energy to rotate the ball. If some of the 1000 J goes into rotation, less is available for linear kinetic energy, so the rotating ball will travel slower.

9. (*b*) If you do not take into account the energy of rotation, you would answer that the two objects would rise to the same height. Another common misconception is that the mass and/or diameter of the objects will affect how high they travel. When using conservation of energy to relate the total initial kinetic energy (translational and rotational) to the final potential energy, the mass and radius of the objects cancel out. The thin hoop has a larger moment of inertia (for a given mass and radius) than the solid sphere. It will therefore have a greater total initial kinetic energy and will travel to a greater height on the ramp.

10. (*a*) Because there is no external torque, students might think that the angular speed would remain constant. But with no external torque, the angular momentum must remain constant. The angular momentum is the product of the moment of inertia and the angular speed. As the string is shortened, the moment of inertia of the block decreases. Thus, the angular speed increases.

11. (*a*) Work is done on the object, so its kinetic energy increases. Thus the tangential velocity has to increase. Another way to consider the problem is that $\text{KE} = \sqrt{L^2/2I}$. As in Question 10, the angular momentum is constant and the rotation inertial decreases. Thus the kinetic energy (and the speed) has to increase.

12. (*a*) No net torque acts on the Earth, so the angular momentum is conserved. As people move toward the equator their distance from the Earth's axis increases. This increases the moment of inertia of the Earth. For angular momentum to be conserved, the angular speed must decrease, and it will take longer for the Earth to complete a full rotation.

13. (*c*) Students might mistakenly reason that since no net torque acts on you and your moment of inertia decreases as the masses are released, your angular speed should increase. This reasoning is erroneous because the angular momentum of the system of you and the masses is conserved. As the masses fall, they carry angular momentum with them. If you consider you and the masses as two separate systems, each with angular momentum from their moments of inertia and angular speed, it is easy to see that by dropping the masses, no net external torque acts on you and your moment of inertia does not change, so your angular speed will not change. The angular momentum of the masses also does not change until they hit the ground and friction (external torque) stops their motion.

Solutions to Problems

1. (*a*) $(45.0°)(2\pi \text{ rad}/360°) = \boxed{\pi/4 \text{ rad}} = \boxed{0.785 \text{ rad}}$

   (*b*) $(60.0°)(2\pi \text{ rad}/360°) = \boxed{\pi/3 \text{ rad}} = \boxed{1.05 \text{ rad}}$

   (*c*) $(90.0°)(2\pi \text{ rad}/360°) = \boxed{\pi/2 \text{ rad}} = \boxed{1.57 \text{ rad}}$

   (*d*) $(360.0°)(2\pi \text{ rad}/360°) = \boxed{2\pi \text{ rad}} = \boxed{6.283 \text{ rad}}$

   (*e*) $(445°)(2\pi \text{ rad}/360°) = \boxed{89\pi/36 \text{ rad}} = \boxed{7.77 \text{ rad}}$

2. The subtended angle (in radians) is the diameter of the Sun divided by the Earth–Sun distance.

$$\theta = \frac{\text{diameter of Sun}}{r_{\text{Earth-Sun}}} \rightarrow$$

$$\text{radius of Sun} = \tfrac{1}{2}\theta\, r_{\text{Earth-Sun}} = \tfrac{1}{2}(0.5°)\left(\frac{\pi \text{ rad}}{180°}\right)(1.5\times10^{11}\text{ m}) = 6.545\times10^{8}\text{ m} \approx \boxed{7\times10^{8}\text{ m}}$$

3. We find the diameter of the spot from the definition of radian angle measure.

$$\theta = \frac{\text{diameter}}{r_{\text{Earth-Moon}}} \rightarrow \text{diameter} = \theta\, r_{\text{Earth-Moon}} = (1.4\times10^{-5}\text{ rad})(3.8\times10^{8}\text{ m}) = \boxed{5300\text{ m}}$$

4. The initial angular velocity is $\omega_0 = \left(6500\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 681\text{ rad/s}$. Use the definition of angular acceleration.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0-681\text{ rad/s}}{4.0\text{ s}} = \boxed{-170\text{ rad/s}^2}$$

5. (*a*) We convert rpm to rad/s.

$$\omega = \left(\frac{7200\text{ rev}}{1\text{ min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 753.98\text{ rad/s} \approx \boxed{750\text{ rad/s}}$$

(*b*) To find the speed, we use the radius of the reading head location along with Eq. 8–4.

$$\upsilon = r\omega = (3.00\times10^{-2}\text{ m})(753.98\text{ rad/s}) = 22.62\text{ m/s} \approx \boxed{23\text{ m/s}}$$

(*c*) We convert the speed of the point on the platter from m/s to bits/s, using the distance per bit.

$$(22.62\text{ m/s})\left(\frac{1\text{ bit}}{0.50\times10^{-6}\text{ m}}\right) = \boxed{4.5\times10^{7}\text{ bits/s}}$$

6. The ball rolls $2\pi r = \pi d$ of linear distance with each revolution.

$$12.0\text{ rev}\left(\frac{\pi d\text{ m}}{1\text{ rev}}\right) = 3.5\text{ m} \rightarrow d = \frac{3.5\text{ m}}{12.0\pi} = \boxed{9.3\times10^{-2}\text{ m}}$$

7. (*a*) We convert rpm to rad/s.

$$\omega = \left(\frac{2200\text{ rev}}{1\text{ min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 230.4\text{ rad/s} \approx \boxed{230\text{ rad/s}}$$

(*b*) To find the speed and acceleration, we use the full radius of the wheel, along with Eqs. 8–4 and 8–6.

$$\upsilon = \omega r = (230.4\text{ rad/s})\left(\frac{0.35\text{ m}}{2}\right) = \boxed{4.0\times10^{1}\text{ m/s}}$$

$$a_{\text{R}} = \omega^2 r = (230.4\text{ rad/s})^2\left(\frac{0.35\text{ m}}{2}\right) = \boxed{9300\text{ m/s}^2}$$

8. In each revolution, the wheel moves forward a distance equal to its circumference, πd.

$$\Delta x = N_{\text{rev}}(\pi d) \rightarrow N = \frac{\Delta x}{\pi d} = \frac{9200\text{ m}}{\pi(0.68\text{ m})} = \boxed{4300\text{ rev}}$$

9. The angular velocity is expressed in radians per second. The second hand makes 1 revolution every 60 seconds, the minute hand makes 1 revolution every 60 minutes, and the hour hand makes 1 revolution every 12 hours.

(*a*) Second hand: $\omega = \left(\dfrac{1\text{ rev}}{60\text{ s}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right) = \boxed{\dfrac{\pi}{30}\dfrac{\text{rad}}{\text{s}}} \approx \boxed{1.05\times 10^{-1}\dfrac{\text{rad}}{\text{s}}}$

(*b*) Minute hand: $\omega = \left(\dfrac{1\text{ rev}}{60\text{ min}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right) = \boxed{\dfrac{\pi}{1800}\dfrac{\text{rad}}{\text{s}}} \approx \boxed{1.75\times 10^{-3}\dfrac{\text{rad}}{\text{s}}}$

(*c*) Hour hand: $\omega = \left(\dfrac{1\text{ rev}}{12\text{ h}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = \boxed{\dfrac{\pi}{21{,}600}\dfrac{\text{rad}}{\text{s}}} \approx \boxed{1.45\times 10^{-4}\dfrac{\text{rad}}{\text{s}}}$

(*d*) The angular acceleration in each case is $\boxed{0}$, since the angular velocity is constant.

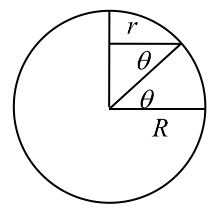
10. The angular speed of the merry-go-round is $2\pi\text{ rad}/4.0\text{ s} = 1.57\text{ rad/s}$.

(*a*) $\upsilon = \omega r = (1.57\text{ rad/s})(1.2\text{ m}) = \boxed{1.9\text{ m/s}}$

(*b*) The acceleration is radial. There is no tangential acceleration.

$$a_R = \omega^2 r = (1.57\text{ rad/s})^2(1.2\text{ m}) = \boxed{3.0\text{ m/s}^2\text{ toward the center}}$$

11. Each location will have the same angular velocity (1 revolution per day), but the radius of the circular path varies with the location. From the diagram, we see $r = R\cos\theta$, where R is the radius of the Earth, and r is the radius at latitude θ.



(*a*) $\upsilon = \omega r = \dfrac{2\pi}{T}r = \left(\dfrac{2\pi\text{ rad}}{1\text{ day}}\right)\left(\dfrac{1\text{ day}}{86{,}400\text{ s}}\right)(6.38\times 10^6\text{ m}) = \boxed{464\text{ m/s}}$

(*b*) $\upsilon = \omega r = \dfrac{2\pi}{T}r = \left(\dfrac{2\pi\text{ rad}}{1\text{ day}}\right)\left(\dfrac{1\text{ day}}{86{,}400\text{ s}}\right)(6.38\times 10^6\text{ m})\cos 66.5^\circ = \boxed{185\text{ m/s}}$

(*c*) $\upsilon = \omega r = \dfrac{2\pi}{T}r = \left(\dfrac{2\pi\text{ rad}}{1\text{ day}}\right)\left(\dfrac{1\text{ day}}{86{,}400\text{ s}}\right)(6.38\times 10^6\text{ m})\cos 42.0^\circ = \boxed{345\text{ m/s}}$

12. (*a*) The Earth makes one orbit around the Sun in one year.

$$\omega_{\text{orbit}} = \frac{\Delta\theta}{\Delta t} = \left(\frac{2\pi\text{ rad}}{1\text{ year}}\right)\left(\frac{1\text{ year}}{3.16\times 10^7\text{ s}}\right) = \boxed{1.99\times 10^{-7}\text{ rad/s}}$$

(*b*) The Earth makes one revolution about its axis in one day.

$$\omega_{\text{rotation}} = \frac{\Delta\theta}{\Delta t} = \left(\frac{2\pi\text{ rad}}{1\text{ day}}\right)\left(\frac{1\text{ day}}{86{,}400\text{ s}}\right) = \boxed{7.27\times 10^{-5}\text{ rad/s}}$$

13. The centripetal acceleration is given by $a_R = \omega^2 r$. Solve for the angular velocity.

$$\omega = \sqrt{\frac{a_R}{r}} = \sqrt{\frac{(100{,}000)(9.80\text{ m/s}^2)}{0.080\text{ m}}} = 3500\frac{\text{rad}}{\text{s}}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{3.3\times 10^4\text{ rpm}}$$

14. Convert the rpm values to angular velocities.

$$\omega_0 = \left(120\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 12.57\ \text{rad/s}$$

$$\omega = \left(280\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 29.32\ \text{rad/s}$$

(*a*) The angular acceleration is found from Eq. 8–9a.

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{29.32\ \text{rad/s} - 12.57\ \text{rad/s}}{4.0\ \text{s}} = 4.188\ \text{rad/s}^2 \approx \boxed{4.2\ \text{rad/s}^2}$$

(*b*) To find the components of the acceleration, the instantaneous angular velocity is needed.

$$\omega = \omega_0 + \alpha t = 12.57\ \text{rad/s} + (4.188\ \text{rad/s}^2)(2.0\ \text{s}) = 20.95\ \text{rad/s}$$

The instantaneous radial acceleration is given by $a_R = \omega^2 r$.

$$a_R = \omega^2 r = (20.95\ \text{rad/s})^2\left(\frac{0.61\ \text{m}}{2}\right) = \boxed{130\ \text{m/s}^2}$$

The tangential acceleration is given by $a_{\tan} = \alpha r$.

$$a_{\tan} = \alpha r = (4.188\ \text{rad/s}^2)\left(\frac{0.61\ \text{m}}{2}\right) = \boxed{1.3\ \text{m/s}^2}$$

15. (*a*) The angular acceleration can be found from Eq. 8–3a. The initial angular frequency is 0 and the final frequency is 1 rpm.

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{\left(1.0\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1.0\ \text{min}}{60\ \text{s}}\right) - 0}{(12\ \text{min})\left(\frac{60\ \text{s}}{1.0\ \text{min}}\right)} = 1.454\times 10^{-4}\ \text{rad/s}^2 \approx \boxed{1.5\times 10^{-4}\ \text{rad/s}^2}$$

(*b*) After 6.0 min (360 s), the angular speed is as follows:

$$\omega = \omega_0 + \alpha t = 0 + (1.454\times 10^{-4}\ \text{rad/s}^2)(360\ \text{s}) = 5.234\times 10^{-2}\ \text{rad/s}$$

Find the components of the acceleration of a point on the outer skin from the angular speed and the radius.

$$a_{\tan} = \alpha R = (1.454\times 10^{-4}\ \text{rad/s}^2)(4.25\ \text{m}) = \boxed{6.2\times 10^{-4}\ \text{m/s}^2}$$

$$a_R = \omega^2 R = (5.234\times 10^{-2}\ \text{rad/s})^2(4.25\ \text{m}) = \boxed{1.2\times 10^{-2}\ \text{m/s}^2}$$

16. The tangential speed of the turntable must be equal to the tangential speed of the roller, if there is no slippage.

$$\upsilon_1 = \upsilon_2 \quad\rightarrow\quad \omega_1 R_1 = \omega_2 R_2 \quad\rightarrow\quad \boxed{\omega_1/\omega_2 = R_2/R_1}$$

17. (*a*) For constant angular acceleration:

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{1200\ \text{rev/min} - 3500\ \text{rev/min}}{2.5\ \text{s}} = \frac{-2300\ \text{rev/min}}{2.5\ \text{s}}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{mi n}}{60\ \text{s}}\right)$$

$$= -96.34\ \text{rad/s}^2 \approx \boxed{-96\ \text{rad/s}^2}$$

(b) For the angular displacement, we assume constant angular acceleration.

$$\theta = \tfrac{1}{2}(\omega_0 + \omega)t = \tfrac{1}{2}(3500\text{ rev/min} + 1200\text{ rev/min})(2.5\text{ s})\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{98\text{ rev}}$$

18. The angular displacement can be found from Eq. 8–9d.

$$\theta = \bar{\omega}t = \tfrac{1}{2}(\omega_0 + \omega)t = \tfrac{1}{2}(0 + 15{,}000\text{ rev/min})(240\text{ s})(1\text{ min}/60\text{ s}) = \boxed{3.0\times 10^4\text{ rev}}$$

19. (a) The angular acceleration can be found from Eq. 8–9b with $\omega_0 = 0$.

$$\alpha = \frac{2\theta}{t^2} = \frac{2(23\text{ rev})}{(1.0\text{ min})^2} = \boxed{46\text{ rev/min}^2}$$

(b) The final angular speed can be found from $\theta = \tfrac{1}{2}(\omega_0 + \omega)t$, with $\omega_0 = 0$.

$$\omega = \frac{2\theta}{t} - \omega_0 = \frac{2(23\text{ rev})}{1.0\text{ min}} = \boxed{46\text{ rpm}}$$

20. (a) The angular acceleration can be found from Eq. 8–9c.

$$\alpha = \frac{\omega^2 - \omega_0^2}{2\theta} = \frac{0 - (850\text{ rev/min})^2}{2(1250\text{ rev})} = \left(-289\frac{\text{rev}}{\text{min}^2}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)^2 = \boxed{-0.50\frac{\text{rad}}{\text{s}^2}}$$

(b) The time to come to a stop can be found from $\theta = \tfrac{1}{2}(\omega_0 + \omega)t$.

$$t = \frac{2\theta}{\omega_0 + \omega} = \frac{2(1250\text{ rev})}{850\text{ rev/min}}\left(\frac{60\text{ s}}{1\text{ min}}\right) = 176.5\text{ s} \approx \boxed{180\text{ s}}$$

21. Use Eq. 8–9d combined with Eq. 8–2a.

$$\bar{\omega} = \frac{\omega + \omega_0}{2} = \frac{240\text{ rpm} + 360\text{ rpm}}{2} = 300\text{ rpm}$$

$$\theta = \bar{\omega}\, t = \left(300\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)(6.8\text{ s}) = 34\text{ rev}$$

Each revolution corresponds to a circumference of travel distance.

$$34\text{ rev}\left[\frac{\pi(0.31\text{ m})}{1\text{ rev}}\right] = \boxed{33\text{ m}}$$

$\boxed{22}$. (a) The angular acceleration can be found from $\omega^2 = \omega_0^2 + 2\alpha\theta$, with the angular velocities being found from $\omega = \upsilon/r$.

$$\alpha = \frac{\omega^2 - \omega_0^2}{2\theta} = \frac{(\upsilon^2 - \upsilon_0^2)}{2r^2\theta} = \frac{\left[(55\text{ km/h})^2 - (95\text{ km/h})^2\right]\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)^2}{2(0.40\text{ m})^2(75\text{ rev})\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)}$$

$$= -3.070\text{ rad/s}^2 \approx \boxed{-3.1\text{ rad/s}^2}$$

(*b*) The time to stop can be found from $\omega = \omega_0 + \alpha t$, with a final angular velocity of 0.

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{\upsilon - \upsilon_0}{r\alpha} = \frac{-(55\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)}{(0.40\text{ m})(-3.070\text{ rad/s}^2)} = 12.44\text{ s} \approx \boxed{12\text{ s}}$$

(*c*) We first find the total angular displacement of the tires as they slow from 55 km/h to rest, and then convert the angular displacement to a linear displacement, assuming that the tires are rolling without slipping.

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta \quad \rightarrow$$

$$\Delta\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{0 - \left(\frac{\upsilon_0}{r}\right)^2}{2\alpha} = -\frac{\left[\frac{(55\text{ km/h})}{0.40\text{ m}}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2}{2(-3.070\text{ rad/s}^2)} = 237.6\text{ rad}$$

$$\Delta x = r\Delta\theta = (0.40\text{ m})(237.6\text{ rad}) = \boxed{95\text{ m}}$$

For the total distance, add the distance moved during the time the car slows from 95 km/h to 55 km/h. The tires made 75 revolutions, so that distance is as follows:

$$\Delta x = r\Delta\theta = (0.40\text{ m})(75\text{ rev})\left(\frac{2\pi\text{ rad}}{\text{rev}}\right) = 188\text{ m}$$

The total distance would be the sum of the two distances, 283 m.

23. Since there is no slipping between the wheels, the tangential component of the linear acceleration of each wheel must be the same.

(*a*) $a_{\substack{\text{tan}\\\text{small}}} = a_{\substack{\text{tan}\\\text{large}}} \quad \rightarrow \quad \alpha_{\text{small}} r_{\text{small}} = \alpha_{\text{large}} r_{\text{large}} \quad \rightarrow$

$$\alpha_{\text{large}} = \alpha_{\text{small}} \frac{r_{\text{small}}}{r_{\text{large}}} = (7.2\text{ rad/s}^2)\left(\frac{2.0\text{ cm}}{27.0\text{ cm}}\right) = 0.5333\text{ rad/s}^2 \approx \boxed{0.53\text{ rad/s}^2}$$

(*b*) Assume the pottery wheel starts from rest. Convert the speed to an angular speed, and then use Eq. 8–9a.

$$\omega = \left(65\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 6.807\text{ rad/s}$$

$$\omega = \omega_0 + \alpha t \quad \rightarrow \quad t = \frac{\omega - \omega_0}{\alpha} = \frac{6.807\text{ rad/s}}{0.5333\text{ rad/s}^2} = 12.76\text{ s} \approx \boxed{13\text{ s}}$$

24. (*a*) The maximum torque will be exerted by the force of her weight, pushing tangential to the circle in which the pedal moves.

$$\tau = r_\perp F = r_\perp mg = (0.17\text{ m})(52\text{ kg})(9.80\text{ m/s}^2) = 86.6\text{ m}\cdot\text{N} \approx \boxed{87\text{ m}\cdot\text{N}}$$

(*b*) She could exert more torque by pushing down harder with her legs, raising her center of mass. She could also pull upward on the handle bars as she pedals, which will increase the downward force of her legs.

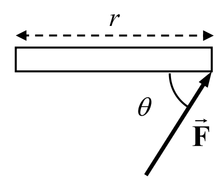
$\boxed{25.}$ Each force is oriented so that it is perpendicular to its lever arm. Call counterclockwise torques positive. The torque due to the three applied forces is given by the following:

$$\tau_{\substack{\text{applied}\\\text{forces}}} = (28\text{ N})(0.24\text{ m}) - (18\text{ N})(0.24\text{ m}) - (35\text{ N})(0.12\text{ m}) = -1.8\text{ m}\cdot\text{N}$$

Since this torque is clockwise, we assume the wheel is rotating clockwise, so the frictional torque is counterclockwise. Thus the net torque is as follows:

$$\tau_{\text{net}} = (28\text{ N})(0.24\text{ m}) - (18\text{ N})(0.24\text{ m}) - (35\text{ N})(0.12\text{ m}) + 0.60\ \text{m}\cdot\text{N} = -1.2\ \text{m}\cdot\text{N}$$
$$= \boxed{1.2\text{ m}\cdot\text{N, clockwise}}$$

26. The torque is calculated by $\tau = rF\sin\theta$. See the diagram, from the top view.



(*a*) For the first case, $\theta = 90°$.

$$\tau = rF\sin\theta = (0.96\text{ m})(42\text{ N})\sin 90° = 40.32\text{ m}\cdot\text{N} \approx \boxed{4.0\times10^{1}\text{ m}\cdot\text{N}}$$

(*b*) For the second case, $\theta = 60.0°$.

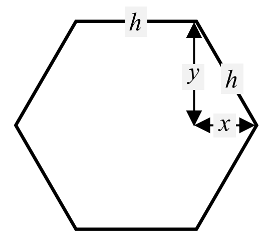
$$\tau = rF\sin\theta = (0.96\text{ m})(42\text{ N})\sin 60.0° = 34.92\text{ m}\cdot\text{N} \approx \boxed{35\text{ m}\cdot\text{N}}$$

27. There is a counterclockwise torque due to the force of gravity on the left block and a clockwise torque due to the force of gravity on the right block. Call clockwise the positive direction.

$$\sum\tau = mg\ell_2 - mg\ell_1 = \boxed{mg(\ell_2 - \ell_1)\text{, clockwise}}$$

28. The force required to produce the torque can be found from $\tau = rF\sin\theta$. The force is applied perpendicularly to the wrench, so $\theta = 90°$.

$$F = \frac{\tau}{r} = \frac{95\text{ m}\cdot\text{N}}{0.28\text{ m}} = 339.3\text{ N} \approx \boxed{340\text{ N}}$$



The net torque still must be $95\text{ m}\cdot\text{N}$. This is produced by six forces, one at each of the six points. We assume for our estimate that those forces are also perpendicular to their lever arms. From the diagram, we estimate the lever arm as follows, and then calculate the force at each point:

$$\text{Lever arm} = r = \tfrac{1}{2}h + x = \tfrac{1}{2}\left(\frac{y}{\cos 30°}\right) + y\tan 30°$$
$$= y\left(\frac{1}{2\cos 30°} + \tan 30°\right) = (7.5\times10^{-3}\text{ m})(1.15)$$

$$\tau_{\text{net}} = (6F_{\text{point}})r \quad\rightarrow\quad F_{\text{point}} = \frac{\tau}{6r} = \frac{95\text{ m}\cdot\text{N}}{6(7.5\times10^{-3}\text{ m})(1.15)} = 1835.7\text{ N} \approx \boxed{1800\text{ N}}$$

29. For each torque, use Eq. 8–10c. Take counterclockwise torques to be positive.

(*a*) Each force has a lever arm of 1.0 m.

$$\tau_{\substack{\text{about}\\ \text{C}}} = -(1.0\text{ m})(56\text{ N})\sin 32° + (1.0\text{ m})(52\text{ N})\sin 58° = 14.42\text{ m}\cdot\text{N} \approx \boxed{14\text{ m}\cdot\text{N}}$$

(*b*) The force at C has a lever arm of 1.0 m, and the force at the top has a lever arm of 2.0 m.

$$\tau_{\substack{\text{about}\\ \text{P}}} = -(2.0\text{ m})(56\text{ N})\sin 32° + (1.0\text{ m})(65\text{ N})\sin 45° = -13.39\text{ m}\cdot\text{N} \approx \boxed{-13\text{ m}\cdot\text{N}}$$

The negative sign indicates a clockwise torque.

30. For a sphere rotating about an axis through its center, the moment of inertia is as follows:

$$I = \tfrac{2}{5}MR^2 = \tfrac{2}{5}(10.8\text{ kg})(0.648\text{ m})^2 = \boxed{1.81\text{ kg}\cdot\text{m}^2}$$

31. Since all of the significant mass is located at the same distance from the axis of rotation, the moment of inertia is given by $I = MR^2$.

$$I = MR^2 = (1.1\text{ kg})\left(\tfrac{1}{2}(0.67\text{ m})\right)^2 = \boxed{0.12\text{ kg}\cdot\text{m}^2}$$

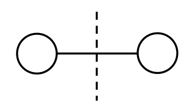
The hub mass can be ignored because its distance from the axis of rotation is very small, so it has a very small rotational inertia.

32. The torque required is equal to the angular acceleration times the moment of inertia. The angular acceleration is found using Eq. 8–9a. Use the moment of inertia of a solid cylinder.

$$\omega = \omega_0 + \alpha t \quad \rightarrow \quad \alpha = \omega/t$$

$$\tau = I\alpha = \left(\tfrac{1}{2}MR_0^2\right)\left(\frac{\omega}{t}\right) = \frac{MR_0^2\omega}{2t} = \frac{(31{,}000\text{ kg})(7.0\text{ m})^2(0.68\text{ rad/s})}{2(34\text{ s})} = \boxed{1.5\times 10^4\text{ m}\cdot\text{N}}$$

33. The oxygen molecule has a "dumbbell" geometry, as though it rotates about the dashed line shown in the diagram. If the total mass is M, then each atom has a mass of $M/2$. If the distance between them is d, then the distance from the axis of rotation to each atom is $d/2$. Treat each atom as a particle for calculating the moment of inertia.



$$I = (M/2)(d/2)^2 + (M/2)(d/2)^2 = 2(M/2)(d/2)^2 = \tfrac{1}{4}Md^2 \quad \rightarrow$$

$$d = \sqrt{4I/M} = \sqrt{4(1.9\times10^{-46}\text{ kg}\cdot\text{m}^2)/(5.3\times10^{-26}\text{ kg})} = \boxed{1.2\times10^{-10}\text{ m}}$$

34. (*a*) The moment of inertia of a cylinder is $\tfrac{1}{2}MR^2$.

$$I = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(0.380\text{ kg})(0.0850\text{ m})^2 = 1.373\times10^{-3}\text{ kg}\cdot\text{m}^2 \approx \boxed{1.37\times10^{-3}\text{ kg}\cdot\text{m}^2}$$

(*b*) The wheel slows down "on its own" from 1500 rpm to rest in 55.0 s. This is used to calculate the frictional torque.

$$\tau_{\text{fr}} = I\alpha_{\text{fr}} = I\frac{\Delta\omega}{\Delta t} = (1.373\times10^{-3}\text{ kg}\cdot\text{m}^2)\frac{(0-1500\text{ rev/min})(2\pi\text{ rad/rev})(1\text{ min/60 s})}{55.0\text{ s}}$$

$$= -3.921\times10^{-3}\text{ m}\cdot\text{N}$$

The net torque causing the angular acceleration is the applied torque plus the (negative) frictional torque.

$$\sum\tau = \tau_{\text{applied}} + \tau_{\text{fr}} = I\alpha \quad \rightarrow \quad \tau_{\text{applied}} = I\alpha - \tau_{\text{fr}} = I\frac{\Delta\omega}{\Delta t} - \tau_{\text{fr}}$$

$$= (1.373\times10^{-3}\text{ kg}\cdot\text{m}^2)\frac{(1750\text{ rev/min})(2\pi\text{ rad/rev})(1\text{ min/60 s})}{5.00\text{ s}} - (-3.921\times10^{-3}\text{ m}\cdot\text{N})$$

$$= \boxed{5.42\times10^{-2}\text{ m}\cdot\text{N}}$$

35. (*a*) The torque gives angular acceleration to the ball only, since the arm is considered massless. The angular acceleration of the ball is found from the given tangential acceleration.

$$\tau = I\alpha = MR^2\alpha = MR^2\frac{a_{\tan}}{R} = MRa_{\tan} = (3.6\text{ kg})(0.31\text{ m})(7.0\text{ m/s}^2)$$

$$= 7.812\text{ m}\cdot\text{N} \approx \boxed{7.8\text{ m}\cdot\text{N}}$$

(*b*) The triceps muscle must produce the torque required, but with a lever arm of only 2.5 cm, perpendicular to the triceps muscle force.

$$\tau = Fr_{\perp} \quad \rightarrow \quad F = \tau / r_{\perp} = 7.812\ \text{m}\cdot\text{N}/(2.5\times 10^{-2}\ \text{m}) = \boxed{310\ \text{N}}$$

36. (*a*) The angular acceleration can be found from the following:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega}{t} = \frac{\upsilon/r}{t} = \frac{(8.5\ \text{m/s})/(0.31\ \text{m})}{0.38\ \text{s}} = 72.16\ \text{rad/s}^2 \approx \boxed{72\ \text{rad/s}^2}$$

(*b*) The force required can be found from the torque, since $\tau = Fr\sin\theta$. In this situation the force is perpendicular to the lever arm, so $\theta = 90°$. The torque is also given by $\tau = I\alpha$, where I is the moment of inertia of the arm–ball combination. Equate the two expressions for the torque, and solve for the force.

$$Fr\sin\theta = I\alpha$$

$$F = \frac{I\alpha}{r\sin\theta} = \frac{m_{\text{ball}}d_{\text{ball}}^2 + \frac{1}{3}m_{\text{arm}}\ell_{\text{arm}}^2}{r\sin 90°}\alpha$$

$$= \frac{(1.00\ \text{kg})(0.31\ \text{m})^2 + \frac{1}{3}(3.7\ \text{kg})(0.31\ \text{m})^2}{(0.025\ \text{m})}(72.16\ \text{rad/s}^2) = 619.5\ \text{N} \approx \boxed{620\ \text{N}}$$

37. The torque is calculated from $\tau = I\alpha$. The rotational inertia of a rod about its end is $I = \frac{1}{3}M\ell^2$.

$$\tau = I\alpha = \frac{1}{3}M\ell^2\frac{\Delta\omega}{\Delta t} = \frac{1}{3}(0.90\ \text{kg})(0.95\ \text{m})^2\frac{(2.6\ \text{rev/s})(2\pi\ \text{rad/rev})}{0.20\ \text{s}} = 22.12\ \text{m}\cdot\text{N} \approx \boxed{22\ \text{m}\cdot\text{N}}$$

38. (*a*) The small ball can be treated as a particle for calculating its moment of inertia.

$$I = MR^2 = (0.350\ \text{kg})(1.2\ \text{m})^2 = 0.504\ \text{kg}\cdot\text{m}^2 \approx \boxed{0.50\ \text{kg}\cdot\text{m}^2}$$

(*b*) To keep a constant angular velocity, the net torque must be zero, so the torque needed is the same magnitude as the torque caused by friction.

$$\sum\tau = \tau_{\text{applied}} - \tau_{\text{fr}} = 0 \quad \rightarrow \quad \tau_{\text{applied}} = \tau_{\text{fr}} = F_{\text{fr}}r = (0.020\ \text{N})(1.2\ \text{m}) = \boxed{2.4\times 10^{-2}\ \text{m}\cdot\text{N}}$$

39. (*a*) To calculate the moment of inertia about the *y* axis (vertical), use the following:

$$I = \sum M_i R_{ix}^2 = m(0.50\ \text{m})^2 + M(0.50\ \text{m})^2 + m(1.00\ \text{m})^2 + M(1.00\ \text{m})^2$$

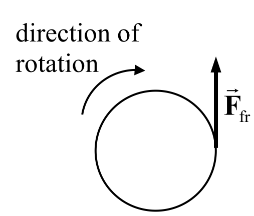
$$= (m+M)\left[(0.50\ \text{m})^2 + (1.00\ \text{m})^2\right] = (5.6\ \text{kg})\left[(0.50\ \text{m})^2 + (1.00\ \text{m})^2\right] = \boxed{7.0\ \text{kg}\cdot\text{m}^2}$$

(*b*) To calculate the moment of inertia about the *x* axis (horizontal), use the following:

$$I = \sum M_i R_{iy}^2 = (2m + 2M)(0.25\ \text{m})^2 = 2(5.6\ \text{kg})(0.25\ \text{m})^2 = \boxed{0.70\ \text{kg}\cdot\text{m}^2}$$

(*c*) Because of the larger *I* value, it is ten times harder to accelerate the array about the $\boxed{\text{vertical axis}}$.

40. (*a*) The torque exerted by the frictional force is $\tau = rF_{\text{fr}}\sin\theta$. The force of friction is assumed to be tangential to the clay, so $\theta = 90°$.



$$\tau_{\text{total}} = rF_{\text{fr}}\sin\theta = \left(\tfrac{1}{2}(0.090\ \text{m})\right)(1.5\ \text{N})\sin 90° = 0.0675\ \text{m}\cdot\text{N}$$

$$\approx \boxed{0.068\ \text{m}\cdot\text{N}}$$

(*b*) The time to stop is found from $\omega = \omega_0 + \alpha t,$ with a final angular velocity of 0. The angular acceleration can be found from $\tau_{\text{total}} = I\alpha.$ The net torque (and angular acceleration) is negative since the object is slowing.

$$t = \frac{\omega - \omega_o}{\alpha} = \frac{\omega - \omega_o}{\tau / I} = \frac{0 - (1.6\text{ rev/s})(2\pi\text{ rad/rev})}{(-0.0675\text{ m}\cdot\text{N})/(0.11\text{ kg}\cdot\text{m}^2)} = 16.38\text{ s} \approx \boxed{16\text{ s}}$$

41. The torque supplied is equal to the angular acceleration times the moment of inertia. The angular acceleration is found by using Eq. 8–9b, with $\omega_0 = 0.$ Use the moment of inertia of a sphere.

$$\theta = \omega_0 + \tfrac{1}{2}\alpha t^2 \;\rightarrow\; \alpha = \frac{2\theta}{t^2};\quad \tau = I\alpha = \left(\tfrac{2}{5}Mr_0^2\right)\left(\frac{2\theta}{t^2}\right) \;\rightarrow$$

$$M = \frac{5\tau t^2}{4r_0^2\theta} = \frac{5(10.8\text{ m}\cdot\text{N})(15.0\text{ s})^2}{4(0.36\text{ m})^2(320\pi\text{ rad})} = 23.31\text{ kg} \approx \boxed{23\text{ kg}}$$

42. (*a*) The moment of inertia of a thin rod, rotating about its end, is $\frac{1}{3}M\ell^2.$ There are three blades to add together.

$$I_{\text{total}} = 3\left(\tfrac{1}{3}M\ell^2\right) = M\ell^2 = (135\text{ kg})(3.75\text{ m})^2 = 1898\text{ kg}\cdot\text{m}^2 \approx \boxed{1.90\times10^3\text{ kg}\cdot\text{m}^2}$$

(*b*) The torque required is the rotational inertia times the angular acceleration, assumed constant.

$$\tau = I_{\text{total}}\alpha = I_{\text{total}}\frac{\omega - \omega_0}{t} = (1898\text{ kg}\cdot\text{m}^2)\frac{(6.0\text{ rev/s})(2\pi\text{ rad/rev})}{8.0\text{ s}} = \boxed{8900\text{ m}\cdot\text{N}}$$

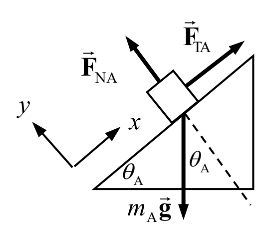
43. The firing force of the rockets will create a net torque but no net force. Since each rocket fires tangentially, each force has a lever arm equal to the radius of the satellite, and each force is perpendicular to the lever arm. Thus, $\tau_{\text{net}} = 4FR.$ This torque will cause an angular acceleration according to $\tau = I\alpha,$ where $I = \frac{1}{2}MR^2 + 4mR^2,$ combining a cylinder of mass M and radius R with four point masses of mass m and lever arm R each. The angular acceleration can be found from the kinematics by $\alpha = \frac{\Delta\omega}{\Delta t}.$ Equating the two expressions for the torque and substituting enables us to solve for the force.

$$4FR = I\alpha = \left(\tfrac{1}{2}M + 4m\right)R^2\frac{\Delta\omega}{\Delta\tau} \;\rightarrow\; F = \frac{\left(\tfrac{1}{2}M + 4m\right)R\Delta\omega}{4\Delta t}$$

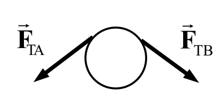
$$= \frac{\left(\tfrac{1}{2}(3600\text{ kg}) + 4(250\text{ kg})\right)(4.0\text{ m})(32\text{ rev/min})(2\pi\text{ rad/rev})(1\text{ min}/60\text{ s})}{4(5.0\text{ min})(60\text{ s/min})} = 31.28\text{ N}$$

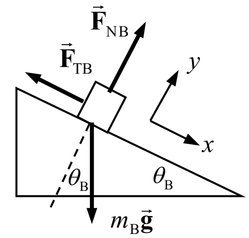
$$\approx \boxed{31\text{ N}}$$

44. (*a*) The free-body diagrams are shown. Note that only the forces producing torque are shown on the pulley. There would also be a gravity force on the pulley (since it has mass) and a normal force from the pulley's suspension, but they are not shown since they do not enter into the solution.



(*b*) Write Newton's second law for the two blocks, taking the positive x direction as shown in the free-body diagrams.





$$m_A: \ \sum F_x = F_{TA} - m_A g \sin\theta_A = m_A a \ \rightarrow$$
$$F_{TA} = m_A(g\sin\theta_A + a)$$
$$= (8.0\text{ kg})\left[(9.80\text{ m/s}^2)\sin 32° + 1.00\text{ m/s}^2\right] = 49.55\text{ N}$$
$$\approx \boxed{50\text{ N}}\ \text{(2 significant figures)}$$
$$m_B: \ \sum F_x = m_B g\sin\theta_B - F_{TB} = m_B a \ \rightarrow$$
$$F_{TB} = m_B(g\sin\theta_B - a)$$
$$= (10.0\text{ kg})\left[(9.80\text{ m/s}^2)\sin 61° - 1.00\text{ m/s}^2\right] = 75.71\text{ N}$$
$$\approx \boxed{76\text{ N}}$$

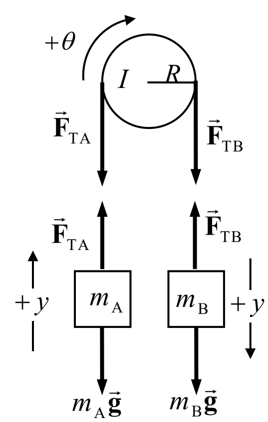
(*c*) The net torque on the pulley is caused by the two tensions. We take clockwise torques as positive.

$$\sum\tau = (F_{TB} - F_{TA})\,R = (75.71\text{ N} - 49.55\text{ N})(0.15\text{ m}) = 3.924\text{ m}\cdot\text{N} \approx \boxed{3.9\text{ m}\cdot\text{N}}$$

Use Newton's second law to find the rotational inertia of the pulley. The tangential acceleration of the pulley's rim is the same as the linear acceleration of the blocks, assuming that the string doesn't slip.

$$\sum\tau = I\alpha = I\frac{a}{R} = (F_{TB} - F_{TA})R \ \rightarrow$$
$$I = \frac{(F_{TB} - F_{TA})R^2}{a} = \frac{(75.71\text{ N} - 49.55\text{ N})(0.15\text{ m})^2}{1.00\text{ m/s}^2} = \boxed{0.59\text{ kg}\cdot\text{m}^2}$$

45. (*a*) Since $m_B > m_A$, m_B will accelerate down, m_A will accelerate up, and the pulley will accelerate clockwise. Call the direction of acceleration the positive direction for each object. The masses will have the same acceleration since they are connected by a cord. The rim of the pulley has that same acceleration since the cord makes it rotate, so $\alpha_{\text{pulley}} = a/R$. From the free-body diagrams, we have the following:



$$\sum F_{yA} = F_{TA} - m_A g = m_A a \ \rightarrow \ F_{TA} = m_A g + m_A a$$
$$\sum F_{yB} = m_B g - F_{TB} = m_B a \ \rightarrow \ F_{TB} = m_B g - m_B a$$
$$\sum\tau = F_{TB}r - F_{TA}r = I\alpha = I\frac{a}{R}$$

We have to assume that the tensions are unequal in order to have a net torque to accelerate the pulley. Substitute the expressions for the tensions into the torque equation, and solve for the acceleration.

$$F_{TB}R - F_{TA}R = I\frac{a}{R} \ \rightarrow \ (m_B g - m_B a)R - (m_A g + m_A a)R = I\frac{a}{R} \ \rightarrow$$
$$a = \frac{(m_B - m_A)}{(m_A + m_B + I/R^2)}g = \frac{(m_B - m_A)}{\left(m_A + m_B + \frac{1}{2}m_P R^2/R^2\right)}g$$
$$= \frac{(75\text{ kg} - 65\text{ kg})}{\left[75\text{ kg} + 65\text{ kg} + \frac{1}{2}(6.0\text{ kg})\right]}(9.80\text{ m/s}^2) = 0.6853\text{ m/s}^2 \approx \boxed{0.69\text{ m/s}^2}$$

(*b*) If the moment of inertia is ignored, then from the torque equation we see that $F_{\text{TB}} = F_{\text{TA}}$, and the acceleration will be $a_{I=0} = \frac{(m_{\text{B}} - m_{\text{A}})}{(m_{\text{A}} + m_{\text{B}})} g = \frac{(75\text{ kg} - 65\text{ kg})}{75\text{ kg} + 65\text{ kg}}(9.80\text{ m/s}^2) = 0.7000\text{ m/s}^2$. We calculate the percent difference, which is small because of the relatively small mass of the pulley.

$$\text{error} = \left(\frac{0.7000\text{ m/s}^2 - 0.6853\text{ m/s}^2}{0.6853\text{ m/s}^2}\right) \times 100 = 2.145\% \approx \boxed{2\%}$$

46. Work can be expressed in rotational quantities as $W = \tau\,\Delta\theta$, so power can be expressed in rotational quantities as $P = \frac{W}{\Delta t} = \tau\frac{\Delta\theta}{\Delta t} = \tau\omega$.

$$P = \tau\omega = (265\text{ m}\cdot\text{N})\left(3350\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{125\text{ hp}}$$

47. The energy required to bring the rotor up to speed from rest is equal to the final rotational kinetic energy of the rotor.

$$\text{KE}_{\text{rot}} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(3.25\times10^{-2}\text{ kg}\cdot\text{m}^2)\left[8750\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]^2 = \boxed{1.36\times10^4\text{ J}}$$

48. Apply conservation of mechanical energy. Take the bottom of the incline to be the zero location for gravitational potential energy. The energy at the top of the incline is then all gravitational potential energy, and at the bottom of the incline, there is both rotational and translational kinetic energy. Since the cylinder rolls without slipping, the angular velocity is given by $\omega = \upsilon/R$.

$$E_{\text{top}} = E_{\text{bottom}} \rightarrow Mgh = \tfrac{1}{2}M\upsilon^2 + \tfrac{1}{2}I_{\text{CM}}\omega^2 = \tfrac{1}{2}M\upsilon^2 + \tfrac{1}{2}\tfrac{1}{2}MR^2\frac{\upsilon^2}{R^2} = \tfrac{3}{4}M\upsilon^2 \rightarrow$$

$$\upsilon = \sqrt{\tfrac{4}{3}gh} = \sqrt{\tfrac{4}{3}(9.80\text{ m/s}^2)(7.20\text{ m})} = \boxed{9.70\text{ m/s}}$$

49. The total kinetic energy is the sum of the translational and rotational kinetic energies. Since the ball is rolling without slipping, the angular velocity is given by $\omega = \upsilon/R$. The rotational inertia of a sphere about an axis through its center is $I = \tfrac{2}{5}mR^2$.

$$\text{KE}_{\text{total}} = \text{KE}_{\text{trans}} + \text{KE}_{\text{rot}} = \tfrac{1}{2}m\upsilon^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}m\upsilon^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mR^2\right)\frac{\upsilon^2}{R^2} = \tfrac{7}{10}m\upsilon^2$$

$$= 0.7(7.25\text{ kg})(3.10\text{ m/s})^2 = \boxed{48.8\text{ J}}$$

50. Maintaining a constant angular speed ω_{steady} will require a torque τ_{motor} to oppose the frictional torque. The power required by the motor is $P = \tau_{\text{motor}}\omega_{\text{steady}} = -\tau_{\text{friction}}\omega_{\text{steady}}$.

$$\tau_{\text{friction}} = I\alpha_{\text{friction}} = \tfrac{1}{2}MR^2\left(\frac{\omega_f - \omega_0}{t}\right) \rightarrow$$

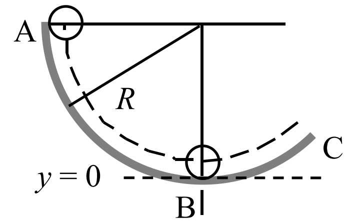
$$P_{\text{motor}} = \tfrac{1}{2}MR^2\left(\frac{\omega_0 - \omega_f}{t}\right)\omega_{\text{steady}} = \tfrac{1}{2}(220\text{ kg})(5.5\text{ m})^2\frac{\left[(3.8\text{ rev/s})\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\right]^2}{16\text{ s}} = 1.186\times10^5\text{ W}$$

$$= 1.186\times10^5\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = 158.9\text{ hp} \approx \boxed{160\text{ hp}}$$

51. The work required is the change in rotational kinetic energy. The initial angular velocity is 0.

$$W = \Delta\text{KE}_{\text{rot}} = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_0^2 = \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega_f^2 = \tfrac{1}{4}(1440\text{ kg})(7.50\text{ m})^2\left(\frac{2\pi\text{ rad}}{7.00\text{ s}}\right)^2 = \boxed{1.63\times10^4\text{ J}}$$

52. Use conservation of mechanical energy to equate the energy at points A and B. Call the zero level for gravitational potential energy the lowest point on which the ball rolls. Since the ball rolls without slipping, $\omega = \upsilon/r$.

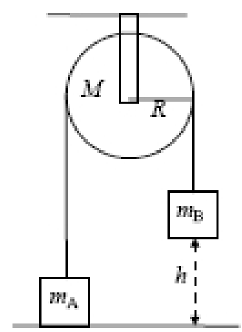


$$E_A = E_B \rightarrow \text{PE}_A = \text{PE}_B + \text{KE}_B = \text{PE}_B + \text{KE}_{B\text{ CM}} + \text{KE}_{B\text{ rot}} \rightarrow$$

$$mgR = mgr + \tfrac{1}{2}m\upsilon_B^2 + \tfrac{1}{2}I\omega_B^2$$

$$= mgr + \tfrac{1}{2}m\upsilon_B^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\left(\frac{\upsilon_B}{r}\right)^2 \rightarrow \upsilon_B = \boxed{\sqrt{\tfrac{10}{7}g(R-r)}}$$

53. The only force doing work in this system is gravity, so mechanical energy is conserved. The initial state of the system is the configuration with m_A on the ground and all objects at rest. The final state of the system has m_B just reaching the ground and all objects in motion. Call the zero level of gravitational potential energy the ground level. Both masses will have the same speed since they are connected by the rope. Assuming that the rope does not slip on the pulley, the angular speed of the pulley is related to the speed of the masses by $\omega = \upsilon/r$. All objects have an initial speed of 0.



$$E_i = E_f \rightarrow$$

$$\tfrac{1}{2}m_A\upsilon_i^2 + \tfrac{1}{2}m_B\upsilon_i^2 + \tfrac{1}{2}I\omega_i^2 + m_Ag\,y_{1i} + m_Bg\,y_{2i} = \tfrac{1}{2}m_A\upsilon_f^2 + \tfrac{1}{2}m_B\upsilon_f^2 + \tfrac{1}{2}I\omega_f^2 + m_Ag\,y_{1f} + m_Bg\,y_{2f}$$

$$m_Bgh = \tfrac{1}{2}m_A\upsilon_f^2 + \tfrac{1}{2}m_B\upsilon_f^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{\upsilon_f^2}{R^2}\right) + m_Agh$$

$$\upsilon_f = \sqrt{\frac{2(m_B - m_A)gh}{\left(m_A + m_B + \tfrac{1}{2}M\right)}} = \sqrt{\frac{2(38.0\text{ kg} - 32.0\text{ kg})(9.80\text{ m/s}^2)(2.5\text{ m})}{\left(38.0\text{ kg} + 32.0\text{ kg} + \left(\tfrac{1}{2}\right)3.1\text{ kg}\right)}} = \boxed{2.0\text{ m/s}}$$

54. Since the lower end of the pole does not slip on the ground, the friction does no work, and mechanical energy is conserved. The initial energy is the potential energy, treating all the mass as though it were at the CM. The final energy is rotational kinetic energy, for rotation about the point of contact with the ground. The linear velocity of the falling tip of the rod is its angular velocity divided by the length.

$$E_{\text{intial}} = E_{\text{final}} \quad \rightarrow \quad \text{PE}_{\text{initial}} = \text{KE}_{\text{final}} \quad \rightarrow \quad mgh = \tfrac{1}{2} I\,\omega^2 \quad \rightarrow \quad mg\,\ell/2 = \tfrac{1}{2}\left(\tfrac{1}{3} m\,\ell^2\right)\left(\upsilon_{\text{end}}/\ell\right)^2 \quad \rightarrow$$

$$\upsilon_{\text{end}} = \sqrt{3g\ell} = \sqrt{3(9.80\text{ m/s}^2)(1.80\text{ m})} = \boxed{7.27\text{ m/s}}$$

55. The angular momentum is given by Eq. 8–18.

$$L = I\omega = MR^2\omega = (0.270\text{ kg})(1.35\text{ m})^2(10.4\text{ rad/s}) = \boxed{5.12\text{ kg}\cdot\text{m}^2/\text{s}}$$

56. (*a*) The angular momentum is given by Eq. 8–18.

$$L = I\omega = \tfrac{1}{2}MR^2\omega = \tfrac{1}{2}(2.8\text{ kg})(0.28\text{ m})^2\left[\left(\frac{1300\text{ rev}}{1\text{ min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]$$

$$= 14.94\text{ kg}\cdot\text{m}^2/\text{s} \approx \boxed{15\text{ kg}\cdot\text{m}^2/\text{s}}$$

(*b*) The torque required is the change in angular momentum per unit time. The final angular momentum is zero.

$$\tau = \frac{L - L_0}{\Delta t} = \frac{0 - 14.94\text{ kg}\cdot\text{m}^2/\text{s}}{6.0\text{ s}} = \boxed{-2.5\text{ m}\cdot\text{N}}$$

The negative sign indicates that the torque is used to oppose the initial angular momentum.

57.(*a*) Consider the person and platform a system for angular momentum analysis. Since the force and torque to raise and/or lower the arms are internal to the system, the raising or lowering of the arms will cause no change in the total angular momentum of the system. However, the rotational inertia increases when the arms are raised. Since angular momentum is conserved, an increase in rotational inertia must be accompanied by a decrease in angular velocity.

(*b*) $L_{\text{i}} = L_{\text{f}} \quad \rightarrow \quad I_{\text{i}}\omega_{\text{i}} = I_{\text{f}}\omega_{\text{f}} \quad \rightarrow \quad I_{\text{f}} = I_{\text{i}}\dfrac{\omega_{\text{i}}}{\omega_{\text{f}}} = I_{\text{i}}\dfrac{0.90\text{ rev/s}}{0.60\text{ rev/s}} = 1.5 I_{\text{i}}$

The rotational inertia has increased by a factor of $\boxed{1.5}$.

58. Since there are no external torques on the system, the angular momentum of the two-disk system is conserved. The two disks have the same final angular velocity.

$$L_{\text{i}} = L_{\text{f}} \quad \rightarrow \quad I\omega + I(0) = 2I\omega_{\text{f}} \quad \rightarrow \quad \boxed{\omega_{\text{f}} = \tfrac{1}{2}\omega}$$

59. There is no net torque on the diver, because the only external force (gravity) passes through the center of mass of the diver. Thus the angular momentum of the diver is conserved. Subscript 1 refers to the tuck position, and subscript 2 refers to the straight position.

$$L_1 = L_2 \quad \rightarrow \quad I_1\omega_1 = I_2\omega_2 \quad \rightarrow \quad \omega_2 = \omega_1\frac{I_1}{I_2} = \left(\frac{2\text{ rev}}{1.5\text{ s}}\right)\left(\frac{1}{3.5}\right) = \boxed{0.38\text{ rev/s}}$$

60. The skater's angular momentum is constant, since no external torques are applied to her.

$$L_{\text{i}} = L_{\text{f}} \quad \rightarrow \quad I_{\text{i}}\omega_{\text{i}} = I_{\text{f}}\omega_{\text{f}} \quad \rightarrow \quad I_{\text{f}} = I_{\text{i}}\frac{\omega_{\text{i}}}{\omega_{\text{f}}} = (4.6\text{ kg}\cdot\text{m}^2)\frac{1.0\text{ rev}/1.5\text{ s}}{2.5\text{ rev/s}} = \boxed{1.2\text{ kg}\cdot\text{m}^2}$$

She accomplishes this by starting with her arms extended (initial angular velocity) and then pulling her arms in to the center of her body (final angular velocity).

61. (*a*) The angular momentum is the moment of inertia (modeling the skater as a cylinder) times the angular velocity.

$$L = I\omega = \tfrac{1}{2}MR^2\omega = \tfrac{1}{2}(48\text{ kg})(0.15\text{ m})^2\left(3.0\,\frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 10.18\text{ kg}\cdot\text{m}^2/\text{s}$$

$$\approx \boxed{1.0\times 10^1\text{ kg}\cdot\text{m}^2/\text{s}}$$

(b) If the rotational inertia does not change, then the change in angular momentum is strictly due to a change in angular velocity.

$$\tau = \frac{\Delta L}{\Delta t} = \frac{I\omega_{\text{final}} - I\omega_0}{\Delta t} = \frac{0 - 10.18\text{ kg}\cdot\text{m}^2/\text{s}}{4.0\text{ s}} = \boxed{-2.5\text{ m}\cdot\text{N}}$$

The negative sign indicates that the torque is in the opposite direction as the initial angular momentum.

62. Since the person is walking radially, no torques will be exerted on the person–platform system, and angular momentum will be conserved. The person is treated as a point mass. Since the person is initially at the center, they have no initial rotational inertia.

(*a*) $L_{\text{i}} = L_{\text{f}} \quad\rightarrow\quad I_{\text{platform}}\omega_{\text{i}} = (I_{\text{platform}} + I_{\text{person}})\omega_{\text{f}}$

$$\omega_{\text{f}} = \frac{I_{\text{platform}}}{I_{\text{platform}} + mR^2}\omega_{\text{i}} = \frac{820\text{ kg}\cdot\text{m}^2}{820\text{ kg}\cdot\text{m}^2 + (75\text{ kg})(3.0\text{ m})^2}(0.95\text{ rad/s}) = 0.5211\text{ rad/s} \approx \boxed{0.52\text{ rad/s}}$$

(*b*) $\text{KE}_{\text{i}} = \tfrac{1}{2}I_{\text{platform}}\omega_{\text{i}}^2 = \tfrac{1}{2}(820\text{ kg}\cdot\text{m}^2)(0.95\text{ rad/s})^2 = \boxed{370\text{ J}}$

$$\text{KE}_{\text{f}} = \tfrac{1}{2}(I_{\text{platform}} + I_{\text{person}})\omega_{\text{f}}^2 = \tfrac{1}{2}(I_{\text{platform}} + m_{\text{person}}r_{\text{person}}^2)\omega_{\text{f}}^2$$

$$= \tfrac{1}{2}[820\text{ kg}\cdot\text{m}^2 + (75\text{ kg})(3.0\text{ m})^2](0.5211\text{ rad/s})^2 = 203\text{ J} \approx \boxed{2.0\times 10^2\text{ J}}$$

63. (*a*) The angular momentum of the combination of merry-go-round (abbreviate mgr) and people will be conserved, because there are no external torques on the combination. This situation is a totally inelastic collision in which the final angular velocity is the same for both the merry-go-round and the people. Subscript 1 represents before the collision, and subscript 2 represents after the collision. The people have no initial angular momentum.

$$L_1 = L_2 \quad\rightarrow\quad I_1\omega_1 = I_2\omega_2 \quad\rightarrow$$

$$\omega_2 = \omega_1\frac{I_1}{I_2} = \omega_1\frac{I_{\text{mgr}}}{I_{\text{mgr}} + I_{\text{people}}} = \omega_1\left[\frac{I_{\text{mgr}}}{I_{\text{mgr}} + 4M_{\text{person}}R^2}\right]$$

$$= (0.80\text{ rad/s})\left[\frac{1360\text{ kg}\cdot\text{m}^2}{1360\text{ kg}\cdot\text{m}^2 + 4(65\text{ kg})(2.1\text{ m})^2}\right] = 0.4341\text{ rad/s} \approx \boxed{0.43\text{ rad/s}}$$

(*b*) If the people jump off the merry-go-round radially, then they exert no torque on the merry-go-round and thus cannot change the angular momentum of the merry-go-round. The merry-go-round would continue to rotate at $\boxed{0.80\text{ rad/s}}$.

64. All parts of the object have the same angular velocity. The moment of inertia is the sum of the rod's moment of inertia and the mass's moment of inertia.

$$L = I\omega = \left[\tfrac{1}{12}M\ell^2 + 2m\left(\tfrac{1}{2}\ell\right)^2\right]\omega = \tfrac{1}{2}\left(\tfrac{1}{6}M + m\right)\ell^2\omega$$

65. The angular momentum of the disk–rod combination will be conserved, because there are no external torques on the combination. This situation is a totally inelastic collision, in which the final angular velocity is the same for both the disk and the rod. Subscript 1 represents before the collision, and subscript 2 represents after the collision. The rod has no initial angular momentum.

$$L_1 = L_2 \;\rightarrow\; I_1\omega_1 = I_2\omega_2 \;\rightarrow$$

$$\omega_2 = \omega_1 \frac{I_1}{I_2} = \omega_1 \frac{I_{\text{disk}}}{I_{\text{disk}} + I_{\text{rod}}} = \omega_1 \left[\frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + \frac{1}{12}M(2R)^2}\right] = (3.3 \text{ rev/s})\left(\frac{3}{5}\right) = \boxed{2.0 \text{ rev/s}}$$

66. Angular momentum will be conserved in the Earth–asteroid system, since all forces and torques are internal to the system. The initial angular velocity of the satellite, just before collision, can be found from $\omega_{\text{asteroid}} = \upsilon_{\text{asteroid}}/R_{\text{Earth}}$. Assuming the asteroid becomes imbedded in the Earth at the surface, the Earth and the asteroid will have the same angular velocity after the collision. We model the Earth as a uniform sphere and the asteroid as a point mass.

$$L_i = L_f \;\rightarrow\; I_{\text{Earth}}\omega_{\text{Earth}} + I_{\text{asteroid}}\omega_{\text{asteroid}} = (I_{\text{Earth}} + I_{\text{asteroid}})\omega_f$$

The moment of inertia of the satellite can be ignored relative to that of the Earth on the right side of the above equation, so the percent change in Earth's angular velocity is found as follows:

$$I_{\text{Earth}}\omega_{\text{Earth}} + I_{\text{asteroid}}\omega_{\text{asteroid}} = I_{\text{Earth}}\omega_f \;\rightarrow\; \frac{(\omega_f - \omega_{\text{Earth}})}{\omega_{\text{Earth}}} = \frac{I_{\text{asteroid}}}{I_{\text{Earth}}}\frac{\omega_{\text{asteroid}}}{\omega_{\text{Earth}}}$$

$$\%\text{ change} = \frac{(\omega_f - \omega_{\text{Earth}})}{\omega_{\text{Earth}}}(100) = \frac{m_{\text{asteroid}}R^2_{\text{Earth}}}{\frac{2}{5}M_{\text{Earth}}R^2_{\text{Earth}}}\frac{\frac{\upsilon_{\text{asteroid}}}{R_{\text{Earth}}}}{\omega_{\text{Earth}}} = \frac{m_{\text{asteroid}}}{\frac{2}{5}M_{\text{Earth}}}\frac{\upsilon_{\text{asteroid}}}{\omega_{\text{Earth}}R_{\text{Earth}}}(100)$$

$$= \frac{(1.0\times10^5 \text{ kg})(3.5\times10^4 \text{ m/s})}{(0.4)(5.97\times10^{24} \text{ kg})\left(\frac{2\pi \text{ rad}}{86,400 \text{ s}}\right)(6.38\times10^6 \text{ m})}(100) = \boxed{(3.2\times10^{-16})\%}$$

67. The angular momentum of the person–turntable system will be conserved. Call the direction of the person's motion the positive rotation direction. Relative to the ground, the person's speed will be $\upsilon + \upsilon_T$, where υ is the person's speed relative to the turntable, and υ_T is the speed of the rim of the turntable with respect to the ground. The turntable's angular speed is $\omega_T = \upsilon_T/R$, and the person's angular speed relative to the ground is $\omega_P = \frac{\upsilon + \upsilon_T}{R} = \frac{\upsilon}{R} + \omega_T$. The person is treated as a point particle for calculation of the moment of inertia.

$$L_i = L_f \;\rightarrow\; 0 = I_T\omega_T + I_P\omega_P = I_T\omega_T + mR^2\left(\omega_T + \frac{\upsilon}{R}\right) \;\rightarrow$$

$$\omega_T = -\frac{mR\upsilon}{I_T + mR^2} = -\frac{(65 \text{ kg})(2.75 \text{ m})(4.0 \text{ m/s})}{1850 \text{ kg}\cdot\text{m}^2 + (65 \text{ kg})(2.75 \text{ m})^2} = -0.3054 \text{ rad/s} \approx \boxed{-0.31 \text{ rad/s}}$$

67. Angular momentum is conserved in the interaction between the child and the merry-go-round.

$$L_{\text{initial}} = L_{\text{final}} \;\rightarrow\; L_{0\,\text{mgr}} = L_{f\,\text{child}} + L_{f\,\text{mgr}} \;\rightarrow\; I_{\text{mgr}}\omega_0 = (I_{\text{mgr}} + I_{\text{child}})\omega = (I_{\text{mgr}} + m_{\text{child}}R^2_{\text{mgr}})\omega \;\rightarrow$$

$$m_{\text{child}} = \frac{I_{\text{mgr}}(\omega_0 - \omega)}{R^2_{\text{mgr}}\omega} = \frac{(1260 \text{ kg}\cdot\text{m}^2)(0.35 \text{ rad/s})}{(2.5 \text{ m})^2(1.35 \text{ rad/s})} = 52.27 \text{ kg} \approx \boxed{52 \text{ kg}}$$

68. The torque is found from $\tau = I\alpha$. The angular acceleration can be found from $\omega = \omega_0 + \alpha t$, and the initial angular velocity is 0. The rotational inertia is that of a cylinder.

$$\tau = I\alpha = \tfrac{1}{2}MR^2\left(\frac{\omega-\omega_0}{t}\right) = 0.5(1.6\text{ kg})(0.20\text{ m})^2\,\frac{(24\text{ rev/s})(2\pi\text{ rad/rev})}{6.0\text{ s}} = \boxed{0.80\text{ m}\cdot\text{N}}$$

69. The linear speed is related to the angular velocity by $\upsilon = \omega R$, and the angular velocity (rad/s) is related to the frequency (rev/s) by $\omega = 2\pi f$. Combine these relationships to find values for the frequency.

$$\omega = 2\pi f = \frac{\upsilon}{R} \;\rightarrow\; f = \frac{\upsilon}{2\pi R};\quad f_1 = \frac{\upsilon}{2\pi R_1} = \frac{1.25\text{ m/s}}{2\pi(0.025\text{ m})}\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{480\text{ rpm}}$$

$$f_2 = \frac{\upsilon}{2\pi R_2} = \frac{1.25\text{ m/s}}{2\pi(0.058\text{ m})}\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{210\text{ rpm}}$$

70. As discussed in Section 8–3 of the textbook, from the reference frame of the axle of the wheel, the points on the wheel are all moving with the same speed of $\upsilon = r\omega$, where υ is the speed of the axle of the wheel relative to the ground. The top of the tire has a velocity of υ to the right relative to the axle, so it has a velocity of 2υ to the right relative to the ground.

$$\vec{\mathbf{v}}_{\substack{\text{top rel}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{top rel}\\\text{center}}} + \vec{\mathbf{v}}_{\substack{\text{center rel}\\\text{ground}}} = (\upsilon\text{ to the right}) + (\upsilon\text{ to the right}) = 2\upsilon\text{ to the right}$$

$$\upsilon_{\substack{\text{top rel}\\\text{ground}}} = 2\upsilon = 2(\upsilon_0 + at) = 2at = 2(1.00\text{ m/s}^2)(2.25\text{ s}) = \boxed{4.50\text{ m/s}}$$

71. Assume that the angular acceleration is uniform. Then the torque required to whirl the rock is the moment of inertia of the rock (treated as a particle) times the angular acceleration.

$$\tau = I\alpha = (mr^2)\left(\frac{\omega-\omega_0}{t}\right) = \frac{(0.60\text{ kg})(1.5\text{ m})^2}{5.0\text{ s}}\left[\left(75\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right] = \boxed{2.1\text{ m}\cdot\text{N}}$$

That torque comes from the arm swinging the sling and is generated by the arm muscles.

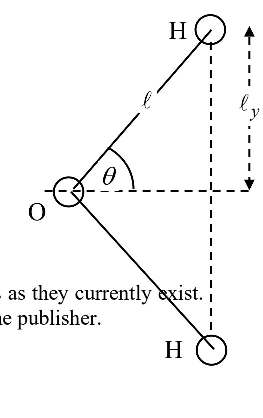
72. (*a*) The linear speed of the chain must be the same as it passes over both sprockets. The linear speed is related to the angular speed by $\upsilon = \omega R$, so $\omega_R R_R = \omega_F R_F$. If the spacing of the teeth on the sprockets is a distance *d*, then the number of teeth on a sprocket times the spacing distance must give the circumference of the sprocket.

$$Nd = 2\pi R \text{ so } R = \frac{Nd}{2\pi}.\text{ Thus } \omega_R\frac{N_R d}{2\pi} = \omega_F\frac{N_F d}{2\pi} \;\rightarrow\; \boxed{\frac{\omega_R}{\omega_F} = \frac{N_F}{N_R}}.$$

(*b*) $\boxed{\omega_R/\omega_F = 52/13 = 4.0}$

(*c*) $\boxed{\omega_R/\omega_F = 42/28 = 1.5}$

73. The mass of a hydrogen atom is 1.01 atomic mass units. The atomic mass unit is 1.66×10^{-27} kg. Since the axis passes through the oxygen atom, the oxygen atom will have no rotational inertia.



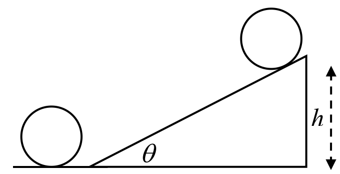
(*a*) If the axis is perpendicular to the plane of the molecule, then each hydrogen atom is a distance ℓ from the axis of rotation.

$$I_{\text{perp}} = 2m_{\text{H}}\ell^2 = 2(1.01)(1.66\times10^{-27}\text{ kg})(0.096\times10^{-9}\text{ m})^2$$
$$= \boxed{3.1\times10^{-47}\text{ kg}\cdot\text{m}^2}$$

(*b*) If the axis is in the plane of the molecule, bisecting the H—O—H bonds, each hydrogen atom is a distance of $\ell_y = \ell \sin\theta = (9.6\times10^{-11}\text{ m})\sin 52° = 7.564\times10^{-11}$ m. Thus the moment of inertia is as follows:

$$I_{\text{plane}} = 2m_H\ell_y^2 = 2(1.01)(1.66\times10^{-27}\text{ kg})(7.564\times10^{-11}\text{ m})^2 = \boxed{1.9\times10^{-47}\text{ kg}\cdot\text{m}^2}$$

74. (*a*) Assuming that there are no dissipative forces doing work, conservation of mechanical energy may be used to find the final height *h* of the hoop. Take the bottom of the incline to be the zero level of gravitational potential energy. We assume that the hoop is rolling without sliding, so that $\omega = \upsilon/R$. Relate the conditions at the bottom of the incline to the conditions at the top by conservation of energy. The hoop has both translational and rotational kinetic energy at the bottom, and the rotational inertia of the hoop is given by $I = mR^2$.



$$E_{\text{bottom}} = E_{\text{top}} \rightarrow \tfrac{1}{2}m\upsilon^2 + \tfrac{1}{2}I\omega^2 = mgh \rightarrow \tfrac{1}{2}m\upsilon^2 + \tfrac{1}{2}mR^2\frac{\upsilon^2}{R^2} = mgh \rightarrow$$

$$h = \frac{\upsilon^2}{g} = \frac{(3.0\text{ m/s})^2}{9.80\text{ m/s}^2} = 0.9184\text{ m}$$

The distance along the plane is given by $d = \dfrac{h}{\sin\theta} = \dfrac{0.9184\text{ m}}{\sin 15°} = 3.548\text{ m} \approx \boxed{3.5\text{ m}}$

(*b*) The time can be found from the constant acceleration of the linear motion.

$$\Delta x = \tfrac{1}{2}(\upsilon + \upsilon_0)t \rightarrow t = \frac{2\Delta x}{\upsilon + \upsilon_0} = \frac{2(3.548\text{ m})}{0 + 3.0\text{ m/s}} = 2.365\text{ s}$$

This is the time to go up the plane. The time to come back down the plane is the same, so the total time is $\boxed{4.7\text{ s}}$.

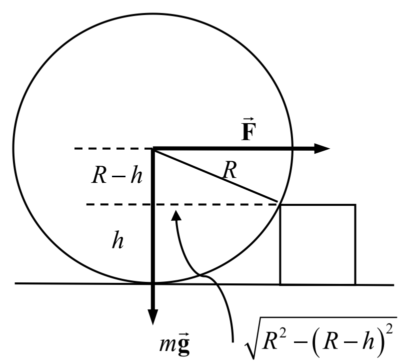
75. (*a*) For the daily rotation about its axis, treat the Earth as a uniform sphere, with an angular frequency of one revolution per day.

$$L_{\text{daily}} = I\omega_{\text{daily}} = \left(\tfrac{2}{5}MR_{\text{Earth}}^2\right)\omega_{\text{daily}}$$
$$= \tfrac{2}{5}(5.98\times10^{24}\text{ kg})(6.38\times10^6\text{ m})^2\left[\left(\frac{2\pi\text{ rad}}{1\text{ day}}\right)\left(\frac{1\text{ day}}{86{,}400\text{ s}}\right)\right] = \boxed{7.08\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s}}$$

(*b*) For the yearly revolution about the Sun, treat the Earth as a particle, with an angular frequency of one revolution per year.

$$L_{\text{daily}} = I\omega_{\text{daily}} = \left(MR_{\substack{\text{Sun-}\\\text{Earth}}}^2\right)\omega_{\text{daily}}$$
$$= (5.98\times10^{24}\text{ kg})(1.496\times10^{11}\text{ m})^2\left[\left(\frac{2\pi\text{ rad}}{365\text{ day}}\right)\left(\frac{1\text{ day}}{86{,}400\text{ s}}\right)\right] = \boxed{2.67\times10^{40}\text{ kg}\cdot\text{m}^2/\text{s}}$$

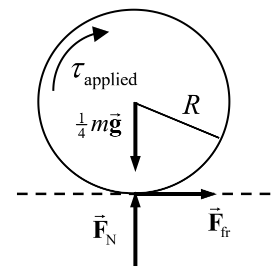
76. The wheel is rolling about the point of contact with the step, so all torques are to be taken about that point. As soon as the wheel is off the floor, there will be only two forces that can exert torques on the wheel: the pulling force and the force of gravity. There will not be a normal force of contact between the wheel and the floor once the wheel is off the floor, and any force on the wheel from the point of the step cannot exert a torque about that very point. Calculate the net torque on the wheel, with clockwise torques positive. The minimum force occurs when the net torque is 0.



$$\sum \tau = F(R-h) - mg\sqrt{R^2-(R-h)^2} = 0$$

$$F = \frac{Mg\sqrt{R^2-(R-h)^2}}{R-h} = \boxed{\frac{Mg\sqrt{2Rh-h^2}}{R-h}}$$

77. Each wheel supports ¼ of the weight of the car. For rolling without slipping, there will be static friction between the wheel and the pavement. For the wheel to be on the verge of slipping, there must be an applied torque that is equal to the torque supplied by the static frictional force. We take counterclockwise torques to the right in the diagram. The bottom wheel would be moving to the left relative to the pavement if it started to slip, so the frictional force is to the right. See the free-body diagram.



$$\tau_{\substack{\text{applied}\\\text{min}}} = \tau_{\substack{\text{static}\\\text{friction}}} = RF_{\text{fr}} = R\mu_{\text{s}}F_{\text{N}} = R\mu_{\text{s}}\,\tfrac{1}{4}mg$$

$$= \tfrac{1}{4}(0.33\text{ m})(0.65)(1080\text{ kg})(9.80\text{ m/s}^2) = \boxed{570\text{ m}\cdot\text{N}}$$

78. (*a*) The kinetic energy of the system is the kinetic energy of the two masses, since the rod is treated as massless. Let A represent the heavier mass and B the lighter mass.

$$\text{KE} = \tfrac{1}{2}I_{\text{A}}\omega_{\text{A}}^2 + \tfrac{1}{2}I_{\text{B}}\omega_{\text{B}}^2 = \tfrac{1}{2}m_{\text{A}}r_{\text{A}}^2\omega_{\text{A}}^2 + \tfrac{1}{2}m_{\text{B}}r_{\text{B}}^2\omega_{\text{A}}^2 = \tfrac{1}{2}r^2\omega^2(m_{\text{A}}+m_{\text{B}})$$

$$= \tfrac{1}{2}(0.210\text{ m})^2(5.60\text{ rad/s})^2(7.00\text{ kg}) = \boxed{4.84\text{ J}}$$

(*b*) The net force on each object produces centripetal motion so can be expressed as $mr\omega^2$.

$$F_{\text{A}} = m_{\text{A}}r_{\text{A}}\omega_{\text{A}}^2 = (4.00\text{ kg})(0.210\text{ m})(5.60\text{ rad/s})^2 = \boxed{26.3\text{ N}}$$

$$F_{\text{B}} = m_{\text{B}}r_{\text{B}}\omega_{\text{B}}^2 = (3.00\text{ kg})(0.210\text{ m})(5.60\text{ rad/s})^2 = \boxed{19.8\text{ N}}$$

These forces are exerted by the rod. Since they are unequal, there would be a net horizontal force on the rod (and hence the axle) due to the masses. This horizontal force would have to be counteracted by the mounting for the rod and axle in order for the rod not to move horizontally. There is also a gravity force on each mass, balanced by a vertical force from the rod so that there is no net vertical force on either mass.

79. Note the similarity between this problem and MisConceptual Questions 10 and 11. There is no torque applied to the block, so its angular momentum would remain constant. The angular velocity is the speed of the block divided by the radius of the string. The moment of inertia of the block about the center of its motion is $\frac{1}{2}mr^2$.

$$I_1\omega_1 = I_2\omega_2 \quad\rightarrow\quad \tfrac{1}{2}mr_1^2\frac{\upsilon_1}{r_1} = \tfrac{1}{2}mr_2^2\frac{\upsilon_2}{r_2} \quad\rightarrow\quad r_2\upsilon_2 = r_1\upsilon_1 \quad\rightarrow$$

$$\upsilon_2 = \upsilon_1\frac{r_1}{r_2} = (2.4\text{ m/s})\left(\frac{0.80\text{ m}}{0.48\text{ m}}\right) = \boxed{4.0\text{ m/s}}$$

80. (*a*) The force of gravity acting through the CM will cause a clockwise torque, which produces an angular acceleration. At the moment of release, the force of gravity is perpendicular to the lever arm from the hinge to the CM.

$$\tau = I\alpha \quad \rightarrow \quad \alpha = \frac{\tau_{\text{gravity}}}{I_{\text{rod about end}}} = \frac{Mg\ell/2}{\frac{1}{3}M\ell^2} = \boxed{\frac{3g}{2\ell}}$$

(*b*) At the end of the rod, there is a tangential acceleration equal to the angular acceleration times the distance from the hinge. There is no radial acceleration, because at the moment of release, the speed of the end of the rod is 0. Thus, the tangential acceleration is the entire linear acceleration.

$$a_{\text{linear}} = a_{\tan} = \alpha\ell = \boxed{\tfrac{3}{2}g}$$

Note that this is bigger than the free-fall acceleration of *g*.

81. (*a*) We assume that no angular momentum is in the thrown-off mass, so the final angular momentum of the neutron star is equal to the angular momentum before collapse.

$$L_0 = L_f \quad \rightarrow \quad I_0\omega_0 = I_f\omega_f \quad \rightarrow \quad \left[\tfrac{2}{5}(8.0M_{\text{Sun}})R_{\text{Sun}}^2\right]\omega_0 = \left[\tfrac{2}{5}\left(\tfrac{1}{4}8.0M_{\text{Sun}}\right)R_f^2\right]\omega_f \quad \rightarrow$$

$$\omega_f = \frac{\left[\tfrac{2}{5}(8.0M_{\text{Sun}})R_{\text{Sun}}^2\right]}{\left[\tfrac{2}{5}\left(\tfrac{1}{4}8.0M_{\text{Sun}}\right)R_f^2\right]}\omega_0 = \frac{4R_{\text{Sun}}^2}{R_f^2}\omega_0 = \frac{4(6.96\times10^8\text{ m})^2}{(12\times10^3\text{ m})^2}\left(\frac{1.0\text{ rev}}{9.0\text{ days}}\right)$$

$$= (1.495\times10^9\text{ rev/day})\left(\frac{1\text{ day}}{86{,}400\text{ s}}\right) = 1.730\times10^4\text{ rev/s} \approx \boxed{17{,}000\text{ rev/s}}$$

(*b*) Now we assume that the final angular momentum of the neutron star is only $\frac{1}{4}$ of the angular momentum before collapse. Since the rotation speed is directly proportional to angular momentum, the final rotation speed will be $\frac{1}{4}$ of that found in part (*a*).

$$\omega_f = \tfrac{1}{4}(1.730\times10^4\text{ rev/s}) = \boxed{4300\text{ rev/s}}$$

82. Since the spool rolls without slipping, each point on the edge of the spool moves with a speed of $\upsilon = r\omega = \upsilon_{\text{CM}}$ relative to the center of the spool, where υ_{CM} is the speed of the center of the spool relative to the ground. Since the spool is moving to the right relative to the ground, and the top of the spool is moving to the right relative to the center of the spool, the top of the spool is moving with a speed of $2\upsilon_{\text{CM}}$ relative to the ground. This is the speed of the rope, assuming it is unrolling without slipping and is at the outer edge of the spool. The speed of the rope is the same as the speed of the person, since the person is holding the rope. So the person is walking with a speed of twice that of the center of the spool. Thus if the person moves forward a distance ℓ, in the same time the center of the spool, traveling with half the speed, moves forward a distance $\boxed{\ell/2}$. The rope, to stay connected both to the person and to the spool, must therefore unwind by an amount $\boxed{\ell/2}$ also.

83. The spin angular momentum of the Moon can be calculated by $L_{\text{spin}} = I_{\text{spin}}\omega_{\text{spin}} = \frac{2}{5}MR_{\text{Moon}}^2\omega_{\text{spin}}$. The orbital angular momentum can be calculated by $L_{\text{orbit}} = I_{\text{orbit}}\omega_{\text{orbit}} = MR_{\text{orbit}}^2\omega_{\text{orbit}}$. Because the same side of the Moon always faces the Earth, $\omega_{\text{spin}} = \omega_{\text{orbit}}$.

$$\frac{L_{\text{spin}}}{L_{\text{orbit}}} = \frac{\frac{2}{5}MR_{\text{Moon}}^2\omega_{\text{spin}}}{MR_{\text{orbit}}^2\omega_{\text{orbit}}} = \frac{2}{5}\left(\frac{R_{\text{Moon}}}{R_{\text{orbit}}}\right)^2 = 0.4\left(\frac{1.74\times10^6\text{ m}}{3.84\times10^8\text{ m}}\right)^2 = \boxed{8.21\times10^{-6}}$$

84. We calculate spin angular momentum for the Sun and orbital angular momentum for the planets, treating them as particles relative to the size of their orbits. The angular velocities are calculated from $\omega = \frac{2\pi}{T}$.

$$L_{\text{Sun}} = I_{\text{Sun}}\omega_{\text{Sun}} = \tfrac{2}{5}M_{\text{Sun}}R_{\text{Sun}}^2\frac{2\pi}{T_{\text{Sun}}} = \tfrac{2}{5}(1.99\times 10^{30}\text{ kg})(6.96\times 10^{8}\text{ m})^2\frac{2\pi}{(25\text{ days})}\left(\frac{1\text{ day}}{86{,}400\text{ s}}\right)$$

$$= 1.1217\times 10^{42}\text{ kg}\cdot\text{m/s}$$

$$L_{\text{Jupiter}} = M_{\text{Jupiter}}R_{\substack{\text{Jupiter}\\\text{orbit}}}^2\frac{2\pi}{T_{\text{Jupiter}}} = (190\times 10^{25}\text{ kg})(778\times 10^{9}\text{ m})^2\frac{2\pi}{(11.9\text{ yr})}\left(\frac{1\text{ yr}}{3.156\times 10^{7}\text{ s}}\right)$$

$$= 1.9240\times 10^{43}\text{ kg}\cdot\text{m/s}$$

In a similar fashion, we calculate the other planetary orbital angular momenta.

$$L_{\text{Saturn}} = M_{\text{Saturn}}R_{\substack{\text{Saturn}\\\text{orbit}}}^2\frac{2\pi}{T_{\text{Saturn}}} = 7.806\times 10^{42}\text{ kg}\cdot\text{m/s}$$

$$L_{\text{Uranus}} = M_{\text{Uranus}}R_{\substack{\text{Uranus}\\\text{orbit}}}^2\frac{2\pi}{T_{\text{Uranus}}} = 1.695\times 10^{42}\text{ kg}\cdot\text{m/s}$$

$$L_{\text{Neptune}} = M_{\text{Neptune}}R_{\substack{\text{Neptune}\\\text{orbit}}}^2\frac{2\pi}{T_{\text{Neptune}}} = 2.492\times 10^{42}\text{ kg}\cdot\text{m/s}$$

$$f = \frac{L_{\text{planets}}}{L_{\text{planets}} + L_{\text{Sun}}} = \frac{(19.240 + 7.806 + 1.695 + 2.492)\times 10^{42}\text{ kg}\cdot\text{m/s}}{(19.240 + 7.806 + 1.695 + 2.492 + 1.122)\times 10^{42}\text{ kg}\cdot\text{m/s}} = \boxed{0.965}$$

85. (*a*) The angular momentum delivered to the waterwheel is that lost by the water.

$$\Delta L_{\text{wheel}} = -\Delta L_{\text{water}} = L_{\substack{\text{initial}\\\text{water}}} - L_{\substack{\text{final}\\\text{water}}} = m\upsilon_1 R - m\upsilon_2 R \quad\rightarrow$$

$$\frac{\Delta L_{\text{wheel}}}{\Delta t} = \frac{m\upsilon_1 R - m\upsilon_2 R}{\Delta t} = \frac{mR}{\Delta t}(\upsilon_1 - \upsilon_2) = (85\text{ kg/s})(3.0\text{ m})(3.2\text{ m/s}) = 816\text{ kg}\cdot\text{m}^2/\text{s}^2$$

$$\approx \boxed{820\text{ kg}\cdot\text{m}^2/\text{s}^2}$$

(*b*) The torque is the rate of change of angular momentum, from Eq. 8–19.

$$\tau_{\substack{\text{on}\\\text{wheel}}} = \frac{\Delta L_{\text{wheel}}}{\Delta t} = 816\text{ kg}\cdot\text{m}^2/\text{s}^2 = 816\text{ m}\cdot\text{N} \approx \boxed{820\text{ m}\cdot\text{N}}$$

(*c*) Power is given by $P = \tau\omega$. See the text immediately after Eq. 8–17.

$$P = \tau\omega = (816\text{ m}\cdot\text{N})\left(\frac{2\pi\text{ rev}}{5.5\text{ s}}\right) = \boxed{930\text{ W}}$$

Solutions to Search and Learn Problems

1. The radian is defined as the ratio of the distance traveled along an arc divided by the radius of the arc. When an angle in radians is multiplied by the radius the result is a distance. Therefore, when angular speed (which is angular displacement divided by time) is multiplied by the radius the result is the

displacement along the arc divided by time, which is a linear speed. Degrees and revolutions are not defined in terms of arc lengths and cannot be used in the same way.

2. The angle in radians is the diameter of the object divided by the distance to the object.

$$\Delta\theta_{\text{Sun}} = \frac{2R_{\text{Sun}}}{r_{\text{Earth-Sun}}} = \frac{2(6.96\times10^5\text{ km})}{149.6\times10^6\text{ km}} = \boxed{9.30\times10^{-3}\text{ rad}}$$

$$\Delta\theta_{\text{Moon}} = \frac{2R_{\text{Moon}}}{r_{\text{Earth-Moon}}} = \frac{2(1.74\times10^3\text{ km})}{384\times10^3\text{ km}} = \boxed{9.06\times10^{-3}\text{ rad}}$$

Since these angles are practically the same (only a 2.6% difference), solar eclipses can occur. Based on these values, the Sun would never be completely obscured. But since the orbits are not perfect circles but are ellipses, the above values are just averages. Full (total) solar eclipses do occur.

3. (*a*) We use conservation of energy to determine the speed of each sphere as a function of position on the incline. The sphere with the greater speed would reach the bottom of the incline first. Potential energy will be zero at the base of the incline ($y = 0$) and the initial height will be H. We take position 1 to be at the top of the incline and position 2 to be at a generic location along the incline.

$$\text{KE}_1 + \text{PE}_1 = \text{KE}_2 + \text{PE}_2 \quad \rightarrow \quad 0 + mgH = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mgy$$

$$mg(H-y) = \tfrac{1}{2}\left(mv^2 + \tfrac{1}{2}mr^2\omega^2\right) = \tfrac{1}{2}\left(mv^2 + \tfrac{2}{5}mr^2\frac{v^2}{r^2}\right) = \tfrac{1}{2}\left(\tfrac{7}{5}mv^2\right) \rightarrow$$

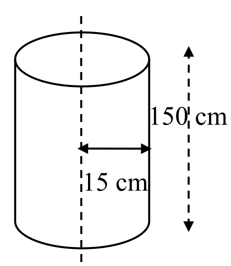
$$v = \sqrt{\tfrac{10}{7}g(H-y)}$$

The velocity along the incline does not depend upon either the mass or the radius of the sphere. Therefore, both spheres have the same speed at each point along the incline, and $\boxed{\text{both will reach the bottom of the incline at the same time.}}$

(*b*) As shown in part (*a*), both spheres will have the same speed at each point along the incline, so $\boxed{\text{both will have the same speed at the bottom of the incline.}}$

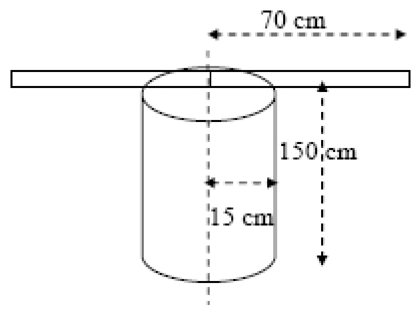
(*c*) By conservation of energy, the total kinetic energy at the bottom of the incline will equal the potential energy at the top of the incline. The initial potential energy is proportional to the mass of each sphere, $\boxed{\text{so the more massive sphere will have the greater kinetic energy}}$. The total kinetic energy is independent of the spheres' radii.

4. Assume a mass of 50 kg, corresponding to a weight of about 110 lb. From Table 7–1, we find that the total arm and hand mass is about 12.5% of the total mass, so the rest of the body is about 87.5% of the total mass. Model the skater as a cylinder of mass 44 kg, and model each arm as a thin rod of mass 3 kg. Estimate the body as 150 cm tall with a radius of 15 cm. Estimate the arm dimension as 70 cm long.



With the arms held tightly, we approximate that the arms are part of the body cylinder. A sketch of the skater in this configuration is then as shown in the first diagram (not to scale). In this configuration, the rotational inertia is

$$I_{\text{in}} = I_{\text{cylinder}} = \tfrac{1}{2}M_{\substack{\text{total}\\\text{body}}}R_{\text{body}}^2$$



With the skater's arms extended, the second diagram applies. In this configuration, the rotational inertia is

$$I_{\text{out}} = I_{\text{body}} + I_{\text{arms}} = \tfrac{1}{2}M_{\text{body}}R_{\text{body}}^2 + 2\left(\tfrac{1}{3}M_{\text{arm}}\right)L_{\text{arm}}^2$$

The forces and torques involved in changing the configuration of the skater are internal to the skater, so the skater's angular momentum is conserved during a configuration change. Thus,

$$L_{\text{in}} = L_{\text{out}} \quad \to \quad I_{\text{in}}\omega_{\text{in}} = I_{\text{out}}\omega_{\text{out}} \quad \to$$

$$\frac{\omega_{\text{out}}}{\omega_{\text{in}}} = \frac{I_{\text{in}}}{I_{\text{out}}} = \frac{\tfrac{1}{2}M_{\substack{\text{total}\\\text{body}}}R_{\text{body}}^2}{\tfrac{1}{2}M_{\text{body}}R_{\text{body}}^2 + 2\left(\tfrac{1}{3}M_{\text{arm}}\right)L_{\text{arm}}^2} = \frac{\tfrac{1}{2}(50\text{ kg})(0.15\text{ m})^2}{\tfrac{1}{2}(44\text{ kg})(0.15\text{ m})^2 + 2\left(\tfrac{1}{3}\right)(3\text{ kg})(0.70\text{ m})^2}$$

$$= 0.381 \approx \boxed{0.4}$$

Alternatively, we would have that $\omega_{\text{in}}/\omega_{\text{out}} = (0.381)^{-1} = 2.6,$ so the skater spins about 2.6 × faster with the arms pulled in.

5. (*a*) The initial energy of the flywheel is used for two purposes: to give the car translational kinetic energy 30 times, and to replace the energy lost due to friction, from air resistance and from braking. The statement of the problem leads us to ignore any gravitational potential energy changes.

$$W_{\text{fr}} = \text{KE}_{\text{final}} - \text{KE}_{\text{initial}} \quad \to \quad F_{\text{fr}}\Delta x\,\cos 180^\circ = \tfrac{1}{2}M_{\text{car}}v_{\text{car}}^2 - \text{KE}_{\text{flywheel}} \quad \to$$

$$\text{KE}_{\text{flywheel}} = F_{\text{fr}}\Delta x + \tfrac{1}{2}M_{\text{car}}v_{\text{car}}^2$$

$$= (450\text{ N})(3.5\times10^5\text{ m}) + (30)\tfrac{1}{2}(1100\text{ kg})\left[(95\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2$$

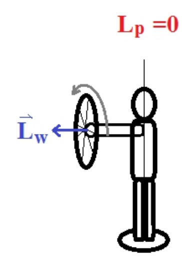
$$= 1.690\times10^8\text{ J} \approx \boxed{1.7\times10^8\text{ J}}$$

(*b*) $\text{KE}_{\text{flywheel}} = \tfrac{1}{2}I\omega^2,$ so

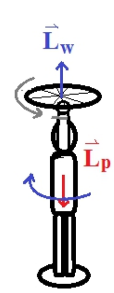
$$\omega = \sqrt{\frac{2\text{ KE}}{\text{I}}} = \sqrt{\frac{2\text{ KE}}{\tfrac{1}{2}M_{\text{flywheel}}R_{\text{flywheel}}^2}} = \sqrt{\frac{2(1.690\times10^8\text{ J})}{\tfrac{1}{2}(270\text{ kg})(0.75\text{ m})^2}} = 2110\text{ rad/s} \approx \boxed{2100\text{ rad/s}}$$

(*c*) To find the time, use the relationship that $\text{power} = \dfrac{\text{work}}{t},$ where the work done by the motor will be equal to the kinetic energy of the flywheel.

$$P = \frac{W}{t} \quad \to \quad t = \frac{W}{P} = \frac{(1.690\times10^8\text{ J})}{(150\text{ hp})(746\text{ W/hp})} = 1.510\times10^3\text{ s}\left(\frac{1\text{ min}}{60\text{ s}}\right) \approx \boxed{25\text{ min}}$$



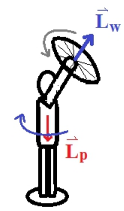
6. When the person and the platform rotate, they do so about the vertical axis. Initially there is no angular momentum pointing along the vertical axis, so any change that the person–wheel–platform undergoes must result in no net angular momentum along the vertical axis. The first diagram shows this condition.



(*a*) Now consider the next diagram. If the wheel is moved so that its angular momentum points upward, then the person and platform must get an equal but opposite angular momentum, which will point downward. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.

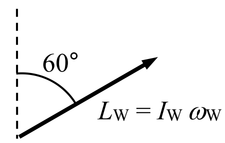
$$L_i = L_f \quad \to \quad 0 = I_W \omega_W + I_P \omega_P \quad \to \quad \boxed{\omega_P = -\frac{I_W}{I_P}\omega_W}$$

The negative sign means that the platform is rotating in the opposite direction of the wheel. If the wheel is spinning counterclockwise when viewed from above, the platform is spinning clockwise.



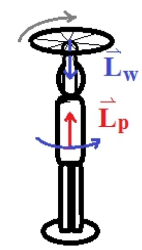
(*b*) Now consider the next diagram. If the wheel is pointing at a 60° angle to the vertical, then the component of its angular momentum that is along the vertical direction is $I_W \omega_W \cos 60°$. Also see the simple vector diagram below the adjacent diagram. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.

$$L_i = L_f \quad \to \quad 0 = I_W \omega_W \cos 60° + I_P \omega_P \quad \to \quad \boxed{\omega_P = -\frac{I_W}{2I_P}\omega_W}$$



Again, the negative sign means that the platform is rotating in the opposite direction of the wheel.

(*c*) Consider the final diagram. If the wheel is moved so that its angular momentum points downward, then the person and platform must get an equal but opposite angular momentum, which will point upward. Write the angular momentum conservation condition for the vertical direction to solve for the angular velocity of the platform.



$$L_i = L_f \quad \to \quad 0 = I_W \omega_W + I_P \omega_P \quad \to \quad \boxed{\omega_P = -\omega_W I_W / I_P}$$

The platform is again rotating in the opposite direction of the wheel. If the wheel is now spinning clockwise when viewed from above, the platform is spinning counterclockwise.

(*d*) Since the total angular momentum is 0, if the wheel is stopped from rotating, the platform will also stop. Thus $\boxed{\omega_P = 0}$.