7

LINEAR MOMENTUM

Responses to Questions

1. For momentum to be conserved, the system under analysis must be "closed"—not have any forces on it from outside the system. A coasting car has air friction and road friction on it, for example, which are "outside" or "external" forces and thus reduce the momentum of the car. If the ground and the air were considered part of the system and their velocities analyzed, then the momentum of the entire system would be conserved, but not necessarily the momentum of any single component, like the car.

2. The momentum of an object can be expressed in terms of its kinetic energy, as follows:

$$p = m\upsilon = \sqrt{m^2\upsilon^2} = \sqrt{m(m\upsilon^2)} = \sqrt{2m\left(\tfrac{1}{2}m\upsilon^2\right)} = \sqrt{2m\,\mathrm{KE}}$$

   Thus if two objects have the same kinetic energy, then the one with more mass has the greater momentum.

3. Consider this problem as a very light object hitting and sticking to a very heavy object. The large object–small object combination (Earth + jumper) would have some momentum after the collision, but due to the very large mass of the Earth, the velocity of the combination is so small that it is not measurable. Thus the jumper lands on the Earth, and nothing more happens.

4. When you release an inflated but untied balloon at rest, the gas inside the balloon (at high pressure) rushes out the open end of the balloon. That escaping gas and the balloon form a closed system, so the momentum of the system is conserved. The balloon and remaining gas acquire a momentum equal and opposite to the momentum of the escaping gas, so they move in the opposite direction to the escaping gas.

5. As the fish swishes its tail back and forth, it moves some water backward, away from the fish. If we consider the system to be the fish and the water, then, from conservation of momentum, the fish must move forward.

6. (*d*) The girl moves in the opposite direction at 2.0 m/s. Since there are no external forces on the pair, momentum is conserved. The initial momentum of the system (boy and girl) is zero. The final momentum of the girl must be the same in magnitude and opposite in direction to the final momentum of the boy so that the net final momentum is also zero.

7. The air bag greatly increases the amount of time over which the stopping force acts on the driver. If a hard object like a steering wheel or windshield is what stops the driver, then a large force is exerted over a very short time. If a soft object like an air bag stops the driver, then a much smaller force is exerted over a much longer time. For instance, if the air bag is able to increase the time of stopping by a factor of 10, then the average force on the person will be decreased by a factor of 10. This greatly reduces the possibility of serious injury or death.

8. Yes. In a perfectly elastic collision, kinetic energy is conserved. In the Earth–ball system, the kinetic energy of the Earth after the collision is negligible, so the ball has the same kinetic energy leaving the floor as it had hitting the floor. The height from which the ball is released determines its potential energy, which is converted to kinetic energy as the ball falls. If it leaves the floor with this same amount of kinetic energy and a velocity upward, it will rise to the same height as it originally had as the kinetic energy is converted back into potential energy.

9. In order to conserve momentum, when the boy dives off the back of the rowboat the boat will move forward.

10. He could have thrown the coins in the direction opposite the shore he was trying to reach. Since the lake is frictionless, momentum would be conserved and he would "recoil" from the throw with a momentum equal in magnitude and opposite in direction to the coins. Since his mass is greater than the mass of the coins, his speed would be less than the speed of the coins, but, since there is no friction, he would maintain this small speed until he hit the shore.

11. When the tennis ball rebounds from a stationary racket, it reverses its component of velocity perpendicular to the racket with very little energy loss. If the ball is hit straight on, and the racket is actually moving forward, the ball can be returned with an energy (and a speed) equal to (or even greater than) the energy it had when it was served.

12. Yes. Impulse is the product of the force and the time over which it acts. A small force acting over a longer time could impart a greater impulse than a large force acting over a shorter time.

13. The collision in which the two cars rebound would probably be more damaging. In the case of the cars rebounding, the change in momentum of each car is greater than in the case in which they stick together, because each car is not only brought to rest but also sent back in the direction from which it came. A greater impulse results from a greater force, so most likely more damage would occur.

14. (*a*) The momentum of the ball is not conserved during any part of the process, because there is an external force acting on the ball at all times—the force of gravity. And there is an upward force on the ball during the collision. So considering the ball as the system, there are always external forces on it, so its momentum is not conserved.
    (*b*) With this definition of the system, all of the forces are internal, so the momentum of the Earth–ball system is conserved during the entire process.
    (*c*) For a piece of putty falling and sticking to a steel plate, if the system is the putty and the Earth, momentum is conserved for the entire path.

15. "Crumple zones" are similar to air bags in that they increase the time of interaction during a collision, and therefore lower the average force required for the change in momentum that the car undergoes in the collision.

16. For maximum power, the turbine blades should be designed so that the water rebounds. The water has a greater change in momentum if it rebounds than if it just stops at the turbine blade. If the water has a greater change in momentum, then, by conservation of momentum, the turbine blades also have a greater change in momentum and will therefore spin faster.

17. (*a*) The direction of the change in momentum of the ball is perpendicular to the wall and away from it, or to the left in the figure.
    (*b*) Since the force on the wall is opposite that on the ball, the force on the wall is to the right.

18. From Eq. 7–7 for a 1-D elastic collision, $\upsilon_A - \upsilon_B = \upsilon'_B - \upsilon'_A$. Let A represent the bat, and let B represent the ball. The positive direction will be the (assumed horizontal) direction that the bat is moving when the ball is hit. We assume that the batter can swing the bat with equal strength in either case, so that υ_A is the same in both pitching situations. Because the bat is so much heavier than the ball, we assume that $\upsilon'_A \approx \upsilon_A$ —the speed of the bat doesn't change significantly during the collision. Then the velocity of the baseball after being hit is $\upsilon'_B = \upsilon'_A + \upsilon_A - \upsilon_B \approx 2\upsilon_A - \upsilon_B$. If $\upsilon_B = 0$, the ball tossed up into the air by the batter, then $\upsilon'_B \approx 2\upsilon_A$ —the ball moves away with twice the speed of the bat. But if $\upsilon_B < 0$, the pitched ball situation, we see that the magnitude of $\upsilon'_B > 2\upsilon_A$, so the ball moves away with greater speed. If, for example, the pitching speed of the ball was about twice the speed at which the batter could swing the bat, then we would have $\upsilon'_B \approx 4\upsilon_A$. Thus the ball has greater speed after being struck, so the ball will travel farther after being hit. This is similar to the "gravitational slingshot" effect discussed in Search and Learn 4.

19. A perfectly inelastic collision between two objects that initially had momenta equal in magnitude but opposite in direction would result in all the kinetic energy being lost. For instance, imagine sliding two clay balls with equal masses and speeds toward each other across a frictionless surface. Since the initial momentum of the system is zero, the final momentum must be zero as well. The balls stick together, so the only way the final momentum can be zero is if they are brought to rest. In this case, all the kinetic energy would be lost. A simpler situation is dropping a ball of clay onto the floor. The clay doesn't rebound after the collision with the floor, and all of the kinetic energy is lost.

20. Passengers may be told to sit in certain seats in order to balance the plane. If they move during the flight, they could change the position of the center of mass of the plane and affect its stability in flight.

21. In order to maintain balance, your CM must be located directly above your feet. If you have a heavy load in your arms, your CM will be out in front of your body and not above your feet. So you lean backward to get your CM directly above your feet. Otherwise, you might fall over forward.

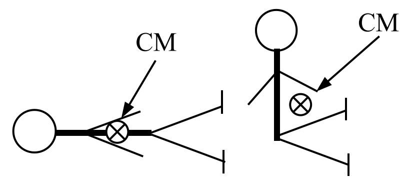
22. The 1-m length of pipe is uniform—it has the same density throughout, so its CM is at its geometric center, which is its midpoint. The arm and leg are not uniform—they are more dense where there is muscle, primarily in the parts that are closest to the body. Thus the CM of the arm or leg is closer to the body than the geometric center. The CM is located closer to the more massive part of the arm or leg.

23. When a rocket expels gas in a given direction, it puts a force on that gas. The momentum of the gas-rocket system stays constant, so if the gas is pushed to the left, the rocket will be pushed to the right due to Newton's third law. So the rocket must carry some kind of material to be ejected (it could be exhaust from some kind of engine, or it could be compressed gas) in order to change direction.

24. Consider Bob, Jim, and the rope as a system. The center of mass of the system is closer to Bob, because he has more mass. Because there is no net external force on the system, the center of mass will stay stationary. As the two men pull hand-over-hand on the rope they will move toward each other, eventually colliding at the center of mass. Since the CM is on Bob's side of the midline, Jim will cross the midline and lose.

25. If there were only two particles as decay products, then by conservation of momentum, the momenta of the two decay products would have to be equal in magnitude and opposite in direction, so that the momenta would be required to lie along a line. If the momenta of the recoil nucleus and the electron do not lie along a line, then some other particle (the neutrino) must have some of the momentum.

26. When you are lying flat on the floor, your CM is inside of the volume of your body. When you sit up on the floor with your legs extended, your CM is outside of the volume of your body. The CM is higher when you sit up, and is slightly in front of your midsection.



27. The engine does not directly accelerate the car. The engine puts a force on the driving wheels, making them rotate. The wheels then push backward on the roadway as they spin. The Newton's third law reaction to this force is the forward pushing of the roadway on the wheels, which accelerates the car. So it is the (external) road surface that accelerates the car.

28. The motion of the center of mass of the rocket will follow the original parabolic path, both before and after explosion. Each individual piece of the rocket will follow a separate path after the explosion, but since the explosion was internal to the system (consisting of the rocket), the center of mass of all the exploded pieces will follow the original path.

Responses to MisConceptual Questions

1. (*d*) Students frequently have one of two common misconceptions. One idea is that since the truck has more mass, it has more momentum and will have a greater momentum change. Alternatively, some students think that since the smaller object has a greater change in speed, it will have the greater change in momentum. In the absence of external net forces, momentum is a conserved quantity. Therefore, momentum lost by one of the vehicles is gained by the other, and the magnitude of the change in momentum is the same for both vehicles.

2. (*b*) A common misconception in this problem is the belief that since the sand is dropped onto the boat, it does not exert a force on the boat and therefore does not accelerate the boat. However, when dropped, the sand has no initial horizontal velocity. For the sand to be at rest on the deck of the boat it must be accelerated from rest to the final speed of the boat. This acceleration is provided by the force of friction between the boat and sand. By Newton's third law, the sand exerts an equal but opposite force on the boat, which will cause the boat to slow down.

3. (*c*) Students may have the misconception that by doubling the mass the final speed will decrease. However, the momentum and kinetic energy are proportional to the mass. So, if each mass is doubled, then every term in the conservation of momentum and conservation of kinetic energy equations is doubled. This factor of two can be divided out to return to the initial equation. Therefore, doubling the masses will have no effect on the final motion.

4. (*a*) Since the net momentum of the astronaut and wrench is zero, the only way for the astronaut to move toward the space station is for the wrench to move away from the station. If the astronaut throws the wrench in any other direction, the astronaut will move away from the wrench but not toward the station. If the astronaut throws the wrench toward the station but does not let go of it, neither the wrench nor the astronaut will move.

5. (*a*) Since the asteroid ends up in the shuttle storage bay, the asteroid and shuttle have the same final speed. This is a completely inelastic collision, so only momentum is conserved.

6. (*a*) A common error is to ignore the vector nature of momentum and impulse. The bean bag and golf ball have the same momentum just before they hit the ground. The bean bag comes to rest when it hits the ground, so the ground has exerted an upward impulse equal to the magnitude of bean bag's momentum. The golf ball rebounds upward with the same magnitude momentum, but in the opposite direction. The ground therefore exerted an upward impulse equal to twice the magnitude of the momentum.

7. (*a*) Students may consider that the superball and clay have the same momentum and as such would be equally effective. However, since the clay and superball interact with the door differently, this is incorrect. The clay sticks to the door, exerting an impulse on the door equal to its momentum. The superball bounces off of the door, exerting an impulse about equal to twice its momentum. Since the superball imparts a greater impulse to the door, it will be more effective.

8. (*c*) This problem requires the student to understand the vector nature of momentum. The ball initially has a momentum toward the batter. If the ball is stopped by the catcher, the change in momentum has the same magnitude as the initial momentum. If the ball is hit straight back to the pitcher, the magnitude of the change in momentum is equal to twice the initial momentum. If the ball is hit straight up at the same speed, the change in momentum has a horizontal and a vertical vector component with the magnitude of each component equal to the initial momentum. Since the two components are perpendicular to each other, the magnitude of the change in momentum will be less than the sum of their magnitudes. As such, the greatest change in momentum occurs when the ball is hit straight back toward the pitcher.

9. (*d*) To solve this question a student should understand the relationships between force, time, momentum, work, and kinetic energy. Impulse is the product of the force and the time over which the force acts. For an object starting at rest, the impulse is also equal to the final momentum. Since the same force acts over the same time on both vehicles, they will have the same momentum. The lighter vehicle will have the greater speed and will therefore have traveled a greater distance in the same time. Since both vehicles start from rest with the same force acting on them, the work-energy theorem shows that the vehicle that travels the greater distance will have the greater final kinetic energy.

10. (*e*) Since the same force acts on both vehicles over the same distance, the work done on both vehicles is the same. From the work-energy theorem both vehicles will have the same final kinetic energy. The lighter vehicle will travel the distance in a shorter amount of time and will therefore experience a smaller impulse and have a smaller final momentum.

11. (*c*) A common misconception is that as the milk drains from the tank car and its mass decreases, the tank car's speed increases. For the tank car's speed to change, a horizontal force would have to act on the car. As the milk drains, it falls vertically, so no horizontal force exits, and the tank car travels at constant speed. As the mass of the tank car decreases, the momentum decreases proportionately, as the milk carries its momentum with it.

12. (*c*) The height to which the bowling ball rises depends upon the impulse exerted on it by the putty and by the rubber ball. The putty sticks to the bowling ball and therefore continues to move forward at the new speed of the bowling ball $(\Delta \upsilon < 5.0 \text{ m/s})$. The rubber bounces backward and therefore has a greater change in velocity $(\Delta \upsilon \approx 10.0 \text{ m/s})$. Since the putty and rubber have the same mass, the rubber exerts a greater impulse onto the bowling ball, causing the bowling ball to travel higher than when it is hit by the putty.

Solutions to Problems

1. Momentum is defined in Eq. 7–1. We use the magnitude.

$$p = m\upsilon = (0.028\ \text{kg})(8.4\ \text{m/s}) = \boxed{0.24\ \text{kg}\cdot\text{m/s}}$$

2. From Eq. 7–2 for a single force, $\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}\Delta t$. For an object of constant mass, $\Delta\vec{\mathbf{p}} = m\Delta\vec{\mathbf{v}}$. Equate the two expressions for $\Delta\vec{\mathbf{p}}$.

$$\vec{\mathbf{F}}\Delta t = m\Delta\vec{\mathbf{v}} \quad\rightarrow\quad \Delta\vec{\mathbf{v}} = \frac{\vec{\mathbf{F}}\Delta t}{m}$$

If the skier moves to the right, then the speed will decrease, because the friction force is to the left.

$$\Delta\upsilon = -\frac{F\Delta t}{m} = -\frac{(25\ \text{N})(15\ \text{s})}{65\ \text{kg}} = \boxed{-5.8\ \text{m/s}}$$

The skier loses $7.7\ \text{m/s}$ of speed.

3. Consider the horizontal motion of the objects. The momentum in the horizontal direction will be conserved. Let A represent the car and B represent the load. The positive direction is the direction of the original motion of the car.

$$p_{\text{initial}} = p_{\text{final}} \quad\rightarrow\quad m_A\upsilon_A + m_B\upsilon_B = (m_A + m_B)\upsilon' \quad\rightarrow$$
$$\upsilon' = \frac{m_A\upsilon_A + m_B\upsilon_B}{m_A + m_B} = \frac{(7150\ \text{kg})(15.0\ \text{m/s}) + 0}{7150\ \text{kg} + 3350\ \text{kg}} = \boxed{10.2\ \text{m/s}}$$

4. The tackle will be analyzed as a one-dimensional momentum-conserving situation. Let A represent the halfback and B represent the tackler. We take the direction of the halfback to be the positive direction, so $\upsilon_A > 0$ and $\upsilon_B < 0$.

$$p_{\text{initial}} = p_{\text{final}} \quad\rightarrow\quad m_A\upsilon_A + m_B\upsilon_B = (m_A + m_B)\upsilon' \quad\rightarrow$$
$$\upsilon' = \frac{m_A\upsilon_A + m_B\upsilon_B}{m_A + m_B} = \frac{(82\ \text{kg})(5.0\ \text{m/s}) + (110\ \text{kg})(-2.5\ \text{m/s})}{82\ \text{kg} + 110\ \text{kg}} = 0.703\ \text{m/s} \approx \boxed{0.70\ \text{m/s}}$$

They will be moving it the direction that the halfback was running before the tackle.

5. The force on the gas can be found from its change in momentum. The speed of 1300 kg of the gas changes from rest to $4.5\times10^4\ \text{m/s}$, over the course of one second. Use Eq. 7–2.

$$F_{\text{gas}} = \frac{\Delta p}{\Delta t} = \frac{m\Delta\upsilon}{\Delta t} = \Delta\upsilon\frac{m}{\Delta t} = (4.5\times10^4\ \text{m/s})(1300\ \text{kg/s})$$
$$= 5.9\times10^7\ \text{N, in the direction of the velocity of the gas}$$

The force on the rocket is the Newton's third law pair (equal and opposite) to the force on the gas, so the force on the rocket is $\boxed{5.9\times10^7\ \text{N in the opposite direction of the velocity of the gas}}$.

6. Consider the motion in one dimension, with the positive direction being the direction of motion of the first car. Let A represent the first car and B represent the second car. Momentum will be conserved in the collision. Note that $\upsilon_B = 0$. Use Eq. 7–3.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_A \upsilon_A + m_B \upsilon_B = (m_A + m_B)\upsilon' \rightarrow$$

$$m_B = \frac{m_A(\upsilon_A - \upsilon')}{\upsilon'} = \frac{(7700\text{ kg})(14\text{ m/s} - 5.0\text{ m/s})}{5.0\text{ m/s}} = 13{,}860\text{ kg} \approx \boxed{14{,}000\text{ kg}}$$

[7.] The throwing of the package is a momentum-conserving action, if the water resistance is ignored. Let A represent the boat and child together, and let B represent the package. Choose the direction that the package is thrown as the positive direction. Apply conservation of momentum, with the initial velocity of both objects being 0. Use Eq. 7–3 in one dimension.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow (m_A + m_B)\upsilon = m_A \upsilon'_A + m_B \upsilon'_B = 0 \rightarrow$$

$$\upsilon'_A = 2\,\frac{m_B \upsilon'_B}{m_A} = -\frac{(5.30\text{ kg})(10.0\text{ m/s})}{(24.0\text{ kg} + 35.0\text{ kg})} = \boxed{-0.898\text{ m/s}}$$

The boat and child move in the opposite direction as the thrown package, as indicated by the negative velocity.

8. Consider the motion in one dimension, with the positive direction being the direction of motion of the alpha particle. Let A represent the alpha particle, with a mass of m_A, and let B represent the daughter nucleus, with a mass of $57m_A$. The total momentum must be 0 since the nucleus decayed at rest. Use Eq. 7–3, in one dimension.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow 0 = m_A \upsilon'_A + m_B \upsilon'_B \rightarrow$$

$$\upsilon'_B = -\frac{m_A \upsilon'_A}{m_B} = -\frac{m_A(2.8 \times 10^5\text{ m/s})}{57m_A} \rightarrow |\upsilon'_B| = \boxed{4900\text{ m/s}}$$

Note that the masses do not have to be converted to kg, since all masses are in the same units, and a ratio of masses is what is significant.

9. Consider the motion in one dimension, with the positive direction being the direction of motion of the original nucleus. Let A represent the alpha particle, with a mass of 4 u, and let B represent the new nucleus, with a mass of 218 u. Use Eq. 7–3 for momentum conservation.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow (m_A + m_B)\upsilon = m_A \upsilon'_A + m_B \upsilon'_B \rightarrow$$

$$\upsilon'_A = \frac{(m_A + m_B)\upsilon - m_B \upsilon'_B}{m_A} = \frac{(222\text{ u})(320\text{ m/s}) - (218\text{ u})(280\text{ m/s})}{4.0\text{ u}} = \boxed{2500\text{ m/s}}$$

Note that the masses do not have to be converted to kg, since all masses are in the same units, and a ratio of masses is what is significant.

10. Momentum will be conserved in one dimension in the explosion. Let A represent the fragment with the larger kinetic energy. Use Eq. 7–3.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow 0 = m_A \upsilon'_A + m_B \upsilon'_B \rightarrow \upsilon'_B = -\frac{m_A \upsilon'_A}{m_B}$$

$$\text{KE}_A = 2\text{KE}_B \rightarrow \tfrac{1}{2} m_A \upsilon'^2_A = 2\left(\tfrac{1}{2} m_B \upsilon'^2_B\right) = m_B\left(-\frac{m_A \upsilon'_A}{m_B}\right)^2 \rightarrow \frac{m_A}{m_B} = \boxed{\frac{1}{2}}$$

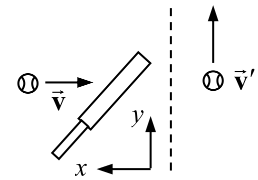
The fragment with the larger kinetic energy has half the mass of the other fragment.

11. Consider the motion in one dimension with the positive direction being the direction of motion of the bullet. Let A represent the bullet and B represent the block. Since there is no net force outside of the block–bullet system (like friction with the table), the momentum of the block and bullet combination is conserved. Use Eq. 7–3, and note that $\upsilon_B = 0$.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A\upsilon_A + m_B\upsilon_B = m_A\upsilon'_A + m_B\upsilon'_B \quad \rightarrow$$

$$\upsilon'_B = \frac{m_A\upsilon_A - m_A\upsilon'_A}{m_B} = \frac{(0.022\text{ kg})(240\text{ m/s}) - (0.022\text{ kg})(150\text{ m/s})}{2.0\text{ kg}} = \boxed{0.99\text{ m/s}}$$

12. To find the average force, we use Eq. 7–2 and divide the change in momentum by the time over which the momentum changes. Choose the x direction to be the opposite of the baseball's incoming direction, so to the left in the diagram. The velocity with which the ball is moving after hitting the bat can be found from conservation of energy and from knowing the height to which the ball rises.



$$(\text{KE}_{\text{initial}} = \text{PE}_{\text{final}})_{\substack{\text{after}\\\text{collision}}} \quad \rightarrow \quad \tfrac{1}{2}m\upsilon'^2 = mg\Delta y \quad \rightarrow$$

$$\upsilon' = \sqrt{2g\Delta y} = \sqrt{2(9.80\text{ m/s}^2)(31.5\text{ m})} = 24.85\text{ m/s}$$

The average force can be calculated from the change in momentum and the time of contact.

$$\bar{F}_x = \frac{\Delta p_x}{\Delta t} = \frac{m(\upsilon'_x - \upsilon_x)}{\Delta t} = \frac{(0.145\text{ kg})(0 - -27.0\text{ m/s})}{2.5\times10^{-3}\text{ s}} = 1566\text{ N}$$

$$\bar{F}_y = \frac{\Delta p_y}{\Delta t} = \frac{m(\upsilon'_y - \upsilon_y)}{\Delta t} = \frac{(0.145\text{ kg})(24.85\text{ m/s} - 0)}{2.5\times10^{-3}\text{ s}} = 1441\text{ N}$$

$$\bar{F} = \sqrt{\bar{F}_x^2 + \bar{F}_y^2} = \sqrt{(1566\text{ N})^2 + (1441\text{ N})^2} = 2128\text{ N} \approx \boxed{2100\text{ N}}$$

$$\theta = \tan^{-1}\frac{\bar{F}_y}{\bar{F}_x} = \tan^{-1}\frac{1441}{1566} = 42.6° \approx \boxed{43°}$$

13. The air is moving with an initial speed of $120\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 33.33\text{ m/s}$. Thus, in one second, a volume of air measuring $45\text{ m}\times 75\text{ m}\times 33.33\text{ m}$ will have been brought to rest. By Newton's third law, the average force on the building will be equal in magnitude to the force causing the change in momentum of the air. The mass of the stopped air is its volume times its density.

$$F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t}\upsilon = \frac{V\rho\Delta\upsilon}{\Delta t} = \frac{(45\text{ m})(75\text{ m})(33.33\text{ m})(1.3\text{ kg/m}^3)(33.33\text{ m/s} - 0)}{1\text{ s}}$$

$$= \boxed{4.9\times10^6\text{ N}}$$

14. (*a*) Consider the motion in one dimension with the positive direction being the direction of motion before the separation. Let A represent the upper stage (that moves away faster) and B represent the lower stage. It is given that $m_A = m_B$, $\upsilon_A = \upsilon_B = \upsilon$, and $\upsilon'_B = \upsilon'_A - \upsilon_{\text{rel}}$. Use Eq. 7–3 for momentum conservation.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad (m_A + m_B)\upsilon = m_A \upsilon_A' + m_B \upsilon_B' = m_A \upsilon_A' + m_B (\upsilon_A' - \upsilon_{\text{rel}}) \quad \rightarrow$$

$$\upsilon_A' = \frac{(m_A + m_B)\upsilon + m_B \upsilon_{\text{rel}}}{m_A + m_B} = \frac{(725\text{ kg})(6.60\times 10^3\text{ m/s}) + \frac{1}{2}(725\text{ kg})(2.80\times 10^3\text{ m/s})}{725\text{ kg}}$$

$$= \boxed{8.00\times 10^3\text{ m/s, away from Earth}}$$

$$\upsilon_B' = \upsilon_A' - \upsilon_{\text{rel}} = 8.007\times 10^3\text{ m/s} - 2.80\times 10^3\text{ m/s} = \boxed{5.20\times 10^3\text{ m/s, away from Earth}}$$

(*b*) The change in kinetic energy was supplied by the explosion.

$$\Delta\text{KE} = \text{KE}_{\text{final}} - \text{KE}_{\text{initial}} = \left(\tfrac{1}{2} m_A \upsilon_A'^2 + \tfrac{1}{2} m_B \upsilon_B'^2\right) - \tfrac{1}{2}(m_A + m_B)\upsilon^2$$

$$= \tfrac{1}{2}[\tfrac{1}{2}(725\text{ kg})][(8.00\times 10^3\text{ m/s})^2 + (5.20\times 10^3\text{ m/s})^2] - \tfrac{1}{2}(72\,5\text{ kg})(6.60\times 10^3\text{ m/s})^2$$

$$= \boxed{7.11\times 10^8\text{ J}}$$

15. Choose the direction from the batter to the pitcher to be the positive direction. Calculate the average force from the change in momentum of the ball.

$$\Delta p = F\Delta t = m\Delta\upsilon \quad \rightarrow$$

$$F = m\frac{\Delta\upsilon}{\Delta t} = (0.145\text{ kg})\left(\frac{46.0\text{ m/s} - -31.0\text{ m/s}}{5.00\times 10^{-3}\text{ s}}\right) = \boxed{2230\text{ N, toward the pitcher}}$$

16. (*a*) The impulse is the change in momentum. The direction of travel of the struck ball is the positive direction.

$$\Delta p = m\Delta\upsilon = (4.5\times 10^{-2}\text{ kg})(38\text{ m/s} - 0) = 1.71\text{ kg}\cdot\text{m/s} \approx \boxed{1.7\text{ kg}\cdot\text{m/s}}$$

(*b*) The average force is the impulse divided by the interaction time.

$$\overline{F} = \frac{\Delta p}{\Delta t} = \frac{1.71\text{ kg}\cdot\text{m/s}}{3.5\times 10^{-3}\text{ s}} = \boxed{490\text{ N}}$$

17. (*a*) The impulse given to the nail is the opposite of the impulse given to the hammer. This is the change in momentum. Call the direction of the initial velocity of the hammer the positive direction.

$$\Delta p_{\text{nail}} = -\Delta p_{\text{hammer}} = [m\upsilon_{\text{initial}} - m\upsilon_{\text{final}}]_{\text{hammer}} = (12\text{ kg})(7.5\text{ m/s}) - 0 = \boxed{9.0\times 10^1\text{ kg}\cdot\text{m/s}}$$

(*b*) The average force is the impulse divided by the time of contact.

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{9.0\times 10^1\text{ kg}\cdot\text{m/s}}{8.0\times 10^{-3}\text{ s}} = \boxed{1.1\times 10^4\text{ N}}$$

18. The impulse given the ball is the change in the ball's momentum. From the symmetry of the problem, the vertical momentum of the ball does not change, so there is no vertical impulse. Call the direction AWAY from the wall the positive direction for momentum perpendicular to the wall.

$$\Delta_{p_\perp} = m\upsilon_{\perp\,\text{final}} - m\upsilon_{\perp\,\text{initial}} = m(\upsilon\sin 45^\circ - \upsilon\sin 45^\circ) = 2m\upsilon\sin 45^\circ$$

$$= 2(6.0\times 10^{-2}\text{ km})(28\text{ m/s})\sin 45^\circ = \boxed{2.4\text{ kg}\cdot\text{m/s, to th e left}}$$

19. (*a*) The momentum of the astronaut–space capsule combination will be conserved since the only forces are "internal" to that system. Let A represent the astronaut and B represent the space capsule, and let the direction the astronaut moves be the positive direction. Due to the choice of reference frame, $\upsilon_A = \upsilon_B = 0.$ We also have $\upsilon'_A = 2.50$ m/s.

$$p_{\text{initial}} = p_{\text{final}} \quad\rightarrow\quad m_A\upsilon_A + m_B\upsilon_B = 0 = m_A\upsilon'_A + m_B\upsilon'_B \quad\rightarrow$$

$$\upsilon'_B = -\upsilon'_A\frac{m_A}{m_B} = -(2.50\text{ m/s})\frac{125\text{ kg}}{1900\text{ kg}} = -0.1645\text{ m/s} \approx \boxed{-0.16\text{ m/s}}$$

The negative sign indicates that the space capsule is moving in the opposite direction to the astronaut.

(*b*) The average force on the astronaut is the astronaut's change in momentum, divided by the time of interaction.

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{m(\upsilon'_A - \upsilon_A)}{\Delta t} = \frac{(125\text{ kg})(2.50\text{ m/s}-0)}{0.600\text{ s}} = \boxed{521\text{ N}}$$

(*c*) $\text{KE}_{\text{astronaut}} = \frac{1}{2}(125\text{ kg})(2.50\text{ m/s})^2 = \boxed{391\text{ J}}, \ \text{KE}_{\text{capsule}} = \frac{1}{2}(1900\text{ kg})(-0.1645\text{ m/s})^2 = \boxed{26\text{ J}}$

20. If the rain does not rebound, then the final speed of the rain is 0. By Newton's third law, the force on the pan due to the rain is equal in magnitude to the force on the rain due to the pan. The force on the rain can be found from the change in momentum of the rain. The mass striking the pan is calculated as volume times density.

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{(m\upsilon_f - m\upsilon_0)}{\Delta t} = -\frac{m}{\Delta t}(\upsilon_f - \upsilon_0) = \frac{\rho V}{\Delta t}\upsilon_0 = \frac{\rho Ah}{\Delta t}\upsilon_0 = \frac{h}{\Delta t}\rho A\upsilon_0$$

$$= \frac{(2.5\times10^{-2}\text{ m})}{1\text{ h}\left(\frac{3600\text{ s}}{1\text{ h}}\right)}(1.00\times10^3\text{ kg/m}^3)(1.0\text{ m}^2)(8.0\text{ m/s}) = \boxed{0.056\text{ N}}$$

$\boxed{21.}$ Call east the positive direction.

(*a*) $p_{\substack{\text{original}\\\text{fullback}}} = m\upsilon_{\substack{\text{original}\\\text{fullback}}} = (95\text{ kg})(3.0\text{ m/s}) = 285\text{ kg}\cdot\text{m/s} \approx \boxed{290\text{ kg}\cdot\text{m/s, to the east}}$

(*b*) The impulse on the fullback is the change in the fullback's momentum.

$$\Delta p_{\text{fullback}} = m(\upsilon_{\substack{\text{final}\\\text{fullback}}} - \upsilon_{\substack{\text{final}\\\text{fullback}}}) = (95\text{ kg})(0-3.0\text{ m/s}) = -285\text{ kg}\cdot\text{m/s} \approx \boxed{-290\text{ kg}\cdot\text{m/s}}$$

The negative sign indicates the impulse is to the west.

(*c*) The impulse on the tackler is the opposite of the impulse on the fullback.

$\boxed{290\text{ kg}\cdot\text{m/s, to the east}}$

(*d*) The average force on the tackler is the impulse on the tackler divided by the time of interaction.

$$\overline{F} = \frac{\Delta p}{\Delta t} = \frac{285\text{ kg}\cdot\text{m/s}}{0.85\text{ s}} = \boxed{340\text{ N, to the east}}$$

22. Impulse is the change of momentum, Eq. 7–5. This is a one-dimensional configuration.

$$\Delta_p = m(\upsilon_{\text{final}} - \upsilon_0) = (0.50\text{ kg})(3.0\text{ m/s}) = \boxed{1.5\text{ kg}\cdot\text{m/s}}$$

23. (*a*) The impulse given the ball is the area under the *F* vs. *t* graph. Approximate the area as a triangle of "height" 250 N, and "width" 0.04 s.

$$\Delta p = \tfrac{1}{2}(250\text{ N})(0.04\text{ s}) \approx \boxed{5\text{ N}\cdot\text{s}}$$

We could also count "boxes" under the graph, where each "box" has an "area" of $(50\text{ N})(0.01\text{ s}) = 0.5\text{ N}\cdot\text{s}$. There are almost seven whole boxes and the equivalent of about three whole boxes in the partial boxes. Ten boxes would be about $\boxed{5\text{ N}\cdot\text{s}}$.

(*b*) The velocity can be found from the change in momentum. Call the positive direction the direction of the ball's travel after being served.

$$\Delta p = m\Delta\upsilon = m(\upsilon_f - \upsilon_i) \quad\rightarrow\quad \upsilon_f = \upsilon_i + \frac{\Delta p}{m} = 0 + \frac{5\text{ N}\cdot\text{s}}{6.0\times10^{-2}\text{ kg}} \approx \boxed{80\text{ m/s}}$$

24. (*a*) The impulse is the change in momentum. Take upward to be the positive direction. The velocity just before reaching the ground is found from conservation of mechanical energy.

$$E_{\text{initial}} = E_{\text{final}} \quad\rightarrow\quad mgh = \tfrac{1}{2}m\upsilon_y^2 \quad\rightarrow$$

$$\upsilon_y = \pm\sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(2.8\text{ m})} = 7.408\text{ m/s, down}$$

$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}} = m(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_0) = (55\text{ kg})(0 - -7.408\text{ m/s}) = 407\text{ kg}\cdot\text{m/s} \approx \boxed{410\text{ kg}\cdot\text{m/s, upward}}$$

(*b*) The net force on the person is the sum of the upward force from the ground, plus the downward force of gravity.

$$F_{\text{net}} = F_{\text{ground}} - mg = ma \quad\rightarrow$$

$$F_{\text{ground}} = m(g+a) = m\left(g + \frac{(v_f^2 - v_0^2)}{2\Delta x}\right) = (55\text{ kg})\left((9.80\text{ m/s}^2) + \frac{0-(-7.408\text{ m/s})^2}{2(-0.010\text{ m})}\right)$$

$$= \boxed{1.5\times10^5\text{ N, upward}}$$

This is about 280 times the jumper's weight.

(*c*) We do this the same as part (*b*), but for the longer distance.

$$F_{\text{ground}} = m\left(g + \frac{(\upsilon_f^2 - \upsilon_0^2)}{2\Delta x}\right) = (55\text{ kg})\left((9.80\text{ m/s}^2) + \frac{0-(-7.408\text{ m/s})^2}{2(-0.5\text{ m})}\right)$$

$$= 3557\text{ N} \approx \boxed{4000\text{ N, upward}}$$

This is about 6.5 times the jumper's weight.

25. Let A represent the 0.440-kg ball and B represent the 0.220-kg ball. We have $\upsilon_A = 3.80\text{ m/s}$ and $\upsilon_B = 0$. Use Eq. 7–7 to obtain a relationship between the velocities.

$$\upsilon_A - \upsilon_B = -(\upsilon'_A - \upsilon'_B) \quad\rightarrow\quad \upsilon'_B = \upsilon_A + \upsilon'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A\upsilon_A + m_B\upsilon_B = m_A\upsilon'_A + m_B\upsilon'_B \quad\rightarrow\quad m_A\upsilon_A = m_A\upsilon'_A + m_B(\upsilon_A + \upsilon'_A) \quad\rightarrow$$

$$\upsilon'_A = \frac{(m_A - m_B)}{(m_A + m_B)}\upsilon_A = \frac{0.220\text{ kg}}{0.660\text{ kg}}(3.80\text{ m/s}) = 1.267\text{ m/s} \approx \boxed{1.27\text{ m/s (east)}}$$

$$\upsilon'_B = \upsilon_A + \upsilon'_A = 3.80\text{ m/s} + 1.27\text{ m/s} = \boxed{5.07\text{ m/s (east)}}$$

26. Let A represent the 0.450-kg puck, and let B represent the 0.900-kg puck. The initial direction of puck A is the positive direction. We have $\upsilon_A = 5.80\text{ m/s}$ and $\upsilon_B = 0.$ Use Eq. 7–7 to obtain a relationship between the velocities.

$$\upsilon_A - \upsilon_B = -(\upsilon'_A - \upsilon'_B) \;\rightarrow\; \upsilon'_B = \upsilon_A + \upsilon'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A\upsilon_A + m_B\upsilon_B = m_A\upsilon'_A + m_B\upsilon'_B \;\rightarrow\; m_A\upsilon_A = m_A\upsilon'_A\ (m_A\upsilon'_A\ ) \;\rightarrow$$

$$\upsilon'_A = \frac{(m_A - m_B)}{(m_A + m_B)}\upsilon_A = \frac{-0.450\text{ kg}}{1.350\text{ kg}}(5.80\text{ m/s}) = -1.933\text{ m/s} \approx \boxed{1.93\text{ m/s (west)}}$$

$$\upsilon'_B = \upsilon_A + \upsilon'_A = 5.80\text{ m/s} - 1.93\text{ m/s} = \boxed{3.87\text{ m/s (east)}}$$

27. Let A represent the 0.060-kg tennis ball, and let B represent the 0.090-kg ball. The initial direction of the balls is the positive direction. We have $\upsilon_A = 5.50\text{ m/s}$ and $\upsilon_B = 3.00\text{ m/s}.$ Use Eq. 7–7 to obtain a relationship between the velocities.

$$\upsilon_A - \upsilon_B = -(\upsilon'_A - \upsilon'_B) \;\rightarrow\; \upsilon'_B = 2.50\text{ m/s} + \upsilon'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A\upsilon_A + m_B\upsilon_B = m_A\upsilon'_A + m_B\upsilon'_B \;\rightarrow\; m_A\upsilon_A + m_B\upsilon_B = m_A\upsilon'_A + m_B\,(2.50\text{ m/s} + \upsilon'_A) \;\rightarrow$$

$$\upsilon'_A = \frac{m_A\upsilon_A + m_B(\upsilon_B - 2.50\text{ m/s})}{m_A + m_B} = \frac{(0.060\text{ kg})(5.50\text{ m/s}) + (0.090\text{ kg})(3.00\text{ m/s} - 2.50\text{ m/s})}{0.150\text{ kg}}$$

$$= \boxed{2.50\text{ m/s}}$$

$$\upsilon'_B = 2.50\text{ m/s} + \upsilon'_A = \boxed{5.00\text{ m/s}}$$

Both balls move in the direction of the tennis ball's initial motion.

28. Let A represent the ball moving at 2.00 m/s, and call that direction the positive direction. Let B represent the ball moving at 3.60 m/s in the opposite direction. Thus, $\upsilon_A = 2.00\text{ m/s}$ and $\upsilon_B = -3.60\text{ m/s}.$ Use Eq. 7–7 to obtain a relationship between the velocities.

$$\upsilon_A - \upsilon_B = -(\upsilon'_A - \upsilon'_B) \;\rightarrow\; \upsilon'_B = 5.60\text{ m/s} + \upsilon'_A$$

Substitute this relationship into the momentum conservation equation for the collision, noting that $m_A = m_B.$

$$m_A\upsilon_A + m_B\upsilon_B = m_A\upsilon'_A + m_B\upsilon'_B \;\rightarrow\; \upsilon_A + \upsilon_B = \upsilon'_A + \upsilon'_B \;\rightarrow$$

$$-1.60\text{ m/s} = \upsilon'_A + (\upsilon'_A + 5.60\text{ m/s}) \;\rightarrow\; 2\upsilon'_A = -7.20\text{ m/s} \;\rightarrow\; \upsilon'_A = \boxed{-3.60\text{ m/s}}$$

$$\upsilon'_B = 5.60\text{ m/s} + \upsilon'_A = \boxed{2.00\text{ m/s}}$$

The two balls have exchanged velocities. This will always be true for 1-D elastic collisions of objects of equal mass.

29. Let A represent the incoming ball and B represent the target ball. We have $\upsilon_B = 0$ and $\upsilon'_A = -0.450\upsilon_A.$ Use Eq. 7–7 to obtain a relationship between the velocities.

$$\upsilon_A - \upsilon_B = -(\upsilon'_A - \upsilon'_B) \;\rightarrow\; \upsilon'_B = \upsilon_A + \upsilon'_A = 0.550\upsilon_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m_A v_A = m_A v_A' + m_B v_B'$$
$$= m_A(-0.450 v_A) + m_B(0.550 v_A) \rightarrow \boxed{m_B = 2.64 m_A}$$

30. Let A represent the moving ball, and let B represent the ball initially at rest. The initial direction of the ball is the positive direction. We have $v_A = 5.5\text{ m/s}$, $v_B = 0$, and $v_A' = -3.8\text{ m/s}$.

(*a*) Use Eq.7–7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v_A' - v_B') \rightarrow v_B' = v_A - v_B + v_A' = 5.5\text{ m/s} - 0 - 3.8\text{ m/s} = \boxed{1.7\text{ m/s}}$$

(*b*) Use momentum conservation to solve for the mass of the target ball.

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \rightarrow$$
$$m_B = m_A \frac{(v_A - v_A')}{(v_B' - v_B)} = (0.220\text{ kg})\frac{(5.5\text{ m/s} - -3.8\text{ m/s})}{1.7\text{ m/s}} = \boxed{1.2\text{ kg}}$$

31. The one-dimensional stationary target elastic collision is analyzed in Search and Learn 5. The algebraic details can be found there, and also in Example 7–8. The kinetic energy lost by the neutron is equal to the kinetic energy gained by the target particle. The fraction of kinetic energy lost is found as follows:

$$\frac{\text{KE}_{\text{A initial}} - \text{KE}_{\text{A final}}}{\text{KE}_{\text{A initial}}} = \frac{\text{KE}_{\text{B final}}}{\text{KE}_{\text{A initial}}} = \frac{\frac{1}{2} m_B v_B'^2}{\frac{1}{2} m_A v_A^2} = \frac{m_B\left[v_A\left(\frac{2m_A}{m_A + m_B}\right)\right]^2}{m_A v_A^2} = \frac{4m_A m_B}{(m_A + m_B)^2}$$

(*a*) $$\frac{4m_A m_B}{(m_A + m_B)^2} = \frac{4(1.01)(1.01)}{(1.01+1.01)^2} = \boxed{1.00}$$

All of the initial kinetic energy is lost by the neutron, as expected for the target mass equal to the incoming mass.

(*b*) $$\frac{4m_A m_B}{(m_A + m_B)^2} = \frac{4(1.01)(2.01)}{(1.01+2.01)^2} = \boxed{0.890}$$

(*c*) $$\frac{4m_A m_B}{(m_A + m_B)^2} = \frac{4(1.01)(12.00)}{(1.01+12.00)^2} = \boxed{0.286}$$

(*d*) $$\frac{4m_A m_B}{(m_A + m_B)^2} = \frac{4(1.01)(208)}{(1.01+208)^2} = \boxed{0.0192}$$

Since the target is quite heavy, almost no kinetic energy is lost. The incoming particle "bounces off" the heavy target, much as a rubber ball bounces off a wall with approximately no loss in speed.

32. From the analysis in Example 7–9, the initial projectile speed is given by $v = \frac{m+M}{m}\sqrt{2gh}$.

Compare the two speeds with the same masses.

$$\frac{\upsilon_2}{\upsilon_1}=\frac{\frac{m+M}{m}\sqrt{2gh_2}}{\frac{m+M}{m}\sqrt{2gh_1}}=\frac{\sqrt{h_2}}{\sqrt{h_1}}=\sqrt{\frac{h_2}{h_1}}=\sqrt{\frac{5.2}{2.6}}=\sqrt{2}\ \rightarrow\ \boxed{\upsilon_2=\sqrt{2}\upsilon_1}$$

33. (*a*) In Example 7–9, $\text{KE}_\text{i}=\frac{1}{2}m\upsilon^2$ and $\text{KE}_\text{f}=\frac{1}{2}(m+M)\upsilon'^2$. The speeds are related by

$$\upsilon'=\frac{m}{m+M}\upsilon.$$

$$\frac{\Delta\text{KE}}{\text{KE}_\text{i}}=\frac{\text{KE}_\text{f}-\text{KE}_\text{i}}{\text{KE}_\text{i}}=\frac{\frac{1}{2}(m+M)\upsilon'^2-\frac{1}{2}m\upsilon^2}{\frac{1}{2}m\upsilon^2}=\frac{(m+M)\left(\frac{m}{m+M}\upsilon\right)^2-m\upsilon^2}{m\upsilon^2}$$

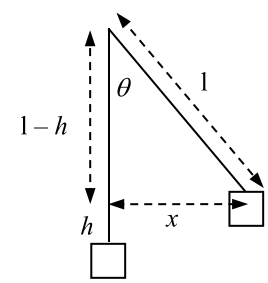
$$=\frac{\frac{m^2\upsilon^2}{m+M}-m\upsilon^2}{m\upsilon^2}=\frac{m}{m+M}-1=\boxed{\frac{-M}{m+M}}$$

(*b*) For the given values, $\frac{-M}{m+M}=\frac{-380\text{ g}}{398\text{ g}}=-0.95$. Thus 95% of the energy is lost.

34. From the analysis in the Example 7–9, we know the following:

$$h=\frac{1}{2g}\left(\frac{m\upsilon}{m+M}\right)^2=\frac{1}{2(9.80\text{ m/s}^2)}\left(\frac{(0.028\text{ kg})(190\text{ m/s})}{0.028\text{ kg}+3.1\text{ kg}}\right)^2$$

$$=0.1476\text{ m}\approx\boxed{0.15\text{ m}}$$



From the diagram we see the following:

$$\ell^2=(\ell-h)^2+x^2$$

$$x=\sqrt{\ell^2-(\ell-h)^2}=\sqrt{(2.8\text{ m})^2-(2.8\text{ m}-0.1476\text{ m})^2}=\boxed{0.90\text{ m}}$$

35. Use conservation of momentum in one dimension, since the particles will separate and travel in opposite directions. Call the direction of the heavier particle's motion the positive direction. Let A represent the heavier particle and B represent the lighter particle. We have $m_\text{A}=1.5m_\text{B}$, and $\upsilon_\text{A}=\upsilon_\text{B}=0$.

$$p_\text{initial}=p_\text{final}\ \rightarrow\ 0=m_\text{A}\upsilon'_\text{A}+m_\text{B}\upsilon'_\text{B}\ \rightarrow\ \upsilon'_\text{A}=2\frac{m_\text{B}\upsilon'_\text{B}}{m_\text{A}}=-\frac{2}{3}\upsilon'_\text{B}$$

The negative sign indicates direction. Since there was no mechanical energy before the explosion, the kinetic energy of the particles after the explosion must equal the energy released.

$$E_\text{released}=\text{KE}'_\text{A}+\text{KE}'_\text{B}=\frac{1}{2}m_\text{A}\upsilon'^2_\text{A}+\frac{1}{2}m_\text{B}\upsilon'^2_\text{B}=\frac{1}{2}(1.5m_\text{B})(\frac{2}{3}\upsilon'_\text{B})^2+\frac{1}{2}m_\text{B}\upsilon'^2_\text{B}=\frac{5}{3}(\frac{1}{2}m_\text{B}\upsilon'^2_\text{B})=\frac{5}{3}\text{KE}'_\text{B}$$

$$\text{KE}'_\text{B}=\frac{3}{5}E_\text{released}=\frac{3}{5}(5500\text{ J})=3300\text{ J}\quad \text{KE}'_\text{A}=E_\text{released}-\text{KE}'_\text{B}=5500\text{ J}-3300\text{ J}=2200\text{ J}$$

Thus $\boxed{\text{KE}'_\text{A}=2200\text{ J};\quad \text{KE}_\text{B}=3300\text{ J}}$.

36. Use conservation of momentum in one dimension. Call the direction of the sports car's velocity the positive *x* direction. Let A represent the sports car and B represent the SUV. We have $\upsilon_B = 0$ and $\upsilon'_A = \upsilon'_B$. Solve for υ_A.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A\upsilon_A + 0 = (m_A + m_B)\upsilon'_A \quad \rightarrow \quad \upsilon_A = \frac{m_A + m_B}{m_A}\upsilon'_A$$

The kinetic energy that the cars have immediately after the collision is lost due to negative work done by friction. The work done by friction can also be calculated using the definition of work. We assume the cars are on a level surface, so that the normal force is equal to the weight. The distance the cars slide forward is Δx. Equate the two expressions for the work done by friction, solve for υ'_A, and use that to find υ_A.

$$W_{\text{fr}} = (\text{KE}_{\text{final}} - \text{KE}_{\text{initial}})_{\substack{\text{after}\\ \text{collision}}} = 0 - \tfrac{1}{2}(m_A + m_B)\upsilon'^2_A$$

$$W_{\text{fr}} = F_{\text{fr}}\Delta x \cos 180° = -\mu_k(m_A + m_B)g\Delta x$$

$$-\tfrac{1}{2}(m_A + m_B)\upsilon'^2_A = -\mu_k(m_A + m_B)g\Delta x \quad \rightarrow \quad \upsilon'_A = \sqrt{2\mu_k g\Delta x}$$

$$\upsilon_A = \frac{m_A + m_B}{m_A}\upsilon'_A = \frac{m_A + m_B}{m_A}\sqrt{2\mu_k g\Delta x} = \frac{980\text{ kg} + 2300\text{ kg}}{980\text{ kg}}\sqrt{2(0.80)(9.80\text{ m/s}^2)(2.6\text{ m})}$$

$$= 21.37\text{ m/s} \approx \boxed{21\text{ m/s}}$$

37. The impulse on the ball is its change in momentum. Call upward the positive direction, so that the final velocity is positive and the initial velocity is negative. The speeds immediately before and immediately after the collision can be found from conservation of energy. Take the floor to be the zero level for gravitational potential energy.

Falling: $\text{KE}_{\text{bottom}} = \text{PE}_{\text{top}} \quad \rightarrow \quad \tfrac{1}{2}m\upsilon^2_{\text{down}} = mgh_{\text{down}} \quad \rightarrow \quad \upsilon_{\text{down}} = -\sqrt{2gh_{\text{down}}}$

Rising: $\text{KE}_{\text{bottom}} = \text{PE}_{\text{top}} \quad \rightarrow \quad \tfrac{1}{2}m\upsilon^2_{\text{up}} = mgh_{\text{up}} \quad \rightarrow \quad \upsilon_{\text{up}} = \sqrt{2gh_{\text{up}}}$

$$\text{Impulse} = \Delta p = m\Delta\upsilon = m(\upsilon_{\text{up}} - \upsilon_{\text{down}}) = m\left(\sqrt{2gh_{\text{up}}} - -\sqrt{2gh_{\text{down}}}\right) = m\sqrt{2g}\left(\sqrt{h_{\text{up}}} + \sqrt{h_{\text{down}}}\right)$$

$$= (0.014\text{ kg})\sqrt{2(9.80\text{ m/s}^2)}\left(\sqrt{0.85\text{ m}} + \sqrt{1.5\text{ m}}\right) = 0.13\text{ kg}\cdot\text{m/s}$$

The direction of the impulse is upwards, so the complete specification of the impulse is $\boxed{0.13\text{ kg}\cdot\text{m/s, upward}}$.

38. $$\text{Fraction KE lost} = \frac{\text{KE}_{\text{initial}} - \text{KE}_{\text{final}}}{\text{KE}_{\text{initial}}} = \frac{\tfrac{1}{2}m_A\upsilon^2_A - \tfrac{1}{2}m_B\upsilon'^2_B}{\tfrac{1}{2}m_A\upsilon^2_A} = \frac{\upsilon^2_A - \upsilon'^2_B}{\upsilon^2_A} = \frac{(38\text{ m/s})^2 - (15\text{ m/s})^2}{(38\text{ m/s})^2} = \boxed{0.84}$$

39. (*a*) Momentum is conserved in the one-dimensional collision. Let A represent the baseball and let B represent the brick.

$$m_A\upsilon_A = m_A\upsilon'_A + m_B\upsilon'_B \quad \rightarrow$$

$$\upsilon'_A = \frac{m_A\upsilon_A - m_B\upsilon'_B}{m_A} = \frac{(0.144\text{ kg})(28.0\text{ m/s}) - (5.25\text{ kg})(1.10\text{ m/s})}{0.144\text{ kg}} = -12.10\text{ m/s}$$

So the baseball's speed in the reverse direction is $\boxed{12.1\text{ m/s}}$.

(*b*) $\text{KE}_{\text{before}} = \tfrac{1}{2}m_A\upsilon^2_A = \tfrac{1}{2}(0.144\text{ kg})(28.0\text{ m/s})^2 = \boxed{56.4\text{ J}}$

$$\text{KE}_{\text{after}} = \tfrac{1}{2}m_A\upsilon_A'^2 + \tfrac{1}{2}m_A\upsilon_B'^2 = \tfrac{1}{2}(0.144 \text{ kg})(1.21 \text{ m/s})^2 + \tfrac{1}{2}(5.25 \text{ kg})(1.10 \text{ m/s})^2 = \boxed{13.7 \text{ J}}$$

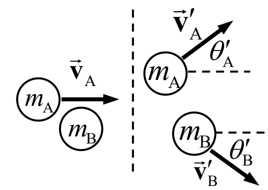
40. The swinging motion will conserve mechanical energy. Take the zero level for gravitational potential energy to be at the bottom of the arc. For the pendulum to swing exactly to the top of the arc, the potential energy at the top of the arc must be equal to the kinetic energy at the bottom.

$$\text{KE}_{\text{bottom}} = \text{PE}_{\text{top}} \rightarrow \tfrac{1}{2}(m+M)\upsilon_{\text{bottom}}^2 = (m+M)g(2L) \rightarrow \upsilon_{\text{bottom}} = 2\sqrt{gL}$$

Momentum will be conserved in the totally inelastic collision at the bottom of the arc. We assume that the pendulum does not move during the collision process.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow m\upsilon = (m+M)\upsilon_{\text{bottom}} \rightarrow \upsilon = \frac{m+M}{m}\upsilon_{\text{bottom}} = \boxed{2\frac{m+M}{m}\sqrt{gL}}$$

41. (*a*) See the diagram.



p_x: $m_A\upsilon_A = m_A\upsilon_A' \cos\theta_A' + m_B\upsilon_B' \cos\theta_B'$

p_y: $0 = m_A\upsilon_A' \sin\theta_A' - m_B\upsilon_B' \sin\theta_B'$

(*b*) Solve the *x* equation for $\cos\theta_B'$ and the *y* equation for $\sin\theta_B'$, and then find the angle from the tangent function.

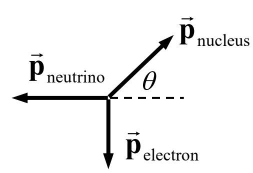
$$\tan\theta_B' = \frac{\sin\theta_B'}{\cos\theta_B'} = \frac{\dfrac{m_A\upsilon_A' \sin\theta_A'}{m_B\upsilon_B'}}{\dfrac{m_A(\upsilon_A - \upsilon_A' \cos\theta_A')}{m_B\upsilon_B'}} = \frac{\upsilon_A' \sin\theta_A'}{(\upsilon_A - \upsilon_A' \cos\theta_A')}$$

$$\theta_B' = \tan^{-1}\frac{\upsilon_A' \sin\theta_A'}{\upsilon_A - \upsilon_A' \cos\theta_A'} = \tan^{-1}\frac{(2.10 \text{ m/s}) \sin 30.0°}{2.80 \text{ m/s} - (2.10 \text{ m/s}) \cos 30.0°} = \boxed{46.9°}$$

With the value of the angle, solve the *y* equation for the velocity.

$$\upsilon_B' = \frac{m_A\upsilon_A' \sin\theta_A'}{m_B \sin\theta_B'} = \frac{(0.120 \text{ kg})(2.10 \text{ m/s}) \sin 30.0°}{(0.140 \text{ kg}) \sin 46.9°} = \boxed{1.23 \text{ m/s}}$$

42. Use this diagram for the momenta after the decay. Since there was no momentum before the decay, the three momenta shown must add to 0 in both the *x* and *y* directions.



$$(p_{\text{nucleus}})_x = p_{\text{neutrino}} \qquad (p_{\text{nucleus}})_y = p_{\text{electron}}$$

$$p_{\text{nucleus}} = \sqrt{(p_{\text{nucleus}})_x^2 + (p_{\text{nucleus}})_y^2} = \sqrt{(p_{\text{neutrino}})^2 + (p_{\text{electron}})^2}$$

$$= \sqrt{(6.2\times10^{-23} \text{ kg}\cdot\text{m/s})^2 + (9.6\times10^{-23} \text{ kg}\cdot\text{m/s})^2} = \boxed{1.14\times10^{-22} \text{ kg}\cdot\text{m/s}}$$

$$\theta = \tan^{-1}\frac{(p_{\text{nucleus}})_y}{(p_{\text{nucleus}})_x} = \tan^{-1}\frac{(p_{\text{electron}})}{(p_{\text{neutrino}})} = \tan^{-1}\frac{(9.6\times10^{-23} \text{ kg}\cdot\text{m/s})}{(6.2\times10^{-23} \text{ kg}\cdot\text{m/s})} = 57°$$

The momentum of the second nucleus is directed $\boxed{\text{147° from the electron's momentum}}$ and is directed $\boxed{\text{123° from the neutrino's momentum}}$.

43. Write momentum conservation in the x and y directions and KE conservation. Note that both masses are the same. We allow $\vec{\mathbf{v}}_{A}$ to have both x and y components.

$$p_x: \quad m\upsilon_B = m\upsilon'_{Ax} \quad \rightarrow \quad \upsilon_B = \upsilon'_{Ax}$$
$$p_y: \quad m\upsilon_A = m\upsilon'_{Ay} + m\upsilon'_B \quad \rightarrow \quad \upsilon_A = \upsilon'_{Ay} + \upsilon'_B$$
$$\text{KE:} \quad \tfrac{1}{2}m\upsilon_A^2 + \tfrac{1}{2}m\upsilon_B^2 = \tfrac{1}{2}m\upsilon_A'^2 + \tfrac{1}{2}m\upsilon_B'^2 \quad \rightarrow \quad \upsilon_A^2 + \upsilon_B^2 = \upsilon_A'^2 + \upsilon_B'^2$$

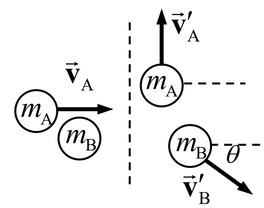
Substitute the results from the momentum equations into the KE equation.

$$(\upsilon'_{Ay} + \upsilon'_B)^2 + (\upsilon'_{Ax})^2 = \upsilon_A'^2 + \upsilon_B'^2 \quad \rightarrow \quad \upsilon_{Ay}'^2 + 2\upsilon_{Ay}'^2\upsilon'_B + \upsilon_B'^2 + \upsilon_{Ay}'^2 = \upsilon_A'^2 + \upsilon_B'^2 \quad \rightarrow$$
$$\upsilon_A'^2 + 2\upsilon_{Ay}'^2\upsilon'_B + \upsilon_B'^2 = \upsilon_A'^2 + \upsilon_B'^2 \quad \rightarrow \quad 2\upsilon_{Ay}'^2\upsilon'_B = 0 \quad \rightarrow \quad \upsilon'_{Ay} = 0 \text{ or } \upsilon'_B = 0$$

Since we are given that $\upsilon'_B \neq 0,$ we must have $\upsilon'_{Ay} = 0.$ This means that the final direction of A is the $\boxed{x \text{ direction}}$. Put this result into the momentum equations to find the final speeds.

$$\upsilon'_A = \upsilon'_{Ax} = \upsilon_B = \boxed{3.7 \text{ m/s}} \qquad \upsilon'_B = \upsilon_A = \boxed{2.0 \text{ m/s}}$$

44. (*a*) Let A represent the incoming nucleus and B represent the target particle. Take the x direction to be in the direction of the initial velocity of mass A (to the right in the diagram) and the y direction to be up in the diagram Momentum is conserved in two dimensions and gives the following relationships:



$$p_x: \quad m_A\upsilon_A = m_B\upsilon'_B \cos\theta \quad \rightarrow \quad \upsilon = 2\upsilon'_B \cos\theta$$
$$p_y: \quad 0 = m_A\upsilon'_A - m_B\upsilon'_B \sin\theta \quad \rightarrow \quad \upsilon'_A = 2\upsilon'_B \sin\theta$$

The collision is elastic, so kinetic energy is also conserved.

$$\text{KE:} \quad \tfrac{1}{2}m_A\upsilon_A^2 = \tfrac{1}{2}m_A\upsilon_A'^2 + \tfrac{1}{2}m_B\upsilon_B'^2 \quad \rightarrow \quad \upsilon^2 = \upsilon_A'^2 + 2\upsilon_B'^2 \quad \rightarrow \quad \upsilon^2 - \upsilon_A'^2 = 2\upsilon_B'^2$$

Square the two momentum equations and add them together.

$$\upsilon = 2\upsilon'_B \cos\theta;\ \upsilon'_A = 2\upsilon'_B \sin\theta \quad \rightarrow \quad \upsilon^2 = 4\upsilon_B'^2\cos^2\theta;$$
$$\upsilon_A'^2 = 4\upsilon_B'^2\sin^2\theta \quad \rightarrow \quad \upsilon^2 + \upsilon_A'^2 = 4\upsilon_B'^2$$

Add these two results together and use them in the x momentum expression to find the angle.

$$\upsilon^2 - \upsilon_A'^2 = 2\upsilon_B'^2;\quad \upsilon^2 + \upsilon_A'^2 = 4\upsilon_B'^2 \quad \rightarrow \quad 2\upsilon^2 = 6\upsilon_B'^2 \quad \rightarrow \quad \upsilon'_B = \frac{\upsilon}{\sqrt{3}}$$

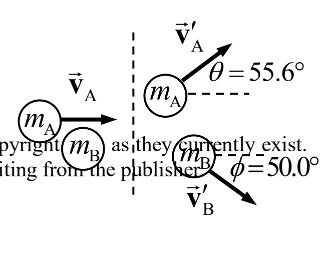
$$\cos\theta = \frac{\upsilon}{2\upsilon'_B} = \frac{\upsilon}{2\frac{\upsilon}{\sqrt{3}}} = \frac{\sqrt{3}}{2} \quad \rightarrow \quad \boxed{\theta = 30^\circ}$$

(*b*) From above, we already have $\boxed{\upsilon'_B = \frac{\upsilon}{\sqrt{3}}}$. Use that in the y momentum equation.

$$\upsilon'_A = 2\upsilon'_B \sin\theta = 2\frac{\upsilon}{\sqrt{3}}\sin 30^\circ = \boxed{\upsilon'_A = \frac{\upsilon}{\sqrt{3}}}$$

(*c*) The fraction transferred is the final energy of the target particle divided by the original kinetic energy.

$$\frac{\text{KE}_{\text{target}}}{\text{KE}_{\text{original}}} = \frac{\tfrac{1}{2}m_B\upsilon_B'^2}{\tfrac{1}{2}m_A\upsilon_A^2} = \frac{\tfrac{1}{2}(2m_A)(\upsilon^2/3)}{\tfrac{1}{2}m_A\upsilon^2} = \boxed{\frac{2}{3}}$$



45. Choose the carbon atom as the origin of coordinates. Use Eq. 7–9a.

$$x_{CM} = \frac{m_C x_C + m_O x_O}{m_C + m_O} = \frac{(12\text{ u})(0) + (16\text{ u})(1.13 \times 10^{-10}\text{ m})}{12\text{ u} + 16\text{ u}} = \boxed{6.5 \times 10^{-11}\text{ m}} \text{ from the C atom}$$

46. Use Eq. 7–9a, extended to three particles.

$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C} = \frac{(1.00\text{ kg})(0) + (1.50\text{ kg})(0.50\text{ m}) + (1.10\text{ kg})(0.75\text{ m})}{1.00\text{ kg} + 1.50\text{ kg} + 1.10\text{ kg}}$$
$$= \boxed{0.438\text{ m}}$$

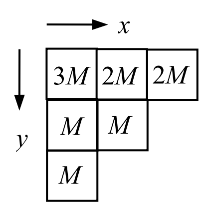
47. Find the CM relative to the front of the car. Use Eq. 7–9a.

$$x_{CM} = \frac{m_{car} x_{car} + m_{front} x_{front} + m_{back} x_{back}}{m_{car} + m_{front} + m_{back}}$$
$$= \frac{(1250\text{ kg})(2.40\text{ m}) + 2(65.0\text{ kg})(2.80\text{ m}) + 3(65.0\text{ kg})(3.90\text{ m})}{1250\text{ kg} + 5(65.0\text{ kg})} = \boxed{2.62\text{ m}}$$

48. By the symmetry of the problem, since the centers of the cubes are along a straight line, the vertical CM coordinate will be 0, and the depth CM coordinate will be 0. The only CM coordinate to calculate is the one along the straight line joining the centers. The mass of each cube will be the volume times the density, so $m_1 = \rho(\ell_0)^3, m_2 = \rho(2\ell_0)^3, m_3 = \rho(3\ell_0)^3$. Measuring from the left edge of the smallest block, the locations of the CMs of the individual cubes are $x_1 = \frac{1}{2}\ell_0, x_2 = 2\ell_0, x_3 = 4.5\ell_0$. Use Eq. 7–9a to calculate the CM of the system.

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{\rho\ell_0^3\left(\frac{1}{2}\ell_0\right) + 8\rho\ell_0^3(2\ell_0) + 27\rho\ell_0^3(4.5\ell_0)}{\rho\ell_0^3 + 8\rho\ell_0^3 + 27\rho\ell_0^3} = \frac{138}{36}\ell_0 = \frac{23}{6}\ell_0$$
$$= \boxed{3.8\ell_0 \text{ from the left edge of the smallest cube}}$$

49. Let each case have a mass M. A top view of the pallet is shown, with the total mass of each stack listed. Take the origin to be the back left corner of the pallet. Use Eqs. 7–9a and 7–9b.



$$x_{CM} = \frac{(5M)(\ell/2) + (3M)(3\ell/2) + (2M)(5\ell/2)}{10M} = \boxed{1.2\ell}$$
$$y_{CM} = \frac{(7M)(\ell/2) + (2M)(3\ell/2) + (1M)(5\ell/2)}{10M} = \boxed{0.9\ell}$$

50. Because the brace is uniform, the mass of each "leg" is proportional to its area. Since each "leg" has the same width of 0.20 m, each leg's mass is proportional to its length. We calculate the center of mass relative to the origin of coordinates as given in the diagram. Let the total mass be M.

$$m_{\substack{\text{horizontal}\\ \text{leg}}} = \frac{2.06}{2.06+1.48}M = 0.5819M\ ; \quad m_{\substack{\text{vertical}\\ \text{leg}}} = \frac{1.48}{2.06+1.48}M = 0.4181M$$

$$x_{\text{CM}} = \frac{m_{\text{horiz}}x_{\text{horiz}} + m_{\text{vert}}x_{\text{vert}}}{m_{\text{horiz}} + m_{\text{vert}}} = \frac{(0.5819M)(1.03\text{ m}) + (0.4181M)(1.96\text{ m})}{M} = \boxed{1.42\text{ m}}$$

$$y_{\text{CM}} = \frac{m_{\text{horiz}}y_{\text{horiz}} + m_{\text{vert}}y_{\text{vert}}}{m_{\text{horiz}} + m_{\text{vert}}} = \frac{(0.5819M)(0.10\text{ m}) + (0.4181M)(-0.74\text{ m})}{M} = \boxed{-0.25\text{ m}}$$

51. Take the upper leg and lower leg together. Note that Table 7–1 gives the relative mass of BOTH legs, so a factor of 1/2 is needed. Assume a person of mass 70 kg.

$$(70\text{ kg})\frac{(21.5+9.6)}{100}\tfrac{1}{2} = 10.885\text{ kg} \approx \boxed{11\text{ kg}}$$

52. With the shoulder as the origin of coordinates for measuring the center of mass, we have the following relative locations from Table 7–1 for the arm components, as percentages of the height. Down is positive.

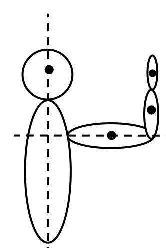
$$x_{\substack{\text{upper}\\ \text{arm}}} = 81.2 - 71.7 = 9.5 \quad x_{\substack{\text{lower}\\ \text{arm}}} = 81.2 - 55.3 = 25.9 \quad x_{\text{hand}} = 81.2 - 43.1 = 38.1$$

To find the CM, we can also use relative mass percentages. Since the expression includes the total mass in the denominator, there is no need to divide all masses by 2 to find single component masses. Simply use the relative mass percentages given in the table.

$$x_{\text{CM}} = \frac{x_{\substack{\text{upper}\\ \text{arm}}}m_{\substack{\text{upper}\\ \text{arm}}} + x_{\substack{\text{lower}\\ \text{arm}}}m_{\substack{\text{lower}\\ \text{arm}}} + x_{\text{hand}}m_{\text{hand}}}{m_{\substack{\text{upper}\\ \text{arm}}} + m_{\substack{\text{lower}\\ \text{arm}}} + m_{\text{hand}}} = \frac{(9.5)(6.6) + (25.9)(4.2) + (38.1)(1.7)}{6.6 + 4.2 + 1.7}$$

$$= \boxed{\text{19\% of the person's height along the line from the shoulder to the hand}}$$

53. Take the shoulder to be the origin of coordinates. We assume that the arm is held with the upper arm parallel to the floor and the lower arm and hand extended upward. Measure *x* horizontally from the shoulder and *y* vertically. Since the expression includes the total mass in the denominator, there is no need to divide all masses by 2 to find single component masses. Use the relative mass percentages given in the table.



$$x_{\text{CM}} = \frac{x_{\substack{\text{upper}\\ \text{arm}}}m_{\substack{\text{upper}\\ \text{arm}}} + x_{\substack{\text{lower}\\ \text{arm}}}m_{\substack{\text{lower}\\ \text{arm}}} + x_{\text{hand}}m_{\text{hand}}}{m_{\substack{\text{upper}\\ \text{arm}}} + m_{\substack{\text{lower}\\ \text{arm}}} + m_{\text{hand}}}$$

$$= \frac{(81.2-71.7)(6.6) + (81.2-62.2)(4.2+1.7)}{6.6+4.2+1.7} = 14.0$$

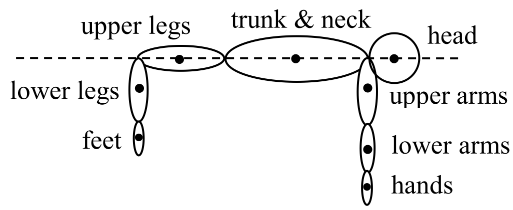
$$y_{\text{CM}} = \frac{y_{\substack{\text{upper}\\ \text{arm}}}m_{\substack{\text{upper}\\ \text{arm}}} + y_{\substack{\text{lower}\\ \text{arm}}}m_{\substack{\text{lower}\\ \text{arm}}} + y_{\text{hand}}m_{\text{hand}}}{m_{\substack{\text{upper}\\ \text{arm}}} + m_{\substack{\text{lower}\\ \text{arm}}} + m_{\text{hand}}}$$

$$= \frac{(0)(6.6) + (62.2-55.3)(4.2) + (62.2-43.1)(1.7)}{6.6+4.2+1.7} = 4.92$$

Convert the distance percentages to actual distance by using the person's height.

$x_{CM} = (14.0\%)(155 \text{ cm}) = \boxed{21.7 \text{ cm}}$ $\quad y_{CM} = (4.92\%)(155 \text{ cm}) = \boxed{7.6 \text{ cm}}$

54. See the diagram of the person. The head, trunk, neck, and thighs are all lined up so that their CMs are on the torso's median line. Call down the positive y direction. The y displacements of the CM of each body part from the median line, in terms of percentage of full height, are shown below, followed by the percentage each body part is of the full body mass.



On median line:	head (h):	0	6.9% body mass
	trunk & neck (t n):	0	46.1% body mass
	upper legs (u l):	0	21.5% body mass
From shoulder hinge point:	upper arms (u a):	$81.2 - 71.7 = 9.5$	6.6% body mass
	lower arms (l a):	$81.2 - 55.3 = 25.9$	4.2% body mass
	hands (ha):	$81.2 - 43.1 = 38.1$	1.7% body mass
From knee hinge point:	lower legs (l l):	$28.5 - 18.2 = 10.3$	9.6% body mass
	feet (f):	$28.5 - 1.8 = 26.7$	3.4% body mass

Using this data, calculate the vertical location of the CM.

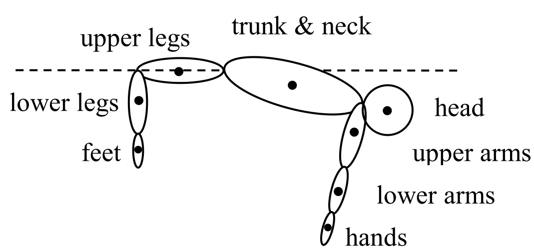
$$y_{CM} = \frac{y_h m_h + y_{tn} m_{tn} + y_{ul} m_{ul} + y_{ua} m_{ua} + y_{la} m_{la} + y_{ha} m_{ha} + y_{ll} m_{ll} + y_f m_f}{m_{\substack{\text{full}\\\text{body}}}}$$

$$= \frac{0 + 0 + 0 + (9.5)(6.6) + (25.9)(4.2) + (38.1)(1.7) + (10.3)(9.6) + (26.7)(3.4)}{100}$$

$$= 4.2591 \approx 4.3$$

So the center of mass is $\boxed{\text{4.3\% of the full body height below the torso's median line}}$. For a person of height 1.7 m, this is about 7.2 cm, which is less than 3 inches. That is most likely $\boxed{\text{inside the body}}$.

55. Based on Fig. 7–27, we place the upper legs parallel to the bar, the lower legs and feet hanging vertically, and the trunk and neck, head, arms, and hands all tilted down by 15°. Call down the positive y direction. The y distances of the CM of each body part from the median line, in terms of percentage of full height, are shown below, followed by the percentage each body part is of the full body mass. The calculations for the lower legs and feet are the same as for Problem 59. Here are the calculations for the angled parts of the body.



Trunk & neck: Hip joint: 52.1% from the floor, center of trunk at 71.1%, difference = 19.0%. CM of trunk & neck $= 19.0(\sin 15.0°) = 4.92$

Head: Hip joint: 52.1%, center of head at 93.5%, difference = 41.4% CM of head $= 41.4(\sin 15.0°) = 10.72$

Shoulder: Hip joint: 52.1%, shoulder at 81.2%, difference = 29.1% $= 29.1(\sin 15.0°) = 7.53$

Upper arms: Shoulder: 81.2%, center of upper arms at 71.7%, difference = 9.5% CM of upper arms = 7.53 (due to shoulder) $+9.5(\cos 15.0°)=16.71$

Lower arms: Shoulder: 81.2%, center of lower arms at 55.3%, difference = 25.9% CM of lower arms = 7.53 (due to shoulder) $+25.9(\cos\ 15.0°)=32.55$

Hands: Shoulder: 81.2%, center of hands at 43.1%, difference = 38.1% CM of hands = 7.53 (due to shoulder) $+38.1(\cos 15.0°)=44.33$

On horizontal line:	upper legs (u l):	0	21.5% body mass
From waist hinge point:	trunk & neck (t n):	4.92	46.1% body mass
	head (h):	10.72	6.9% body mass
From shoulder hinge point:	upper arms (u a):	16.71	6.6% body mass
	lower arms (l a):	32.55	4.2% body mass
	hands (ha):	44.33	1.7% body mass
From knee hinge point:	lower legs (l l):	10.3	9.6% body mass
	feet (f):	26.7	3.4% body mass

Using this data, calculate the vertical location of the CM.

$$y_{\text{CM}}=\frac{y_{\text{h}}m_{\text{h}}+y_{\text{t n}}m_{\text{t n}}+y_{\text{u l}}m_{\text{u l}}+y_{\text{u a}}m_{\text{u a}}+y_{\text{l a}}m_{\text{l a}}+y_{\text{ha}}m_{\text{ha}}+y_{\text{l l}}m_{\text{l l}}+y_{\text{f}}m_{\text{f}}}{m_{\substack{\text{full}\\ \text{body}}}}$$

$$=\frac{\begin{bmatrix}10.72(6.9)+4.92(46.1)+0+(16.71)(6.6)+(32.55)(4.2)\\ +(44.33)(1.7)+(10.3)(9.6)+(26.7)(3.4)\end{bmatrix}}{100}$$

$$=8.128\approx 8.1$$

Thus the center of mass is 8.1% of the full body height below the torso's median line. For a person of height 1.7 m, this is about 14 cm. That is about 5.5 inches, and it is most likely slightly outside the body.

56. (*a*) Find the CM relative to the center of the Earth.

$$x_{\text{CM}}=\frac{m_{\text{E}}x_{\text{E}}+m_{\text{M}}x_{M}}{m_{\text{E}}+m_{\text{M}}}=\frac{(5.98\times 10^{24}\ \text{kg})(0)+(7.35\times 10^{22}\ \text{kg})(3.84\times 10^{8}\ \text{m})}{5.98\times 10^{24}\ \text{kg}+7.35\times 10^{22}\ \text{kg}}$$

$$=\boxed{4.66\times 10^{6}\ \text{m from the center of the Earth}}$$

This is actually inside the volume of the Earth, since $R_{\text{E}}=6.38\times 10^{6}$ m.

(*b*) It is this Earth–Moon CM location that actually traces out the orbit, as discussed in Chapter 5. The Earth and Moon will orbit about this location in (approximately) circular orbits. The motion of the Moon, for example, around the Sun would then be a sum of two motions: (i) the motion of the Moon about the Earth–Moon CM and (ii) the motion of the Earth–Moon CM about the Sun. To an external observer, the Moon's motion would appear to be a small radius, higher frequency circular motion (motion about the Earth–Moon CM) combined with a large radius, lower frequency circular motion (motion about the Sun).

57. (*a*) Measure all distances from the <u>original</u> position of the woman.

$$x_{CM} = \frac{m_W x_W + m_M x_M}{m_W + m_M} = \frac{(52\text{ kg})(0) + (72\text{ kg})(10.0\text{ m})}{124\text{ kg}} = 5.806\text{ m}$$

$$\approx \boxed{5.8\text{ m from the woman}}$$

(*b*) Since there is no force external to the man–woman system, the CM will not move, relative to the <u>original</u> position of the woman. The woman's distance will no longer be 0, and the man's distance has changed to 7.5 m.

$$x_{CM} = \frac{m_W x_W + m_M x_M}{m_W + m_M} = \frac{(52\text{ kg})x_W + (72\text{ kg})(7.5\text{ m})}{124\text{ kg}} = 5.806\text{ m} \rightarrow$$

$$x_W = \frac{(5.806\text{ m})(124\text{ kg}) - (72\text{ kg})(7.5\text{ m})}{52\text{ kg}} = 3.460\text{ m}$$

$$x_M - x_W = 7.5\text{ m} - 3.460\text{ m} = 4.040\text{ m} \approx \boxed{4.0\text{ m}}$$

(*c*) When the man collides with the woman, he will be at the original location of the center of mass.

$$x_{M_{\text{final}}} - x_{M_{\text{initial}}} = 5.806\text{ m} - 10.0\text{ m} = -4.194\text{ m}$$

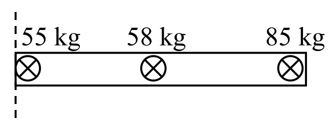
He has moved $\boxed{4.2\text{ m}}$ from his original position.

58. The CM of the system will follow the same path regardless of the way the mass splits and will still be $2d$ from the launch point when the parts land. Assume that the explosion is designed so that m_I still is stopped in midair and falls straight down.

(*a*) $$x_{CM} = \frac{m_I x_I + m_{II} x_{II}}{m_I + m_{II}} \rightarrow 2d = \frac{m_I d + 3m_I x_{II}}{4m_I} = \frac{d + 3x_{II}}{4} \rightarrow x_{II} = \boxed{\tfrac{7}{3}d}$$

(*b*) $$x_{CM} = \frac{m_I x_I + m_{II} x_{II}}{m_I + m_{II}} \rightarrow 2d = \frac{3m_{II} d + m_{II} x_{II}}{4m_{II}} = \frac{3d + x_{II}}{4} \rightarrow x_{II} = \boxed{5d}$$

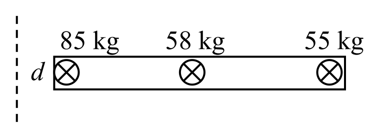
59. Calculate the CM relative to the 55-kg person's seat, at one end of the boat. See the first diagram. Be sure to include the boat's mass.



$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C}$$

$$= \frac{(55\text{ kg})(0) + (58\text{ kg})(1.5\text{ m}) + (85\text{ kg})(3.0\text{ m})}{198\text{ kg}} = 1.727\text{ m}$$

Now, when the passengers exchange positions, the boat will move some distance *d* as shown, but the CM will not move. We measure the location of the CM from the same place as before, but now the boat has moved relative to that origin.



$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C}$$

$$1.727\text{ m} = \frac{(85\text{ kg})(d) + (58\text{ kg})(1.5\text{ m} + d) + (55\text{ kg})(3.0\text{ m} + d)}{218\text{ kg}} = \frac{198d\text{ kg}\cdot\text{m} + 252\text{ kg}\cdot\text{m}}{198\text{ kg}}$$

$$d = \frac{(1.727\text{ m})(198\text{ kg}) - 252\text{ kg}\cdot\text{m}}{198\text{ kg}} = 0.4543\text{ m}$$

Thus the boat will move $\boxed{0.45\text{ m toward the initial position of the 85-kg person}}$.

60. Call the origin of coordinates the CM of the balloon, gondola, and passenger at rest. Since the CM is at rest, the total momentum of the system relative to the ground is 0. The passenger sliding down the rope cannot change the total momentum of the system, so the CM must stay at rest. Call the upward direction positive. Then the velocity of the passenger with respect to the balloon is $-\upsilon$. Call the velocity of the balloon with respect to the ground υ_{BG}. Then the velocity of the passenger with respect to the ground is $\upsilon_{MG} = -\upsilon + \upsilon_{BG}$. Apply Eq. 7–10.

$$0 = m\upsilon_{MG} + M\upsilon_{BG} = m(-\upsilon + \upsilon_{BG}) + M\upsilon_{BG} \quad \rightarrow \quad \boxed{\upsilon_{BG} = \upsilon\frac{m}{m+M}\text{, upward}}$$

If the passenger stops, $\boxed{\text{the balloon also stops}}$, and the CM of the system remains at rest.

61. The only forces on the astronauts are internal to the two-astronaut system, so their CM will not change. Call the CM location the origin of coordinates. That is also the initial location of the astronauts.

$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} \quad \rightarrow \quad 0 = \frac{(55\text{ kg})(12\text{ m}) + (85\text{ kg})x_B}{140\text{ kg}} \quad \rightarrow \quad x = -7.76\text{ m}$$

Their distance apart is $x_A - x_B = 12\text{ m} - (-7.76\text{ m}) = \boxed{2.0\times10^1\text{ m}}$.

62. This is a totally inelastic collision in one dimension. Call the direction of asteroid A the positive direction, and use conservation of momentum.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A\upsilon_A + m_B\upsilon_B = (m_A + m_B)\upsilon' \quad \rightarrow$$

$$\upsilon' = \frac{m_A\upsilon_A + m_B\upsilon_B}{m_A + m_B} = \frac{(7.5\times10^{12}\text{ kg})(3.3\text{ km/s}) + (1.45\times10^{13}\text{ kg})(-1.4\text{ km/s})}{7.5\times10^{12}\text{ kg} + 1.45\times10^{13}\text{ kg}}$$

$$= 0.0203\text{ km/s} \approx \boxed{0.2\text{ km/s, in the original direction of asteroid A}}$$

63. Consider conservation of energy during the rising and falling of the ball, between contacts with the floor. The gravitational potential energy at the top of a path will be equal to the kinetic energy at the start and the end of each rising-falling cycle. Thus $mgh = \frac{1}{2}m\upsilon^2$ for any particular bounce cycle. Thus for an interaction with the floor, the ratio of the energies before and after the bounce is $\frac{\text{KE}_{\text{after}}}{\text{KE}_{\text{before}}} = \frac{mgh'}{mgh} = \frac{1.20\text{ m}}{1.60\text{ m}} = 0.75.$ We assume that each bounce will further reduce the energy to 75% of its pre-bounce amount. The number of bounces to lose 90% of the energy can be expressed as follows:

$$(0.75)^n = 0.1 \quad \rightarrow \quad n = \frac{\log 0.1}{\log 0.75} = 8.004$$

Thus after $\boxed{\text{8 bounces}}$, 90% of the energy is lost.

As an alternate method, after each bounce, 75% of the available energy is left. So after 1 bounce, 75% of the original energy is left. After the second bounce, only 75% of 75%, or 56%, is left. After the third bounce, 42%. After the fourth bounce, 32%. After the fifth bounce, 24%. After the sixth bounce, 18%. After the seventh bounce, 13%. After the eight bounce, 10%. So it takes 8 bounces.

64. Momentum will be conserved in the horizontal direction. Let A represent the railroad car and B represent the snow. For the horizontal motion, $\upsilon_B = 0$ and $\upsilon'_B = \upsilon'_A$. Momentum conservation in the horizontal direction gives the following.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A\upsilon_A = (m_A + m_B)\upsilon'_A$$

$$\upsilon'_A = \frac{m_A\upsilon_A}{m_A + m_B} = \frac{(4800 \text{ kg})(7.60 \text{ m/s})}{4800 \text{ kg} + \left(\frac{3.80 \text{ kg}}{\text{min}}\right)(60.0 \text{ min})} = 7.255 \text{ m/s} \approx \boxed{7.3 \text{ m/s}}$$

65. Let the original direction of the cars be the positive direction. We have $\upsilon_A = 4.50$ m/s and $\upsilon_B = 3.70$ m/s.

(*a*) Use Eq. 7–7 to obtain a relationship between the velocities.

$$\upsilon_A - \upsilon_B = -(\upsilon'_A - \upsilon'_B) \quad \rightarrow \quad \upsilon'_B = \upsilon_A - \upsilon_B + \upsilon'_A = 0.80 \text{ m/s} + \upsilon'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A\upsilon_A + m_B\upsilon_B = m_A\upsilon'_A + m_B\upsilon'_B \quad \rightarrow \quad m_A\upsilon_A + m_B\upsilon_B = m_A\upsilon'_A + m_B\,(0.80 \text{ m/s} + \upsilon'_A) \quad \rightarrow$$

$$\upsilon'_A = \frac{m_A\upsilon_A + m_B(\upsilon_B - 0.80 \text{ m/s})}{m_A + m_B} = \frac{(435 \text{ kg})(4.50 \text{ m/s}) + (495 \text{ kg})(2.90 \text{ m/s})}{930 \text{ kg}} = 3.648 \text{ m/s}$$

$$\approx \boxed{3.65 \text{ m/s}}; \quad \upsilon'_B = 0.80 \text{ m/s} + \upsilon'_A = 4.448 \text{ m/s} \approx \boxed{4.45 \text{ m/s}}$$

(*b*) Calculate $\Delta p = p' - p$ for each car.

$$\Delta p_A = m_A\upsilon'_A - m_A\upsilon_A = (435 \text{ kg})(3.648 \text{ m/s} - 4.50 \text{ m/s}) = -370.62 \text{ kg}\cdot\text{m/s}$$

$$\approx \boxed{-370 \text{ kg}\cdot\text{m/s}}$$

$$\Delta p_B = m_B\upsilon'_B - m_B\upsilon_B = (495 \text{ kg})(4.448 \text{ m/s} - 3.70 \text{ m/s}) = 370.26 \text{ kg}\cdot\text{m/s}$$

$$\approx \boxed{370 \text{ kg}\cdot\text{m/s}}$$

The two changes are equal and opposite because momentum was conserved. The slight difference is due to round-off error on the calculations.

66. This is a ballistic "pendulum" of sorts, similar to Example 7–9. The only difference is that the block and bullet are moving vertically instead of horizontally. The collision is still totally inelastic and conserves momentum, and the energy is still conserved in the rising of the block and embedded bullet after the collision. So we simply quote the equation from that example.

$$\upsilon = \frac{m+M}{m}\sqrt{2gh} \quad \rightarrow$$

$$h = \frac{1}{2g}\left(\frac{m\upsilon}{m+M}\right)^2 = \frac{1}{2(9.80\text{ m/s}^2)}\left(\frac{(0.0250\text{ kg})(230\text{ m/s})}{0.0250\text{ kg}+1.40\text{ kg}}\right)^2 = 0.8307\text{ m} \approx \boxed{0.83\text{ m}}$$

67. We assume that all motion is along a single direction. The distance of sliding can be related to the change in the kinetic energy of a car, as follows:

$$W_{\text{fr}} = \Delta\text{KE} = \tfrac{1}{2}m(\upsilon_{\text{f}}^2 - \upsilon_{\text{i}}^2) \qquad W_{\text{fr}} = F_{\text{fr}}\Delta x\cos 180°\theta = -\mu_{\text{k}}F_{\text{N}}\Delta x = -\mu_{\text{k}}mg\Delta x \quad \rightarrow$$

$$-\mu_{\text{k}}g\Delta x = \tfrac{1}{2}(\upsilon_{\text{f}}^2 - \upsilon_{\text{i}}^2)$$

For post-collision sliding, $\upsilon_{\text{f}} = 0$ and υ_{i} is the speed immediately after the collision, υ'. Use this relationship to find the speed of each car immediately after the collision.

Car A: $-\mu_{\text{k}}g\Delta x'_{\text{A}} = -\tfrac{1}{2}\upsilon'^2_{\text{A}} \quad \rightarrow \quad \upsilon'_{\text{A}} = \sqrt{2\mu_{\text{k}}g\Delta x'_{\text{A}}} = \sqrt{2(0.60)(9.80\text{ m/s}^2)(18\text{ m})} = 14.55\text{ m/s}$

Car B: $-\mu_{\text{k}}g\Delta x'_{\text{B}} = 2\ \tfrac{1}{2}\upsilon'^2_{\text{B}} \quad \rightarrow \quad \upsilon'_{\text{B}} = \sqrt{2\mu_{\text{k}}g\Delta x'_{\text{B}}} = \sqrt{2(0.60)(9.80\text{ m/s}^2)(30\text{ m})} = 18.78\text{ m/s}$

During the collision, momentum is conserved in one dimension. Note that $\upsilon_{\text{B}} = 0$.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_{\text{A}}\upsilon_{\text{A}} = m_{\text{A}}\upsilon'_{\text{A}} + m_{\text{B}}\upsilon'_{\text{B}}$$

$$\upsilon_{\text{A}} = \frac{m_{\text{A}}\upsilon'_{\text{A}} + m_{\text{B}}\upsilon'_{\text{B}}}{m_{\text{A}}} = \frac{(1500\text{ kg})(14.55\text{ m/s}) + (1100\text{ kg})(18.78\text{ m/s})}{1500\text{ kg}} = 28.32\text{ m/s}$$

For pre-collision sliding, again apply the friction–energy relationship, with $\upsilon_{\text{f}} = \upsilon_{\text{A}}$ and υ_{i} the speed when the brakes were first applied.

$$-\mu_{\text{k}}g\Delta x_{\text{A}} = \tfrac{1}{2}(\upsilon_{\text{A}}^2 - \upsilon_i^2) \quad \rightarrow \quad \upsilon_{\text{i}} = \sqrt{\upsilon_{\text{A}}^2 + 2\mu_{\text{k}}g\Delta x_{\text{A}}} = \sqrt{(28.32\text{ m/s})^2 + 2(0.60)(9.80\text{ m/s}^2)(15\text{ m})}$$

$$= 31.28\text{ m/s}\left(\frac{1\text{ mi/h}}{0.447\text{ m/s}}\right) = \boxed{70\text{ mi/h}}$$

Car A was definitely over the speed limit.

68. (*a*) The meteor striking and coming to rest in the Earth is a totally inelastic collision. Let A represent the Earth and B represent the meteor. Use the frame of reference in which the Earth is at rest before the collision, so $\upsilon_{\text{A}} = 0$. Write momentum conservation for the collision.

$$m_{\text{B}}\upsilon_{\text{B}} = (m_{\text{A}} + m_{\text{B}})\upsilon' \quad \rightarrow$$

$$\upsilon' = \upsilon_{\text{B}}\frac{m_{\text{B}}}{m_{\text{A}} + m_{\text{B}}} = (2.5\times10^4\text{ m/s})\frac{1.5\times10^8\text{ kg}}{6.0\times10^{24}\text{ kg} + 1.5\times10^8\text{ kg}} = 6.25\times10^{-13}\text{ m/s}$$

$$\approx \boxed{6.3\times10^{-13}\text{ m/s}}$$

(*b*) The fraction of the meteor's KE transferred to the Earth is the final KE of the Earth divided by the initial KE of the meteor.

$$\frac{KE_{\substack{\text{final}\\ \text{Earth}}}}{KE_{\substack{\text{initial}\\ \text{meteor}}}} = \frac{\frac{1}{2}m_A\upsilon'^2}{\frac{1}{2}m_B\upsilon_B^2} = \frac{\frac{1}{2}(6.0\times10^{24}\text{ kg})(6.25\times10^{-13}\text{ m/s})^2}{\frac{1}{2}(1.5\times10^{8}\text{ kg})(2.5\times10^{4}\text{ m/s})^2} = \boxed{2.5\times10^{-17}}$$

(*c*) The Earth's change in KE can be calculated directly.

$$\Delta KE_{\text{Earth}} = KE_{\substack{\text{final}\\ \text{Earth}}} - KE_{\substack{\text{initial}\\ \text{Earth}}} = \frac{1}{2}m_A\upsilon'^2 - 0 = \frac{1}{2}(6.0\times10^{24}\text{ kg})(6.25\times10^{-13}\text{ m/s})^2 = \boxed{1.2\text{ J}}$$

69. This is a ballistic "pendulum" of sorts, similar to Example 7–9. The mass of the bullet is m, and the mass of the block of wood is M. The speed of the bullet before the collision is υ, and the speed of the combination after the collision is υ'. Momentum is conserved in the totally inelastic collision, so $m\upsilon = (m+M)\upsilon'$. The kinetic energy present immediately after the collision is lost due to negative work being done by friction.

$$W_{\text{fr}} = \Delta KE = \frac{1}{2}m(\upsilon_f^2 - \upsilon_i^2)_{\substack{\text{after}\\ \text{collision}}} \qquad W_{\text{fr}} = F_{\text{fr}}\Delta x\cos 180°\theta = -\mu_k F_N\Delta x = -\mu_k mg\Delta x \;\rightarrow$$

$$-\mu_k g\Delta x = \frac{1}{2}(\upsilon_f^2 - \upsilon_i^2) = -\frac{1}{2}\upsilon'^2 \;\rightarrow\; \upsilon' = \sqrt{2\mu_k g\Delta x}$$

Use this expression for υ' in the momentum equation in order to solve for υ.

$$m\upsilon = (m+M)\upsilon' = (m+M)\sqrt{2\mu_k g\Delta x} \;\rightarrow$$

$$\upsilon = \left(\frac{m+M}{m}\right)\sqrt{2\mu_k g\Delta x} = \left(\frac{0.028\text{ kg}+1.35\text{ kg}}{0.028\text{ kg}}\right)\sqrt{2(0.28)(9.80\text{ m/s}^2)(8.5\text{ m})} = \boxed{340\text{ m/s}}$$

70. (*a*) The average force is the momentum change divided by the elapsed time.

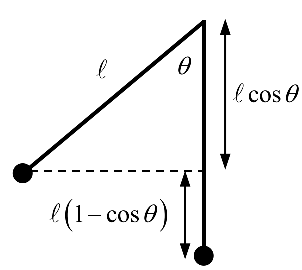
$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{m\Delta\upsilon}{\Delta t} = \frac{(1500\text{ kg})(0-45\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)}{0.15\text{ s}} = -1.25\times10^5\text{ N} \approx \boxed{-1.3\times10^5\text{ N}}$$

The negative sign indicates direction—that the force is in the opposite direction to the original direction of motion.

(*b*) Use Newton's second law. We use the absolute value of the force because the problem asked for the deceleration.

$$F_{\text{avg}} = ma_{\text{avg}} \;\rightarrow\; a_{\text{avg}} = \frac{F_{\text{avg}}}{m} = \frac{1.25\times10^5\text{ N}}{1500\text{ kg}} = 83.33\text{ m/s}^2\left(\frac{1\,g}{9.80\text{ m/s}^2}\right) \approx \boxed{8.5\,g\text{'s}}$$

71. For the swinging balls, their velocity at the bottom of the swing and the height to which they rise are related by conservation of energy. If the zero of gravitational potential energy is taken to be the lowest point of the swing, then the kinetic energy at the low point is equal to the potential energy at the highest point of the swing, where the speed is zero. Thus we have $\frac{1}{2}m\upsilon_{\text{bottom}}^2 = mgh$ for any swinging ball, so the relationship between speed and height is $\upsilon_{\text{bottom}}^2 = 2gh$. From the diagram we see that $h = \ell(1-\cos\theta)$.



(*a*) Calculate the speed of the lighter ball at the bottom of its swing.

$$\upsilon_A = \sqrt{2gh_A} = \sqrt{2(9.80\text{ m/s}^2)(0.35\text{ m})(1-\cos 66°)} = 2.017\text{ m/s} \approx \boxed{2.0\text{ m/s}}$$

(*b*) Assume that the collision is elastic, and use the results of Search and Learn 5. Take the direction that ball A is moving just before the collision as the positive direction.

$$\upsilon'_A = \frac{(m_A - m_B)}{(m_A + m_B)}\upsilon_A = \frac{(0.045\text{ kg} - 0.065\text{ kg})}{(0.045\text{ kg} + 0.065\text{ kg})}(2.017\text{ m/s}) = -0.3667\text{ m/s} \approx \boxed{-0.37\text{ m/s}}$$

$$\upsilon'_B = \frac{2m_A}{(m_A + m_B)}\upsilon_A = \frac{2(0.045\text{ kg})}{(0.045\text{ kg} + 0.065\text{ kg})}(2.017\text{ m/s}) = 1.650\text{ m/s} \approx \boxed{1.7\text{ m/s}}$$

Notice that ball A has rebounded backward.

(*c*) After each collision, use the conservation of energy relationship again.

$$h'_A = \frac{\upsilon'^2_A}{2g} = \frac{(-0.3667\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = \boxed{6.9\times 10^{-3}\text{ m}} \quad h'_B = \frac{\upsilon'^2_B}{2g} = \frac{(1.650\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = \boxed{1.4\times 10^{-1}\text{ m}}$$

72. (*a*) Momentum is conserved in the *x* direction. The initial *x* momentum is 0.

$$p_{x\,\text{before}} = p_{x\,\text{after}} \rightarrow 0 = m_{\text{satellite}}\upsilon_{x\text{ satellite}} + m_{\text{shuttle}}\upsilon_{x\text{ shuttle}} \rightarrow$$

$$\upsilon_{x\text{ shuttle}} = -\frac{m_{\text{satellite}}\upsilon_{x\text{ satellite}}}{m_{\text{shuttle}}} = 2\,\frac{850\text{ kg}}{92,000\text{ kg}}(0.30\text{ m/s}) = -2.8\times 10^{-3}\text{ m/s}$$

So the component in the minus *x* direction is $\boxed{2.8\times10^{-3}\text{ m/s}}$.

(*b*) The average force is the change in momentum per unit time. The force on the satellite is in the positive *x* direction.

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{m\Delta\upsilon}{\Delta t} = \frac{(850\text{ kg})(0.30\text{ m/s})}{4.8\text{ s}} = \boxed{53\text{ N}}$$

73. (*a*) In the reference frame of the Earth, the final speed of the Earth–asteroid system is essentially 0, because the mass of the Earth is so much greater than the mass of the asteroid. It is like throwing a ball of mud at the wall of a large building—the smaller mass stops, and the larger mass doesn't move appreciably. Thus all of the asteroid's original kinetic energy can be released as destructive energy.

$$\text{KE}_{\text{orig}} = \tfrac{1}{2}m\upsilon_0^2 = \tfrac{1}{2}[(3200\text{ kg/m}^3)\tfrac{4}{3}\pi(1.0\times10^3\text{ m})^3](1.5\times10^4\text{ m/s})^2 = 1.507\times10^{21}\text{ J}$$

$$\approx \boxed{1.5\times10^{21}\text{ J}}$$

(*b*) $1.507\times10^{21}\text{ J}\left(\frac{1\text{ bomb}}{4.0\times10^{16}\text{ J}}\right) = \boxed{38,000\text{ bombs}}$

74. The initial momentum of the astronaut and the gas in the jet pack is 0, so the final momentum of the astronaut and the gas ejected from his jet pack must also be 0. We let A refer to the astronaut and B refer to the gas. The velocity of the astronaut is taken to be the positive direction. We also assume that gas is ejected very quickly, so that the 35 m/s is relative to the astronaut's rest frame.

$$0 = m_A\upsilon'_A + m_B\upsilon'_B = (210\text{ kg} - m_B)(2.0\text{ m/s}) + m_B(-35\text{ m/s}) \rightarrow$$

$$m_B = \frac{210\text{ kg}(2.0\text{ m/s})}{-37\text{ m/s}} = 11.35\text{ kg} \approx \boxed{11\text{ kg}}$$

75. (*a*) $\boxed{\text{No}}$, there is no net external force on the system. In particular, the spring force is internal to the system.

(*b*) Use conservation of momentum to determine the ratio of speeds. Note that the two masses will be moving in opposite directions. The initial momentum, when the masses are released, is 0.

$$p_{\text{initial}} = p_{\text{later}} \rightarrow 0 = m_A\upsilon_A - m_B\upsilon_B \rightarrow \upsilon_A/\upsilon_B = \boxed{m_B/m_A}$$

(*c*) $$\frac{KE_A}{KE_B} = \frac{\frac{1}{2}m_A\upsilon_A^2}{\frac{1}{2}m_B\upsilon_B^2} = \frac{m_A}{m_B}\left(\frac{\upsilon_A}{\upsilon_B}\right)^2 = \frac{m_A}{m_B}\left(\frac{m_B}{m_A}\right)^2 = \boxed{m_B/m_A}$$

(*d*) The center of mass was initially at rest. Since there is no net external force on the system, the center of mass does not move, and the system stays at rest.

76. Because all of the collisions are perfectly elastic, no energy is lost in the collisions. With each collision, the horizontal velocity is constant, and the vertical velocity reverses direction. So, after each collision, the ball rises again to the same height from which it dropped. Thus, after five bounces, the bounce height will be 4.00 m, the same as the starting height.

77. In this interaction, energy is conserved (initial potential energy of mass–compressed spring system = final kinetic energy of moving blocks) and momentum is conserved, since the net external force is 0. Use these two relationships to find the final speeds.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow 0 = m\upsilon_m - 3m\upsilon_{3m} \rightarrow \upsilon_m = 3\upsilon_{3m}$$

$$E_{\text{initial}} = E_{\text{final}} \rightarrow PE_{\substack{\text{spring}\\\text{initial}}} = KE_{\text{final}} \rightarrow$$

$$\tfrac{1}{2}kD^2 = \tfrac{1}{2}m\upsilon_m^2 + \tfrac{1}{2}3m\upsilon_{3m}^2 = \tfrac{1}{2}m(3\upsilon_{3m})^2 + \tfrac{1}{2}3m\upsilon_{3m}^2 = 6m\upsilon_{3m}^2$$

$$\tfrac{1}{2}kD^2 = 6m\upsilon_{3m}^2 \rightarrow \boxed{\upsilon_{3m} = D\sqrt{\frac{k}{12m}};\ \upsilon_m = 3D\sqrt{\frac{k}{12m}}}$$

Solutions to Search and Learn Problems

1. It is best to use $\sum\vec{\mathbf{F}}_{\text{ext}} = 0$ and $\sum\vec{\mathbf{p}}_i = \sum\vec{\mathbf{p}}_f$ when the system can be broken up into two or more objects for which only the forces between the objects are significant. The principle of impulse, $\sum\vec{\mathbf{F}}_{\text{ext}}\Delta t = \Delta\vec{\mathbf{p}},$ is useful in cases where the time over which the force acts is known and when examining the change in momentum of a single object due to external forces.

2. In each case, use momentum conservation. Let A represent the 6.0-kg object, and let B represent the 8.0-kg object. Then we have $m_A = 6.0\text{ kg},\ \ \upsilon_A = 6.5\text{ m/s}, m_B = 8.0\text{ kg},$ and $\upsilon_B = -4.0\text{ m/s}.$

(*a*) In this case, the objects stick together, so $\upsilon'_A = \upsilon'_B$.

$$m_A\upsilon_A + m_B\upsilon_B = (m_A + m_B)\upsilon'_A \rightarrow$$

$$\upsilon'_B = \upsilon'_A = \frac{m_A\upsilon_A + m_B\upsilon_B}{m_A + m_B} = \frac{(6.0\text{ kg})(6.5\text{ m/s}) + (8.0\text{ kg})(-4.0\text{ m/s})}{14.0\text{ kg}} = \boxed{0.5\text{ m/s}}$$

(*b*) In this case, use Eq. 7–7 to find a relationship between the velocities.

$$\upsilon_A - \upsilon_B = -(v'_A - v'_B) \rightarrow \upsilon'_B = \upsilon_A - \upsilon_B + \upsilon'_A$$

$$m_A\upsilon_A + m_B\upsilon_B = m_A\upsilon'_A + m_B\upsilon'_B = m_A\upsilon'_A + m_B(\upsilon_A - \upsilon_B + \upsilon'_A) \rightarrow$$

$$\upsilon'_A = \frac{(m_A - m_B)\upsilon_A + 2m_B\upsilon_B}{m_A + m_B} = \frac{(-2.0\text{ kg})(6.5\text{ m/s}) + 2(8.0\text{ kg})(-4.0\text{ m/s})}{14.0\text{ kg}} = \boxed{-5.5\text{ m/s}}$$

$$\upsilon'_B = \upsilon_A - \upsilon_B + \upsilon'_A = 6.5\text{ m/s} - (-4.0\text{ m/s}) - 5.5\text{ m/s} = \boxed{5.0\text{ m/s}}$$

(*c*) In this case, $\upsilon'_A = 0$.

$$m_{\text{A}}\upsilon_{\text{A}} + m_{\text{B}}\upsilon_{\text{B}} = m_{\text{B}}\upsilon'_{\text{B}} \quad \rightarrow$$

$$\upsilon'_{\text{B}} = \frac{m_{\text{A}}\upsilon_{\text{A}} + m_{\text{B}}\upsilon_{\text{B}}}{m_{\text{B}}} = \frac{(6.0\text{ kg})(6.5\text{ m/s}) + (8.0\text{ kg})(-4.0\text{ m/s})}{8.0\text{ kg}} = 0.875\text{ m/s} \approx \boxed{0.9\text{ m/s}}$$

To check for "reasonableness," first note the final directions of motion. A has stopped, and B has reversed direction. This is reasonable. Secondly, calculate the change in kinetic energy.

$$\Delta\text{KE} = \tfrac{1}{2}m_{\text{B}}\upsilon'^2_{\text{B}} - \left(\tfrac{1}{2}m_{\text{A}}\upsilon_{\text{A}}^2 + \tfrac{1}{2}m_{\text{B}}\upsilon_{\text{B}}^2\right)$$

$$= \tfrac{1}{2}(8.0\text{ kg})(0.875\text{ m/s})^2 - \left[\tfrac{1}{2}(6.0\text{ kg})(6.5\text{ m/s})^2 + \tfrac{1}{2}(8.0\text{ kg})(-4.0\text{ m/s})^2\right] = -188\text{ J}$$

Since the system has lost kinetic energy and the directions are possible, this interaction is reasonable.

(*d*) In this case, $\upsilon'_{\text{B}} = 0$.

$$m_{\text{A}}\upsilon_{\text{A}} + m_{\text{B}}\upsilon_{\text{B}} = m_{\text{A}}\upsilon'_{\text{A}} \quad \rightarrow$$

$$\upsilon'_{\text{A}} = \frac{m_{\text{A}}\upsilon_{\text{A}} + m_{\text{B}}\upsilon_{\text{B}}}{m_{\text{A}}} = \frac{(6.0\text{ kg})(6.5\text{ m/s}) + (8.0\text{ kg})(-4.0\text{ m/s})}{6.0\text{ kg}} = 1.167\text{ m/s} \approx \boxed{1\text{ m/s}}$$

This answer is not reasonable, because A continues to move in its original direction while B has stopped. Thus A has somehow "passed through" B. If B has stopped, A should have rebounded and would have had a negative velocity.

(*e*) In this case, $\upsilon'_{\text{A}} = -4.0$ m/s.

$$m_{\text{A}}\upsilon_{\text{A}} + m_{\text{B}}\upsilon_{\text{B}} = m_{\text{A}}\upsilon'_{\text{A}} + m_{\text{B}}\upsilon'_{\text{B}} \quad \rightarrow$$

$$\upsilon'_{\text{B}} = \frac{(6.0\text{ kg})(6.5\text{ m/s} - -4.0\text{ m/s}) + (8.0\text{ kg})(-4.0\text{ m/s})}{8.0\text{ kg}} = 3.875\text{ m/s} \approx \boxed{4\text{ m/s}}$$

The directions are reasonable, in that each object rebounds. Since the speed of both objects is smaller than in the perfectly elastic case (*b*), the system has lost kinetic energy. This interaction is reasonable.

(*f*) As quoted above, the results for (*c*) and (*e*) are reasonable, but (*d*) is not reasonable.

3. (*a*) Use Eq. 7–7, along with $\upsilon_B = 0$, to obtain a relationship between the velocities.

$$\upsilon_{\text{A}} - \upsilon_{\text{B}} = -(\upsilon'_{\text{A}} - \upsilon'_{\text{B}}) \quad \rightarrow \quad \upsilon'_{\text{B}} = \upsilon_{\text{A}} + \upsilon'_{\text{A}}$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_{\text{A}}\upsilon_{\text{A}} + m_{\text{B}}\upsilon_{\text{B}} = m_{\text{A}}\upsilon'_{\text{A}} + m_{\text{B}}\upsilon'_{\text{B}} = m_{\text{A}}\upsilon'_{\text{A}} + m_{\text{B}}(\upsilon_{\text{A}} + \upsilon'_{\text{A}}) = m_{\text{A}}\upsilon'_{\text{A}} + m_{\text{B}}\upsilon_{\text{A}} + m_{\text{B}}\upsilon'_{\text{A}} \quad \rightarrow$$

$$m_{\text{A}}\upsilon_{\text{A}} - m_{\text{B}}\upsilon_{\text{A}} = m_{\text{A}}\upsilon'_{\text{A}} + m_{\text{B}}\upsilon'_{\text{A}} \quad \rightarrow \quad (m_{\text{A}} - m_{\text{B}})\upsilon_{\text{A}} = (m_{\text{A}} + m_{\text{B}})\upsilon'_{\text{A}} \quad \rightarrow \quad \upsilon'_{\text{A}} = \frac{(m_{\text{A}} - m_{\text{B}})}{(m_{\text{A}} + m_{\text{B}})}\upsilon_{\text{A}}$$

Substitute this result into the result of Eq. 7–7.

$$\upsilon'_{\text{B}} = \upsilon_{\text{A}} + \upsilon'_{\text{A}} = \upsilon_{\text{A}} + \frac{(m_{\text{A}} - m_{\text{B}})}{(m_{\text{A}} + m_{\text{B}})}\upsilon_{\text{A}} = \upsilon_{\text{A}}\frac{(m_{\text{A}} + m_{\text{B}})}{(m_{\text{A}} + m_{\text{B}})} + \frac{(m_{\text{A}} - m_{\text{B}})}{(m_{\text{A}} + m_{\text{B}})}\upsilon_{\text{A}} = \upsilon_{\text{A}}\frac{2m_{\text{A}}}{(m_{\text{A}} + m_{\text{B}})}$$

Thus we have $\boxed{\upsilon'_{\text{A}} = \frac{(m_{\text{A}} - m_{\text{B}})}{(m_{\text{A}} + m_{\text{B}})}\upsilon_{\text{A}}}$ and $\boxed{\upsilon'_{\text{B}} = \upsilon_{\text{A}}\frac{2m_{\text{A}}}{(m_{\text{A}} + m_{\text{B}})}}$.

(*b*) If $m_{\text{A}} \quad m_{\text{B}}$, then approximate $m_{\text{A}} = 0$ when added to or subtracted from m_{B}.

$$\upsilon'_{\text{A}} = \frac{(m_{\text{A}} - m_{\text{B}})}{(m_{\text{A}} + m_{\text{B}})}\upsilon_{\text{A}} = \frac{(-m_{\text{B}})}{(+m_{\text{B}})}\upsilon_{\text{A}} = -\upsilon_{\text{A}} \qquad \upsilon'_{\text{B}} = \frac{2m_{\text{A}}\upsilon_{\text{A}}}{(m_{\text{A}} + m_{\text{B}})} = 0$$

The result is $\boxed{\upsilon'_A = -\upsilon_A;\ \upsilon'_B = 0}$. An example of this is a ball bouncing off of the floor. The massive floor has no speed after the collision, and the velocity of the ball is reversed (if dissipative forces are not present).

(*c*) If $m_A \quad m_B$, then approximate $m_B = 0$ when added to or subtracted from m_A.

$$\upsilon'_A = \frac{(m_A - m_B)}{(m_A + m_B)}\upsilon_A = \frac{(m_A)}{(m_A)}\upsilon_A = \upsilon_A \qquad \upsilon'_B = \frac{2m_A\upsilon_A}{(m_A + m_B)} = \frac{2m_A\upsilon_A}{(m_A)} = 2\upsilon_A$$

The result is $\boxed{\upsilon'_A = \upsilon_A;\ \upsilon'_B = 2\upsilon_A}$. An example of this would be a golf club hitting a golf ball. The speed of the club immediately after the collision is essentially the same as its speed before the collision, and the golf ball takes off with twice the speed of the club.

(*d*) If $m_A = m_B$, then set $m_A = m_B = m$.

$$\upsilon'_A = \frac{(m - m)}{(m + m)}\upsilon_A = 0 \qquad \upsilon'_B = \frac{2m\upsilon_A}{(m + m)} = \frac{2m\upsilon_A}{2m} = \upsilon_A$$

The result is $\boxed{\upsilon'_A = 0;\ \upsilon'_B = \upsilon_A}$. An example of this is one billiard ball making a head-on collision with another. The first ball stops, and the second ball takes off with the same speed that the first one had.

4. The interaction between the planet and the spacecraft is elastic, because the force of gravity is conservative. Thus kinetic energy is conserved in the interaction. Consider the problem a one-dimensional collision, with A representing the spacecraft and B representing Saturn. Because the mass of Saturn is so much bigger than the mass of the spacecraft, Saturn's speed is not changed appreciably during the interaction. Use Eq. 7–7, with $\upsilon_A = 10.4$ km/s and $\upsilon_B = \upsilon'_B = -9.6$ km/s.

$$\upsilon_A - \upsilon_B = -\upsilon'_A + \upsilon'_B \quad \rightarrow \quad \upsilon'_A = 2\upsilon_B - \upsilon_A = 2(-9.6\text{ km/s}) - 10.4\text{ km/s} = \boxed{-29.6\text{ km/s}}$$

Thus there is almost a threefold increase in the spacecraft's speed.