5

CIRCULAR MOTION; GRAVITATION

Responses to Questions

1. The three major "accelerators" are the accelerator pedal, the brake pedal, and the steering wheel. The accelerator pedal (or gas pedal) can be used to increase speed (by depressing the pedal) or to decrease speed in combination with friction (by releasing the pedal). The brake pedal can be used to decrease speed by depressing it. The steering wheel is used to change direction, which also is an acceleration. There are some other controls that could also be considered accelerators. The parking brake can be used to decrease speed by depressing it. The gear shift lever can be used to decrease speed by downshifting. If the car has a manual transmission, then the clutch can be used to decrease speed by depressing it (friction will slow the car), or, if on a steep downward incline, depressing the clutch can allow the car to increase speed. Finally, shutting the engine off can be used to decrease the car's speed. Any change in speed or direction means that an object is accelerating.

2. Yes, the centripetal acceleration will be greater when the speed is greater since centripetal acceleration is proportional to the square of the speed (when the radius is constant): $a_R = \frac{v^2}{r}$. When the speed is higher, the acceleration has a larger magnitude.

3. No, the acceleration will not be the same. The centripetal acceleration is inversely proportion to the radius (when the speed is constant): $a_R = \frac{v^2}{r}$. Traveling around a sharp curve, with a smaller radius, will require a larger centripetal acceleration than traveling around a gentle curve, with a larger radius.

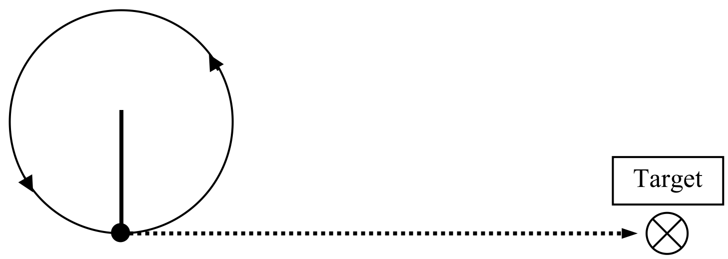
4. The three main forces on the child are the downward force of gravity (the child's weight), the normal force up on the child from the horse, and the static frictional force on the child from the surface of the horse. The frictional force provides the centripetal acceleration. If there are other forces, such as contact forces between the child's hands or legs and the horse, which have a radial component, they will contribute to the centripetal acceleration.

5. On level ground, the normal force on the child would be the same magnitude as his weight. This is the "typical" situation. But as the child and sled come over the crest of the hill, they are moving in a curved path, which can at least be approximated by a circle. There must be a centripetal force, pointing inward toward the center of the arc. The combination of gravity (acting downward) and the normal force on his body (acting upward when the sled is at the top of the hill) provides this centripetal force,

which must be greater than zero. At the top of the hill, if downward is the positive direction, Newton's second law says $F_y = mg - F_N = m\dfrac{\upsilon^2}{r}$. Thus the normal force must be less than the child's weight.

6. No. The barrel of the dryer provides a centripetal force on the clothes to keep them moving in a circular path. A water droplet on the solid surface of the drum will also experience this centripetal force and move in a circle. However, as soon as the water droplet is at the location of a hole in the drum there will be no centripetal force on it and it will therefore continue moving in a path in the direction of its tangential velocity, which will take it out of the drum. There is no centrifugal force throwing the water outward; there is rather a lack of centripetal force to keep the water moving in a circular path.

$\boxed{7.}$ She should let go of the string at the moment that the tangential velocity vector is directed exactly at the target. This would also be when the string is perpendicular to the desired direction of motion of the ball. See the "top view" diagram. Also see Fig. 5–6 in the textbook.



$\boxed{8.}$ At the top of the bucket's arc, the gravitational force and normal forces from the bucket, both pointing downward, must provide the centripetal force needed to keep the water moving in a circle. In the limiting case of no normal force, Newton's second law would give $F_{\text{net}} = mg = m\dfrac{\upsilon^2}{r}$, which means that the bucket must be moving with a tangential speed of $\upsilon \geq \sqrt{gr}$ or the water will spill out of the bucket. At the top of the arc, the water has a horizontal velocity. As the bucket passes the top of the arc, the velocity of the water develops a vertical component. But the bucket is traveling with the water, with the same velocity, and contains the water as it falls through the rest of its path.

$\boxed{9.}$ For objects (including astronauts) on the inner surface of the cylinder, the normal force provides a centripetal force, which points inward toward the center of the cylinder. This normal force simulates the normal force we feel when on the surface of Earth.

(*a*) Falling objects are not in contact with the floor, so when released they will continue to move with constant velocity until they reach the shell. From the frame of reference of the astronaut inside the cylinder, it will appear that the object "falls" in a curve, rather than straight down.

(*b*) The magnitude of the normal force on the astronaut's feet will depend on the radius and speed of the cylinder. If these are such that $\dfrac{\upsilon^2}{r} = g$ (so that $m\dfrac{\upsilon^2}{r} = mg$ for all objects), then the normal force will feel just like it does on the surface of Earth.

(*c*) Because of the large size of Earth compared to humans, we cannot tell any difference between the gravitational force at our heads and at our feet. In a rotating space colony, the difference in the simulated gravity at different distances from the axis of rotation could be significant, perhaps producing dizziness or other adverse effects. Also, playing "catch" with a ball could be difficult since the normal parabolic paths as experienced on Earth would not occur in the rotating cylinder.

10. (*a*) The normal force on the car is largest at point C. In this case, the centripetal force keeping the car in a circular path of radius R is directed upward, so the normal force must be greater than the weight to provide this net upward force.

(*b*) The normal force is smallest at point A, the crest of the hill. At this point the centripetal force must be directed downward (toward the center of the circle), so the normal force must be less than the weight. (Notice that the normal force is equal to the weight at point B.)

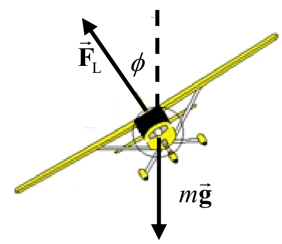
(*c*) The driver will feel heaviest where the normal force is greatest, or at point C.

(*d*) The driver will feel lightest at point A, where the normal force is the least.

(*e*) At point A, the centripetal force is weight minus normal force, or $mg - F_N = \frac{mv^2}{R}$. The point at which the car just loses contact with the road corresponds to a normal force of zero, which is the maximum speed without losing contact. Setting $F_N = 0$ gives $mg = \frac{mv_{max}^2}{R} \rightarrow v_{max} = \sqrt{Rg}$.

11. Yes, a particle with constant speed can be accelerating. A particle traveling around a curve while maintaining a constant speed is accelerating because its direction is changing. However, a particle with a constant velocity cannot be accelerating, since the velocity is not changing in magnitude or direction, and to have an acceleration the velocity must be changing.

12. When an airplane is in level flight, the downward force of gravity is counteracted by the upward lift force, analogous to the upward normal force on a car driving on a level road. The lift on an airplane is perpendicular to the plane of the airplane's wings, so when the airplane banks, the lift vector has both vertical and horizontal components (similar to the vertical and horizontal components of the normal force on a car on a banked turn). Assuming that the plane has no vertical acceleration, then the vertical component of the lift balances the weight and the horizontal component of the lift provides the centripetal force. If F_L is the total lift and $\phi =$ the banking angle, measured from the vertical, then $F_L \cos\phi = mg$ and $F_L \sin\phi = m\frac{v^2}{r}$, so $\phi = \tan^{-1}(v^2/gr)$.



13. Whether the apple is (*a*) attached to a tree or (*b*) falling, it exerts a gravitational force on the Earth equal to the force the Earth exerts on it, which is the weight of the apple (Newton's third law). That force is independent of the motion of the apple.

14. Since the Earth's mass is much greater than the Moon's mass, the point at which the net gravitational pull on the spaceship is zero is closer to the Moon. It is shown in Problem 30 that this occurs at about 90% of the way from the Earth to the Moon. So, a spaceship traveling from the Earth toward the Moon must therefore use fuel to overcome the net pull backward for 90% of the trip. Once it passes that point, the Moon will exert a stronger pull than the Earth and accelerate the spacecraft toward the Moon. However, when the spaceship is returning to the Earth, it reaches the zero point at only 10% of the way from the Moon to the Earth. Therefore, for most of the trip toward the Earth, the spacecraft is "helped" by the net gravitational pull in the direction of travel, so less fuel is used.

15. The satellite needs a certain speed with respect to the center of the Earth to achieve orbit. The Earth rotates toward the east so it would require less speed (with respect to the Earth's surface) to launch a satellite (*a*) toward the east. Before launch, the satellite is moving with the surface of the Earth so already has a "boost" in the right direction.

16. If the antenna becomes detached from a satellite in orbit, the antenna will continue in orbit around the Earth with the satellite. If the antenna were given a component of velocity toward the Earth (even a very small one), it would eventually spiral in and hit the Earth. If the antenna were somehow slowed down, it would also fall toward the Earth.

17. Yes, we are heavier at midnight. At noon, the gravitational force on a person due to the Sun and the gravitational force due to the Earth are in the opposite directions. At midnight, the two forces point in the same direction. Therefore, your apparent weight at midnight is greater than your apparent weight at noon.

18. Your apparent weight will be greatest in case (*b*), when the elevator is accelerating upward. The scale reading (your apparent weight) indicates your force on the scale, which, by Newton's third law, is the same as the normal force of the scale on you. If the elevator is accelerating upward, then the net force must be upward, so the normal force (up) must be greater than your actual weight (down). When in an elevator accelerating upward, you "feel heavy."

    Your apparent weight will be least in case (*c*), when the elevator is in free fall. In this situation your apparent weight is zero since you and the elevator are both accelerating downward at the same rate and the normal force is zero.

    Your apparent weight will be the same as when you are on the ground in case (*d*), when the elevator is moving upward at a constant speed. If the velocity is constant, acceleration is zero and $N = mg$. (Note that it doesn't matter if the elevator is moving up or down or even at rest, as long as the velocity is constant.)

19. If the Earth were a perfect, nonrotating sphere, then the gravitational force on each droplet of water in the Mississippi would be the same at the headwaters and at the outlet, and the river wouldn't flow. Since the Earth is rotating, the droplets of water experience a centripetal force provided by a part of the component of the gravitational force perpendicular to the Earth's axis of rotation. The centripetal force is smaller for the headwaters, which are closer to the North Pole, than for the outlet, which is closer to the equator. Since the centripetal force is equal to $mg - N$ (apparent weight) for each droplet, N is smaller at the outlet, and the river will flow. This effect is large enough to overcome smaller effects on the flow of water due to the bulge of the Earth near the equator.

20. The satellite remains in orbit because it has a velocity. The instantaneous velocity of the satellite is tangent to the orbit. The gravitational force provides the centripetal force needed to keep the satellite in orbit, acting like the tension in a string when twirling a rock on a string. A force is not needed to keep the satellite "up"; a force is needed to bend the velocity vector around in a circle. The satellite can't just have any speed at any radius, though. For a perfectly circular orbit, the speed is determined by the orbit radius, or vice versa, through the relationship $v_{\text{orbit}} = \sqrt{rg}$, where r is the radius of the orbit and g is the acceleration due to gravity at the orbit position.

21. The centripetal acceleration of Mars in its orbit around the Sun is smaller than that of the Earth. For both planets, the centripetal force is provided by gravity, so the centripetal acceleration is inversely proportional to the square of the distance from the planet to the Sun:

$$\frac{m_{\text{p}}v^2}{r} = \frac{Gm_{\text{s}}m_{\text{p}}}{r^2} \quad \text{so} \quad a_{\text{R}} = \frac{v^2}{r} = \frac{Gm_{\text{s}}}{r^2}$$

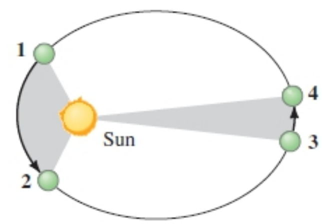
    Since Mars is at a greater distance from the Sun than is Earth, it has a smaller centripetal acceleration. Note that the mass of the planet does not appear in the equation for the centripetal acceleration.

22. For Pluto's moon, we can equate the gravitational force from Pluto on the moon to the centripetal force needed to keep the moon in orbit:

$$\frac{m_{\mathrm{m}}\upsilon^2}{r} = \frac{Gm_{\mathrm{p}}m_{\mathrm{m}}}{r^2} \rightarrow m_{\mathrm{p}} = \frac{\upsilon^2 r}{G} = \frac{4\pi^2 r^3}{GT^2}$$

This allows us to solve for the mass of Pluto (m_{p}) if we know G, the radius of the moon's orbit, and the velocity of the moon, which can be determined from the period T and orbital radius. Note that the mass of the moon cancels out.

$\boxed{23.}$ The Earth is closer to the Sun in January. See Fig. 5–29 and the accompanying discussion about Kepler's second law. The caption in the textbook says: "Planets move fastest when closest to the Sun." So in the (greatly exaggerated) figure, the time between points 1 and 2 would be during January, and the time between points 3 and 4 would be July.



Responses to MisConceptual Questions

1. (*b*) As you turn, you feel the force between yourself and the car door. A common misconception is that a centrifugal force is pushing you into the door (answer (*a*)). Actually, your inertia tries to keep you moving in a straight line. As the car (and door) turn right, the door accelerates into you, pushing you away from your straight-line motion and toward the right.

2. (*e*) In circular motion, the velocity is always perpendicular to the radius of the circle, so (*b*), (*c*), and (*d*) are incorrect. The net force is always in the same direction as the acceleration, so if the acceleration points toward the center, the net force must also. Therefore, (*e*) is a better choice than (*a*).

3. (*c*) A common misconception is that the ball will continue to move in a curved path after it exits the tube (answers (*d*) or (*e*)). However, for the ball to move in a curved path, a net force must be acting on the ball. When it is inside the tube, the normal force from the tube wall provides the centripetal force. After the ball exits the tube, there is no net force, so the ball must travel in a straight-line path in the same direction it was traveling as it exited the tube.

4. (*d*) The phrase "steady speed" is not the same as "constant velocity," as velocity also includes direction. A common misconception is that if a car moves at steady speed, the acceleration and net force are zero (answers (*a*) or (*b*)). However, since the path is circular, a radially inward force must cause the centripetal acceleration. If this force (friction between the tires and road) were not present, the car would move in a straight line. It would not accelerate outward.

5. (*b*) A common error in this problem is to ignore the contribution of gravity in the centripetal force. At the top of the loop gravity assists the tension in providing the centripetal force, so the tension is less than the centripetal force. At the bottom of the loop gravity opposes the tension, so the tension is greater than the centripetal force. At all other points in the loop the tension is between the maximum at the bottom and the minimum at the top.

6. (*a*) The forces acting on the child are gravity (downward), the normal force (away from the wall), and the force of friction (parallel to the wall and in this case opposing gravity). In particular, there is nothing "pushing" outward on the rider, so answers (*b*), (*d*), and (*e*) cannot be correct.

7. (*d*) If the net force on the Moon were zero (answer (*a*)), the Moon would move in a straight line and not orbit about the Earth. Gravity pulls the Moon away from the straight-line motion. The large tangential velocity is what keeps the Moon from crashing into the Earth. The gravitational force of the Sun also acts on the Moon, but this force causes the Earth and Moon to orbit the Sun.

8. (*f*) A common misconception is that since the Earth is more massive than the Moon, it must exert more force. However, the force is an interaction between the Earth and Moon, so by Newton's third law, the forces must be equal. Since the Moon is less massive than the Earth and the forces are equal, the Moon has the greater acceleration.

9. (*c*) The nonzero gravitational force on the ISS is responsible for it orbiting the Earth instead of moving is a straight line through space. Astronauts aboard the ISS experience the same centripetal acceleration (free fall toward the Earth) as the station and as a result do not experience a normal force (apparent weightlessness).

10. (*b*) A common misconception is that the mass of an object affects its orbital speed. However, as with all objects in free fall, when calculating the acceleration the object's mass is divided out of the gravitational force. All objects at the same radial distance from the Earth experience the same centripetal acceleration, and by Eq. 5–1 they have the same orbital speed.

11. (*c*) Each of the incorrect answers assumes the presence of an external force to change the orbital motion of the payload. When the payload is attached to the arm, it is orbiting the Earth at the same distance and speed as the shuttle. When it is released, the only force acting on the payload is the force of gravity, which due to the speed of the payload keeps it in orbital motion. For the payload to fall straight down or to follow a curved path that hits the Earth, a force would need to slow down the payload's speed, but no such force is present. To drift out into deep space a force would be needed to overcome the gravity that is keeping it in orbit, but no such force is present.

12. (*d*) Since the penny is rotating around the turntable it experiences a centripetal force toward the center of the turntable, as in (*c*). The rotation is also slowing down, so the penny experiences a decelerating force opposite its velocity, as in (*a*). These two forces are vectors and must be added together to give a net force in the direction of (*d*).

Solutions to Problems

1. (*a*) Find the centripetal acceleration from Eq. 5–1.

$$a_{\mathrm{R}} = \frac{v^2}{r} = (1.10\ \mathrm{m/s})^2/1.20\ \mathrm{m} = 1.008\ \mathrm{m/s^2} \approx \boxed{1.01\ \mathrm{m/s^2}}$$

(*b*) The net horizontal force is causing the centripetal motion, so it will be the centripetal force.

$$F_{\mathrm{R}} = ma_{\mathrm{R}} = (22.5\ \mathrm{kg})(1.008\ \mathrm{m/s^2}) = 22.68\ \mathrm{N} \approx \boxed{22.7\ \mathrm{N}}$$

2. Find the centripetal acceleration from Eq. 5–1.

$$a_R = \frac{v^2}{r} = \frac{(525\ \mathrm{m/s})^2}{5.20\times10^3\ \mathrm{m}} = (53.00\ \mathrm{m/s^2})\left(\frac{1\ g}{9.80\ \mathrm{m/s^2}}\right) = \boxed{5.41\, g\text{'s}}$$

3. Find the speed from Eq. 5–3.

$$F_{\mathrm{R}} = \frac{mv^2}{r} \quad\rightarrow\quad v = \sqrt{\frac{F_{\mathrm{R}}r}{\mathrm{m}}} = \sqrt{\frac{(310\ \mathrm{N})(0.90\ \mathrm{m})}{2.0\ \mathrm{kg}}} = 11.81\ \mathrm{m/s} \approx \boxed{12\ \mathrm{m/s}}$$

4. To find the period, take the reciprocal of the rotational speed (in rev/min) to get min/rev, and then convert to s/rev. Use the period to find the speed, and then the centripetal acceleration.

$$T=\left(\frac{1\text{ min}}{45\text{ rev}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right)=1.333\frac{\text{s}}{\text{rev}}\quad r=0.175\text{ m}\quad \upsilon=\frac{2\pi r}{T}=\frac{2\pi(0.175\text{ m})}{1.333\text{ s}}=0.8249\text{ m/s}$$

$$a_{\text{R}}=\frac{\upsilon^2}{r}=\frac{(0.8249\text{ m/s})^2}{0.175\text{ m}}=3.888\text{ m/s}^2\approx\boxed{3.9\text{ m/s}^2}$$

5. The centripetal force that the tension provides is given by Eq. 5–3. Solve that for the speed.

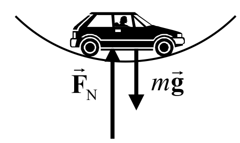
$$F_{\text{R}}=\frac{m\upsilon^2}{r}\quad\rightarrow\quad\upsilon=\sqrt{\frac{F_{\text{R}}r}{m}}=\sqrt{\frac{(75\text{ N})(1.3\text{ m})}{0.55\text{ kg}}}=13.31\text{ m/s}\approx\boxed{13\text{ m/s}}$$

6. The centripetal acceleration of a rotating object is given by Eq. 5–1. Solve that for the velocity.

$$\upsilon=\sqrt{a_{\text{R}}r}=\sqrt{(1.25\times10^5g)r}=\sqrt{(1.25\times10^5)(9.80\text{ m/s}^2)(7.00\times10^{22}\text{ m})}=2.928\times10^2\text{ m/s}$$

$$(2.928\times10^2\text{ m/s})\left(\frac{1\text{ rev}}{2\pi(7.00\times10^{2\,2}\text{ m})}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right)=\boxed{3.99\times10^4\text{ rpm}}$$

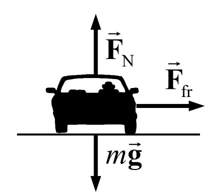
7. A free-body diagram for the car is shown. Write Newton's second law for the car in the vertical direction, assuming that up is positive. The normal force is twice the weight.



$$\sum F=F_{\text{N}}-mg=ma\quad\rightarrow\quad 2mg-mg=m\frac{\upsilon^2}{r}\quad\rightarrow$$

$$\upsilon=\sqrt{rg}=\sqrt{(115\text{ m})(9.80\text{ m/s}^2)}=33.57\text{ m/s}\approx\boxed{34\text{ m/s}}$$

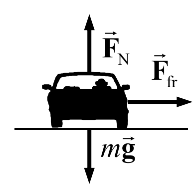
8. In the free-body diagram, the car is coming out of the page, and the center of the circular path is to the right of the car, in the plane of the page. The vertical forces (gravity and normal force) are of the same magnitude, because the car is not accelerating vertically. We assume that friction is the force causing the circular motion. At maximum speed, the car would be on the verge of slipping, and static friction would be at its maximum value.



$$F_{\text{R}}=F_{\text{fr}}\quad\rightarrow\quad m\frac{\upsilon^2}{r}=\mu_{\text{s}}F_{\text{N}}=\mu_{\text{s}}mg\quad\rightarrow\quad\mu_{\text{s}}=\frac{\upsilon^2}{rg}=\frac{\left[(95\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2}{(125\text{ m})(9.80\text{ m/s}^2)}=\boxed{0.57}$$

Notice that the result is independent of the car's mass.

9. A free-body diagram for the car at one instant is shown, as though the car is coming out of the page. The center of the circular path is to the right of the car, in the plane of the page. At maximum speed, the car would be on the verge of slipping, and static friction would be at its maximum value. The vertical forces (gravity and normal force) are of the same magnitude, because the car is not accelerating vertically. We assume that friction is the force causing the circular motion.

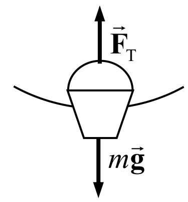


$$F_{\text{R}}=F_{\text{fr}}\quad\rightarrow\quad m\frac{\upsilon^2}{r}=\mu_{\text{s}}F_{\text{N}}=\mu_{\text{s}}mg\quad\rightarrow$$

$$\upsilon=\sqrt{\mu_{\text{s}}rg}=\sqrt{(0.65)(90.0\text{ m})(9.80\text{ m/s}^2)}=23.94\text{ m/s}\approx\boxed{24\text{ m/s}}$$

Notice that the result is $\boxed{\text{independent of the car's mass}}$.

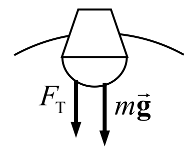
10. (a) At the bottom of the motion, a free-body diagram of the bucket would be as shown. Since the bucket is moving in a circle, there must be a net force on it toward the center of the circle and a centripetal acceleration. Write Newton's second law for the bucket, Eq. 5–3, with up as the positive direction.



$$\sum F_R = F_T - mg = ma = m\upsilon^2/r \rightarrow$$

$$\upsilon = \sqrt{\frac{r(F_T - mg)}{m}} = \sqrt{\frac{(1.20\text{ m})[25.0\text{ N} - (2.00\text{ kg})(9.80\text{ m/s}^2)]}{2.00\text{ kg}}} = \boxed{1.8\text{ m/s}}$$

(b) A free-body diagram of the bucket at the top of the motion is shown. The bucket is moving in a circle, so there must be a net force on it toward the center of the circle, and a centripetal acceleration. Write Newton's second law for the bucket, Eq. 5–3, with down as the positive direction.



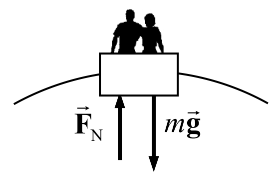
$$\sum F_R = F_T + mg = ma = m\frac{\upsilon^2}{r} \quad \rightarrow \quad \upsilon = \sqrt{\frac{r(F_T + mg)}{m}}$$

If the tension is to be zero, then

$$\upsilon = \sqrt{\frac{r(0 + mg)}{m}} = \sqrt{rg} = \sqrt{(1.20\text{ m})(9.80\text{ m/s}^2)} = \boxed{3.43\text{ m/s}}$$

The bucket must move faster than 3.43 m/s in order for the rope not to go slack.

$\boxed{11.}$ The free-body diagram for passengers at the top of a Ferris wheel is as shown. F_N is the normal force of the seat pushing up on the passengers. The sum of the forces on the passengers is producing the centripetal motion and must be a centripetal force. Call the downward direction positive, and write Newton's second law for the passengers, Eq. 5–3.



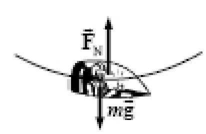
$$\sum F_R = mg - F_N = ma = m\frac{\upsilon^2}{r}$$

Since the passengers are to feel "weightless," they must lose contact with their seat, and the normal force will be 0. The diameter is 25 m, so the radius is 12.5 m.

$$mg = m\frac{\upsilon^2}{r} \quad \rightarrow \quad \upsilon = \sqrt{gr} = \sqrt{(9.80\text{ m/s}^2)(12.5\text{ m})} = 11.07\text{ m/s}$$

$$(11.07\text{ m/s})\left(\frac{1\text{ rev}}{2\pi(12.5\text{ m})}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 8.457\text{ rpm} \approx \boxed{8.5\text{ rpm}}$$

$\boxed{12.}$ (a) See the free-body diagram for the pilot in the jet at the bottom of the loop.



We have $a_R = \frac{\upsilon^2}{r} = 6.0\,g$.

$$\frac{\upsilon^2}{r} = 6.0\,g \quad \rightarrow \quad r = \frac{\upsilon^2}{6.0\,g} = \frac{\left[(840\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2}{6.0(9.80\text{ m/s}^2)} = 925.9\text{ m} \approx \boxed{930\text{ m}}$$

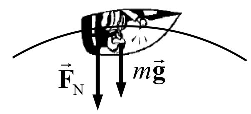
(b) The net force must be centripetal, to make the pilot go in a circle. Write Newton's second law for the vertical direction, with up as positive. The normal force is the apparent weight.

$$\sum F_R = F_N - mg = m\frac{\upsilon^2}{r}$$

The centripetal acceleration is to be $\frac{v^2}{r} = 6.0g$.

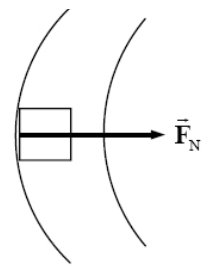
$$F_N = mg + m\frac{v^2}{r} = 7mg = 7(78\text{ kg})(9.80\text{ m/s}^2) = 5350\text{ N} = \boxed{5400\text{ N}}$$

(*c*) See the free-body diagram for the pilot at the top of the loop. The normal force is down, because the pilot is upside down. Write Newton's second law in the vertical direction, with down as positive.



$$\sum F_R = F_N + mg = mv^2/r = 6mg \quad \to \quad F_N = 5mg = \boxed{3800\text{ N}}$$

13. To experience a gravity-type force, objects must be on the inside of the outer wall of the tube, so that there can be a centripetal force to move the objects in a circle. See the free-body diagram for an object on the inside of the outer wall and a portion of the tube. The normal force of contact between the object and the wall must be maintaining the circular motion. Write Newton's second law for the radial direction.



$$\sum F_R = F_N = ma = m\frac{v^2}{r}$$

This is to have nearly the same effect as Earth gravity, with $F_N = 0.90\,mg$. Equate the two expressions for normal force and solve for the speed.

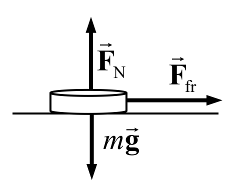
$$F_N = m\frac{v^2}{r} = 0.90mg \quad \to \quad v = \sqrt{0.90gr} = \sqrt{(0.90)(9.80\text{ m/s}^2)(550\text{ m})} = 69.65\text{ m/s}$$

$$(69.65\text{ m/s})\left(\frac{1\text{ rev}}{2\pi(550\text{ m})}\right)\left(\frac{86{,}400\text{ s}}{1\text{ day}}\right) = 1741\text{ rev/day} \approx \boxed{1700\text{ rev/day}}$$

14. The radius of either skater's motion is 0.80 m, and the period is 2.5 s. Thus their speed is given by $v = 2\pi r/T = \frac{2\pi(0.80\text{ m})}{2.5\text{ s}} = 2.0\text{ m/s}$. Since each skater is moving in a circle, the net radial force on each one is given by Eq. 5–3.

$$F_R = m\frac{v^2}{r} = \frac{(55.0\text{ kg})(2.0\text{ m/s})^2}{0.80\text{ m}} = 275\text{ N} \approx \boxed{2.8\times 10^2\text{ N}}$$

15. The force of static friction is causing the circular motion—it is the centripetal force. The coin slides off when the static frictional force is not large enough to move the coin in a circle. The maximum static frictional force is the coefficient of static friction times the normal force, and the normal force is equal to the weight of the coin as seen in the free-body diagram, since there is no vertical acceleration. In the free-body diagram, the coin is coming out of the page and the center of the circle is to the right of the coin, in the plane of the page.

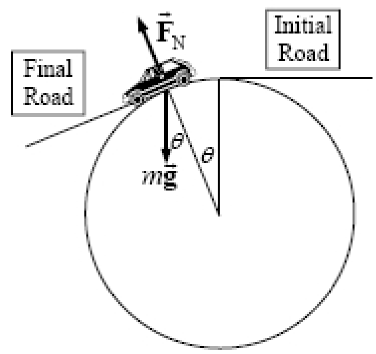


The rotational speed must be changed into a linear speed.

$$v = \left(38.0\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi(0.130\text{ m})}{1\text{ rev}}\right) = 0.5173\text{ m/s}$$

$$F_R = F_{fr} \quad \to \quad m\frac{v^2}{r} = \mu_s F_N = \mu_s mg \quad \to \quad \mu_s = \frac{v^2}{rg} = \frac{(0.5173\text{ m/s})^2}{(0.130\text{ m})(9.80\text{ m/s}^2)} = \boxed{0.210}$$

16. For the car to stay on the road, the normal force must be greater than 0. See the free-body diagram, write the net radial force, and solve for the radius.

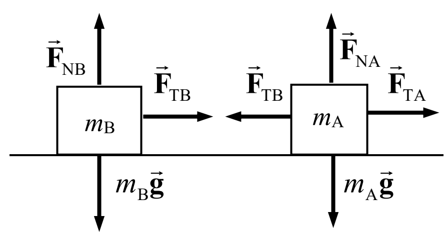


$$F_{\mathrm{R}} = mg\cos\theta - F_{\mathrm{N}} = \frac{mv^2}{r} \quad \rightarrow \quad r = \frac{mv^2}{mg\cos\theta - F_{\mathrm{N}}}$$

For the car to be on the verge of leaving the road, the normal force would be 0, so $r_{\mathrm{critical}} = \dfrac{mv^2}{mg\cos\theta} = \dfrac{v^2}{g\cos\theta}$. This expression gets larger as the angle increases, so we must evaluate at the largest angle to find a radius that is good for all angles in the range.

$$r_{\substack{\mathrm{critical}\\ \mathrm{maximum}}} = \frac{v^2}{g\cos\theta_{\mathrm{max}}} = \frac{\left[95\ \mathrm{km/h}\left(\dfrac{1\ \mathrm{m/s}}{3.6\ \mathrm{km/h}}\right)\right]^2}{(9.80\ \mathrm{m/s^2})\cos 18^\circ} = 74.7\ \mathrm{m} \approx \boxed{75\ \mathrm{m}}$$

17. If the masses are in line and both have the same frequency of rotation, then they will always stay in line. Consider a free-body diagram for both masses, from a side view, at the instant that they are to the left of the post. Note that the same tension that pulls inward on mass 2 pulls outward on mass 1, by Newton's third law. Also notice that since there is no vertical acceleration, the normal force on each mass is equal to its weight. Write Newton's second law for the horizontal direction for both masses, noting that they are in uniform circular motion.



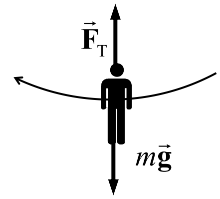
$$\sum F_{\mathrm{RA}} = F_{\mathrm{TA}} - F_{\mathrm{TB}} = m_{\mathrm{A}}a_{\mathrm{A}} = m_{\mathrm{A}}\frac{v_{\mathrm{A}}^2}{r_{\mathrm{A}}} \qquad \sum F_{\mathrm{RB}} = F_{\mathrm{TB}} = m_{\mathrm{B}}a_{\mathrm{B}} = m_{\mathrm{B}}\frac{v_{\mathrm{B}}^2}{r_{\mathrm{B}}}$$

The speeds can be expressed in terms of the frequency as follows: $v = \left(f\,\dfrac{\mathrm{rev}}{\mathrm{s}}\right)\left(\dfrac{2\pi r}{1\ \mathrm{rev}}\right) = 2\pi rf$.

$$F_{\mathrm{TB}} = m_{\mathrm{B}}\frac{v_{\mathrm{B}}^2}{r_{\mathrm{B}}} = m_{\mathrm{B}}(2\pi r_{\mathrm{B}}f)^2/r_{\mathrm{B}} = \boxed{4\pi^2 m_{\mathrm{B}}r_{\mathrm{B}}f^2}$$

$$F_{\mathrm{TA}} = F_{\mathrm{TB}} + m_{\mathrm{A}}\frac{v_{\mathrm{A}}^2}{r_{\mathrm{A}}} = 4\pi m_{\mathrm{B}}r_{\mathrm{B}}f^2 + m_{\mathrm{A}}(2\pi r_{\mathrm{A}}f)^2/r_{\mathrm{A}} = \boxed{4\pi^2 f^2(m_{\mathrm{A}}r_{\mathrm{A}} + m_{\mathrm{B}}r_{\mathrm{B}}}$$

18. A free-body diagram of Tarzan at the bottom of his swing is shown. The upward tension force is created by his pulling down on the vine. Write Newton's second law in the vertical direction. Since he is moving in a circle, his acceleration will be centripetal and will point upward when he is at the bottom.



$$\sum F = F_{\mathrm{T}} - mg = ma = m\frac{v^2}{r} \quad \rightarrow \quad v = \sqrt{\frac{(F_{\mathrm{T}} - mg)r}{m}}$$

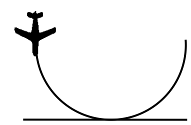
The maximum speed will be obtained with the maximum tension.

$$\upsilon_{\max} = \sqrt{\frac{(\vec{\mathbf{F}}_{\text{T max}} - mg)r}{m}} = \sqrt{\frac{[1150\ \text{N} - (78\ \text{kg})(9.80\ \text{m/s}^2)](4.7\ \text{m})}{78\ \text{kg}}} = \boxed{4.8\ \text{m/s}}$$

19. The speed is 50 km/h, the curve is unbanked, and the static friction coefficient for rubber on wet concrete is 0.7. If the car is just at the point of slipping, the static frictional force, which is providing the acceleration, would be at its maximum.

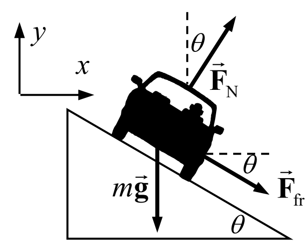
$$\frac{m\upsilon^2}{r} = \mu_s mg \rightarrow r = \frac{\upsilon^2}{\mu_s g} = \frac{\left[50\ \text{km/h}\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right)\right]^2}{(0.7)(9.80\ \text{m/s}^2)} = 28.12\ \text{m} \approx \boxed{30\ \text{m}}$$

20. The fact that the pilot can withstand 8.0 g's without blacking out, along with the speed of the aircraft, will determine the radius of the circle that he must fly as he pulls out of the dive. To just avoid crashing into the sea, he must begin to form that circle (pull out of the dive) at a height equal to the radius of that circle.



$$a_R = \frac{\upsilon^2}{r} = 8.0\,g \quad \rightarrow \quad r = \frac{\upsilon^2}{8.0\,g} = \frac{(270\ \text{m/s})^2}{8.0(9.80\ \text{m/s}^2)} = \boxed{930\ \text{m}}$$

21. Since the curve is designed for 65 km/h, traveling at a higher speed with the same radius means that more centripetal force will be required. That extra centripetal force will be supplied by a force of static friction, downward along the incline. See the free-body diagram for the car on the incline. Note that from Example 5–7 in the textbook, the no-friction banking angle is given by the following:



$$\theta = \tan^{-1}\frac{\upsilon^2}{rg} = \tan^{-1}\frac{\left[(65\ \text{km/h})\left(\frac{1.0\ \text{m/s}}{3.6\ \text{km/h}}\right)\right]^2}{(95\ \text{m})(9.80\ \text{m/s}^2)} = 19.3^\circ$$

Write Newton's second law in both the x and y directions. The car will have no acceleration in the y direction and centripetal acceleration in the x direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{fr} = \mu_s F_N$. Solve each equation for the normal force.

$$\sum F_y = F_N\cos\theta - mg - F_{fr}\sin\theta = 0 \quad \rightarrow \quad F_N\cos\theta - \mu_s F_N\sin\theta = mg \quad \rightarrow$$

$$F_N = \frac{mg}{(\cos\theta - \mu_s\sin\theta)}$$

$$\sum F_x = F_N\sin\theta + F_{fr}\cos\theta = F_R = m\frac{\upsilon^2}{r} \quad \rightarrow \quad F_N\sin\theta + \mu_s F_N\cos\theta + = m\frac{\upsilon^2}{r} \quad \rightarrow$$

$$F_N = \frac{m\upsilon^2/r}{(\sin\theta + \mu_s\cos\theta)}$$

Equate the two expressions for F_N and solve for the coefficient of friction. The speed of rounding the curve is given by $\upsilon = (95 \text{ km/h})\left(\frac{1.0 \text{ m/s}}{3.6 \text{ km/h}}\right) = 26.39 \text{ m/s}.$

$$\frac{mg}{(\cos\theta - \mu_s \sin\theta)} = \frac{m\upsilon^2/r}{(\sin\theta + \mu_s \cos\theta)} \rightarrow$$

$$\mu_s = \frac{\left(\frac{\upsilon^2}{r}\cos\theta - g\sin\theta\right)}{\left(g\cos\theta + \frac{\upsilon^2}{r}\sin\theta\right)} = \frac{\left(\frac{\upsilon^2}{r} - g\tan\theta\right)}{\left(g + \frac{\upsilon^2}{r}\tan\theta\right)} = \frac{\left(\frac{(26.39 \text{ m/s})^2}{95 \text{ m}} - (9.80 \text{ m/s}^2)\tan 19.3°\right)}{\left(9.80 \text{ m/s}^2 + \frac{(26.39 \text{ m/s})^2}{95 \text{ m}}\tan 19.3°\right)} = \boxed{0\ 32}$$

22. From Example 5–8, we are given that the track radius is 500 m (assumed to have two significant figures), and the tangential acceleration is 3.2 m/s^2. Thus the tangential force is

$$F_{\tan} = ma_{\tan} = (950 \text{ kg})(3.2 \text{ m/s}^2) = 3040 \text{ N} \approx \boxed{3.0\times10^3 \text{ N}}$$

The centripetal force is given by Eq. 5–3.

$$F_R = m\frac{\upsilon^2}{r} = (950 \text{ kg})(15 \text{ m/s})^2/(500 \text{ m}) = 427.5 \text{ N} \approx \boxed{430 \text{ N}}$$

23. The car has constant tangential acceleration, which is the acceleration that causes the speed to change. Thus use constant-acceleration equations to calculate the tangential acceleration. The initial speed is 0, the final speed is $270 \text{ km/h}\left(\frac{1.0 \text{ m/s}}{3.6 \text{ km/h}}\right) = 75 \text{ m/s},$ and the distance traveled is one-half of a circular arc of radius 220 m, so $\Delta x_{\tan} = 220\pi$ m. Find the tangential acceleration using Eq. 2–11c.

$$\upsilon_{\tan}^2 - \upsilon_{0\tan}^2 = 2a_{\tan}\Delta x_{\tan} \rightarrow a_{\tan} = \frac{\upsilon_{\tan}^2 - \upsilon_{0\tan}^2}{2\Delta x_{\tan}} = \frac{(75 \text{ m/s})^2}{2(220\pi \text{ m})} = 4.069 \text{ m/s}^2 \approx \boxed{4.1 \text{ m/s}^2}$$

With this tangential acceleration, we can find the speed that the car has halfway through the turn, using Eq. 2–11c, and then calculate the radial acceleration.

$$\upsilon_{\tan}^2 - \upsilon_{0\tan}^2 = 2a_{\tan}\Delta x_{\tan} \rightarrow \upsilon_{\tan} = \sqrt{\upsilon_{0\tan}^2 + 2a_{\tan}\Delta x_{\tan}} = \sqrt{2(4.069 \text{ m/s}^2)(110\pi \text{ m})} = 53.03 \text{ m/s}$$

$$a_R = \frac{\upsilon^2}{r} = \frac{(53.03 \text{ m/s})^2}{220 \text{ m}} = 12.78 \text{ m/s}^2 \approx \boxed{13 \text{ m/s}^2}$$

The total acceleration is given by the Pythagorean combination of the tangential and centripetal accelerations, $a_{\text{total}} = \sqrt{a_R^2 + a_{\tan}^2}$. If static friction is to provide the total acceleration, then $F_{\text{fr}} = ma_{\text{total}} = m\sqrt{a_R^2 + a_{\tan}^2}$. We assume that the car is on the verge of slipping and is on a level surface, so the static frictional force has its maximum value of $F_{\text{fr}} = \mu_s F_N = \mu_s mg$. If we equate these two expressions for the frictional force, we can solve for the coefficient of static friction.

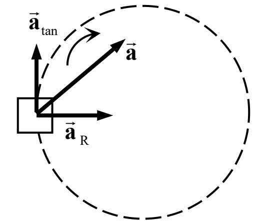
$$F_{\text{fr}} = ma_{\text{total}} = m\sqrt{a_R^2 + a_{\tan}^2} = \mu_s mg \rightarrow$$

$$\mu_s = \frac{\sqrt{a_R^2 + a_{\tan}^2}}{g} = \frac{\sqrt{(12.78 \text{ m/s}^2)^2 + (4.069 \text{ m/s}^2)^2}}{9.80 \text{ m/s}^2} = 1.37 \approx \boxed{1.4}$$

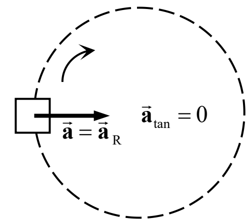
This is an exceptionally large coefficient of friction, so the curve had better be banked.

24. In all cases, we draw a view from above, and the car is moving clockwise around the circular path.

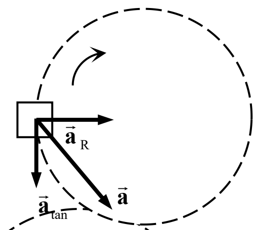
(*a*) In this case, the car is gaining speed, so it has a tangential acceleration in the direction of its velocity, as well as a centripetal acceleration. The total acceleration vector is somewhat "forward."



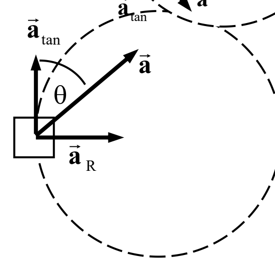
(*b*) In this case, the car has a constant speed, so there is no tangential acceleration. The total acceleration is equal to the radial acceleration.



(*c*) In this case, the car is slowing down, so its tangential acceleration is in the opposite direction as the velocity. It also has a centripetal acceleration. The total acceleration vector is somewhat "backward."



25. We show a top view of the particle in circular motion, traveling clockwise. Because the particle is in circular motion, there must be a radially inward component of the acceleration.



(*a*) $a_R = a\sin\theta = \dfrac{v^2}{r} \rightarrow$

$$v = \sqrt{ar\sin\theta} = \sqrt{(1.05\text{ m/s}^2)(1.95\text{ m})\sin 25.0^\circ} = \boxed{0.930\text{ m/s}}$$

(*b*) The particle's speed change comes from the tangential acceleration, which is given by $a_{tan} = a\cos\theta$. Since the tangential acceleration is constant, we use Eq. 2–11a.

$$v_{tan} - v_{0\,tan} = a_{tan}t \rightarrow$$

$$v_{tan} = v_{0\,tan} + a_{tan}t = 0.930\text{ m/s} + (1.05\text{ m/s}^2)(\cos 25.0^\circ)(2.00\text{ s}) = \boxed{2.83\text{ m/s}}$$

26. The spacecraft is at 3.00 Earth radii from the center of the Earth, or three times as far from the Earth's center as when at the surface of the Earth. Therefore, since the force of gravity decreases as the square of the distance, the force of gravity on the spacecraft will be one-ninth of its weight at the Earth's surface.

$$F_G = \tfrac{1}{9}mg_{\substack{\text{Earth's}\\ \text{surface}}} = \frac{(1850\text{ kg})(9.80\text{ m/s}^2)}{9} = \boxed{2010\text{ N}}$$

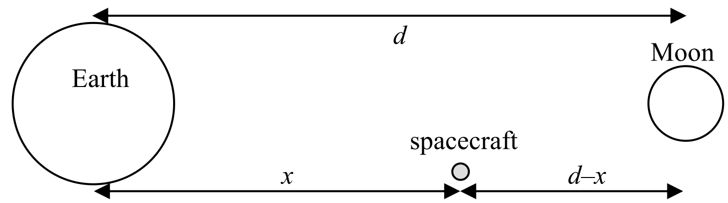
This could also have been found using Eq. 5–4, Newton's law of universal gravitation.

27. (*a*) Mass is independent of location, so the mass of the ball is $\boxed{24.0\text{ kg}}$ on both the Earth and the planet.

(*b*) The weight is found by using $W = mg$.

$$W_{\text{Earth}} = mg_{\text{Earth}} = (24.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{235\text{ N}}$$
$$W_{\text{Planet}} = mg_{\text{Planet}} = (24.0\text{ kg})(12.0\text{ m/s}^2) = \boxed{288\text{ N}}$$

28. For the net force to be zero means that the gravitational force on the spacecraft due to the Earth must be the same as that due to the Moon. Write the gravitational forces on the spacecraft, equate them, and solve for the distance x. We measure from the center of the bodies.



$$F_{\substack{\text{Earth-}\\\text{spacecraft}}} = G\frac{M_{\text{Earth}}m_{\text{spacecraft}}}{x^2}; \quad F_{\substack{\text{Moon}\\\text{spacecraft}}} = G\frac{M_{\text{Moon}}m_{\text{spacecraft}}}{(d-x)^2}$$

$$G\frac{M_{\text{Earth}}m_{\text{spacecraft}}}{x^2} = G\frac{M_{\text{Moon}}m_{\text{spacecraft}}}{(d-x)^2} \rightarrow \frac{x^2}{M_{\text{Earth}}} = \frac{(d-x)^2}{M_{\text{Moon}}} \rightarrow \frac{x}{\sqrt{M_{\text{Earth}}}} = \frac{d-x}{\sqrt{M_{\text{Moon}}}}$$

$$x = d\frac{\sqrt{M_{\text{Earth}}}}{\left(\sqrt{M_{\text{Moon}}}+\sqrt{M_{\text{Earth}}}\right)} = (3.84\times10^8\text{ m})\frac{\sqrt{5.97\times10^{24}\text{ kg}}}{\left(\sqrt{7.35\times10^{22}\text{ kg}}+\sqrt{5.97\times10^{24}\text{ kg}}\right)} = \boxed{3.46\times10^8\text{ m}}$$

This is only about 22 Moon radii away from the Moon. Alternatively, it is about 90% of the distance from the center of the Earth to the center of the Moon.

29. Assume that the two objects can be treated as point masses, with $m_1 = m$ and $m_2 = 4.00\text{ kg} - m$. The gravitational force between the two masses is given by the following:

$$F = G\frac{m_1m_2}{r^2} = G\frac{m(4.00-m)}{r^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{4.00m-m^2}{(0.25\text{ m})^2} = 2.5\times10^{-10}\text{ N}$$

This can be rearranged into a quadratic form of $m^2 - 4.00m + 0.234 = 0$. Use the quadratic formula to solve for m, resulting in two values, which are the two masses.

$$\boxed{m_1 = 3.94\text{ kg}, m_2 = 0.06\text{ kg}}$$

30. The acceleration due to gravity at any location on or above the surface of a planet is given by $g_{\text{planet}} = GM_{\text{planet}}/r^2$, where r is the distance from the center of the planet to the location in question.

$$g_{\text{planet}} = G\frac{M_{\text{Planet}}}{r^2} = G\frac{M_{\text{Earth}}}{(2.0\,R_{\text{Earth}})^2} = \frac{1}{(2.0)^2}G\frac{M_{\text{Earth}}}{R^2_{\text{Earth}}} = \frac{1}{4.0}g_{\text{Earth}} = \frac{9.80\text{ m/s}^2}{4.0} = \boxed{2\ 5\text{ m/s}^2}$$

31. The force of gravity on an object at the surface of a planet is given by Newton's law of universal gravitation, Eq. 5–4, using the mass and radius of the planet. If that is the only force on an object, then the acceleration of a freely falling object is acceleration due to gravity.

$$F_G = G\frac{M_{\text{Moon}}m}{r^2_{\text{Moon}}} = mg_{\text{Moon}} \quad \rightarrow$$

$$g_{\text{Moon}} = G\frac{M_{\text{Moon}}}{r^2_{\text{Moon}}} = (6.67\times10^{2\ 11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(7.35\times10^{22}\text{ kg})}{(1.74\times10^6\text{ m})^2} = \boxed{1.62\text{ m/s}^2}$$

32. The expression for the acceleration due to gravity at the surface of a body is $g_{\text{body}} = G\dfrac{M_{\text{body}}}{R_{\text{body}}^2}$, where R_{body} is the radius of the body. For Mars, $g_{\text{Mars}} = 0.38g_{\text{Earth}}$.

$$G\frac{M_{\text{Mars}}}{R_{\text{Mars}}^2} = 0.38G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2} \rightarrow$$

$$M_{\text{Mars}} = 0.38M_{\text{Earth}}\left(\frac{R_{\text{Mars}}}{\text{R}_{\text{Earth}}}\right)^2 = 0.38(5.98\times10^{24}\text{ kg})\left(\frac{3400\text{ km}}{6380\text{ km}}\right)^2 = \boxed{6.5\times10^{23}\text{ kg}}$$

33. We assume that the distance from the Moon to the Sun is the same as the distance from the Earth to the Sun.

$$F_{\text{ME}} = F_x = G\frac{M_{\text{Moon}}M_{\text{Earth}}}{r_{\text{Moon-Earth}}^2} \qquad F_{\text{MS}} = F_y = G\frac{M_{\text{Moon}}M_{\text{Sun}}}{r_{\text{Moon-Sun}}^2}$$

$$F_{\text{net}} = \sqrt{F_x^2 + F_y^2} = \sqrt{\left(G\frac{M_{\text{Moon}}M_{\text{Earth}}}{r_{\text{Moon-Earth}}^2}\right)^2 + \left(G\frac{M_{\text{Moon}}M_{\text{Sun}}}{r_{\text{Moon-Sun}}^2}\right)^2} = GM_{\text{Moon}}\sqrt{\left(\frac{M_{\text{Earth}}}{r_{\text{Moon-Earth}}^2}\right)^2 + \left(\frac{M_{\text{Sun}}}{r_{\text{Moon-Sun}}^2}\right)^2}$$

$$= (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(7.35\times10^{22}\text{ kg})\sqrt{\left(\frac{(5.98\times10^{24}\text{ kg})}{(384\times10^6\text{ m})^2}\right)^2 + \left(\frac{(1.99\times10^{30}\text{ kg})}{(149.6\times10^9\text{ m})^2}\right)^2}$$

$$= \boxed{4.79\times10^{20}\text{ N}}$$

$$\theta = \tan^{-1}\frac{F_x}{F_y} = \tan^{-1}\frac{\left(G\dfrac{M_{\text{Moon}}M_{\text{Earth}}}{r_{\text{Moon-Earth}}^2}\right)}{\left(G\dfrac{M_{\text{Moon}}M_{\text{Sun}}}{r_{\text{Moon-Sun}}^2}\right)} = \tan^{-1}\frac{\left(\dfrac{M_{\text{Earth}}}{r_{\text{Moon-Earth}}^2}\right)}{\left(\dfrac{M_{\text{Sun}}}{r_{\text{Moon-Sun}}^2}\right)} = \tan^{-1}\left(\frac{M_{\text{Earth}}}{r_{\text{Moon-Earth}}^2}\frac{r_{\text{Moon-Sun}}^2}{M_{\text{Sun}}}\right)$$

$$= \tan^{-1}\left[\frac{(5.98\times10^{24}\text{ kg})}{(384\times10^6\text{ m})^2}\frac{(149.6\times10^9\text{ m})^2}{(1.99\times10^{30}\text{ kg})}\right] = \tan^{-1}\left[\frac{(5.98\times10^{24}\text{ kg})}{(384\times10^6\text{ m})^2}\frac{(149.6\times10^9\text{ m})^2}{(1.99\times10^{30}\text{ kg})}\right]$$

$$= \tan^{-1}0.456 = \boxed{24.5^\circ}$$

34. The acceleration due to gravity at any location at or above the surface of a planet is given by $g_{\text{planet}} = GM_{\text{Planet}}/r^2$, where r is the distance from the center of the planet to the location in question.

$$g_{\text{planet}} = G\frac{M_{\text{Planet}}}{r^2} = G\frac{2.80M_{\text{Earth}}}{R_{\text{Earth}}^2} = 2.80\left(G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2}\right) = 2.80g_{\text{Earth}} = 2.80(9.80\text{ m/s}^2) = \boxed{27.4\text{ m/s}^2}$$

35. The acceleration due to gravity is determined by the mass of the Earth and the radius of the Earth.

$$g_0 = \frac{GM_0}{r_0^2} \qquad g_{\text{new}} = \frac{GM_{\text{new}}}{r_{\text{new}}^2} = \frac{G2M_0}{(3r_0)^2} = \frac{2}{9}\frac{GM_0}{r_0^2} = \tfrac{2}{9}g_0$$

So g is multiplied by a factor of $\boxed{2/9}$.

36. The acceleration due to gravity at any location at or above the surface of a planet is given by $g_{\text{planet}} = GM_{\text{Planet}}/r^2$, where r is the distance from the center of the planet to the location in question. For this problem, $M_{\text{Planet}} = M_{\text{Earth}} = 5.97\times10^{24}$ kg.

(*a*) $r = R_{\text{Earth}} + 6400\text{ m} = 6.38\times10^6\text{ m} + 6400\text{ m}$

$$g = G\frac{M_{\text{Earth}}}{r^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(5.98\times10^{24}\text{ kg})}{(6.38\times10^6\text{ m}+6400\text{ m})^2} = \boxed{9.78\text{ m/s}^2}$$

(*b*) $r = R_{\text{Earth}} + 6400\text{ km} = 6.38\times10^6\text{ m} + 6.4\times10^6\text{ m} = 12.78\times10^6\text{ m}$ (3 significant figures)

$$g = G\frac{M_{\text{Earth}}}{r^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(5.98\times10^{24}\text{ kg})}{(12.78\times10^6\text{ m})^2} = \boxed{2.44\text{ m/s}^2}$$

37. We are to calculate the force on Earth, so we need the distance of each planet from Earth.

$$r_{\substack{\text{Earth}\\\text{Venus}}} = (150-108)\times10^6\text{ km} = 4.2\times10^{10}\text{ m} \qquad r_{\substack{\text{Earth}\\\text{Jupiter}}} = (778-150)\times10^6\text{ km} = 6.28\times10^{11}\text{ m}$$

$$r_{\substack{\text{Earth}\\\text{Saturn}}} = (1430-150)\times10^6\text{ km} = 1.28\times10^{12}\text{ m}$$

Jupiter and Saturn will exert a rightward force, and Venus will exert a leftward force. Take the right direction as positive.

$$F_{\substack{\text{Earth-}\\\text{planets}}} = G\frac{M_{\text{Earth}}M_{\text{Jupiter}}}{r^2_{\substack{\text{Earth}\\\text{Jupiter}}}} + G\frac{M_{\text{Earth}}M_{\text{Saturn}}}{r^2_{\substack{\text{Earth}\\\text{Saturn}}}} - G\frac{M_{\text{Earth}}M_{\text{Venus}}}{r^2_{\substack{\text{Earth}\\\text{Venus}}}}$$

$$= GM^2_{\text{Earth}}\left(\frac{318}{(6.28\times10^{11}\text{ m})^2} + \frac{95.1}{(1.28\times10^{12}\text{ m})^2} - \frac{0.815}{(4.2\times10^{10}\text{ m})^2}\right)$$

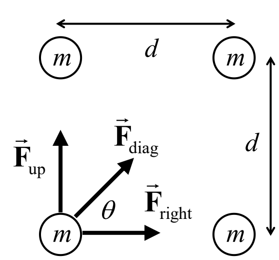
$$= (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})^2(4.02\times10^{-22}\text{ m}^{-2}) = 9.56\times10^{17}\text{ N} \approx 9.6\times10^{17}\text{ N}$$

The force of the Sun on the Earth is as follows:

$$F_{\substack{\text{Earth-}\\\text{Sun}}} = G\frac{M_{\text{Earth}}M_{\text{Sun}}}{r^2_{\substack{\text{Earth}\\\text{Sun}}}} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(5.97\times10^{24}\text{ kg})(1.99\times10^{30}\text{ kg})}{(1.50\times10^{11}\text{ m})^2} = 3.52\times10^{22}\text{ N}$$

So the ratio is $F_{\substack{\text{Earth-}\\\text{planets}}}\Big/F_{\substack{\text{Earth-}\\\text{Sun}}} = 9.56\times10^{17}\text{ N}/3.52\times10^{22}\text{ N} = \boxed{2.7\times10^{-5}}$, which is 27 millionths.

38. Calculate the force on the sphere in the lower left corner, using the free-body diagram shown. From the symmetry of the problem, the net forces in the x and y directions will be the same. Note $\theta = 45°$.



$$F_x = F_{\text{right}} + F_{\text{diag}}\cos\theta = G\frac{m^2}{d^2} + G\frac{m^2}{(\sqrt{2}d)^2}\frac{1}{\sqrt{2}} = G\frac{m^2}{d^2}\left(1+\frac{1}{2\sqrt{2}}\right)$$

Thus $F_y = F_x = G\dfrac{m^2}{d^2}\left(1+\dfrac{1}{2\sqrt{2}}\right)$. The net force can be found by the

Pythagorean combination of the two component forces. Due to the symmetry of the arrangement, the net force will be along the diagonal of the square.

$$F=\sqrt{F_x^2+F_y^2}=\sqrt{2F_x^2}=F_x\sqrt{2}=G\frac{m^2}{d^2}\left(1+\frac{1}{2\sqrt{2}}\right)\sqrt{2}=G\frac{m^2}{d^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

$$=(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(7.5\ \text{kg})^2}{(0.80\ \text{m})^2}\left(\sqrt{2}+\frac{1}{2}\right)=\boxed{1.1\times10^{-8}\ \text{N at }45^\circ}$$

The force points toward the center of the square.

39. In general, the acceleration due to gravity of the Earth is given by $g=GM_{\text{Earth}}/r^2$, where r is the distance from the center of the Earth to the location in question. So, for the location in question,

$$g=\tfrac{1}{10}g_{\text{surface}}\ \rightarrow\ G\frac{M_{\text{Earth}}}{r^2}=\tfrac{1}{10}G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2}\ \rightarrow\ r^2=10\,R_{\text{Earth}}^2$$

$$r=\sqrt{10}R_{\text{Earth}}=\sqrt{10}(6.38\times10^6\ \text{m})=\boxed{2.02\times10^7\ \text{m}}$$

40. The acceleration due to gravity at any location at or above the surface of a star is given by $g_{\text{star}}=GM_{\text{star}}/r^2$, where r is the distance from the center of the star to the location in question.

$$g_{\text{star}}=G\frac{M_{\text{star}}}{r^2}=G\frac{5M_{\text{Sun}}}{r^2}=(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{5(1.99\times10^{30}\ \text{kg})}{(1\times10^4\ \text{m})^2}=\boxed{7\times10^{12}\ \text{m/s}^2}$$

41. The shuttle must be moving at "orbit speed" in order for the satellite to remain in the orbit when released. The speed of a satellite in circular orbit around the Earth is given in Example 5–12.

$$\upsilon_{\text{orbit}}=\sqrt{G\frac{M_{\text{Earth}}}{r}}$$

$$\upsilon=\sqrt{G\frac{M_{\text{Earth}}}{r}}=\sqrt{G\frac{M_{\text{Earth}}}{(R_{\text{Earth}}+780\ \text{km})}}=\sqrt{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(5.98\times10^{24}\ \text{kg})}{(6.38\times10^6\ \text{m}+7.8\times10^5\ \text{m})}}$$

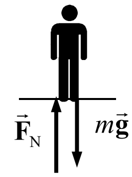
$$=\boxed{7.46\times10^3\ \text{m/s}}$$

42. The speed of a satellite in a circular orbit around a body is given in Example 5–12 as $\upsilon_{\text{orbit}}=\sqrt{GM_{\text{body}}/r}$, where r is the distance from the satellite to the center of the body.

$$\upsilon=\sqrt{G\frac{M_{\text{body}}}{r}}=\sqrt{G\frac{M_{\text{Earth}}}{R_{\text{Earth}}+4.8\times10^6\ \text{m}}}=\sqrt{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(5.98\times10^{24}\ \text{kg})}{(6.38\times10^6\ \text{m}+4.8\times10^6\ \text{m})}}$$

$$=\boxed{5.97\times10^3\ \text{m/s}}$$

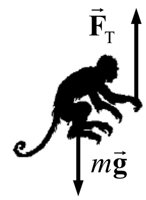
43. Consider a free-body diagram of yourself in the elevator. $\vec{\mathbf{F}}_N$ is the force of the scale pushing up on you and reads the normal force. Since the scale reads 77 kg, if it were calibrated in newtons, the normal force would be $F_N = (77\text{ kg})(9.80\text{ m/s}^2) = 754.6\text{ N}.$
Write Newton's second law in the vertical direction, with upward as positive.



$$\sum F = F_N - mg = ma \quad \to \quad a = \frac{F_N - mg}{m} = \frac{754.6\text{ N} - (62\text{ kg})(9.80\text{ m/s}^2)}{62\text{ kg}} = \boxed{2.4\text{ m/s}^2\text{ upward}}$$

Since the acceleration is positive, the acceleration is upward.

44. Draw a free-body diagram of the monkey. Then write Newton's second law for the vertical direction, with up as positive.



$$\sum F = F_T - mg = ma \quad \to \quad a = \frac{F_T - mg}{m}$$

For the maximum tension of 185 N,

$$a = \frac{185\text{ N} - (12.0\text{ kg})(9.80\text{ m/s}^2)}{(12.0\text{ kg})} = 5.62\text{ m/s}^2 \approx 5.6\text{ m/s}^2$$

Thus the elevator must have an $\boxed{\text{upward acceleration greater than } a = 5.6\text{ m/s}^2}$ for the cord to break. Any downward acceleration would result in a tension less than the monkey's weight.

45. The speed of an object in a circular orbit of radius r around mass M is given in Example 5–12 by $\upsilon = \sqrt{GM/r}$ and is also given by $\upsilon = 2\pi r/T$, where T is the period of the orbiting object. Equate the two expressions for the speed and solve for T.

$$\sqrt{G\frac{M}{r}} = \frac{2\pi r}{T} \quad \to$$

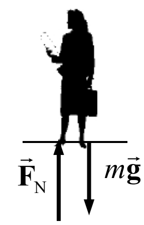
$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(1.74\times10^6\text{ m} + 9.5\times10^4\text{ m})^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(7.35\times10^{22}\text{ m})}} = \boxed{7.05\times10^3\text{ s} \approx 118\text{ min}}$$

46. The speed of a satellite in circular orbit around the Earth is shown in Example 5–12 to be $\upsilon_{\text{orbit}} = \sqrt{G\frac{M_{\text{Earth}}}{r}}$. Thus the velocity is inversely related to the radius, so the closer satellite will be orbiting faster.

$$\frac{\upsilon_{\text{close}}}{\upsilon_{\text{far}}} = \frac{\sqrt{\dfrac{GM_{\text{Earth}}}{r_{\text{close}}}}}{\sqrt{\dfrac{GM_{\text{Earth}}}{r_{\text{far}}}}} = \sqrt{\frac{r_{\text{far}}}{r_{\text{close}}}} = \sqrt{\frac{R_{\text{Earth}} + 1.5\times10^7\text{ m}}{R_{\text{Earth}} + 7.5\times10^6\text{m}}} = \sqrt{\frac{6.38\times10^6\text{m} + 1.5\times10^7\text{ m}}{6.38\times10^6\text{m} + 7.5\times10^6\text{m}}} = 1.24$$

So $\boxed{\text{the close satellite is moving 1.2 times faster}}$ than the far satellite.

47. Consider a free-body diagram for the woman in the elevator. $\vec{\mathbf{F}}_N$ is the upward force the spring scale exerts, providing a normal force. Write Newton's second law for the vertical direction, with up as positive.



$$\sum F = F_N - mg = ma \quad \rightarrow \quad F_N = m(g+a)$$

(a, b) For constant-speed motion in a straight line, the acceleration is 0, so the normal force is equal to the weight.

$$F_N = mg = (58.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{568\text{ N}}$$

(c) Here $a = +0.23\,g$, so $F_N = 1.23\,mg = 1.23(58.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{699\text{ N}}$.

(d) Here $a = -0.23\,g$, so $F_N = 0.77\,mg = 0.77(58.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{440\text{ N}}$.

(e) Here $a = -g$, so $F_N = \boxed{0}$.

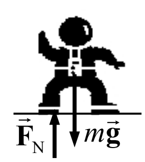
48. The speed of an object in an orbit of radius r around the Earth is given in Example 5–12 by $\upsilon = \sqrt{GM_{\text{Earth}}/r}$ and is also given by $\upsilon = 2\pi r/T$, where T is the period of the object in orbit. Equate the two expressions for the speed and solve for T. Also, for a "near-Earth" orbit, $r = R_{\text{Earth}}$.

$$\sqrt{G\frac{M_{\text{Earth}}}{r}} = \frac{2\pi r}{T} \quad \rightarrow \quad T = 2\pi\sqrt{\frac{r^3}{GM_{\text{Earth}}}}$$

$$T = 2\pi\sqrt{\frac{R_{\text{Earth}}^3}{GM_{\text{Earth}}}} = 2\pi\sqrt{\frac{(6.38\times10^6\text{ m})^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ m})}} = \boxed{5070\text{ s} = 84.5\text{ min}}$$

$\boxed{\text{No}}$, the result does not depend on the mass of the satellite.

49. Consider the free-body diagram for the astronaut in the space vehicle. The Moon is below the astronaut in the figure. We assume that the astronaut is touching the inside of the space vehicle, or in a seat, or strapped in somehow, so a force will be exerted on the astronaut by the spacecraft. That force has been labeled $\vec{\mathbf{F}}_N$. The magnitude of that force is the apparent weight of the astronaut. Take down as the positive direction.



(a) If the spacecraft is moving with a constant velocity, then the acceleration of the astronaut must be 0, so the net force on the astronaut is 0.

$$\sum F = mg - F_N = 0 \quad \rightarrow$$

$$F_N = mg = G\frac{mM_{\text{Moon}}}{r^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(75\text{ kg})(7.4\times10^{22}\text{ kg})}{(2.5\times10^6\text{ m})^2} = 59.23\text{ N}$$

Since the value here is positive, the normal force points in the original direction as shown on the free-body diagram. The apparent weight is $\boxed{59\text{ N, away from the Moon}}$.

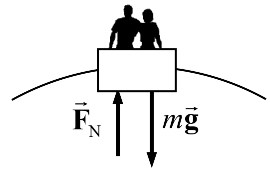
(b) Now the astronaut has an acceleration toward the Moon. Write Newton's second law for the astronaut, with down as the positive direction.

$$\sum F = mg - F_N = ma \quad \rightarrow \quad F_N = mg - ma = 59.23\text{ N} - (75\text{ kg})(1.8\text{ m/s}^2) = -76\text{ N}$$

Because of the negative value, the normal force points in the opposite direction from what is shown on the free-body diagram—it is pointing toward the Moon. So perhaps the astronaut is pinned against the "ceiling" of the spacecraft, or safety belts are pulling down on the astronaut. The apparent weight is $\boxed{76\text{ N, toward the Moon}}$.

50. The apparent weight is the normal force on the passenger. For a person at rest, the normal force is equal to the actual weight. If there is acceleration in the vertical direction, either up or down, then the normal force (and hence the apparent weight) will be different than the actual weight. The speed of the Ferris wheel is $\upsilon = 2\pi r/T = 2\pi(11.0\text{ m})/12.5\text{ s} = 5.529\text{ m/s}$.

(*a*) See the free-body diagram for the highest point of the motion. We assume the passengers are right-side up, so that the normal force of the Ferris wheel seat is upward. The net force must point to the center of the circle, so write Newton's second law with downward as the positive direction. The acceleration is centripetal since the passengers are moving in a circle.

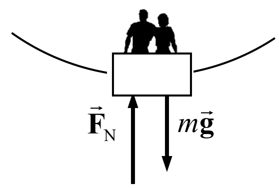


$$\sum F = F_R = mg - F_N = ma = m\frac{\upsilon^2}{r} \quad \rightarrow \quad F_N = mg - m\frac{\upsilon^2}{r}$$

The ratio of apparent weight to real weight is given by the following:

$$\frac{mg - m\dfrac{\upsilon^2}{r}}{mg} = \frac{g - \dfrac{\upsilon^2}{r}}{g} = 1 - \frac{\upsilon^2}{rg} = 1 - \frac{(5.529\text{ m/s})^2}{(11.0\text{ m})(9.80\text{ m/s}^2)} = \boxed{0.716}$$

(*b*) At the bottom, consider the free-body diagram shown. We assume the passengers are right-side up, so that the normal force of the Ferris wheel seat is upward. The net force must point to the center of the circle, so write Newton's second law with upward as the positive direction. The acceleration is centripetal since the passengers are moving in a circle.



$$\sum F = F_R = F_N - mg = ma = m\frac{\upsilon^2}{r} \quad \rightarrow \quad F_N = mg + m\frac{\upsilon^2}{r}$$

The ratio of apparent weight to real weight is given by the following:

$$\frac{mg + m\dfrac{\upsilon^2}{r}}{mg} = 1 + \frac{\upsilon^2}{rg} = 1 + \frac{(5.529\text{ m/s})^2}{(11.0\text{ m})(9.80\text{ m/s}^2)} = \boxed{1.284}$$

$\boxed{51.}$ The centripetal acceleration will simulate gravity. Thus $\dfrac{\upsilon^2}{r} = 0.70\,g \rightarrow \upsilon = \sqrt{0.70gr}$. Also for a rotating object, the speed is given by $\upsilon = 2\pi r/T$. Equate the two expressions for the speed and solve for the period.

$$\upsilon = \sqrt{0.70gr} = \frac{2\pi r}{T} \quad \rightarrow \quad T = \frac{2\pi r}{\sqrt{0.70gr}} = \frac{2\pi(16\text{ m})}{\sqrt{(0.70)(9.80\text{ m/s}^2)(16\text{ m})}} = \boxed{9.6\text{ s}}$$

52. (*a*) The speed of an object in near-surface orbit around a planet is given in Example 5–12 to be $\upsilon = \sqrt{GM/R}$, where M is the planet mass and R is the planet radius. The speed is also given by $\upsilon = 2\pi R/T$, where T is the period of the object in orbit. Equate the two expressions for the speed.

$$\sqrt{G\frac{M}{R}} = \frac{2\pi R}{T} \quad \rightarrow \quad G\frac{M}{R} = \frac{4\pi^2 R^2}{T^2} \quad \rightarrow \quad \frac{M}{R^3} = \frac{4\pi^2}{GT^2}$$

The density of a uniform spherical planet is given by $\rho = \dfrac{M}{\text{Volume}} = \dfrac{M}{\frac{4}{3}\pi R^3}$. Thus

$$\rho = \frac{3M}{4\pi R^3} = \frac{3}{4\pi}\frac{4\pi^2}{GT^2} = \boxed{\frac{3\pi}{GT^2}}$$

(*b*) For Earth, we have the following:

$$\rho = \frac{3\pi}{GT^2} = \frac{3\pi}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)[(85\ \text{min})(60\ \text{s/min})]^2} = \boxed{5.4\times10^3\ \text{kg/m}^3}$$

53. Use Kepler's third law for objects orbiting the Sun.

$$\left(T_{\text{Neptune}}/T_{\text{Earth}}\right)^2 = \left(r_{\text{Neptune}}/r_{\text{Earth}}\right)^3 \quad\rightarrow$$

$$T_{\text{Neptune}} = T_{\text{Earth}}\left(\frac{r_{\text{Neptune}}}{r_{\text{Earth}}}\right)^{3/2} = (1\ \text{year})\left(\frac{4.5\times10^9\ \text{km}}{1.50\times10^8\ \text{km}}\right)^{3/2} = \boxed{160\ \text{years}}$$

54. Use Kepler's third law for objects orbiting the Sun.

$$\left(\frac{r_{\text{Icarus}}}{r_{\text{Earth}}}\right)^3 = \left(\frac{T_{\text{Icarus}}}{T_{\text{Earth}}}\right)^2 \quad\rightarrow\quad r_{\text{Icarus}} = r_{\text{Earth}}\left(\frac{T_{\text{Icarus}}}{T_{\text{Earth}}}\right)^{2/3} = (1.50\times10^{11}\ \text{m})\left(\frac{410\ \text{d}}{365\ \text{d}}\right)^{2/3} = \boxed{1.6\times10^{11}\ \text{m}}$$

55. Use Kepler's third law for objects orbiting the Earth. The following are given:

$$T_2 = \text{period of Moon} = (27.4\ \text{day})\left(\frac{86{,}400\ \text{s}}{1\ \text{day}}\right) = 2.367\times10^6\ \text{s}$$

$$r_2 = \text{radius of Moon's orbit} = 3.84\times10^8\ \text{m}$$

$$r_1 = \text{radius of near}-\text{Earth orbit} = R_{\text{Earth}} = 6.38\times10^6\ \text{m}$$

$$\left(T_1/T_2\right)^2 = \left(r_1/r_2\right)^3 \quad\rightarrow$$

$$T_1 = T_2\left(r_1/r_2\right)^{3/2} = (2.367\times10^6\ \text{s})\left(\frac{6.38\times10^6\ \text{m}}{3.84\times10^8\ \text{m}}\right)^{3/2} = \boxed{5.07\times10^3\ \text{s}}\ (= 84.5\ \text{min})$$

56. Knowing the period of the Moon and the distance to the Moon, we can calculate the speed of the Moon by $\upsilon = 2\pi r/T$. But the speed can also be calculated for any Earth satellite by $\upsilon = \sqrt{GM_{\text{Earth}}/r}$, as derived in Example 5–12. Equate the two expressions for the speed, and solve for the mass of the Earth.

$$\sqrt{GM_{\text{Earth}}/r} = 2\pi r/T \quad\rightarrow$$

$$M_{\text{Earth}} = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(3.84\times10^8\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)[(27.4\ \text{d})(86{,}400\ \text{s/d})]^2} = \boxed{5.98\times10^{24}\ \text{kg}}$$

57. There are two expressions for the velocity of an object in circular motion around a mass M: $\upsilon = \sqrt{GM/r}$ and $\upsilon = 2\pi r/T$. Equate the two expressions and solve for T.

$$\sqrt{GM/r} = 2\pi r/T \quad\rightarrow$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{\left((3\times10^4\ \text{ly})\dfrac{(3\times10^8\ \text{m/s})(3.16\times10^7\ \text{s})}{1\ \text{ly}}\right)^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(4\times10^{41}\ \text{kg})}} = 5.8\times10^{15}\ \text{s} = 1.8\times10^8\ \text{yr}$$

$$\approx \boxed{2\times10^8\ \text{yr}}$$

58. Use Kepler's third law, Eq. 5–7b, to find the radius of each moon, using Io's data for r_2 and T_2.

$$(r_1/r_2)^3 = (T_1/T_2)^2 \quad \rightarrow \quad r_1 = r_2(T_1/T_2)^{2/3}$$

$$r_{\text{Europa}} = r_{\text{Io}}\left(T_{\text{Europa}}/T_{\text{Io}}\right)^{2/3} = (422\times10^3 \text{ km})(3.55 \text{ d}/1.77 \text{ d})^{2/3} = \boxed{671\times10^3 \text{ km}}$$

$$r_{\text{Ganymede}} = (422\times10^3 \text{ km})(7.16 \text{ d}/1.77 \text{ d})^{2/3} = \boxed{1070\times10^3 \text{ km}}$$

$$r_{\text{Callisto}} = (422\times10^3 \text{ km})(16.7 \text{ d}/1.77 \text{ d})^{2/3} = \boxed{1880\times10^3 \text{ km}}$$

The agreement with the data in the table is excellent.

59. As found in Example 5–12, the speed for an object orbiting a distance r around a mass M is given by $\upsilon = \sqrt{GM/r}$.

$$\frac{\upsilon_A}{\upsilon_B} = \frac{\sqrt{\dfrac{GM_{\text{star}}}{r_A}}}{\sqrt{\dfrac{GM_{\text{star}}}{r_B}}} = \sqrt{\frac{r_B}{r_A}} = \sqrt{\frac{1}{7.0}} = \boxed{0.38}$$

60. Use Kepler's third law to relate the orbits of Earth and Halley's comet around the Sun.

$$\left(r_{\text{Halley}}/r_{\text{Earth}}\right)^3 = \left(T_{\text{Halley}}/T_{\text{Earth}}\right)^2 \quad \rightarrow$$

$$r_{\text{Halley}} = r_{\text{Earth}}\left(T_{\text{Halley}}/T_{\text{Earth}}\right)^{2/3} = (150\times10^6 \text{ km})(76 \text{ yr}/1 \text{ yr})^{2/3} = 2690\times10^6 \text{ km}$$

This value is half the sum of the nearest and farthest distances of Halley's comet from the Sun. Since the nearest distance is very close to the Sun, we will approximate that nearest distance as 0. Then the farthest distance is twice the value above, or $5380\times10^6 \text{ km} = \boxed{5.4\times10^{12} \text{ m}}$. This distance approaches the mean orbit distance of Pluto, which is 5.9×10^{12} m. $\boxed{\text{It is still in the solar system, nearest to Pluto's orbit.}}$

61. The centripetal acceleration is $a_R = \dfrac{\upsilon^2}{R_{\substack{\text{Earth}\\ \text{orbit}}}} = \dfrac{\left(2\pi R_{\substack{\text{Earth}\\ \text{orbit}}}/T\right)^2}{R_{\substack{\text{Earth}\\ \text{orbit}}}} = \dfrac{4\pi^2 R_{\substack{\text{Earth}\\ \text{orbit}}}}{T^2}$. The force (from Newton's second law) is $F_R = m_{\text{Earth}}a_R$. The period is one year, converted into seconds.

$$a_R = \frac{4\pi^2 R_{\substack{\text{Earth}\\ \text{orbit}}}}{T^2} = \frac{4\pi^2(1.50\times10^{11} \text{ m})}{(3.15\times10^7 \text{ s})^2} = \boxed{5.97\times10^{-3} \text{ m/s}^2}$$

$$F_R = ma = (5.97\times10^{24} \text{ kg})(5.97\times10^{-3} \text{ m/s}^2) = \boxed{3.56\times10^{22} \text{ N}}$$

$\boxed{\text{The Sun}}$ exerts this force on the Earth. It is a gravitational force.

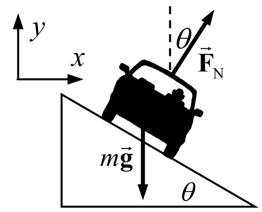
62. Since mass m is dangling, the tension in the cord must be equal to the weight of mass m, so $F_T = mg$. That same tension is in the other end of the cord, maintaining the circular motion of mass M, so $F_T = F_R = Ma_R = M\dfrac{\upsilon^2}{r}$. Equate the expressions for tension and solve for the velocity.

$$M\frac{\upsilon^2}{r} = mg \quad \rightarrow \quad \upsilon = \boxed{\sqrt{mgR/M}}$$

63. The force is a centripetal force, and is of magnitude 7.45 mg. Use Eq. 5–3 for centripetal force.

$$F = m\frac{\upsilon^2}{r} = 7.45\,mg \quad \rightarrow \quad \upsilon = \sqrt{7.45\,rg} = \sqrt{7.45(11.0\text{ m})(9.80\text{ m/s}^2)} = 28.3$$

$$(28.34\text{ m/s}) \times \frac{1\text{ rev}}{2\pi(11.0\text{ m})} = \boxed{0.410\text{ rev/s}}$$



64. The car moves in a horizontal circle, so there must be a net horizontal centripetal force. The car is not accelerating vertically. Write Newton's second law for both the x and y directions.

$$\sum F_y = F_N \cos\theta - mg = 0 \quad \rightarrow \quad F_N = \frac{mg}{\cos\theta}$$

$$\sum F_x = \sum F_R = F_N \sin\theta = ma_x$$

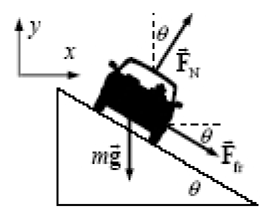
The amount of centripetal force needed for the car to round the curve is as follows:

$$F_R = m\frac{\upsilon^2}{r} = (1050\text{ kg})\frac{\left[(85\text{ km/h})\left(\frac{1.0\text{ m/s}}{3.6\text{ km/h}}\right)\right]^2}{72\text{ m}} = 8.130\times10^3\text{ N}$$

The actual horizontal force available from the normal force is as follows:

$$F_N \sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = (1050\text{ kg})(9.80\text{ m/s}^2)\tan 14^\circ = 2.566\times10^3\text{ N}$$

Thus more force is necessary for the car to round the curve than can be supplied by the normal force. That extra force will have to have a horizontal component to the right in order to provide the extra centripetal force. Accordingly, we add a frictional force pointed down the plane. That corresponds to the car not being able to make the curve without friction.



Again write Newton's second law for both directions, and again the y acceleration is zero.

$$\sum F_y = F_N \cos\theta - mg - F_{fr}\sin\theta = 0 \quad \rightarrow \quad F_N = \frac{mg + F_{fr}\sin\theta}{\cos\theta}$$

$$\sum F_x = F_N \sin\theta + F_{fr}\cos\theta = m\frac{\upsilon^2}{r}$$

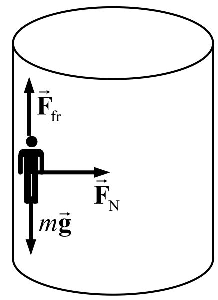
Substitute the expression for the normal force from the y equation into the x equation, and solve for the friction force.

$$\frac{mg + F_{\text{fr}} \sin\theta}{\cos\theta}\sin\theta + F_{\text{fr}}\cos\theta = m\frac{v^2}{r} \rightarrow (mg + F_{\text{fr}}\sin\theta)\sin\theta + F_{\text{fr}}\cos^2\theta = m\frac{v^2}{r}\cos\theta$$

$$F_{\text{fr}} = m\frac{v^2}{r}\cos\theta - mg\sin\theta = (8.130\times10^3\ \text{N})\cos 14° - (1050\ \text{kg})(9.80\ \text{m/s}^2)\sin 14°$$
$$= 5.399\times10^3\ \text{N}$$

So a frictional force of $\boxed{5.4\times10^3\ \text{N down the plane}}$ is needed to provide the necessary centripetal force to round the curve at the specified speed.

65. Consider the free-body diagram for a person in the "Rotor-ride." $\vec{\mathbf{F}}_{\text{N}}$ is the normal force of contact between the rider and the wall, and $\vec{\mathbf{F}}_{\text{fr}}$ is the static frictional force between the back of the rider and the wall. Write Newton's second law for the vertical forces, noting that there is no vertical acceleration.



$$\sum F_y = F_{\text{fr}} - mg = 0 \rightarrow F_{\text{fr}} = mg$$

If we assume that the static friction force is a maximum, then

$$F_{\text{fr}} = \mu_s F_{\text{N}} = mg \rightarrow F_{\text{N}} = mg/\mu_s$$

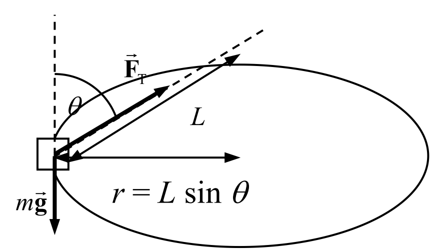
But the normal force must be the force causing the centripetal motion—it is the only force pointing to the center of rotation. Thus $F_{\text{R}} = F_{\text{N}} = m\frac{v^2}{r}$. Using $v = 2\pi r/T$, we have $F_{\text{N}} = \frac{4\pi^2 mr}{T^2}$. Equate the two expressions for the normal force and solve for the coefficient of friction. Note that since there are 0.50 revolutions per second, the period is 2.0 s.

$$F_{\text{N}} = \frac{4\pi^2 mr}{T^2} = \frac{mg}{\mu_s} \rightarrow \mu_s = \frac{gT^2}{4\pi^2 r} = \frac{(9.80\ \text{m/s}^2)(2.0\ \text{s})^2}{4\pi^2(5.5\ \text{m})} = \boxed{0.18}$$

Any larger value of the coefficient of friction would mean that the normal force could be smaller to achieve the same frictional force, so the period could be longer or the cylinder radius smaller.

There is no force pushing outward on the riders. Rather, the wall pushes against the riders. By Newton's third law, the riders therefore push against the wall. This gives the sensation of being pressed into the wall.

66. A free-body diagram for the sinker weight is shown. L is the length of the string actually swinging the sinker. The radius of the circle of motion is moving is $r = L\sin\theta$. Write Newton's second law for the vertical direction, noting that the sinker is not accelerating vertically. Take up to be positive.



$$\sum F_y = F_{\text{T}}\cos\theta - mg = 0 \rightarrow F_{\text{T}} = \frac{mg}{\cos\theta}$$

The radial force is the horizontal portion of the tension. Write Newton's second law for the radial motion.

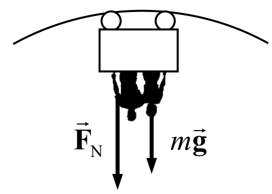
$$\sum F_R = F_T \sin\theta = ma_R = m\frac{\upsilon^2}{r}$$

Substitute the tension from the vertical equation and the relationships $r = L\sin\theta$ and $\upsilon = 2\pi r/T$.

$$F_T \sin\theta = m\frac{\upsilon^2}{r} \rightarrow \frac{mg}{\cos\theta}\sin\theta = \frac{4\pi^2 mL\sin\theta}{T^2} \rightarrow \cos\theta = \frac{gT^2}{4\pi^2 L}$$

$$\theta = \cos^{-1}\frac{gT^2}{4\pi^2 L} = \cos^{-1}\frac{(9.80\text{ m/s}^2)(0.75\text{ s})^2}{4\pi^2(0.25\text{ m})} = \boxed{56^\circ}$$

67. At the top of a circle, a free-body diagram for the passengers would be as shown, assuming the passengers are upside down. Then the car's normal force would be pushing DOWN on the passengers, as shown in the diagram. We assume no safety devices are present. Choose the positive direction to be down, and write Newton's second law for the passengers.



$$\sum F = F_N + mg = ma = m\frac{\upsilon^2}{r} \rightarrow F_N = m\left(\frac{\upsilon^2}{r} - g\right)$$

We see from this expression that for a high speed, the normal force is positive, meaning the passengers are in contact with the car. But as the speed decreases, the normal force also decreases. If the normal force becomes 0, the passengers are no longer in contact with the car—they are in free fall. The limiting condition is as follows:

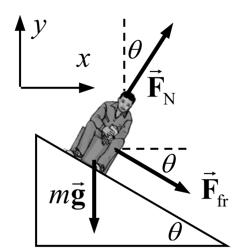
$$\frac{\upsilon_{min}^2}{r} - g = 0 \rightarrow \upsilon_{min} = \sqrt{rg} = \sqrt{(9.80\text{ m/s}^2)(8.6\text{ m})} = \boxed{9.2\text{ m/s}}$$

68. The speed of the train is $(160\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 44.44\text{ m/s}$.

(*a*) If there is no tilt, then the friction force must supply the entire centripetal force on the passenger.

$$F_R = m\frac{\upsilon^2}{R} = \frac{(55\text{ kg})(44.44\text{ m/s})^2}{(570\text{ m})} = 190.6\text{ N} \approx \boxed{1.9\times 10^2\text{ N}}$$

(*b*) For the banked case, the normal force will contribute to the radial force needed. Write Newton's second law for both the *x* and *y* directions. The *y* acceleration is zero, and the *x* acceleration is radial.



$$\sum F_y = F_N\cos\theta - mg - F_{fr}\sin\theta = 0 \rightarrow F_N = \frac{mg + F_{fr}\sin\theta}{\cos\theta}$$

$$\sum F_x = F_N\sin\theta + F_{fr}\cos\theta = m\frac{\upsilon^2}{r}$$

Substitute the expression for the normal force from the *y* equation into the *x* equation, and solve for the friction force.

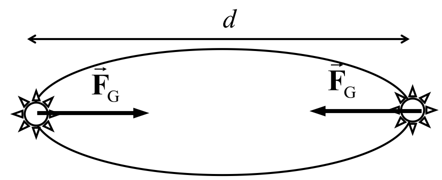
$$\frac{mg + F_{fr}\sin\theta}{\cos\theta}\sin\theta + F_{fr}\cos\theta = m\frac{\upsilon^2}{r} \rightarrow$$

$$(mg + F_{fr}\sin\theta)\sin\theta + F_{fr}\cos^2\theta = m\frac{\upsilon^2}{r}\cos\theta \rightarrow$$

$$F_{\text{fr}} = m\left(\frac{\upsilon^2}{r}\cos\theta - g\sin\theta\right)$$

$$= (55\text{ kg})\left[\frac{(44.44\text{ m/s})^2}{570\text{ m}}\cos 8.0° - (9.80\text{ m/s}^2)\sin 8.0°\right] = 113.7\text{ N} \approx \boxed{1.1\times10^2\text{ N}}$$

69. See the diagram for the two stars.



(*a*) The two stars don't crash into each other because of their circular motion. The force on them is centripetal and maintains their circular motion. Another way to consider it is that the stars have a velocity, and the gravity force causes CHANGE in velocity, not actual velocity. If the stars were somehow brought to rest and then released under the influence of their mutual gravity, they would crash into each other.

(*b*) Set the gravity force on one of the stars equal to the centripetal force, using the relationship that $\upsilon = 2\pi r/T = \pi d/T$, and solve for the mass.

$$F_G = G\frac{M^2}{d^2} = F_R = M\frac{\upsilon^2}{d/2} = M\frac{2(\pi d/T)^2}{d} = \frac{2\pi^2 Md}{T^2} \rightarrow G\frac{M^2}{d^2} = \frac{2\pi^2 Md}{T^2} \rightarrow$$

$$M = \frac{2\pi^2 d^3}{GT^2} = \frac{2\pi^2(8.0\times10^{11}\text{ m})^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\left(12.6\text{ yr}\times\frac{3.15\times10^7\text{ s}}{1\text{ yr}}\right)^2} = \boxed{9.6\times10^{29}\text{ kg}}$$

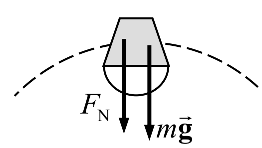
70. The acceleration due to the Earth's gravity at a location at or above the surface is given by $g = GM_{\text{Earth}}/r^2$, where r is the distance from the center of the Earth to the location in question. Find the location where $g = \frac{1}{2}g_{\text{surface}}$.

$$\frac{GM_{\text{Earth}}}{r^2} = \frac{1}{2}\frac{GM_{\text{Earth}}}{R_{\text{Earth}}^2} \rightarrow r^2 = 2R_{\text{Earth}}^2 \rightarrow r = \sqrt{2}R_{\text{Earth}}$$

The distance above the Earth's surface is as follows:

$$r - R_{\text{Earth}} = \left(\sqrt{2}-1\right)R_{\text{Earth}} = \left(\sqrt{2}-1\right)(6.38\times10^6\text{ m}) = \boxed{2.64\times10^6\text{ m}} = 0.414\,R_{\text{Earth}}$$

71. We assume the water is rotating in a vertical circle of radius r. When the bucket is at the top of its motion, there would be two forces on the water (considering the water as a single mass). The weight of the water would be directed down, and the normal force of the bottom of the bucket pushing on the water would also be down. See the free-body diagram. If the water is moving in a circle, then the net downward force would be a centripetal force.



$$\sum F = F_N + mg = ma = m\frac{\upsilon^2}{r} \rightarrow F_N = m\left(\frac{\upsilon^2}{r} - g\right)$$

The limiting condition of the water falling out of the bucket means that the water loses contact with the bucket, so the normal force becomes 0.

$$F_N = m\left(\frac{\upsilon^2}{r} - g\right) \rightarrow m\left(\frac{\upsilon_{\text{critical}}^2}{r} - g\right) = 0 \rightarrow \upsilon_{\text{critical}} = \sqrt{rg}$$

From this, we see that $\boxed{\text{yes}}$, it is possible to whirl the bucket of water fast enough. The minimum speed is $\boxed{\sqrt{rg}}$. All you really need to know is the radius of the circle in which you will be swinging the bucket. It would be approximately the length of your arm, plus the height of the bucket.

72. For an object to be apparently weightless, the object would have a centripetal acceleration equal to g. This is the same as asking what the orbital period would be for an object orbiting the Earth with an orbital radius equal to the Earth's radius. To calculate, use $g = a_R = \dfrac{\upsilon^2}{R_{\text{Earth}}}$, along with $\upsilon = 2\pi R_{\text{Earth}}/T$, and solve for T.

$$g = \frac{\upsilon^2}{R_{\text{Earth}}} = \frac{4\pi^2 R_{\text{Earth}}}{T^2} \rightarrow T = 2\pi\sqrt{\frac{R_{\text{Earth}}}{g}} = 2\pi\sqrt{\frac{6.38\times10^6\ \text{m}}{9.80\ \text{m/s}^2}} = \boxed{5.07\times10^3\ \text{s}}\ (\approx 84.5\ \text{min})$$

73. The speed of an object in an orbit of radius r around a planet is given in Example 5–12 as $\upsilon = \sqrt{GM_{\text{planet}}/r}$, and is also given by $\upsilon = 2\pi r/T$, where T is the period of the object in orbit. Equate the two expressions for the speed and solve for T.

$$\sqrt{G\frac{M_{\text{Planet}}}{r}} = \frac{2\pi r}{T} \rightarrow T = 2\pi\sqrt{\frac{r^3}{GM_{\text{Planet}}}}$$

For this problem, the inner orbit has radius $r_{\text{inner}} = 7.3\times10^7$ m, and the outer orbit has radius $r_{\text{outer}} = 1.7\times10^8$ m. Use these values to calculate the periods.

$$T_{\text{inner}} = 2\pi\sqrt{\frac{(7.3\times10^7\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.7\times10^{26}\ \text{kg})}} = \boxed{2.0\times10^4\ \text{s}}$$

$$T_{\text{outer}} = 2\pi\sqrt{\frac{(1.7\times10^8\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.7\times10^{26}\ \text{kg})}} = \boxed{7.1\times10^4\ \text{s}}$$

Saturn's rotation period (day) is 10 h 39 min, which is about 3.8×10^4 s. Thus the inner ring will appear to move across the sky faster than the Sun (about twice per Saturn day), while the outer ring will appear to move across the sky slower than the Sun (about once every two Saturn days).

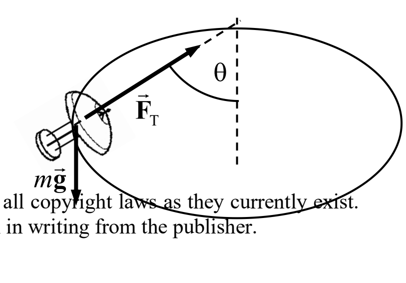
74. The speed of an object in an orbit of radius r around the Moon is given by $\upsilon = \sqrt{GM_{\text{Moon}}/r}$, and is also given by $\upsilon = 2\pi r/T$, where T is the period of the object in orbit. Equate the two expressions for the speed and solve for T.

$$\sqrt{GM_{\text{Moon}}/r} = 2\pi r/T \rightarrow$$

$$T = 2\pi\sqrt{\frac{r^3}{GM_{\text{Moon}}}} = 2\pi\sqrt{\frac{\left(R_{\text{Moon}} + 100\ \text{km}\right)^3}{GM_{\text{Moon}}}} = 2\pi\sqrt{\frac{(1.74\times10^6\ \text{m} + 1\times10^5\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.35\times10^{22}\ \text{kg})}}$$

$$= \boxed{7.1\times10^3\ \text{s}\ \ (\approx 2.0\ \text{h})}$$

75. The lamp must have the same speed and acceleration as the train. The forces on the lamp as the train rounds the corner are shown in the free-body diagram included. The tension in the suspending



cord must not only hold the lamp up, but also provide the
centripetal force needed to make the lamp move in a circle. Write Newton's second law for the vertical direction, noting that the
lamp is not accelerating vertically.

$$\sum F_y = F_T \cos\theta - mg = 0 \quad \rightarrow \quad F_T = \frac{mg}{\cos\theta}$$

The force moving the lamp in a circle is the horizontal portion of the tension. Write Newton's second law for that radial motion.

$$\sum F_R = F_T \sin\theta = ma_R = m\frac{\upsilon^2}{r}$$

Substitute the expression for the tension from the first equation into the second equation, and solve for the speed.

$$F_T \sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = m\frac{\upsilon^2}{r} \quad \rightarrow$$

$$\upsilon = \sqrt{rg\tan\theta} = \sqrt{(215\text{ m})(9.80\text{ m/s}^2)\tan 16.5^\circ} = \boxed{25.0\text{ m/s}}$$

76. The speed of rotation of the Sun about the galactic center, under the assumptions made, is given by $\upsilon = \sqrt{G\frac{M_{\text{galaxy}}}{r_{\text{Sun orbit}}}}$, so $M_{\text{galaxy}} = \frac{r_{\text{Sun orbit}}\upsilon^2}{G}$. Substitute in the relationship that $\upsilon = 2\pi r_{\text{Sun orbit}}/T$.

$$M_{\text{galaxy}} = \frac{4\pi^2(r_{\text{Sun orbit}})^3}{GT^2} = \frac{4\pi^2[(30{,}000)(9.5\times10^{15}\text{ m})]^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\left[(200\times10^6\text{ yr})\left(\frac{3.15\times10^7\text{ s}}{1\text{ yr}}\right)\right]^2}$$

$$= 3.452\times10^{41}\text{ kg} \approx \boxed{3\times10^{41}\text{ kg}}$$

The number of solar masses is found by dividing the result by the solar mass.

$$\#\text{ stars} = \frac{M_{\text{galaxy}}}{M_{\text{Sun}}} = \frac{3.452\times10^{41}\text{ kg}}{2.0\times10^{30}\text{ kg}} = 1.726\times10^{11} \approx \boxed{2\times10^{11}\text{ stars}}$$

77. (*a*) The gravitational force on the satellite is given by $F_{\text{grav}} = G\frac{M_{\text{Earth}}m}{r^2}$, where *r* is the distance of the satellite from the center of the Earth. Since the satellite is moving in circular motion, then the net force on the satellite can be written as $F_{\text{net}} = m\frac{\upsilon^2}{r}$. By substituting $\upsilon = 2\pi r/T$ for a circular orbit, we have $F_{\text{net}} = \frac{4\pi^2 mr}{T^2}$. Then, since gravity is the only force on the satellite, the two expressions for force can be equated and solved for the orbit radius.

$$G\frac{M_{\text{Earth}}m}{r^2} = \frac{4\pi^2 mr}{T^2} \quad \rightarrow$$

$$r = \left(\frac{GM_{\text{Earth}}T^2}{4\pi^2}\right)^{1/3} = \left[\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(6.0\times10^{24}\text{ kg})(6600\text{ s})^2}{4\pi^2}\right]^{1/3}$$

$$= 7.615\times10^6\text{ m} \approx \boxed{7.6\times10^6\text{ m}}$$

(*b*) From this value the gravitational force on the satellite can be calculated.

$$F_{\text{grav}} = G\frac{M_{\text{Earth}}m}{r^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(6.0\times10^{24}\ \text{kg})(5500\ \text{kg})}{(7.615\times10^6\ \text{m})^2} = 3.796\times10^4\ \text{N}$$

$$\approx \boxed{3.8\times10^4\ \text{N}}$$

(*c*) The altitude of the satellite above the Earth's surface is given by the following:

$$r - R_{\text{Earth}} = 7.615\times10^6\ \text{m} - 6.38\times10^6\ \text{m} = \boxed{1.2\times10^6\ \text{m}}$$

78. The speed of an orbiting object is given in Example 5–12 as $\upsilon = \sqrt{GM/r}$, where r is the radius of the orbit, and M is the mass around which the object is orbiting. Solve the equation for M.

$$\upsilon = \sqrt{GM/r} \;\rightarrow\; M = \frac{r\upsilon^2}{G} = \frac{(5.7\times10^{17}\ \text{m})(7.8\times10^5\ \text{m/s})^2}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = \boxed{5.2\times10^{39}\ \text{kg}}$$

The number of solar masses is found by dividing the result by the solar mass.

$$\#\ \text{solar masses} = \frac{M_{\text{galaxy}}}{M_{\text{Sun}}} = \frac{5.2\times10^{39}\ \text{kg}}{2\times10^{30}\ \text{kg}} = \boxed{2.6\times10^9\ \text{solar masses}}$$

79. Find the "new" Earth radius by setting the acceleration due to gravity at the Sun's surface equal to the acceleration due to gravity at the "new" Earth's surface.

$$g_{\substack{\text{Earth}\\ \text{new}}} = g_{\text{Sun}} \;\rightarrow\; \frac{GM_{\text{Earth}}}{r^2_{\substack{\text{Earth}\\ \text{new}}}} = \frac{GM_{\text{Sun}}}{r^2_{\text{Sun}}} \;\rightarrow$$

$$r_{\substack{\text{Earth}\\ \text{new}}} = r_{\text{Sun}}\sqrt{\frac{M_{\text{Earth}}}{M_{\text{Sun}}}} = (6.96\times10^8\ \text{m})\sqrt{\frac{5.98\times10^{24}\ \text{kg}}{1.99\times10^{30}\ \text{kg}}} = \boxed{1.21\times10^6\ \text{m}}$$

This is about 1/5 the actual Earth radius.

80. The speed of an object orbiting a mass is given in Example 5–12 as $\upsilon = \sqrt{\dfrac{GM_{\text{Sun}}}{r}}$.

$$\upsilon_{\text{new}} = 1.5\upsilon \text{ and } \upsilon_{\text{new}} = \sqrt{\frac{GM_{\text{Sun}}}{r_{\text{new}}}} \;\rightarrow\; 1.5\upsilon = \sqrt{\frac{GM_{\text{Sun}}}{r_{\text{new}}}} \;\rightarrow\; 1.5\sqrt{\frac{GM_{\text{Sun}}}{r}} = \sqrt{\frac{GM_{\text{Sun}}}{r_{\text{new}}}} \;\rightarrow$$

$$r_{\text{new}} = \frac{r}{1.5^2} = \boxed{0.44r}$$

Note that the answer doesn't depend on either of the asteroid masses.

81. The goal is to form a quantity that has acceleration units, from the speed of the radius of an object in circular motion. Speed has dimensions $\left[\dfrac{L}{T}\right]$, radius has dimensions $[L]$, and acceleration has dimensions $\left[\dfrac{L}{T^2}\right]$. To get time units squared in the denominator, the speed must be squared. But the dimensions of speed squared are $\left[\dfrac{L^2}{T^2}\right]$. This has one too many powers of length, so to reduce that, divide by the radius.

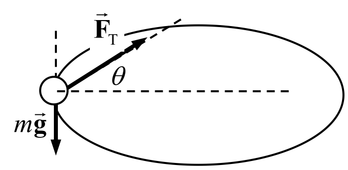
$$\frac{\upsilon^2}{r} \to \frac{\left[\dfrac{L^2}{T^2}\right]}{[L]} = \left[\frac{L}{T^2}\right] \to a$$

Factors such as 2 or π, if needed for the final formula, cannot be determined with dimensional analysis.

Solutions to Search and Learn Problems

1. Ex. 5–1: (i) The ball; (ii) tension in the string acting on the ball.
   Ex. 5–2: (i) The Moon; (ii) gravitational force on the Moon from the Earth.
   Ex. 5–3: (i) The ball; (ii) tension in the string acting on the ball.
   Ex. 5–4: (i) The ball; (ii) at the top it is the sum of the tension and force of gravity; at the bottom it is the difference between tension and gravity.
   Ex. 5–5: (i) The tetherball; (ii) the horizontal component of the tension.
   Ex. 5–6: (i) The car; (ii) the force of static friction between the tires and the road.
   Ex. 5–7: (i) The car; (ii) the horizontal component of the normal force.
   Ex. 5–8: (i) The race car; (ii) the radial component of the static friction.
   Ex. 5–9: No centripetal acceleration
   Ex. 5–10: (i) The spacecraft; (ii) the force of gravity on the spacecraft from the Earth.
   Ex. 5–11: No centripetal acceleration.
   Ex. 5–12: (i) The satellite; (ii) the force of gravity on the satellite from the Earth
   Ex. 5–13: (i) Mars; (ii) the force of gravity on Mars from the Sun.
   Ex. 5–14: (i) Earth; (ii) the force of gravity on Earth from the Sun.

2. A free-body diagram for the ball is shown, similar to Fig. 5–7. The tension in the suspending cord must not only hold the ball up, but also provide the centripetal force needed to make the ball move in a circle. Write Newton's second law for the vertical direction, noting that the ball is not accelerating vertically.



$$\sum F_y = F_T \sin\theta - mg = 0 \quad \to \quad F_T = \frac{mg}{\sin\theta}$$

The force moving the ball in a circle is the horizontal component of the tension. Write Newton's second law for that radial motion.

$$\sum F_R = F_T \cos\theta = ma_R = m\frac{\upsilon^2}{r}$$

Substitute the expression for the tension from the first equation into the second equation, and solve for the angle. Also substitute in the fact that for a rotating object, $\upsilon = 2\pi r/T$. Finally, we recognize that if the string is of length L, then the radius of the circle is $r = L\cos\theta$.

$$F_T \cos\theta = \frac{mg}{\sin\theta}\cos\theta = \frac{mv^2}{r} = \frac{4\pi^2 mr}{T^2} = \frac{4\pi^2 mL\cos\theta}{T^2} \rightarrow$$

$$\sin\theta = \frac{gT^2}{4\pi^2 L} \rightarrow \theta = \sin^{-1}\frac{gT^2}{4\pi^2 L} = \sin^{-1}\frac{(9.80\text{ m/s}^2)(0.500\text{ s})^2}{4\pi^2(0.600\text{ m})} = \boxed{5.94^\circ}$$

The tension is then given by $F_T = \dfrac{mg}{\sin\theta} = \dfrac{(0.150\text{ kg})(9.80\text{ m s}^2)}{\sin 5.94^\circ} = \boxed{14.2\text{ N}}$

3. An object at the Earth's equator is rotating in a circle with a radius equal to the radius of the Earth and a period equal to one day. Use that data to find the centripetal acceleration and then compare it with g.

$$a_R = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} \rightarrow \frac{a_R}{g} = \frac{\frac{4\pi^2(6.38\times10^6\text{ m})}{(86{,}400\text{ s})^2}}{(9.80\text{ m/s}^2)} = 0.00344 \approx \boxed{\frac{3}{1000}}$$

So, for example, if we were to calculate the normal force on an object at the Earth's equator, we could not say $\sum F = F_N - mg = 0$. Instead, we would have the following:

$$\sum F = F_N - mg = -m\frac{v^2}{r} \rightarrow F_N = mg - m\frac{v^2}{r}$$

If we then assumed that $F_N = mg_{\text{eff}} = mg - m\dfrac{v^2}{r}$, then we see that the effective value of g is

$$g_{\text{eff}} = g - \frac{v^2}{r} = g - 0.003g = \boxed{0.997g}.$$

4. (*a*) The acceleration due to gravity at any location at or above the surface of a star is given by $g_{\text{star}} = GM_{\text{star}}/r^2$, where r is the distance from the center of the star to the location in question.

$$g_{\text{star}} = G\frac{M_{\text{sun}}}{R_{\text{Moon}}^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(1.99\times10^{30}\text{ kg})}{(1.74\times10^6\text{ m})^2} = \boxed{4.38\times10^7\text{ m/s}^2}$$

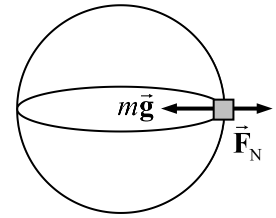
(*b*) $W = mg_{\text{star}} = (65\text{ kg})(4.38\times10^7\text{ m/s}^2) = \boxed{2.8\times10^9\text{ N}}$

(c) Use Eq. 2–11c, with an initial velocity of 0.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow$$

$$v = \sqrt{2a(x-x_0)} = \sqrt{2(4.38\times10^7\text{ m/s}^2)(1.0\text{ m})} = \boxed{9.4\times10^3\text{ m/s}}$$

5. For a body on the equator, the net motion is circular. Consider the free-body diagram as shown. F_N is the normal force, which is the apparent weight. The net force must point to the center of the circle for the object to be moving in a circular path at constant speed. Write Newton's second law with the inward direction as positive.



$$\sum F_R = mg_{\text{Jupiter}} - F_N = m\frac{\upsilon^2}{R_{\text{Jupiter}}} \rightarrow$$

$$F_N = m\left(g_{\text{Jupiter}} - \frac{\upsilon^2}{R_{\text{Jupiter}}}\right) = m\left(G\frac{M_{\text{Jupiter}}}{R^2_{\text{Jupiter}}} - \frac{\upsilon^2}{R_{\text{Jupiter}}}\right)$$

Use the fact that for a rotating object, $\upsilon = 2\pi r/T$.

$$F_N = m\left(G\frac{M_{\text{Jupiter}}}{R^2_{\text{Jupiter}}} - \frac{4\pi^2 R_{\text{Jupiter}}}{T^2_{\text{Jupiter}}}\right)$$

Thus the perceived acceleration of the object on the surface of Jupiter is

$$G\frac{M_{\text{Jupiter}}}{R^2_{\text{Jupiter}}} - \frac{4\pi^2 R_{\text{Jupiter}}}{T^2_{\text{Jupiter}}} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(1.9\times10^{27}\ \text{kg})}{(7.1\times10^7\ \text{m})^2} - \frac{4\pi^2(7.1\times10^7\ \text{m})}{\left[(595\ \text{min})\left(\frac{60\ \text{s}}{1\ \text{min}}\right)\right]^2}$$

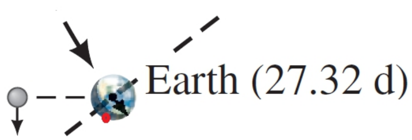
$$= 22.94\ \text{m/s}^2\left(\frac{1\,g}{9.80\ \text{m/s}^2}\right) = \boxed{2.3\ g\text{'s}}$$

Thus you would not be crushed at all. You would certainly feel “heavy” and quite uncomfortable, but not at all crushed.

6. (*a*) The Moon is Full when the Sun and Moon are on opposite sides of the Earth. In this position, a person on the Earth will only be able to see either the Sun or the Moon in the sky at any given time. Therefore, as the Sun sets, the Moon rises and as the Moon sets, the Sun rises.

   (*b*) As the Moon orbits the Earth it moves toward the east 1/29.53 of a synodic orbit, or about 12° every day. Therefore, if the Moon was just rising at 6 PM on the day of the Full Moon, it would be about 12° below the horizon at 6 PM the next day, and therefore would not be visible.

   (*c*) The red dot represents the location of a person on the Earth who sees the Full Moon rise at 6 PM on the day shown as figure (a). A day later (b) the Earth has completed one full rotation and for the person at that location it is again 6 PM. When the next Full Moon arrives, 29.53 days have elapsed. That means the red dot has revolved around the Earth about 29 and a half times. Because of the half revolution, the dot is on the other side of the Earth. To the observer it is now about 6 AM and the Full Moon is setting as the Sun rises. In part (d) the Earth will have completed about 27 and a third revolutions, so the red dot should be about one-third of a counter-clockwise rotation from 6 PM, or about 2 AM.



   (*d*) The Earth completes one full revolution, or 360°, around the Sun every year, or 365.25 days. The angle of the Moon in Fig. 5–31e relative to the “horizontal” (the dashed line in part (a)) is equal to the angle that the Earth moves between consecutive Full Moons:

$$\theta = \left(\frac{29.53\ \text{days}}{365.25\ \text{days}}\right)360° = 29.11°$$

So in 29.53 days the Moon has orbited 360° + 29.11° = 389.11°. The angular speed of the Moon is constant and can be written as the ratio of the orbital angle to orbital period for either sidereal or synodic orbits. Setting the ratios equal, solve for the sidereal period.

$$\omega = \frac{\theta_{\text{sidereal}}}{T_{\text{sidereal}}} = \frac{\theta_{\text{synoptic}}}{T_{\text{synoptic}}} \rightarrow$$

$$T_{\text{sidereal}} = T_{\text{synoptic}}\left(\frac{\theta_{\text{sidereal}}}{\theta_{\text{synoptic}}}\right) = (29.53 \text{ days})\left(\frac{360°}{389.11°}\right) = \boxed{27.32 \text{ days}}$$