3

Kinematics in Two Dimensions; Vectors

Responses to Questions

1. No, the two velocities are not equal. Velocity is a vector quantity, with a magnitude and direction. If two vectors have different directions, they cannot be equal.

2. No. The car may be traveling at a constant *speed* of 60 km/h and going around a curve, in which case it would be accelerating.

3. (i) During one year, the Earth travels a distance equal to the circumference of its orbit but has a displacement of 0 relative to the Sun.
   (ii) Any kind of cross-country "round trip" air travel would result in a large distance traveled but a displacement of 0.
   (iii) The displacement for a race car from the start to the finish of the Indianapolis 500 auto race is 0.

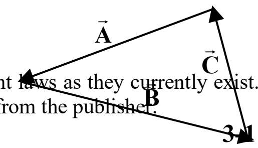
4. The length of the displacement vector is the straight-line distance between the beginning point and the ending point of the trip and therefore the shortest distance between the two points. If the path is a straight line, then the length of the displacement vector is the same as the length of the path. If the path is curved or consists of different straight-line segments, then the distance from beginning to end will be less than the path length. Therefore, the displacement vector can never be longer than the length of the path traveled, but it can be shorter.

5. Since both the batter and the ball started their motion at the same location (where the ball was hit) and ended their motion at the same location (where the ball was caught), the displacement of both was the same.

6. V is the magnitude of the vector $\vec{\mathbf{V}}$; it is not necessarily larger than the magnitudes V_1 and/or V_2. For instance, if $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$ have the same magnitude and are in opposite directions, then V is zero. The magnitude of the sum is determined by the angle between the two contributing vectors.

7. If the two vectors are in the same direction, the magnitude of their sum will be a maximum and will be 7.5 km. If the two vectors are in the opposite direction, the magnitude of their sum will be a minimum and will be 0.5 km. If the two vectors are oriented in any other configuration, the magnitude of their sum will be between 0.5 km and 7.5 km.

8. Two vectors of unequal magnitude can never add to give the zero vector. The



only way that two vectors can add up to give the zero vector is if they have the same magnitude and point in exactly opposite directions. However, three vectors of unequal magnitude can add to give the zero vector. As a one-dimensional example, a vector 10 units long in the positive x direction added to two vectors of 4 and 6 units each in the negative x direction will result in the zero vector. In two dimensions, if their sum using the tail-to-tip method gives a closed triangle, then the vector sum will be zero. See the diagram, in which $\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}=0.$

9. (*a*) The magnitude of a vector can equal the length of one of its components if the other components of the vector are all 0; that is, if the vector lies along one of the coordinate axes.
   (*b*) The magnitude of a vector can never be less than one of its components, because each component contributes a positive amount to the overall magnitude, through the Pythagorean relationship. The square root of a sum of squares is never less than the absolute value of any individual term.

10. The odometer and the speedometer of the car both measure scalar quantities (distance and speed, respectively).

11. To find the initial speed, use the slingshot to shoot the rock directly horizontally (no initial vertical speed) from a height of 1 meter (measured with the meter stick). The vertical displacement of the rock can be related to the time of flight by Eq. 2–11b. Take downward to be positive.

$$y=y_0+\upsilon_{y0}t+\tfrac{1}{2}at^2 \quad\rightarrow\quad 1\text{ m}=\tfrac{1}{2}gt^2 \quad\rightarrow\quad t=\sqrt{2(1\text{ m})/(9.8\text{ m/s}^2)}=0.45\text{ s}$$

Measure the horizontal range R of the rock with the meter stick. Then, if we measure the horizontal range R, we know that $R=\upsilon_x t=\upsilon_x(0.45\text{ s})$, so $\upsilon_x=R/0.45\text{ s}$, which is the speed the slingshot imparts to the rock. The only measurements are the height of fall and the range, both of which can be measured with a meter stick.

$\boxed{12.}$ The arrow should be aimed above the target, because gravity will deflect the arrow downward from a horizontal flight path. The angle of aim (above the horizontal) should increase as the distance from the target increases, because gravity will have more time to act in deflecting the arrow from a straight-line path. If we assume that the arrow was shot from the same height as the target, then the "level horizontal range" formula is applicable: $R=\upsilon_0^2\sin 2\theta_0/g \quad\rightarrow\quad \theta=\tfrac{1}{2}\sin^{-1}(Rg/\upsilon_0^2).$ As the range and hence the argument of the inverse sine function increases, the angle increases.

13. If the bullet was fired from the ground, then the y component of its velocity slowed considerably by the time it reached an altitude of 2.0 km, because of both the downward acceleration due to gravity and air resistance. The x component of its velocity would have slowed due to air resistance as well. Therefore, the bullet could have been traveling slowly enough to be caught.

14. The balloons will hit each other, although not along the line of sight from you to your friend. If there were no acceleration due to gravity, the balloons would hit each other along the line of sight. Gravity causes each balloon to accelerate downward from that "line of sight" path, each with the same acceleration. Thus each balloon falls below the line of sight by the same amount at every instant along their flight, so they collide. This situation is similar to Conceptual Example 3–7 and to Problem 36.

15. The horizontal component of the velocity stays constant in projectile motion, assuming that air resistance is negligible. Thus the horizontal component of velocity 1.0 seconds after launch will be the same as the horizontal component of velocity 2.0 seconds after launch. In both cases the horizontal velocity will be given by $\upsilon_x=\upsilon_0\cos\theta=(30\text{ m/s})(\cos 30°)=26\text{ m/s}.$

16. A projectile has the least speed at the top of its path. At that point the vertical speed is zero. The horizontal speed remains constant throughout the flight, if we neglect the effects of air resistance.

$\boxed{17.}$ (*a*) Cannonball A, with the larger angle, will reach a higher elevation. It has a larger initial vertical velocity, so by Eq. 2–11c it will rise higher before the vertical component of velocity is 0.

(*b*) Cannonball A, with the larger angle, will stay in the air longer. It has a larger initial vertical velocity, so it takes more time to decelerate to 0 and start to fall.

(*c*) The cannonball with a launch angle closest to 45° will travel the farthest. The range is a maximum for a launch angle of 45° and decreases for angles either larger or smaller than 45°.

18. (*a*) The ball lands at the same point from which it was thrown inside the train car—back in the thrower's hand.

(*b*) If the car accelerates, the ball will land behind the thrower's hand.

(*c*) If the car decelerates, the ball will land in front of the thrower's hand.

(*d*) If the car rounds a curve (assume it curves to the right), the ball will land to the left of the thrower's hand.

(*e*) The ball will be slowed by air resistance and will land behind the thrower's hand.

$\boxed{19.}$ Your reference frame is that of the train you are riding. If you are traveling with a relatively constant velocity (not over a hill or around a curve or drastically changing speed), then you will interpret your reference frame as being at rest. Since you are moving forward faster than the other train, the other train is moving backward relative to you. Seeing the other train go past your window from front to rear makes it look like the other train is going backward.

20. Both rowers need to cover the same "cross-river" distance. The rower with the greatest speed in the "cross-river" direction will be the one that reaches the other side first. The current has no bearing on the time to cross the river because the current doesn't help either of the boats move across the river. Thus the rower heading straight across will reach the other side first. All of his "rowing effort" has gone into crossing the river. For the upstream rower, some of his "rowing effort" goes into battling the current, so his "cross-river" speed will be only a fraction of his rowing speed.

21. When you stand still under an umbrella in a vertical rain, you are in a cylinder-shaped volume in which there is no rain. The rain has no horizontal component of velocity, so the rain cannot move from outside that cylinder into it. You stay dry. But as you run, you have a forward horizontal velocity relative to the rain, so the rain has a backward horizontal velocity relative to you. It is the same as if you were standing still under the umbrella but the rain had some horizontal component of velocity toward you. The perfectly vertical umbrella would not completely shield you.

Responses to MisConceptual Questions

1. (*c*) The shortest possible resultant will be 20 units, which occurs when the vectors point in opposite directions. Since 0 units and 18 units are less than 20 units, (*a*) and (*b*) cannot be correct answers. The largest possible result will be 60 units, which occurs when the vectors point in the same direction. Since 64 units and 100 units are greater than 60 units, (*d*) and (*e*) cannot be correct answers. Answer (*c*) is the only choice that falls between the minimum and maximum vector lengths.

2. (*a*) The components of a vector make up the two legs of a right triangle when the vector is the hypotenuse. The legs of a right triangle cannot be longer than the hypotenuse, therefore (*c*) and (*d*) cannot be correct answers. Only when the vector is parallel to the component is the

magnitude of the vector equal to the magnitude of the component, as in (*b*). For all other vectors, the magnitude of the component is less than the magnitude of the vector.

3. (*b*) If you turned 90°, as in (*a*), your path would be that of a right triangle. The distance back would be the hypotenuse of that triangle, which would be longer than 100 m. If you turned by only 30°, as in (*c*), your path would form an obtuse triangle; the distance back would have to be greater than if you had turned 90°, and therefore it too would be greater than 100 m. If you turned 180°, as in (*d*), you would end up back at your starting point, not 100 m away. Three equal distances of 100 m would form an equilateral triangle, so (*b*) is the correct answer.

4. (*a*) The bullet falls due to the influence of gravity, not due to air resistance. Therefore, (*b*) and (*c*) are incorrect. Inside the rifle the barrel prevents the bullet from falling, so the bullet does not begin to fall until it leaves the barrel.

5. (*b*) Assuming that we ignore air resistance, the ball is in free fall after it leaves the bat. If the answer were (*a*), the ball would continue to accelerate forward and would not return to the ground. If the answer were (*c*), the ball would slow to a stop and return backward toward the bat.

6. (*b*) If we ignore air friction, the horizontal and vertical components of the velocity are independent of each other. The vertical components of the two balls will be equal when the balls reach ground level. The ball thrown horizontally will have a horizontal component of velocity in addition to the vertical component. Therefore, it will have the greater speed.

7. (*c*) Both you and the ball have the same constant horizontal velocity. Therefore, in the time it takes the ball to travel up to its highest point and return to ground level, your hand and the ball have traveled the same horizontal distance, and the ball will land back in your hand.

8. (*d*) Both the time of flight and the maximum height are determined by the vertical component of the initial velocity. Since all three kicks reach the same maximum height, they must also have the same time of flight. The horizontal components of the initial velocity are different, which accounts for them traveling different distances.

9. (*c*) The baseball is in projectile motion during the entire flight. In projectile motion the acceleration is downward and constant; it is never zero. Therefore, (*a*) is incorrect. Since the ball was hit high and far, it must have had an initial horizontal component of velocity. For projectile motion the horizontal component of velocity is constant, so at the highest point the magnitude of the velocity cannot be zero, and thus (*b*) is incorrect. However, at the highest point, the vertical component of velocity is zero, so the magnitude of the velocity has a minimum at the highest point. So (*c*) is the correct answer.

10. (*b*) Both the monkey and bullet fall at the same rate due to gravity. If the gun was pointed directly at the monkey and gravity did not act upon either the monkey or bullet, the bullet would hit the monkey. Since both start falling at the same time and fall at the same rate, the bullet will still hit the monkey.

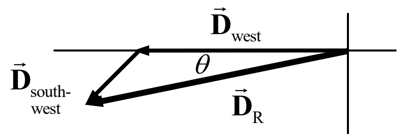
11. (*b*, *e*) In projectile motion the acceleration is vertical, so the x velocity is constant throughout the motion, so (*a*) is valid. The acceleration is that of gravity, which, when up is positive, is a constant negative value, so (*b*) is not valid and (*c*) is valid. At the highest point in the trajectory the vertical velocity is changing from a positive to a negative value. At this point the y component of velocity is zero, so (*d*) is valid. However, the x component of the velocity is constant, but not necessarily zero, so (*e*) is not valid.

12. (*a*) The maximum relative speed between the two cars occurs when the cars travel in opposite directions. This maximum speed would be the sum of their speeds relative to the ground or

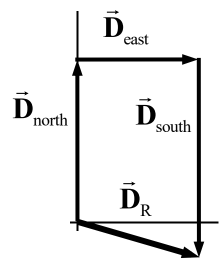
20 m/s. Since the two cars are traveling perpendicular to each other (not in opposite directions), their relative speed must be less than the maximum relative speed.

Solutions to Problems

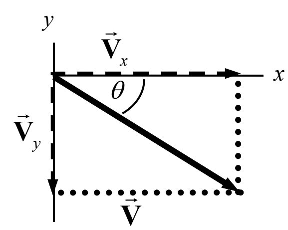
1. The resultant vector displacement of the car is given by $\vec{\mathbf{D}}_R = \vec{\mathbf{D}}_{west} + \vec{\mathbf{D}}_{\substack{south-\\west}}$. The westward displacement is $225\ \text{km} + (98\ \text{km})\cos 45° = 294.3\ \text{km}$ and the southward displacement is $(98\ \text{km})\sin 45° = 69.3\ \text{km}$. The resultant displacement has a magnitude of $\sqrt{294.3^2 + 69.3^2} = \boxed{302\ \text{km}}$. The direction is $\theta = \tan^{-1} 69.3/294.3 = \boxed{13°\ \text{south of west}}$.



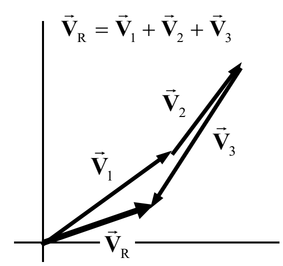
2. The truck has a displacement of $21 + (-26) = -5$ blocks north and 16 blocks east. The resultant has a magnitude of $\sqrt{(-5)^2 + 16^2} = 16.76\ \text{blocks} \approx \boxed{17\ \text{blocks}}$ and a direction of $\tan^{-1} 5/16 = \boxed{17°\ \text{south of east}}$.



3. Given that $V_x = 9.80$ units and $V_y = -6.40$ units, the magnitude of $\vec{\mathbf{V}}$ is given by $V = \sqrt{V_x^2 + V_y^2} = \sqrt{9.80^2 + (-6.40)^2} = \boxed{11.70\ \text{units}}$. The direction is given by $\theta = \tan^{-1}\dfrac{-6.40}{9.80} = \boxed{-33.1°}$, or $33.1°$ below the positive x axis.



4. The vectors for the problem are drawn approximately to scale. The resultant has a length of $\boxed{17.5\ \text{m}}$ and a direction $\boxed{19°}$ north of east. If calculations are done, the actual resultant should be 17 m at 23° north of east. Keeping one more significant figure would give 17.4 m at 22.5°.

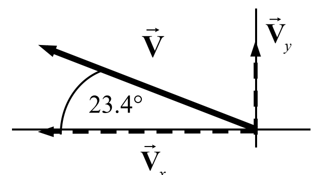


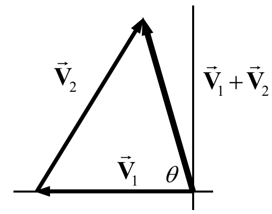
5. (*a*) See the accompanying diagram.

   (*b*) $V_x = -24.8\cos 23.4° = \boxed{-22.8\ \text{units}}$ $V_y = 24.8\sin 23.4° = \boxed{9.85\ \text{units}}$

   (*c*) $V = \sqrt{V_x^2 + V_y^2} = \sqrt{(-22.8)^2 + (9.85)^2} = \boxed{24.8\ \text{units}}$

   $\theta = \tan^{-1}\dfrac{9.85}{22.8} = \boxed{23.4°\ \text{above the} - x\ \text{axis}}$





6. (*a*) $V_{1x} = \boxed{-6.6 \text{ units}}$ $\quad V_{1y} = \boxed{0 \text{ units}}$

$$V_{2x} = 8.5 \cos 55^\circ = 4.875 \text{ units} \approx \boxed{4.9 \text{ units}}$$

$$V_{2y} = 8.5 \sin 55^\circ = 6.963 \text{ units} \approx \boxed{7.0 \text{ units}}$$

(*b*) To find the components of the sum, add the components of the individual vectors.

$$\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 = (V_{1x} + V_{2x},\, V_{1y} + V_{2y}) = (-1.725,\, 6.963)$$

$$\left|\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2\right| = \sqrt{(-1.725)^2 + (6.963)^2} = 7.173 \text{ units} \approx 7.2 \text{ units} \qquad \theta = \tan^{-1}\frac{6.963}{1.725} = 76^\circ$$

The sum has a magnitude of $\boxed{7.2 \text{ units}}$ and is $\boxed{76^\circ \text{ clockwise from the negative } x \text{ axis}}$, or 104° counterclockwise from the positive *x* axis.

7. We see from the diagram that $\vec{\mathbf{A}} = (6.8,\, 0)$ and $\vec{\mathbf{B}} = (-5.5,\, 0)$.

(*a*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = (6.8,\, 0) + (-5.5,\, 0) = (1.3,\, 0)$. The magnitude is $\boxed{1.3 \text{ units}}$, and the direction is $\boxed{+x}$.

(*b*) $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}} = (6.8,\, 0) - (-5.5,\, 0) = (12.3,\, 0)$. The magnitude is $\boxed{12.3 \text{ units}}$, and the direction is $\boxed{+x}$.

(*c*) $\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}} = (-5.5,\, 0) - (6.8,\, 0) = (-12.3,\, 0)$. The magnitude is $\boxed{12.3 \text{ units}}$, and the direction is $\boxed{-x}$.

8. (*a*) $\upsilon_{\text{north}} = (835 \text{ km/h})(\cos 41.5^\circ) = \boxed{625 \text{ km/h}}$ $\quad \upsilon_{\text{west}} = (835 \text{ km/h})(\sin 41.5^\circ) = \boxed{553 \text{ km/h}}$

(*b*) $\Delta d_{\text{north}} = \upsilon_{\text{north}} t = (625 \text{ km/h})(1.75 \text{ h}) = \boxed{1090 \text{ km}}$

$$\Delta d_{\text{west}} = \upsilon_{\text{west}} t = (553 \text{ km/h})(1.75 \text{ h}) = \boxed{968 \text{ km}}$$

$\boxed{9.}$ $A_x = 44.0 \cos 28.0^\circ = 38.85 \qquad A_y = 44.0 \sin 28.0^\circ = 20.66$

$$B_x = -26.5 \cos 56.0^\circ = -14.82 \qquad B_y = 26.5 \sin 56.0^\circ = 21.97$$

$$C_x = 31.0 \cos 270^\circ = 0.0 \qquad C_y = 31.0 \sin 270^\circ = -31.0$$

(*a*) $(\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}})_x = 38.85 + (-14.82) + 0.0 = 24.03 = \boxed{24.0}$

$$(\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}})_y = 20.66 + 21.97 + (-31.0) = 11.63 = \boxed{11.6}$$

(*b*) $\left|\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}\right| = \sqrt{(24.03)^2 + (11.63)^2} = \boxed{26.7} \qquad \theta = \tan^{-1}\frac{11.63}{24.03} = \boxed{25.8^\circ}$

10. $A_x = 44.0 \cos 28.0^\circ = 38.85 \qquad A_y = 44.0 \sin 28.0^\circ = 20.66$

$$B_x = -26.5 \cos 56.0^\circ = -14.82 \qquad B_y = 26.5 \sin 56.0^\circ = 21.97$$

(*a*) $(\vec{\mathbf{B}} - \vec{\mathbf{A}})_x = (-14.82) - 38.85 = -53.67 \quad (\vec{\mathbf{B}} - \vec{\mathbf{A}})_y = 21.97 - 20.66 = 1.31$

Note that since the *x* component is negative and the *y* component is positive, the vector is in the 2nd quadrant.

$$\left|\vec{\mathbf{B}} - \vec{\mathbf{A}}\right| = \sqrt{(-53.67)^2 + (1.31)^2} = \boxed{53.7} \quad \theta_{B-A} = \tan^{-1}\frac{1.31}{-53.67} = \boxed{1.4^\circ \text{ above} -x \text{ axis}}$$

(*b*) $(\vec{\mathbf{A}}-\vec{\mathbf{B}})_x = 38.85-(-14.82) = 53.67 \quad (\vec{\mathbf{A}}-\vec{\mathbf{B}})_y = 20.66-21.97 = -1.31$

Note that since the *x* component is positive and the *y* component is negative, the vector is in the 4th quadrant.

$$\left|\vec{\mathbf{A}}-\vec{\mathbf{B}}\right| = \sqrt{(53.67)^2+(-1.31)^2} = \boxed{53.7} \quad \theta = \tan^{-1}\frac{-1.31}{53.7} = \boxed{1.4° \text{ below } + x \text{ axis}}$$

Comparing the results shows that $\vec{\mathbf{B}}-\vec{\mathbf{A}}$ is the opposite of $\vec{\mathbf{A}}-\vec{\mathbf{B}}$.

11. $A_x = 44.0\cos 28.0° = 38.85 \qquad A_y = 44.0\sin 28.0° = 20.66$

$C_x = 31.0\cos 270° = 0.0 \qquad C_y = 31.0\sin 270° = -31.0$

$(\vec{\mathbf{A}}-\vec{\mathbf{C}})_x = 38.85-0.0 = 38.85 \qquad (\vec{\mathbf{A}}-\vec{\mathbf{C}})_y = 20.66-(-31.0) = 51.66$

$$\left|\vec{\mathbf{A}}-\vec{\mathbf{C}}\right| = \sqrt{(38.85)^2+(51.66)^2} = \boxed{64.6} \qquad \theta = \tan^{-1}\frac{51.66}{38.85} = \boxed{53.1°}$$

12. $A_x = 44.0\cos 28.0° = 38.85 \qquad A_y = 44.0\sin 28.0° = 20.66$

$B_x = -26.5\cos 56.0° = -14.82 \qquad B_y = 26.5\sin 56.0° = 21.97$

$C_x = 31.0\cos 270^{\text{o}} = 0.0 \qquad C_y = 31.0\sin 270° = -31.0$

(*a*) $(\vec{\mathbf{B}}-3\vec{\mathbf{A}})_x = -14.82-3(38.85) = -131.37 \quad (\vec{\mathbf{B}}-3\vec{\mathbf{A}})_y = 21.97-3(20.66) = -40.01$

Note that since both components are negative, the vector is in the 3rd quadrant.

$$\left|\vec{\mathbf{B}}-3\vec{\mathbf{A}}\right| = \sqrt{(-131.37)^2+(-40.01)^2} = 137.33 \approx \boxed{137}$$

$$\theta = \tan^{-1}\frac{-40.01}{-131.37} = \boxed{16.9° \text{ below } - x \text{ axis}}$$

(*b*) $(2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}})_x = 2(38.85)-3(-14.82)+2(0.0) = 122.16$

$(2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}})_y = 2(20.66)-3(21.97)+2(-31.0) = -86.59$

Note that since the *x* component is positive and the *y* component is negative, the vector is in the 4th quadrant.

$$\left|2\vec{\mathbf{A}}-3\vec{\mathbf{B}}+2\vec{\mathbf{C}}\right| = \sqrt{(122.16)^2+(-86.59)^2} = \boxed{149.7} \quad \theta = \tan^{-1}\frac{-86.59}{122.16} = \boxed{35.3° \text{ below } + x \text{ axis}}$$

13. $A_x = 44.0\cos 28.0° = 38.85 \qquad A_y = 44.0\sin 28.0° = 20.66$

$B_x = -26.5\cos 56.0° = -14.82 \qquad B_y = 26.5\sin 56.0° = 21.97$

$C_x = 31.0\cos 270° = 0.0 \qquad C_y = 31.0\sin 270° = -31.0$

(*a*) $(\vec{\mathbf{A}}-\vec{\mathbf{B}}+\vec{\mathbf{C}})_x = 38.85-(-14.82)+0.0 = 53.67$

$(\vec{\mathbf{A}}-\vec{\mathbf{B}}+\vec{\mathbf{C}})_y = 20.66-21.97+(-31.0) = -32.31$

Note that since the *x* component is positive and the *y* component is negative, the vector is in the 4th quadrant.

$$\left|\vec{\mathbf{A}}-\vec{\mathbf{B}}+\vec{\mathbf{C}}\right| = \sqrt{(53.67)^2+(-32.31)^2} = \boxed{62.6} \quad \theta = \tan^{-1}\frac{-32.31}{53.67} = \boxed{31.0° \text{ below } + x \text{ axis}}$$

(*b*) $(\vec{\mathbf{A}}+\vec{\mathbf{B}}-\vec{\mathbf{C}})_x = 38.85+(-14.82)-0.0 = 24.03$

$(\vec{\mathbf{A}}+\vec{\mathbf{B}}-\vec{\mathbf{C}})_y = 20.66+21.97-(-31.0) = 73.63$

$$\left|\vec{\mathbf{A}}+\vec{\mathbf{B}}-\vec{\mathbf{C}}\right| = \sqrt{(24.03)^2+(73.63)^2} = \boxed{77.5} \quad \theta = \tan^{-1}\frac{73.63}{24.03} = \boxed{71.9^\circ}$$

(*c*) $(\vec{\mathbf{C}}-\vec{\mathbf{A}}-\vec{\mathbf{B}})_x = 0.0-38.85-(-14.82) = -24.03$

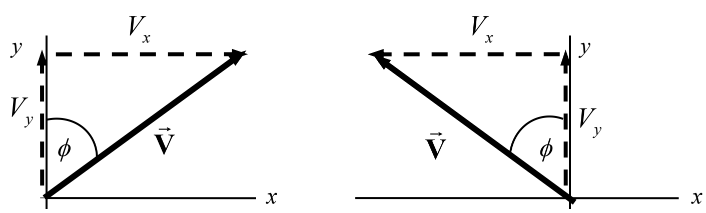
$(\vec{\mathbf{C}}-\vec{\mathbf{A}}-\vec{\mathbf{B}})_y = -31.0-20.66-21.97 = -73.63$

Note that since both components are negative, the vector is in the 3rd quadrant.

$$\left|\vec{\mathbf{C}}-\vec{\mathbf{A}}-\vec{\mathbf{B}}\right| = \sqrt{(-24.03)^2+(-73.63)^2} = \boxed{77.5} \quad \theta = \tan^{-1}\frac{-73.63}{-24.03} = \boxed{71.9^\circ \text{ below } -x \text{ axis}}$$

Note that the answer to (*c*) is the exact opposite of the answer to (*b*).

14. If the angle is in the first quadrant, then $V_x = V\sin\phi$ and $V_y = V\cos\phi$. See the first diagram. If the angle is in the second quadrant, then $V_x = V\sin\phi$ and $V_y = -V\cos\phi$. See the second diagram.



15. The *x* component is negative and the *y* component is positive, since the summit is to the west of north. The angle measured counterclockwise from the positive *x* axis would be 122.4°. Thus the components are found to be the following:

$x = -4580\sin 38.4^\circ = -2845$ m $\quad y = 4580\cos 38.4^\circ = 3589$ m $\quad z = 2450$ m

$$\boxed{\vec{\mathbf{r}} = (-2845\text{ m}, 3589\text{ m}, 2450\text{ m})} \quad \left|\vec{\mathbf{r}}\right| = \sqrt{(-2845)^2+(3589)^2+(2450)^2} = \boxed{5190\text{ m}}$$

16. (*a*) Use the Pythagorean theorem to find the possible *x* components.

$$90.0^2 = x^2+(-65.0)^2 \rightarrow x^2 = 3875 \rightarrow x = \boxed{\pm 62.2 \text{ units}}$$

(*b*) Express each vector in component form, with $\vec{\mathbf{V}}$ the vector to be determined. The answer is given both as components and in magnitude/direction format.

$(62.2, -65.0)+(V_x, V_y) = (-80.0, 0) \rightarrow$

$V_x = (-80.0-62.2) = -142.2 \quad V_y = 65.0 \quad \vec{\mathbf{V}} = \boxed{(-142.2, 65.0)}$

$$\left|\vec{\mathbf{V}}\right| = \sqrt{(-142.2)^2+65.0^2} = \boxed{156 \text{ units}} \quad \theta = \tan^{-1}\frac{65.0}{-142.2} = \boxed{24.6^\circ \text{ above } -x \text{ axis}}$$

17. Choose downward to be the positive *y* direction. The origin will be at the point where the tiger leaps from the rock. In the horizontal direction, $\upsilon_{x0} = 3.0$ m/s and $a_x = 0$. In the vertical direction,

$\upsilon_{y0} = 0$, $a_y = 9.80\text{ m/s}^2$, $y_0 = 0$, and the final location is $y = 7.5\text{ m}$. The time for the tiger to reach the ground is found from applying Eq. 2–11b to the vertical motion.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 7.5\text{ m} = 0 + 0 + \tfrac{1}{2}(9.80\text{ m/s}^2)t^2 \quad \rightarrow \quad t = \sqrt{\frac{2(7.5\text{ m})}{9.80\text{ m/s}^2}} = 1.237\text{ s}$$

The horizontal displacement is calculated from the constant horizontal velocity.

$$\Delta x = \upsilon_x t = (3.0\text{ m/s})(1.237\text{ s}) = \boxed{3.7\text{ m}}$$

18. Choose downward to be the positive y direction. The origin will be at the point where the diver dives from the cliff. In the horizontal direction, $\upsilon_{x0} = 2.5\text{ m/s}$ and $a_x = 0$. In the vertical direction, $\upsilon_{y0} = 0$, $a_y = 9.80\text{ m/s}^2$, $y_0 = 0$, and the time of flight is $t = 3.0\text{ s}$. The height of the cliff is found from applying Eq. 2–11b to the vertical motion.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}(9.80\text{ m/s}^2)(3.0\text{ s})^2 = \boxed{44\text{ m}}$$

The distance from the base of the cliff to where the diver hits the water is found from the horizontal motion at constant velocity:

$$\Delta x = \upsilon_x t = (2.5\text{ m/s})(3.0\text{ s}) = \boxed{7.5\text{ m}}$$

19. Apply the level horizontal range formula derived in the text. If the launching speed and angle are held constant, the range is inversely proportional to the value of g. The acceleration due to gravity on the Moon is one-sixth that on Earth.

$$R_{\text{Earth}} = \frac{\upsilon_0^2 \sin 2\theta_0}{g_{\text{Earth}}} \quad R_{\text{Moon}} = \frac{\upsilon_0^2 \sin 2\theta_0}{g_{\text{Moon}}} \quad \rightarrow \quad R_{\text{Earth}}g_{\text{Earth}} = R_{\text{Moon}}g_{\text{Moon}}$$

$$R_{\text{Moon}} = R_{\text{Earth}}\frac{g_{\text{Earth}}}{g_{\text{Moon}}} = 6R_{\text{Earth}}$$

Thus, on the Moon, the person can jump $\boxed{\text{6 times farther}}$.

20. Choose downward to be the positive y direction. The origin will be at the point where the ball is thrown from the roof of the building. In the vertical direction, $\upsilon_{y0} = 0$, $a_y = 9.80\text{ m/s}^2$, $y_0 = 0$, and the displacement is 7.5 m. The time of flight is found from applying Eq. 2–11b to the vertical motion.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 7.5\text{ m} = \tfrac{1}{2}(9.80\text{ m/s}^2)t^2 \quad \rightarrow \quad t = \sqrt{\frac{2(9.0\text{ m})}{9.80\text{ m/s}^2}} = 1.237\text{ s}$$

The horizontal speed (which is the initial speed) is found from the horizontal motion at constant velocity.

$$\Delta x = \upsilon_x t \quad \rightarrow \quad \upsilon_x = \Delta x / t = 9.5\text{ m}/1.237\text{ s} = \boxed{7.7\text{ m/s}}$$

21. Choose downward to be the positive y direction. The origin is the point where the ball is thrown from the roof of the building. In the vertical direction $\upsilon_{y0} = 0,\ y_0 = 0,$ and $a_y = 9.80\ \text{m/s}^2$. The initial horizontal velocity is 12.2 m/s and the horizontal range is 21.0 m. The time of flight is found from the horizontal motion at constant velocity.

$$\Delta x = \upsilon_x t \quad \rightarrow \quad t = \Delta x/\upsilon_x = (21.0\ \text{m})/(12.2\ \text{m/s}) = 1.721\ \text{s}$$

The vertical displacement, which is the height of the building, is found by applying Eq. 2–11b to the vertical motion.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}(9.80\ \text{m/s}^2)(1.721\ \text{s})^2 = \boxed{14.5\ \text{m}}$$

22. Choose the point at which the football is kicked as the origin, and choose upward to be the positive y direction. When the football reaches the ground again, the y displacement is 0. For the football, $\upsilon_{y0} = (18.0 \sin 31.0°)\ \text{m/s},\ a_y = -9.80\ \text{m/s}^2,$ and the final y velocity will be the opposite of the starting y velocity. Use Eq. 2–11a to find the time of flight.

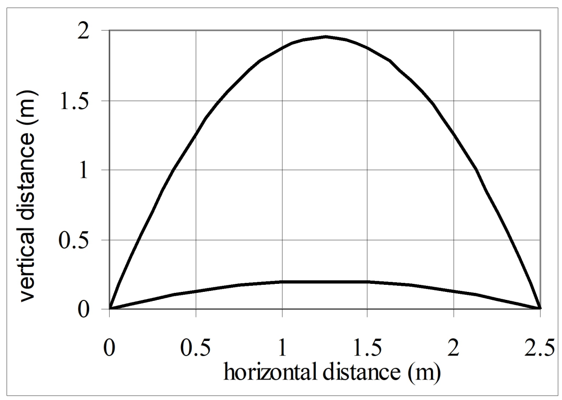
$$\upsilon_y = \upsilon_{y0} + at \quad \rightarrow \quad t = \frac{\upsilon_y - \upsilon_{y0}}{a} = \frac{(-18.0 \sin 31.0°)\ \text{m/s} - (18.0 \sin 31.0°)\ \text{m/s}}{-9.80\ \text{m/s}^2} = \boxed{1.89\ \text{s}}$$

23. Apply the level horizontal range formula derived in the text.

$$R = \frac{\upsilon_0^2 \sin 2\theta_0}{g} \quad \rightarrow$$

$$\sin 2\theta_0 = \frac{Rg}{\upsilon_0^2} = \frac{(2.5\ \text{m})(9.80\ \text{m/s}^2)}{(6.5\ \text{m/s})^2} = 0.5799$$

$$2\theta_0 = \sin^{-1} 0.5799 \quad \rightarrow \quad \theta_0 = \boxed{18°,\ 72°}$$



There are two angles because each angle gives the same range. If one angle is $\theta = 45° + \delta$, then $\theta = 45° - \delta$ is also a solution. The two paths are shown in the graph.

24. When shooting the gun vertically, half the time of flight is spent moving upward. Thus the upward flight takes two seconds. Choose upward as the positive y direction. Since at the top of the flight the vertical velocity is zero, find the launching velocity from Eq. 2–11a.

$$\upsilon_y = \upsilon_{y0} + at \quad \rightarrow \quad \upsilon_{y0} = \upsilon_y - at = 0 = (9.80\ \text{m/s}^2)(2.0\ \text{s}) = 19.6\ \text{m/s}$$

Using this initial velocity and an angle of 45° in the level horizontal range formula derived in the text will give the maximum range for the gun.

$$R = \frac{\upsilon_0^2 \sin 2\theta_0}{g} = \frac{(19.6\ \text{m/s})^2 \sin(2 \times 45°)}{9.80\ \text{m/s}^2} = \boxed{39\ \text{m}}$$

25. The level horizontal range formula derived in the text can be used to find the launching velocity of the grasshopper.

$$R = \frac{\upsilon_0^2 \sin 2\theta_0}{g} \quad \rightarrow \quad \upsilon_0 = \sqrt{\frac{Rg}{\sin 2\theta_0}} = \sqrt{\frac{(0.80\ \text{m})(9.80\ \text{m/s}^2)}{\sin 90°}} = 2.8\ \text{m/s}$$

Since there is no time between jumps, the horizontal velocity of the grasshopper is the horizontal component of the launching velocity.

$$\upsilon_x = \upsilon_0 \cos\theta_0 = (2.8 \text{ m/s}) \cos 45° = \boxed{2.0 \text{ m/s}}$$

26. (*a*) Take the ground to be the $y = 0$ level, with upward as the positive direction. Use Eq. 2–11b to solve for the time, with an initial vertical velocity of 0.

$$y = y_0 + \upsilon_{0y}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 150 \text{ m} = 910 \text{ m} + \tfrac{1}{2}(-9.80 \text{ m/s}^2)t^2 \quad \rightarrow$$

$$t = \sqrt{\frac{2(150-910)}{(-9.80 \text{ m/s}^2)}} = 12.45 \text{ s} \approx \boxed{12 \text{ s}}$$

(*b*) The horizontal motion is at a constant speed, since air resistance is being ignored.

$$\Delta x = \upsilon_x t = (4.0 \text{ m/s})(12.45 \text{ s}) = 49.8 \text{ m} \approx \boxed{5.0\times10^1 \text{ m}}$$

$\boxed{27.}$ Choose the origin to be where the projectile is launched and upward to be the positive *y* direction. The initial velocity of the projectile is υ_0, the launching angle is θ_0, $a_y = -g$, and $\upsilon_{y0} = \upsilon_0 \sin\theta_0$.

(*a*) The maximum height is found from Eq. 2–11c with $\upsilon_y = 0$ at the maximum height.

$$y_{\max} = 0 + \frac{\upsilon_y^2 - \upsilon_{y0}^2}{2a_y} = \frac{-\upsilon_0^2 \sin^2\theta_0}{-2g} = \frac{\upsilon_0^2 \sin^2\theta_0}{2g} = \frac{(36.6 \text{ m/s})^2 \sin^2 42.2°}{2(9.80 \text{ m/s}^2)} = \boxed{30.8 \text{ m}}$$

(*b*) The total time in the air is found from Eq. 2–11b, with a total vertical displacement of 0 for the ball to reach the ground.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = \upsilon_0 \sin\theta_0 t - \tfrac{1}{2}gt^2 \quad \rightarrow$$

$$t = \frac{2\upsilon_0 \sin\theta_0}{g} = \frac{2(36.6 \text{ m/s}) \sin 42.2°}{9.80 \text{ m/s}^2} = 5.0173 \text{ s} \approx \boxed{5.02 \text{ s}} \text{ and } t = 0$$

The time of 0 represents the launching of the ball.

(*c*) The total horizontal distance covered is found from the horizontal motion at constant velocity.

$$\Delta x = \upsilon_x t = (\upsilon_0 \cos\theta_0)t = (36.6 \text{ m/s})(\cos 42.2°)(5.0173 \text{ s}) = \boxed{136 \text{ m}}$$

(*d*) The velocity of the projectile 1.50 s after firing is found as the vector sum of the horizontal and vertical velocities at that time. The horizontal velocity is a constant $\upsilon_0 \cos\theta_0 =$ $(36.6 \text{ m/s})(\cos 42.2°) = 27.11 \text{ m/s}$. The vertical velocity is found from Eq. 2–11a.

$$\upsilon_y = \upsilon_{y0} + at = \upsilon_0 \sin\theta_0 - gt = (36.6 \text{ m/s}) \sin 42.2° - (9.80 \text{ m/s}^2)(1.50 \text{ s}) = 9.885 \text{ m/s}$$

Thus the speed of the projectile is as follows:

$$\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2} = \sqrt{(27.11 \text{ m/s})^2 + (9.885 \text{ m/s})^2} = \boxed{28.9 \text{ m/s}}$$

28. (*a*) Use the level horizontal range formula derived in the text.

$$R = \frac{\upsilon_0^2 \sin 2\theta_0}{g} \quad \rightarrow \quad \upsilon_0 = \sqrt{\frac{Rg}{\sin 2\theta_0}} = \sqrt{\frac{(7.80 \text{ m})(9.80 \text{ m/s}^2)}{\sin 54.0°}} = \boxed{9.72 \text{ m/s}}$$

(*b*) Now increase the speed by 5.0% and calculate the new range. The new speed would be $9.72\text{ m/s}\,(1.05) = 10.21\text{ m/s}$ and the new range would be as follows:

$$R = \frac{\upsilon_0^2 \sin 2\theta_0}{g} = \frac{(10.21\text{ m/s})^2 \sin 54°}{9.80\text{ m/s}^2} = 8.606\text{ m}$$

This is an increase of $\boxed{0.81\text{ m (10\% increase)}}$.

29. Choose the origin to be the point on the ground directly below the point where the baseball was hit. Choose upward to be the positive *y* direction. Then $y_0 = 1.0\text{ m}$, $y = 13.0\text{ m}$ at the end of the motion, $\upsilon_{y0} = (27.0 \sin 45.0°)\text{ m/s} = 19.09\text{ m/s}$, and $a_y = -9.80\text{ m/s}^2$. Use Eq. 2–11b to find the time of flight.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow\quad \tfrac{1}{2}a_y t^2 + \upsilon_{y0}t + (y_0 - y) = 0 \quad\rightarrow$$

$$t = \frac{-\upsilon_{y0} \pm \sqrt{\upsilon_{y0}^2 - 4\left(\tfrac{1}{2}a_y\right)(y_0 - y)}}{2\tfrac{1}{2}a_y} = \frac{-19.09 \pm \sqrt{(19.09)^2 - 2(-9.80)(-12.0)}}{-9.80}$$

$$= 0.788\text{ s}, 3.108\text{ s}$$

The smaller time is the time when the baseball reached the building's height on the way up, and the larger time is the time at which the baseball reached the building's height on the way down. We must choose the larger result, because the baseball cannot land on the roof on the way up. Now calculate the horizontal distance traveled using the horizontal motion at constant velocity.

$$\Delta x = \upsilon_x t = [(27.0 \cos 45.0°)\text{ m/s}](3.108\text{ s}) = \boxed{59.3\text{ m}}$$

30. Choose the origin to be the location on the ground directly below the airplane at the time the supplies are dropped, and choose upward as the positive *y* direction. For the supplies, $y_0 = 235\text{ m}$, $\upsilon_{y0} = 0$, $a_y = -g$, and the final *y* location is $y = 0$. The initial (and constant) *x* velocity of the supplies is $\upsilon_x = 69.4\text{ m/s}$. First the time for the supplies to reach the ground is found from Eq. 2–11b.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow\quad 0 = y_0 + 0 + \tfrac{1}{2}at^2 \quad\rightarrow\quad t = \sqrt{\frac{-2y_0}{a}} = \sqrt{\frac{-2(235\text{ m})}{(-9.80\text{ m/s}^2)}} = 6.925\text{ s}$$

Then the horizontal distance of travel for the package is found from the horizontal constant velocity.

$$\Delta x = \upsilon_x t = (69.4\text{ m/s})(6.925\text{ s}) = \boxed{481\text{ m}}$$

31. We have the same set-up as in Problem 31. Choose the origin to be the location on the ground directly below the airplane at the time the supplies are dropped, and choose upward as the positive *y* direction. For the supplies, $y_0 = 235\text{ m}$, $\upsilon_{y0} = 0$, $a_y = -g$, and the final *y* location is $y = 0$. The initial (and constant) *x* velocity of the supplies is $\upsilon_x = 69.4\text{ m/s}$. The supplies have to travel a horizontal distance of 425 m. The time of flight is found from the horizontal motion at constant velocity.

$$\Delta x = \upsilon_x t \quad\rightarrow\quad t = \Delta x/\upsilon_x = 425\text{ m}/69.4\text{ m/s} = 6.124\text{ s}$$

The *y* motion must satisfy Eq. 2–11b for this time.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \quad\rightarrow$$

$$\upsilon_{y0} = \frac{y - y_0 - \tfrac{1}{2}a_y t^2}{t} = \frac{0 - 235\text{ m} - \tfrac{1}{2}(-9.80\text{ m/s}^2)(6.124\text{ s})^2}{6.124\text{ s}} = \boxed{-8.37\text{ m/s}}$$

Notice that since this is a negative velocity, the object must be projected DOWN.

The horizontal component of the speed of the supplies upon landing is the constant horizontal speed of 69.4 m/s. The vertical speed is found from Eq. 2–11a.

$$\upsilon_y = \upsilon_{y0} + a_y t = -8.37 \text{ m/s} + (-9.80 \text{ m/s}^2)(6.124 \text{ s}) = 68.4 \text{ m/s}$$

Thus the speed is given by $\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2} = \sqrt{(69.4 \text{ m/s})^2 + (68.4 \text{ m/s})^2} = \boxed{97.4 \text{ m/s}}$

32. Choose the origin to be the point of launch and upward to be the positive *y* direction. The initial velocity is υ_0, the launching angle is θ_0, $a_y = -g$, $y_0 = 0$, and $\upsilon_{y0} = \upsilon_0 \sin\theta_0$. Eq. 2–11a is used to find the time required to reach the highest point, at which $\upsilon_y = 0$.

$$\upsilon_y = \upsilon_{y0} + at_{\text{up}} \quad \rightarrow \quad t_{\text{up}} = \frac{\upsilon_y - \upsilon_{y0}}{a} = \frac{0 - \upsilon_0 \sin\theta_0}{-g} = \frac{\upsilon_0 \sin\theta_0}{g}$$

Eq. 2–11c is used to find the height at this highest point.

$$\upsilon_y^2 = \upsilon_{y0}^2 + 2a_y(y_{\max} - y_0) \quad \rightarrow \quad y_{\max} = y_0 + \frac{\upsilon_y^2 - \upsilon_{y0}^2}{2a_y} = 0 + \frac{-\upsilon_0^2 \sin^2\theta_0}{-2g} = \frac{\upsilon_0^2 \sin^2\theta_0}{2g}$$

Eq. 2–11b is used to find the time for the object to fall the same distance with a starting velocity of 0.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = \frac{\upsilon_0^2 \sin^2\theta_0}{2g} + 0\ (t_{\text{down}}) - \tfrac{1}{2}g t_{\text{down}}^2 \quad \rightarrow \quad t_{\text{down}} = \frac{\upsilon_0 \sin\theta_0}{g}$$

A comparison shows that $\boxed{t_{\text{up}} = t_{\text{down}}}$.

33. Choose upward to be the positive *y* direction. The origin is the point from which the football is kicked. The initial speed of the football is $\upsilon_0 = 20.0$ m/s. We have $\upsilon_{y0} = \upsilon_0 \sin 37.0° = 12.04$ m/s, $y_0 = 0$, and $a_y = -9.80 \text{ m/s}^2$. In the horizontal direction, $\upsilon_x = \upsilon_0 \cos 37.0° = 15.97$ m/s, and $\Delta x = 36.0$ m. The time of flight to reach the goalposts is found from the horizontal motion at constant speed.

$$\Delta x = \upsilon_x t \quad \rightarrow \quad t = \Delta x / \upsilon_x = 36.0 \text{ m}/15.97 \text{ m/s} = 2.254 \text{ s}$$

Now use this time with the vertical motion data and Eq. 2–11b to find the height of the football when it reaches the horizontal location of the goalposts.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 = 0 + (12.04 \text{ m/s})(2.254 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(2.254 \text{ s})^2 = 2.24 \text{ m}$$

Since the ball's height is less than 3.05 m, $\boxed{\text{the football does not clear the bar}}$. It is 0.81 m too low when it reaches the horizontal location of the goalposts.

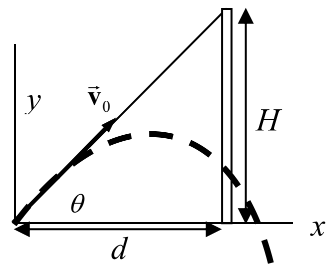
To find the distances from which a score can be made, redo the problem (with the same initial conditions) to find the times at which the ball is exactly 3.05 m above the ground. Those times would correspond with the maximum and minimum distances for making the score. Use Eq. 2–11b.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 3.05 = 0 + (12.04\ \text{m/s})t + \tfrac{1}{2}(-9.80\ \text{m/s}^2)t^2 \quad \rightarrow$$

$$4.90t^2 - 12.04t + 3.05 = 0 \quad \rightarrow \quad t = \frac{12.04 \pm \sqrt{(12.04)^2 - 4(4.90)(3.05)}}{2(4.90)} = 2.1703\ \text{s},\ 0.2868\ \text{s}$$

$$\Delta x_1 = \upsilon_x t = 15.97\ \text{m/s}\ (0.2868\ \text{s}) = 4.580\ \text{m};\ \Delta x_1 = \upsilon_x t = 15.97\ \text{m/s}\ (2.1703\ \text{s}) = 34.660\ \text{m}$$

So the kick must be made in the range from $\boxed{4.6\ \text{m to } 34.7\ \text{m}}$.



34. Choose the origin to be the location from which the balloon is fired, and choose upward as the positive y direction. Assume that the boy in the tree is a distance H up from the point at which the balloon is fired and that the tree is a horizontal distance d from the point at which the balloon is fired. The equations of motion for the balloon and boy are as follows, using constant acceleration relationships:

$$x_{\text{Balloon}} = \upsilon_0 \cos\theta_0 t \qquad y_{\text{Balloon}} = 0 + \upsilon_0 \sin\theta_0 t - \tfrac{1}{2}gt^2 \qquad y_{\text{Boy}} = H - \tfrac{1}{2}gt^2$$

Use the horizontal motion at constant velocity to find the elapsed time after the balloon has traveled d to the right.

$$d = \upsilon_0 \cos\theta_0 t \quad \rightarrow \quad t = \frac{d}{\upsilon_0 \cos\theta_0}$$

Where is the balloon vertically at that time?

$$y_{\text{Balloon}} = \upsilon_0 \sin\theta_0 t - \tfrac{1}{2}gt^2 = \upsilon_0 \sin\theta_0 \upsilon \frac{d}{\upsilon_0 \cos\theta_0} - \tfrac{1}{2}g\left(\frac{d}{\upsilon_0 \cos\theta_0}\right)^2$$

$$= d\tan\theta_0 - \tfrac{1}{2}g\left(\frac{d}{\upsilon_0 \cos\theta_0}\right)^2$$

Where is the boy vertically at that time? Note that $H = d\tan\theta_0$.

$$y_{\text{Boy}} = H - \tfrac{1}{2}gt^2 = H - \tfrac{1}{2}g\left(\frac{d}{\upsilon_0 \cos\theta_0}\right)^2 = d\tan\theta_0 - \tfrac{1}{2}g\left(\frac{d}{\upsilon_0 \cos\theta_0}\right)^2$$

Note that $y_{\text{Balloon}} = y_{\text{Boy}}$, so the boy and the balloon are at the same height and the same horizontal location at the same time. Thus they collide!

35. (*a*) Choose the origin to be the location where the car leaves the ramp, and choose upward to be the positive y direction. At the end of its flight over the 8 cars, the car must be at $y = -1.5$ m. Also for the car, $\upsilon_{y0} = 0$, $a_y = -g$, $\upsilon_x = \upsilon_0$, and $\Delta x = 22$ m. The time of flight is found from the horizontal motion at constant velocity: $\Delta x = \upsilon_x t \rightarrow t = \Delta x/\upsilon_0$. That expression for the time is used in Eq. 2–11b for the vertical motion.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad y = 0 + 0 + \tfrac{1}{2}(-g)(\Delta x/\upsilon_0)^2 \quad \rightarrow$$

$$\upsilon_0 = \sqrt{\frac{-g(\Delta x)^2}{2(y)}} = \sqrt{\frac{-(9.80\ \text{m/s}^2)(22\ \text{m})^2}{2(-1.5\ \text{m})}} = 39.76\ \text{m/s} \approx \boxed{4.0\times 10^1\ \text{m/s}}$$

(*b*) Again, choose the origin to be the location where the car leaves the ramp, and choose upward to be the positive y direction. The y displacement of the car at the end of its flight over the 8 cars must again be $y = -1.5\text{ m}$. For the car, $\upsilon_{y0} = \upsilon_0 \sin\theta_0$, $a_y = -g$, $\upsilon_x = \upsilon_0 \cos\theta_0$, and $\Delta x = 22\text{ m}$. The launch angle is $\theta_0 = 7.0°$. The time of flight is found from the horizontal motion at constant velocity.

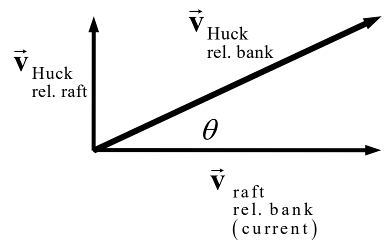
$$\Delta x = \upsilon_x t \quad \rightarrow \quad t = \frac{\Delta x}{\upsilon_0 \cos\theta_0}$$

That expression for the time is used in Eq. 2–11b for the vertical motion.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad y = \upsilon_0 \sin\theta_0 \frac{\Delta x}{\upsilon_0 \cos\theta_0} + \tfrac{1}{2}(-g)\left(\frac{\Delta x}{\upsilon_0 \cos\theta_0}\right)^2 \quad \rightarrow$$

$$\upsilon_0 = \sqrt{\frac{g(\Delta x)^2}{2(\Delta x \tan\theta_0 - y)\cos^2\theta_0}} = \sqrt{\frac{(9.80\text{ m/s}^2)(22\text{ m})^2}{2((22\text{ m})\tan 7.0° + 1.5\text{ m})\cos^2 7.0°}} = \boxed{24\text{ m/s}}$$

36. Call the direction of the flow of the river the x direction and the direction of Huck walking relative to the raft the y direction.



$$\vec{\mathbf{v}}_{\substack{\text{Huck}\\\text{rel. bank}}} = \vec{\mathbf{v}}_{\substack{\text{Huck}\\\text{rel. raft}}} + \vec{\mathbf{v}}_{\substack{\text{raft rel.}\\\text{bank}}} = (0, 0.70)\text{ m/s} + (1.50, 0)\text{ m/s}$$

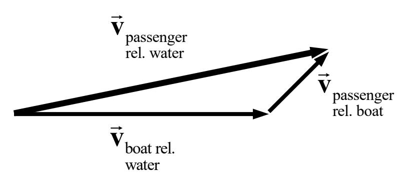
$$= (1.50, 0.70)\text{ m/s}$$

Magnitude: $\upsilon_{\substack{\text{Huck}\\\text{rel. bank}}} = \sqrt{1.50^2 + 0.70^2} = \boxed{1.66\text{ m/s}}$

Direction: $\theta = \tan^{-1}\dfrac{0.70}{1.50} = \boxed{25° \text{ relative to river}}$

37. If each plane has a speed of 780 km/h, then their relative speed of approach is 1560 km/h. If the planes are 10.0 km apart, then the time for evasive action is found as follows:

$$\Delta d = \upsilon t \quad \rightarrow \quad t = \frac{\Delta d}{\upsilon} = \left(\frac{10.0\text{ km}}{1560\text{ km/h}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = \boxed{23.1\text{ s}}$$



38. Call the direction of the boat relative to the water the x direction and upward the y direction. Also see the diagram.

$$\vec{\mathbf{v}}_{\substack{\text{passenger}\\\text{rel. water}}} = \vec{\mathbf{v}}_{\substack{\text{passenger}\\\text{rel. boat}}} + \vec{\mathbf{v}}_{\substack{\text{boat rel.}\\\text{water}}}$$

$$= (0.60\cos 45°, 0.60\sin 45°)\text{ m/s}$$

$$+(1.70, 0)\text{ m/s} = (2.124, 0.424)\text{ m/s}$$

$$\upsilon_{\substack{\text{passenger}\\\text{rel. water}}} = \sqrt{(2.124\text{ m/s})^2 + (0.424\text{ m/s})^2} = \boxed{2.17\text{ m/s}} \quad \theta_{\substack{\text{passenger}\\\text{rel. water}}} = \tan^{-1}\frac{0.424}{2.124} = \boxed{11°}$$

39. (a) Call the upward direction positive for the vertical motion. Then the velocity of the ball relative to a person on the ground is the vector sum of the horizontal and vertical motions. The horizontal velocity is $\upsilon_x = 10.0\text{ m/s}$ and the vertical velocity is $\upsilon_y = 3.0\text{ m/s}$.

$$\vec{\mathbf{v}} = (10.0\text{ m/s}, 3.0\text{ m/s}) \quad \rightarrow \quad \upsilon = \sqrt{(10.0\text{ m/s})^2 + (3.0\text{ m/s})^2} = \boxed{10.4\text{ m/s}}$$

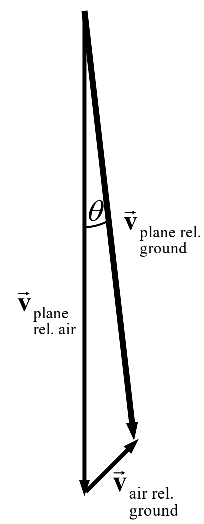
$$\theta = \tan^{-1}\frac{3.0\text{ m/s}}{10.0\text{ m/s}} = \boxed{17° \text{ above the horizontal}}$$

(*b*) The only change is the initial vertical velocity, so $\upsilon_y = -5.0\text{ m/s}$.

$$\vec{\mathbf{v}} = (10.0\text{ m/s},\ -3.0\text{ m/s}) \quad\rightarrow\quad \upsilon = \sqrt{(10.0\text{ m/s})^2 + (-3.0\text{ m/s})^2} = \boxed{10.4\text{ m/s}}$$

$$\theta = \tan^{-1}\frac{-3.0\text{ m/s}}{10.0\text{ m/s}} = \boxed{17^\circ\text{ below the horizontal}}$$

40. Call east the positive x direction and north the positive y direction. Then the following vector velocity relationship exists.



(*a*)
$$\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{plane}\\\text{rel. air}}} + \vec{\mathbf{v}}_{\substack{\text{air rel.}\\\text{ground}}}$$

$$= (0,\ -688)\text{ km/h} + (90.0\cos 45.0^\circ,\ 90.0\sin 45.0^\circ)\text{ km/h}$$

$$= (63.6,\ -624)\text{ km/h}$$

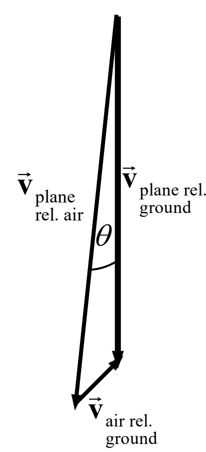
$$\upsilon_{\substack{\text{plane rel.}\\\text{ground}}} = \sqrt{(63.6\text{ km/h})^2 + (-624\text{ km/h})^2} = \boxed{628\text{ km/h}}$$

$$\theta = \tan^{-1}\frac{63.6}{624} = \boxed{5.82^\circ\text{ east of south}}$$

(*b*) The plane is away from its intended position by the distance the air has caused it to move. The wind speed is 90.0 km/h, so after 11.0 min (11/60 h), the plane is off course by this amount.

$$\Delta x = \upsilon_x t = (90.0\text{ km/h})\left(\tfrac{11}{60}\text{h}\right) = \boxed{16.5\text{ km}}$$

41. Call east the positive x direction and north the positive y direction. Then the following vector velocity relationship exists.



$$\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{plane}\\\text{rel. air}}} + \vec{\mathbf{v}}_{\substack{\text{air rel.}\\\text{ground}}} \quad\rightarrow$$

$$\left(0,\ -\upsilon_{\substack{\text{plane rel.}\\\text{ground}}}\right) = (-688\sin\theta,\ 688\cos\theta)\text{ km/h}$$

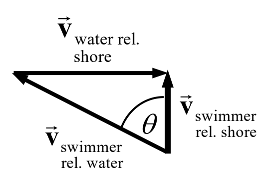
$$+ (90.0\cos 45.0^\circ,\ 90.0\sin 45.0^\circ)\text{ km/h}$$

Equate x components in the above equation.

$$0 = -688\sin\theta + 90.0\cos 45.0^\circ \quad\rightarrow$$

$$\theta = \sin^{-1}\frac{90.0\cos 45.0^\circ}{688} = \boxed{5.31^\circ\text{, west of south}}$$

$\boxed{42.}$ (*a*) Call the direction of the flow of the river the x direction and the direction straight across the river the y direction.



$$\sin\theta = \frac{\upsilon_{\substack{\text{water rel.}\\\text{shore}}}}{\upsilon_{\substack{\text{swimmer}\\\text{rel. water}}}} = \frac{0.50\text{ m/s}}{0.60\text{ m/s}} \quad\rightarrow\quad \theta = \sin^{-1}\frac{0.50}{0.60} = 56.44^\circ \approx \boxed{56^\circ}$$

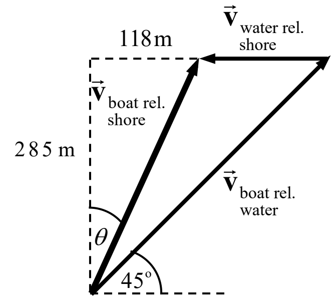
(*b*) From the diagram, her speed with respect to the shore is found as follows:

$$\upsilon_{\substack{\text{swimmer}\\\text{rel. shore}}} = \upsilon_{\substack{\text{swimmer}\\\text{rel. water}}}\cos\theta = (0.60\text{ m/s})\cos 56.44^\circ = 0.332\text{ m/s}$$

The time to cross the river can be found from the constant velocity relationship.

$$\Delta x = \upsilon t \quad \to \quad t = \frac{\Delta x}{\upsilon} = \frac{45\text{ m}}{0.332\text{ m/s}} = 135.5\text{ s} \approx \boxed{140\text{ s} = 2.3\text{ min}}$$

43. Call the direction of the flow of the river the x direction (to the left in the diagram) and the direction straight across the river the y direction (to the top in the diagram). From the diagram, $\theta = \tan^{-1} 118\text{ m}/285\text{ m} = 22.49°$. Equate the vertical components of the velocities to find the speed of the boat relative to the shore.



$$\upsilon_{\substack{\text{boat rel.}\\ \text{shore}}} \cos\theta = \upsilon_{\substack{\text{boat rel.}\\ \text{water}}} \sin 45° \quad \to$$

$$\upsilon_{\substack{\text{boat rel.}\\ \text{shore}}} = (2.50\text{ m/s})\frac{\sin 45°}{\cos 22.49°} = 1.913\text{ m/s}$$

Equate the horizontal components of the velocities to find the speed of the current.

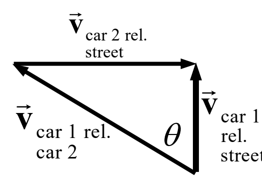
$$\upsilon_{\substack{\text{boat rel.}\\ \text{shore}}} \sin\theta = \upsilon_{\substack{\text{boat rel.}\\ \text{water}}} \cos 45° - \upsilon_{\substack{\text{water}\\ \text{rel. shore}}} \quad \to$$

$$\upsilon_{\substack{\text{water}\\ \text{rel. shore}}} = \upsilon_{\substack{\text{boat rel.}\\ \text{water}}} \cos 45° - \upsilon_{\substack{\text{boat rel.}\\ \text{shore}}} \sin\theta$$

$$= (2.50\text{ m/s}) \cos 45° - (1.913\text{ m/s}) \sin 22.49° = 1.036\text{ m/s} \approx \boxed{1.0\text{ m/s}}$$

44. The lifeguard will be carried downstream at the same rate as the child. Thus only the horizontal motion need be considered. To cover 45 m horizontally at a rate of 2 m/s takes $\frac{45\text{ m}}{2\text{ m/s}} = 22.5\text{ s} \approx \boxed{23\text{ s}}$ for the lifeguard to reach the child. During this time they would both be moving downstream at 1.0 m/s, so they would travel $(1.0\text{ m/s})(22.5\text{ s}) = 22.5\text{ m} \approx \boxed{23\text{ m}}$ downstream.

45. Call east the positive x direction and north the positive y direction. From the first diagram, this relative velocity relationship is seen.

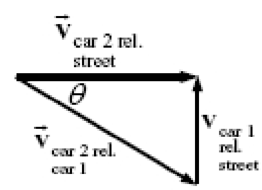


$$\vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\ \text{street}}} = \vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\ \text{car 2}}} + \vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\ \text{street}}} \quad \to \quad \upsilon_{\substack{\text{car 1 rel.}\\ \text{car 2}}} = \sqrt{(-55)^2 + (35)^2}$$

$$= \boxed{65\text{ km/h}}$$

$$\theta = \tan^{-1} 55/35 = \boxed{58° \text{ west of north}}$$

For the other relative velocity relationship:



$$\vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\ \text{street}}} = \vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\ \text{car 1}}} + \vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\ \text{street}}} \quad \to \quad \upsilon_{\substack{\text{car 2 rel.}\\ \text{car 1}}} = \sqrt{(55)^2 + (-35)^2}$$

$$= \boxed{65\text{ km/h}}$$

$$\theta = \tan^{-1} 35/55 = \boxed{32° \text{ south of east}}$$

Notice that the two relative velocities are opposites of each other: $\vec{\mathbf{v}}_{\substack{\text{car 2 rel.}\\ \text{car 1}}} = -\vec{\mathbf{v}}_{\substack{\text{car 1 rel.}\\ \text{car 2}}}$

46. (*a*) For the magnitudes to add linearly, the two vectors must be parallel. $\boxed{\vec{\mathbf{V}}_1 \parallel \vec{\mathbf{V}}_2}$

(*b*) For the magnitudes to add according to the Pythagorean theorem, the two vectors must be at right angles to each other. $\boxed{\vec{\mathbf{V}}_1 \perp \vec{\mathbf{V}}_2}$

(*c*) The magnitude of vector 2 must be 0. $\boxed{\vec{\mathbf{V}}_2 = 0}$

47. The deceleration is along a straight line. The starting velocity is $110\text{ km/h}\left(\dfrac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 30.6\text{ m/s},$ and the ending velocity is 0 m/s. The acceleration is found from Eq. 2–11a.

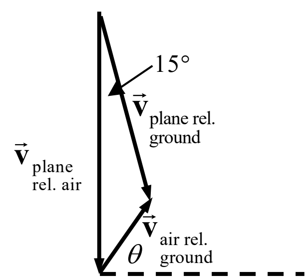
$$\upsilon = \upsilon_0 + at \quad \rightarrow \quad 0 = 30.6\text{ m/s} + a(7.0\text{ s}) \quad \rightarrow \quad a = -\frac{30.6\text{ m/s}}{7.0\text{ s}} = -4.37\text{ m/s}^2$$

The horizontal acceleration is $a_{\text{horiz}} = a\cos\theta = -4.37\text{ m/s}^2(\cos 26°) = \boxed{-3.9\text{ m/s}^2}.$

The vertical acceleration is $a_{\text{vert}} = a\sin\theta = -4.37\text{ m/s}^2(\sin 26°) = \boxed{-1.9\text{ m/s}^2}.$

The horizontal acceleration is to the left in Fig. 3–48, and the vertical acceleration is down.

48. Call east the *x* direction and north the *y* direction. Then this relative velocity relationship follows (see the accompanying diagram).



$$\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{plane}\\\text{rel. air}}} + \vec{\mathbf{v}}_{\substack{\text{air rel.}\\\text{ground}}}$$

Equate the *x* components of the velocity vectors. The magnitude of $\vec{\mathbf{v}}_{\substack{\text{plane rel.}\\\text{ground}}}$ is given as 135 km/h.

$$(135\text{ km/h})\sin 15.0° = 0 + \upsilon_{\text{wind } x} \quad \rightarrow \quad \upsilon_{\text{wind } x} = 34.94\text{ km/h}.$$

From the *y* components of the relative velocity equation, we find $\upsilon_{\text{wind } y}$.

$$-135\cos 15.0° = -185 + \upsilon_{\text{wind } y} \quad \rightarrow \quad \upsilon_{\text{wind } y} = 185 - 135\cos 15.0° = 54.60\text{ km/h}$$

The magnitude of the wind velocity is as follows:

$$\upsilon_{\text{wind}} = \sqrt{\upsilon_{\text{wind } x}^2 + \upsilon_{\text{wind } y}^2} = \sqrt{(34.94\text{ km/h})^2 + (54.60\text{ km/h})^2} = 64.82\text{ km/h} \approx \boxed{65\text{ km/h}}$$

The direction of the wind is $\theta = \tan^{-1}\dfrac{\upsilon_{\text{wind } y}}{\upsilon_{\text{wind } x}} = \tan^{-1}\dfrac{54.60}{34.94} = 57.38° \approx \boxed{57° \text{ north of east}}.$

$\boxed{49.}$ Choose upward to be the positive *y* direction. The origin is the point from which the pebbles are released. In the vertical direction, $a_y = -9.80\text{ m/s}^2,$ the velocity at the window is $\upsilon_y = 0,$ and the vertical displacement is 8.0 m. The initial *y* velocity is found from Eq. 2–11c.

$$\upsilon_y^2 = \upsilon_{y0}^2 + 2a_y(y - y_0) \quad \rightarrow$$

$$\upsilon_{y0} = \sqrt{\upsilon_y^2 - 2a_y(y - y_0)} = \sqrt{0 - 2(-9.80\text{ m/s}^2)(8.0\text{ m})} = 12.5\text{ m/s}$$

Find the time for the pebbles to travel to the window from Eq. 2–11a.

$$v_y = v_{y0} + at \quad \rightarrow \quad t = \frac{v_y - v_{y0}}{a} = \frac{0 - 12.5\ \text{m/s}}{-9.80\ \text{m/s}^2} = 1.28\ \text{s}$$

Find the horizontal speed from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \Delta x/t = 8.5\ \text{m}/1.28\ \text{s} = \boxed{6.6\ \text{m/s}}$$

This is the speed of the pebbles when they hit the window.

50. Assume that the golf ball takes off and lands at the same height, so that the level horizontal range formula derived in the text can be applied. The only variable is to be the acceleration due to gravity.

$$R_{\text{Earth}} = v_0^2 \sin 2\theta_0 / g_{\text{Earth}} \qquad R_{\text{Moon}} = v_0^2 \sin 2\theta_0 / g_{\text{Moon}}$$

$$\frac{R_{\text{Earth}}}{R_{\text{Moon}}} = \frac{v_0^2 \sin 2\theta_0 / g_{\text{Earth}}}{v_0^2 \sin 2\theta_0 / g_{\text{Moon}}} = \frac{1/g_{\text{Earth}}}{1/g_{\text{Moon}}} = \frac{g_{\text{Moon}}}{g_{\text{Earth}}} = \frac{32\ \text{m}}{180\ \text{m}} = 0.1778 \quad \rightarrow$$

$$g_{\text{Moon}} = 0.1778\, g_{\text{Earth}} = 0.1778(9.80\ \text{m/s}^2) = 1.742\ \text{m/s}^2 \approx \boxed{1.7\ \text{m/s}^2}$$

51. (*a*) Use the level horizontal range formula from the text to find her takeoff speed.

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \quad \rightarrow \quad v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.80\ \text{m/s}^2)(8.0\ \text{m})}{\sin 90^\circ}} = 8.854\ \text{m/s} \approx \boxed{8.9\ \text{m/s}}$$

(*b*) Let the launch point be at the $y = 0$ level, and choose upward to be positive. Use Eq. 2–11b to solve for the time to fall to 2.5 m below the starting height, and then calculate the horizontal distance traveled.

$$y = y_0 + v_{0y} t + \tfrac{1}{2} a_y t^2 \quad \rightarrow \quad -2.5\ \text{m} = (8.854\ \text{m/s}) \sin 45^\circ t + \tfrac{1}{2}(-9.80\ \text{m/s}^2) t^2$$

$$4.9t^2 - 6.261t - 2.5\ \text{m} = 0 \quad \rightarrow$$

$$t = \frac{6.261 \pm \sqrt{(6.261)^2 - 4(4.9)(-2.5)}}{2(4.9)} = \frac{6.261 \pm 9.391}{2(4.9)} = -0.319\ \text{s},\ 1.597\ \text{s}$$

Use the positive time to find the horizontal displacement during the jump.

$$\Delta x = v_{0x} t = v_0 \cos 45^\circ t = (8.854\ \text{m/s}) \cos 45^\circ (1.597\ \text{s}) = 10.0\ \text{m}$$

$\boxed{\text{She will land exactly on the opposite bank, neither long nor short.}}$

52. Choose the origin to be at ground level, under the place where the projectile is launched, and upward to be the positive *y* direction. For the projectile, $v_0 = 65.0\ \text{m/s}$, $\theta_0 = 35.0^\circ$, $a_y = -g$, $y_0 = 115\ \text{m}$, and $v_{y0} = v_0 \sin \theta_0$.

(*a*) The time taken to reach the ground is found from Eq. 2–11b, with a final height of 0.

$$y = y_0 + v_{y0} t + \tfrac{1}{2} a_y t^2 \quad \rightarrow \quad 0 = y_0 + v_0 \sin\theta_0 t - \tfrac{1}{2} g t^2 \quad \rightarrow$$

$$t = \frac{-v_0 \sin \theta_0 \pm \sqrt{v_0^2 \sin^2 \theta_0 - 4\left(-\tfrac{1}{2} g\right) y_0}}{2\left(-\tfrac{1}{2} g\right)} = 9.964\ \text{s}, -2.3655\ \text{s} = \boxed{9.96\ \text{s}}$$

Choose the positive time since the projectile was launched at time $t = 0$.

(*b*) The horizontal range is found from the horizontal motion at constant velocity.

$$\Delta x = \upsilon_x t = (\upsilon_0 \cos\theta_0)t = (65.0\text{ m/s})(\cos 35.0^\circ)(9.964\text{ s}) = \boxed{531\text{ m}}$$

(*c*) At the instant just before the particle reaches the ground, the horizontal component of its velocity is the constant $\upsilon_x = \upsilon_0 \cos\theta_0 = (65.0\text{ m/s})\cos 35.0^\circ = \boxed{53.2\text{ m/s}}$. The vertical component is found from Eq. 2–11a.

$$\upsilon_y = \upsilon_{y0} + at = \upsilon_0 \sin\theta_0 - gt = (65.0\text{ m/s})\sin 35.0^\circ - (9.80\text{ m/s}^2)(9.964\text{ s})$$
$$= \boxed{-60.4\text{ m/s}}$$

(*d*) The magnitude of the velocity is found from the *x* and *y* components calculated in part (*c*) above.

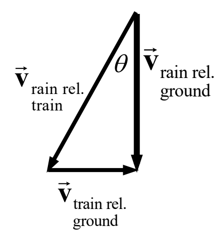
$$\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2} = \sqrt{(53.2\text{ m/s})^2 + (-60.4\text{ m/s})^2} = \boxed{80.5\text{ m/s}}$$

(*e*) The direction of the velocity is $\theta = \tan^{-1}\dfrac{\upsilon_y}{\upsilon_x} = \tan^{-1}\dfrac{-60.4}{53.2} = -48.6^\circ$, so the object is moving $\boxed{48.6^\circ\text{ below the horizontal}}$.

(*f*) The maximum height above the cliff top reached by the projectile will occur when the *y* velocity is 0 and is found from Eq. 2–11c.

$$\upsilon_y^2 = \upsilon_{y0}^2 + 2a_y(y - y_0) \quad\to\quad 0 = \upsilon_0^2 \sin^2\theta_0 - 2gy_{\max}$$
$$y_{\max} = \frac{\upsilon_0^2 \sin^2\theta_0}{2g} = \frac{(65.0\text{ m/s})^2 \sin^2 35.0^\circ}{2(9.80\text{ m/s}^2)} = \boxed{70.9\text{ m}}$$

53. Choose the *x* direction to be the direction of train travel (the direction the passenger is facing), and choose the *y* direction to be up. This relationship exists among the velocities: $\vec{\mathbf{v}}_{\substack{\text{rain rel.}\\ \text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{rain rel.}\\ \text{train}}} + \vec{\mathbf{v}}_{\substack{\text{train rel.}\\ \text{ground}}}$. From the diagram, find the expression for the speed of the raindrops.



$$\tan\theta = \frac{\upsilon_{\substack{\text{train rel.}\\ \text{ground}}}}{\upsilon_{\substack{\text{rain rel.}\\ \text{ground}}}} = \frac{\upsilon_T}{\upsilon_{\substack{\text{rain rel.}\\ \text{ground}}}} \quad\to\quad \boxed{\upsilon_{\substack{\text{rain rel.}\\ \text{ground}}} = \frac{\upsilon_T}{\tan\theta}}$$

54. (*a*) Choose downward to be the positive *y* direction. The origin is the point where the bullet leaves the gun. In the vertical direction, $\upsilon_{y0} = 0$, $y_0 = 0$, and $a_y = 9.80\text{ m/s}^2$. In the horizontal direction, $\Delta x = 38.0\text{ m}$ and $\upsilon_x = 23.1\text{ m/s}$. The time of flight is found from the horizontal motion at constant velocity.

$$\Delta x = \upsilon_x t \quad\to\quad t = \Delta x/\upsilon_x = 38.0\text{ m}/23.1\text{ m/s} = 1.645\text{ s}$$

This time can now be used in Eq. 2–11b to find the vertical drop of the bullet.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \quad\to\quad y = 0 + 0 + \tfrac{1}{2}(9.80\text{ m/s}^2)(1.645\text{ s})^2 = \boxed{13.3\text{ m}}$$

(*b*) For the bullet to hit the target at the same level, the level horizontal range formula derived in the text applies. The range is 38.0 m, and the initial velocity is 23.1 m/s. Solving for the angle of launch results in the following:

$$R = \frac{\upsilon_0^2 \sin 2\theta_0}{g} \quad\to\quad \sin 2\theta_0 = \frac{Rg}{\upsilon_0^2} \quad\to\quad \theta_0 = \tfrac{1}{2}\sin^{-1}\frac{(38.0\text{ m})(9.80\text{ m/s}^2)}{(23.1\text{ m/s})^2} = \boxed{22.1^\circ}$$

Because of the symmetry of the range formula, there is also an answer of the complement of the above answer, which would be $67.9°$. That is an unreasonable answer from a practical physical viewpoint—it is pointing the bow nearly straight up.

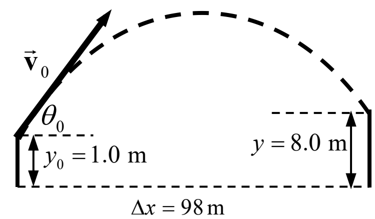
55. Choose downward to be the positive y direction. The origin is at the point from which the divers push off the cliff. In the vertical direction, the initial velocity is $\upsilon_{y0} = 0$, the acceleration is $a_y = 9.80\ \text{m/s}^2$, and the displacement is 35 m. The time of flight is found from Eq. 2–11b.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \rightarrow 35\ \text{m} = 0 + 0 + \tfrac{1}{2}(9.80\ \text{m/s}^2)t^2 \rightarrow t = \sqrt{\frac{2(35\ \text{m})}{9.8\ \text{m/s}^2}} = \boxed{2.7\ \text{s}}$$

The horizontal speed (which is the initial speed) is found from the horizontal motion at constant velocity.

$$\Delta x = \upsilon_x t \rightarrow \upsilon_x = \Delta x/t = 5.0\ \text{m}/2.7\ \text{s} = \boxed{1.9\ \text{m/s}}$$

56. The minimum speed will be that for which the ball just clears the fence; that is, the ball has a height of 8.0 m when it is 98 m horizontally from home plate. The origin is at home plate, with upward as the positive y direction. For the ball, $y_0 = 1.0\ \text{m}$, $y = 8.0\ \text{m}$, $a_y = -g$, $\upsilon_{y0} = \upsilon_0 \sin\theta_0$, $\upsilon_x = \upsilon_0 \cos\theta_0$, and $\theta_0 = 36°$. See the diagram (not to scale). For the constant-velocity horizontal motion, $\Delta x = \upsilon_x t = \upsilon_0 \cos\theta_0 t$, so $t = \dfrac{\Delta x}{\upsilon_0 \cos\theta_0}$.



For the vertical motion, apply Eq. 2–11b.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 = y_0 + \upsilon_0(\sin\theta_0)t - \tfrac{1}{2}gt^2$$

Substitute the value of the time of flight for the first occurrence only in the above equation, and then solve for the time.

$$y = y_0 + \upsilon_0 t \sin\theta_0 - \tfrac{1}{2}gt^2 \rightarrow y = y_0 + \upsilon_0 \sin\theta_0 \frac{\Delta x}{\upsilon_0 \cos\theta_0} - \tfrac{1}{2}gt^2 \rightarrow$$

$$t = \sqrt{2\left(\frac{y_0 - y + \Delta x \tan\theta_0}{g}\right)} = \sqrt{2\left(\frac{1.0\ \text{m} - 8.0\ \text{m} + (98\ \text{m})\tan 36°}{9.80\ \text{m/s}^2}\right)} = 3.620\ \text{s}$$

Finally, use the time with the horizontal range to find the initial speed.

$$\Delta x = \upsilon_0 \cos\theta_0 t \rightarrow \upsilon_0 = \frac{\Delta x}{t\cos\theta_0} = \frac{98\ \text{m}}{(3.620\ \text{s})\cos 36°} = \boxed{33\ \text{m/s}}$$

57. Choose the origin to be the location on the ground directly underneath the ball when served, and choose upward as the positive y direction. Then for the ball, $y_0 = 2.50\ \text{m}$, $\upsilon_{y0} = 0$, $a_y = -g$, and the y location when the ball just clears the net is $y = 0.90\ \text{m}$. The time for the ball to reach the net is calculated from Eq. 2–11b.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0.90\text{ m} = 2.50\text{ m} + 0 + \tfrac{1}{2}(-9.80\text{ m/s}^2)t^2 \quad \rightarrow$$

$$t_{\substack{\text{to}\\\text{net}}} = \sqrt{\frac{2(-1.60\text{ m})}{-9.80\text{ m/s}^2}} = 0.57143\text{ s}$$

The *x* velocity is found from the horizontal motion at constant velocity.

$$\Delta x = \upsilon_x t \quad \rightarrow \quad \upsilon_x = \frac{\Delta x}{t} = \frac{15.0\text{ m}}{0.57143\text{ s}} = 26.25 \approx \boxed{26.3\text{ m/s}}$$

This is the minimum speed required to clear the net.

To find the full time of flight of the ball, set the final *y* location to be $y = 0$, and again use Eq. 2–11b.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = 2.50\text{ m} + \tfrac{1}{2}(-9.80\text{ m/s}^2)t^2 \quad \rightarrow$$

$$t_{\text{total}} = \sqrt{\frac{2(-2.50\text{ m})}{-9.80\text{ m/s}^2}} = 0.7143 \approx \boxed{0.714\text{ s}}$$

The horizontal position where the ball lands is found from the horizontal motion at constant velocity.

$$\Delta x = \upsilon_x t = (26.25\text{ m/s})(0.7143\text{ s}) = 18.75 \approx \boxed{18.8\text{ m}}$$

Since this is between 15.0 and 22.0 m, $\boxed{\text{the ball lands in the "good" region}}$.

58. Work in the frame of reference in which the car is at rest at ground level. In this reference frame, the helicopter is moving horizontally with a speed of $208\text{ km/h} - 156\text{ km/h} = 52\text{ km/h}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) =$ 14.44 m/s. For the vertical motion, choose the level of the helicopter to be the origin and downward to be positive. Then the package's *y* displacement is $y = 78.0\text{ m}$, $\upsilon_{y0} = 0$, and $a_y = g$. The time for the package to fall is calculated from Eq. 2–11b.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 78.0\text{ m} = \tfrac{1}{2}(9.80\text{ m/s}^2)t^2 \quad \rightarrow \quad t = \sqrt{\frac{2(78.0\text{ m})}{9.80\text{ m/s}^2}} = 3.99\text{ s}$$

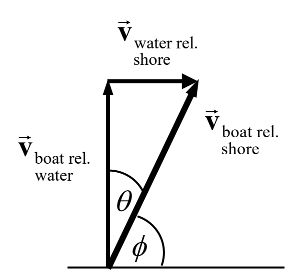
The horizontal distance that the package must move, relative to the "stationary" car, is found from the horizontal motion at constant velocity.

$$\Delta x = \upsilon_x t = (14.44\text{ m/s})(3.99\text{ s}) = 57.6\text{ m}$$

Thus the angle under the horizontal for the package release will be as follows:

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{78.0\text{ m}}{57.6\text{ m}}\right) = 53.6° \approx \boxed{54°}$$

59. Call the direction of the flow of the river the *x* direction, and the direction the boat is headed (which is different from the direction it is moving) the *y* direction.



(*a*) $$\upsilon_{\substack{\text{boat rel.}\\ \text{shore}}} = \sqrt{\upsilon^2_{\substack{\text{water rel.}\\ \text{shore}}} + \upsilon^2_{\substack{\text{boat rel.}\\ \text{water}}}} = \sqrt{(1.20\text{ m/s})^2 + (2.20\text{ m/s})^2}$$

$$= 2.506\text{ m/s} \approx \boxed{2.51\text{ m/s}}$$

$$\theta = \tan^{-1}\frac{1.20}{2.20} = 28.6°,\ \phi = 90° - \theta = \boxed{61.4° \text{ relative to shore}}$$

(*b*) The position of the boat after 3.00 seconds is given by the following:

$$\Delta d = \upsilon_{\substack{\text{boat rel.}\\ \text{shore}}} t = [(1.20\text{ m/s}, 2.20\text{ m/s})](3.00\text{ s})$$

$$= \boxed{3.60\text{ m downstream, } 6.60\text{ m across the river}}$$

As a magnitude and direction, it would be 7.52 m away from the starting point, at an angle of 61.4° relative to the shore.

60. Choose the origin to be the point from which the projectile is launched, and choose upward as the positive *y* direction. The *y* displacement of the projectile is 135 m, and the horizontal range of the projectile is 195 m. The acceleration in the *y* direction is $a_y = -g$, and the time of flight is 6.6 s.

The horizontal velocity is found from the horizontal motion at constant velocity.

$$\Delta x = \upsilon_x t \quad \rightarrow \quad \upsilon_x = \frac{\Delta x}{t} = \frac{195\text{ m}}{6.6\text{ s}} = 29.55\text{ m/s}$$

Calculate the initial *y* velocity from the given data and Eq. 2–11b.

$$y = y_0 + \upsilon_{y0}t + \tfrac{1}{2}a_y t^2 \rightarrow\ 135\text{ m} = \upsilon_{y0}(6.6\text{ s}) + \tfrac{1}{2}(-9.80\text{ m/s}^2)(6.6\text{ s})^2 \rightarrow \upsilon_{y0} = 52.79\text{ m/s}$$

Thus the initial velocity and direction of the projectile are as follows:

$$\upsilon_0 = \sqrt{\upsilon_x^2 + \upsilon_{y0}^2} = \sqrt{(29.55\text{ m/s})^2 + (52.79\text{ m/s})^2} = 60.4978\text{ m/s} \approx \boxed{6.0\times 10^1\text{ m/s}}$$

$$\theta = \tan^{-1}\frac{\upsilon_{y0}}{\upsilon_x} = \tan^{-1}\frac{52.79\text{ m/s}}{29.55\text{ m/s}} = \boxed{61°}$$

61. Find the time of flight from the vertical data, using Eq. 2–11b. Call the floor the *y* = 0 location, and choose upward as positive.

$$y = y_0 + \upsilon_{0y}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 3.05\text{ m} = 2.40\text{ m} + (12\text{ m/s})\sin 35° t + \tfrac{1}{2}(-9.80\text{ m/s}^2)t^2$$

$$4.90t^2 - 6.883t + 0.65\text{ m} = 0 \quad \rightarrow$$

$$t = \frac{6.883 \pm \sqrt{6.883^2 - 4(4.90)(0.65)}}{2(4.90)} = 1.303\text{ s},\ 0.102\text{ s}$$

(*a*) Use the longer time for the time of flight. The shorter time is the time for the ball to rise to the basket height on the way up, while the longer time is the time for the ball to be at the basket height on the way down.

$$x = \upsilon_x t = \upsilon_0(\cos 35°)t = (12\text{ m/s})(\cos 35°)(1.303\text{ s}) = 12.81\text{ m} \approx \boxed{13\text{ m}}$$

(*b*) The angle to the horizontal is determined by the components of the velocity.

$$\upsilon_x = \upsilon_0 \cos\theta_0 = 12\cos 35° = 9.830\text{ m/s}$$

$$\upsilon_y = \upsilon_{y0} + at = \upsilon_0 \sin\theta_0 - gt = 12\sin 35° - 9.80(1.303) = -5.886\text{ m/s}$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{-5.886}{9.830} = -30.9° \approx \boxed{-31°}$$

The negative angle means it is below the horizontal.

62. Let the launch point be the origin of coordinates, with right and upward as the positive directions. The equation of the line representing the ground is $y_{\text{gnd}} = -x$. The equations representing the motion of the rock are $x_{\text{rock}} = v_0 t$ and $y_{\text{rock}} = -\frac{1}{2}gt^2$, which can be combined into $y_{\text{rock}} = -\frac{1}{2}\frac{g}{v_0^2}x_{\text{rock}}^2$.

Find the intersection (the landing point of the rock) by equating the two expressions for y, and thereby find where the rock meets the ground.

$$y_{\text{rock}} = y_{\text{gnd}} \rightarrow -\frac{1}{2}\frac{g}{v_0^2}x^2 = -x \rightarrow x = \frac{2v_0^2}{g} \rightarrow t = \frac{x}{v_0} = \frac{2v_0}{g} = \frac{2(15\text{ m/s})}{9.80\text{ m/s}^2} = \boxed{3.1\text{ s}}$$

63. Choose the origin to be the point at the top of the building from which the ball is shot, and call upward the positive y direction. The initial velocity is $v_0 = 18\text{ m/s}$ at an angle of $\theta_0 = 42°$. The acceleration due to gravity is $a_y = -g$.

(*a*) $v_x = v_0 \cos\theta_0 = (18\text{ m/s})\cos 42° = 13.38\text{ m/s} \approx \boxed{13\text{ m/s}}$

$v_{y0} = v_0 \sin\theta_0 = (18\text{ m/s})\sin 42° = 12.04\text{ m/s} \approx \boxed{12\text{ m/s}}$

(*b*) Since the horizontal velocity is known and the horizontal distance is known, the time of flight can be found from the constant velocity equation for horizontal motion.

$$\Delta x = v_x t \rightarrow t = \frac{\Delta x}{v_x} = \frac{55\text{ m}}{13.38\text{ m/s}} = 4.111\text{ s}$$

With that time of flight, calculate the vertical position of the ball using Eq. 2–11b.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = (12.04\text{ m/s})(4.111\text{ s}) + \tfrac{1}{2}(-9.80\text{ m/s}^2)(4.111\text{ s})^2$$
$$= -33.3\text{ m} = \boxed{-33\text{ m}}$$

So the ball will strike 33 m below the top of the building.

64. First we find the time of flight for the ball. From that time we can calculate the vertical speed of the ball. From that vertical speed we can calculate the total speed of the ball and the % change in the speed. We choose the downward direction to be positive for vertical motion.

$$v_x = v_{0x} = (150\text{ km/h})\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 41.67\text{ m/s}$$

$$\Delta x = v_x t \rightarrow t = \frac{\Delta x}{v_x} = \frac{18\text{ m}}{41.67\text{ m/s}} = 0.432\text{ s}$$

$$v_y = v_{0y} + at = (9.80\text{ m/s}^2)(0.432\text{ s}) = 4.234\text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(41.67\text{ m/s})^2 + (4.234\text{ m/s})^2} = 41.88\text{ m/s}$$

$$\%\text{ change} = \frac{v - v_0}{v_0}\times 100 = \frac{41.88 - 41.67}{41.67}\times 100 = \boxed{0.50\%}$$

Solutions to Search and Learn Problems

1. Consider the downward vertical component of the motion, which will occur in half the total time. Take the starting position to be $y = 0$ and the positive direction to be downward. Use Eq. 2–11b with an initial vertical velocity of 0.

$$y = y_0 + \upsilon_{0y}t + \tfrac{1}{2}a_y t^2 \;\rightarrow\; h = 0 + 0 + \tfrac{1}{2}gt_{\text{down}}^2 = \tfrac{1}{2}g\left(\frac{t}{2}\right)^2 = \frac{9.80}{8}t^2 = 1.225t^2 \approx \boxed{1.2t^2}$$

As can be seen from the equation, by starting the analysis from the "top" point of the motion, the initial vertical speed is 0. This eliminates the need to know the original launch speed or direction in that calculation. We then also realize that the time for the object to rise is the same as the time for it to fall, so we have to analyze only the downward motion.

2. The ranges can be written in terms of the angles and initial velocities using the level horizontal range equation derived in the text. Then setting the two ranges equal, we can obtain the ratio of the initial velocities.

$$R = \frac{\upsilon_0^2 \sin 2\theta}{g} \;\rightarrow\; \frac{\upsilon_A^2 \sin(2\times30^\circ)}{g} = \frac{\upsilon_B^2 \sin(2\times60^\circ)}{g} \;\rightarrow\; \frac{\upsilon_A}{\upsilon_B} = \sqrt{\frac{\sin 120^\circ}{\sin 60^\circ}} = \boxed{\frac{\upsilon_A}{\upsilon_B} = 1}$$

The two projectiles each have the same initial velocity.

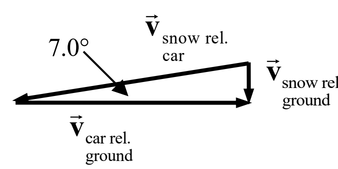
The time of flight is found using Eq. 2–11b with both the initial and final heights equal to 0 and the initial vertical velocities written in terms of the launching angle. We choose the positive vertical direction to be upward.

$$y = y_0 + \upsilon_{y0}t - \tfrac{1}{2}gt^2 \;\rightarrow\; 0 = 0 + \upsilon_{y0}t - \tfrac{1}{2}gt^2 = t\left(\upsilon_{0y} - \tfrac{1}{2}gt\right) \;\rightarrow\; t = \frac{2\upsilon_{0y}}{g} = \frac{2\upsilon_0 \sin\theta}{g}$$

$$\frac{t_B}{t_A} = \frac{2\upsilon_0 \sin 60^\circ/g}{2\upsilon_0 \sin 30^\circ/g} = \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}/2}{1/2} = \boxed{\sqrt{3}}$$

The projectile launched at 60° is in the air 1.73 times as long as the projectile launched at 30°.

3. We have $\upsilon_{\substack{\text{car rel.}\\ \text{ground}}} = 12\text{ m/s}$. Use the diagram, showing the snow falling straight down relative to the ground and the car moving parallel to the ground, and illustrating $\vec{\mathbf{v}}_{\substack{\text{snow rel.}\\ \text{ground}}} = \vec{\mathbf{v}}_{\substack{\text{snow rel.}\\ \text{car}}} + \vec{\mathbf{v}}_{\substack{\text{car rel.}\\ \text{ground}}}$, to calculate the other speeds.



$$\tan 7.0^\circ = \frac{\upsilon_{\substack{\text{snow rel.}\\ \text{ground}}}}{\upsilon_{\substack{\text{car rel.}\\ \text{ground}}}} \;\rightarrow\; \upsilon_{\substack{\text{snow rel.}\\ \text{ground}}} = (12\text{ m/s})\tan 7.0^\circ = 1.473\text{ m/s} \approx \boxed{1.5\text{ m/s}}$$