

## DESCRIBING MOTION: KINEMATICS IN ONE DIMENSION

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### Responses to Questions

1. A car speedometer measures only speed. It does not give any information about the direction, so it does not measure velocity.
2. If the velocity of an object is constant, then the speed and the direction of travel must also be constant. If that is the case, then the average velocity is the same as the instantaneous velocity, because nothing about its velocity is changing. The ratio of displacement to elapsed time will not be changing, no matter the actual displacement or time interval used for the measurement.
3. There is no general relationship between the magnitude of speed and the magnitude of acceleration. For example, one object may have a large but constant speed. The acceleration of that object is then zero. Another object may have a small speed but be gaining speed and therefore have a positive acceleration. So in this case the object with the greater speed has the lesser acceleration.

Consider two objects that are dropped from rest at different times. If we ignore air resistance, then the object dropped first will always have a greater speed than the object dropped second, but both will have the same acceleration of  $9.80 \text{ m/s}^2$ .

4. The accelerations of the motorcycle and the bicycle are the same, assuming that both objects travel in a straight line. Acceleration is the change in velocity divided by the change in time. The magnitude of the change in velocity in each case is the same,  $10 \text{ km/h}$ , so over the same time interval the accelerations will be equal.
5. Yes. For example, a car that is traveling northward and slowing down has a northward velocity and a southward acceleration.
6. The velocity of an object can be negative when its acceleration is positive. If we define the positive direction to be to the right, then an object traveling to the left that is having a reduction in speed will have a negative velocity with a positive acceleration.

If again we define the positive direction to be to the right, then an object traveling to the right that is having a reduction in speed will have a positive velocity and a negative acceleration.

7. If north is defined as the positive direction, then an object traveling to the south and increasing in speed has both a negative velocity and a negative acceleration. Or if up is defined as the positive direction, then an object falling due to gravity has both a negative velocity and a negative acceleration.
8. Yes. Remember that acceleration is a *change in velocity* per unit time, or a *rate of change* in velocity. So velocity can be increasing while the rate of increase goes down. For example, suppose a car is traveling at 40 km/h and one second later is going 50 km/h. One second after that, the car's speed is 55 km/h. The car's speed was increasing the entire time, but its acceleration in the second time interval was lower than in the first time interval. Thus its acceleration was decreasing even as the speed was increasing.

Another example would be an object falling WITH air resistance. Let the downward direction be positive. As the object falls, it gains speed, and the air resistance increases. As the air resistance increases, the acceleration of the falling object decreases, and it gains speed less quickly the longer it falls.

9. If the two cars emerge side by side, then the one moving faster is passing the other one. Thus car A is passing car B. With the acceleration data given for the problem, the ensuing motion would be that car A would pull away from car B for a time, but eventually car B would catch up to and pass car A.
10. If there were no air resistance, the ball's only acceleration during flight would be the acceleration due to gravity, so the ball would land in the catcher's mitt with the same speed it had when it left the bat, 120 km/h. Since the acceleration is the same through the entire flight, the time for the ball's speed to change from 120 km/h to 0 on the way up is the same as the time for its speed to change from 0 to 120 km/h on the way down. In both cases the ball has the same magnitude of displacement.

11. (a) If air resistance is negligible, the acceleration of a freely falling object stays the same as the object falls toward the ground. That acceleration is  $9.80 \text{ m/s}^2$ . Note that the object's speed increases, but since that speed increases at a constant rate, the acceleration is constant.
- (b) In the presence of air resistance, the acceleration decreases. Air resistance increases as speed increases. If the object falls far enough, the acceleration will go to zero and the velocity will become constant. That velocity is often called the terminal velocity.

12. Average speed is the displacement divided by the time. Since the distances from A to B and from B to C are equal, you spend more time traveling at 70 km/h than at 90 km/h, so your average speed should be less than 80 km/h. If the distance from A to B (or B to C) is  $x$  km, then the total distance traveled is  $2x$ . The total time required to travel this distance is  $x/70$  plus  $x/90$ . Then

$$\bar{v} = \frac{d}{t} = \frac{2x}{x/70 + x/90} = \frac{2(90)(70)}{90 + 70} = 78.75 \text{ km/h} \approx 79 \text{ km/h}.$$

13. Yes. For example, a rock thrown straight up in the air has a constant, nonzero acceleration due to gravity for its entire flight. However, at the highest point it momentarily has zero velocity. A car, at the moment it starts moving from rest, has zero velocity and nonzero acceleration.
14. Yes. Any time the velocity is constant, the acceleration is zero. For example, a car traveling at a constant 90 km/h in a straight line has nonzero velocity and zero acceleration.
15. A rock falling from a cliff has a constant acceleration IF we neglect air resistance. An elevator moving from the second floor to the fifth floor making stops along the way does NOT have a constant acceleration. Its acceleration will change in magnitude and direction as the elevator starts and stops. The dish resting on a table has a constant (zero) acceleration.

16. The slope of the position versus time curve is the object's velocity. The object starts at the origin with a constant velocity (and therefore zero acceleration), which it maintains for about 20 s. For the next 10 s, the positive curvature of the graph indicates the object has a positive acceleration; its speed is increasing. From 30 s to 45 s, the graph has a negative curvature; the object uniformly slows to a stop, changes direction, and then moves backwards with increasing speed. During this time interval, the acceleration is negative, since the object is slowing down while traveling in the positive direction and then speeding up while traveling in the negative direction. For the final 5 s shown, the object continues moving in the negative direction but slows down, which gives it a positive acceleration. During the 50 s shown, the object travels from the origin to a point 20 m away, and then back 10 m to end up 10 m from the starting position.
17. Initially, the object moves in the positive direction with a constant acceleration, until about  $t = 45$  s, when it has a velocity of about 37 m/s in the positive direction. The acceleration then decreases, reaching an instantaneous acceleration of 0 at about  $t = 50$  s, when the object has its maximum speed of about 38 m/s. The object then begins to slow down but continues to move in the positive direction. The object stops moving at  $t = 90$  s and stays at rest until about  $t = 108$  s. Then the object begins to move in the positive direction again, at first with a larger acceleration, and then with a lesser acceleration. At the end of the recorded motion, the object is still moving to the right and gaining speed.

### Responses to MisConceptual Questions

1. (*a, b, c, d, e, f, g*) All of these actions should be a part of solving physics problems.
2. (*d*) It is a common misconception that a positive acceleration always increases the speed, as in (*b*) and (*c*). However, when the velocity and acceleration are in opposite directions, the speed will decrease.
3. (*d*) Since the velocity and acceleration are in opposite directions, the object will slow to a stop. However, since the acceleration remains constant, it will stop only momentarily before moving toward the left.
4. (*c*) Students commonly confuse the concepts of velocity and acceleration in free-fall motion. At the highest point in the trajectory, the velocity is changing from positive (upward) to negative (downward) and therefore passes through zero. This changing velocity is due to a constant downward acceleration.
5. (*a*) Since the distance between the rocks increases with time, a common misconception is that the velocities are increasing at different rates. However, both rocks fall under the influence of gravity, so their velocities increase at the same rate.
6. (*c*) Since the distances are the same, a common error is to assume that the average speed will be halfway between the two speeds, or 40 km/h. However, it takes the car much longer to travel the 4 km at 30 km/h than at 50 km/h. Since more time is spent at 30 km/h, the average speed will be closer to 30 km/h than to 50 km/h.
7. (*c*) A common misconception is that the acceleration of an object in free fall depends upon the motion of the object. If there is no air resistance, the accelerations for the two balls have the same magnitude and direction throughout both of their flights.
8. (*b, c*) Each of the given equations is based on Eqs. 2-11a–d. Answer (*a*) has the acceleration replaced properly with  $-g$ , but the initial velocity is downward and as such should be negative. Answer (*d*) is

incorrect because the initial velocity has been inserted for the average velocity. Answers (b) and (c) have the correct signs for each variable and the known values are inserted properly.

9. (a) Increasing speed means that the slope must be getting steeper over time. In graphs (b) and (e), the slope remains constant, so these are cars moving at constant speed. In graph (c), as time increases  $x$  decreases. However, the rate at which it decreases is also decreasing. This is a car slowing down. In graph (d), the car is moving away from the origin, but again it is slowing down. The only graph in which the slope is increasing with time is graph (a).

## Solutions to Problems

1. The distance of travel (displacement) can be found by rearranging Eq. 2-2 for the average velocity. Also note that the units of the velocity and the time are not the same, so the speed units will be converted.

$$\bar{v} = \Delta x / \Delta t \rightarrow \Delta x = \bar{v} \Delta t = (95 \text{ km/h}) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) (2.0 \text{ s}) = 0.053 \text{ km} = \boxed{53 \text{ m}}$$

2. The average speed is given by Eq. 2-1, using  $d$  to represent distance traveled.

$$\bar{v} = d / \Delta t = 235 \text{ km} / 2.75 \text{ h} = \boxed{85.5 \text{ km/h}}$$

3. The average velocity is given by Eq. 2-2.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{8.5 \text{ cm} - 4.8 \text{ cm}}{4.5 \text{ s} - (-2.0 \text{ s})} = \frac{3.7 \text{ cm}}{6.5 \text{ s}} = \boxed{0.57 \text{ cm/s}}$$

The average speed cannot be calculated. To calculate the average speed, we would need to know the actual distance traveled, and it is not given. We only have the displacement.

4. The average velocity is given by Eq. 2-2.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-4.2 \text{ cm} - 8.4 \text{ cm}}{6.1 \text{ s} - 3.0 \text{ s}} = \frac{-12.6 \text{ cm}}{3.1 \text{ s}} = \boxed{-4.1 \text{ cm/s}}$$

The negative sign indicates the direction.

5. The time of travel can be found by rearranging the average velocity equation.

$$\bar{v} = \Delta x / \Delta t \rightarrow \Delta t = \Delta x / \bar{v} = (3.5 \text{ km}) / (25 \text{ km/h}) = \boxed{0.14 \text{ h}} = 8.4 \text{ min}$$

6. (a) The speed of sound is intimated in the problem as 1 mile per 5 seconds. The speed is calculated as follows:

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \left( \frac{1 \text{ mi}}{5 \text{ s}} \right) \left( \frac{1610 \text{ m}}{1 \text{ mi}} \right) = 322 \text{ m/s} \approx \boxed{300 \text{ m/s}}$$

- (b) The speed of 322 m/s would imply the sound traveling a distance of 966 meters (which is approximately 1 km) in 3 seconds. So the rule could be approximated as 1 km every 3 seconds.

7. The time for the first part of the trip is calculated from the initial speed and the first distance, using  $d$  to represent distance.

$$\bar{v}_1 = \frac{d_1}{\Delta t_1} \rightarrow \Delta t_1 = \frac{d_1}{\bar{v}_1} = \frac{180 \text{ km}}{95 \text{ km/h}} = 1.895 \text{ h} = 113.7 \text{ min}$$

The time for the second part of the trip is now calculated.

$$\Delta t_2 = \Delta t_{\text{total}} - \Delta t_1 = 4.5 \text{ h} - 1.895 \text{ h} = 2.605 \text{ h} = 156.3 \text{ min}$$

The distance for the second part of the trip is calculated from the average speed for that part of the trip and the time for that part of the trip.

$$\bar{v}_2 = \frac{d_2}{\Delta t_2} \rightarrow d_2 = \bar{v}_2 \Delta t_2 = (65 \text{ km/h})(2.605 \text{ h}) = 169.3 \text{ km} \approx 170 \text{ km}$$

(a) The total distance is then  $d_{\text{total}} = d_1 + d_2 = 180 \text{ km} + 169.3 \text{ km} = 349.3 \text{ km} \approx \boxed{350 \text{ km}}$

(b) The average speed is NOT the average of the two speeds. Use the definition of average speed, Eq. 2-1.

$$\bar{v} = \frac{d_{\text{total}}}{\Delta t_{\text{total}}} = \frac{349.3 \text{ km}}{4.5 \text{ h}} = 77.62 \text{ km/h} \approx \boxed{78 \text{ km/h}}$$

8. The distance traveled is  $38 \text{ m} + \frac{1}{2}(38 \text{ m}) = 57 \text{ m}$ , and the displacement is  $38 \text{ m} - \frac{1}{2}(38 \text{ m}) = 19 \text{ m}$ . The total time is  $9.0 \text{ s} + 1.8 \text{ s} = 10.8 \text{ s}$ .

(a) Average speed =  $\frac{\text{distance}}{\text{time elapsed}} = \frac{57 \text{ m}}{10.8 \text{ s}} = \boxed{5.3 \text{ m/s}}$

(b) Average velocity =  $v_{\text{avg}} = \frac{\text{displacement}}{\text{time elapsed}} = \frac{19 \text{ m}}{10.8 \text{ s}} = \boxed{1.8 \text{ m/s}}$

9. The distance traveled is 3200 m (8 laps  $\times$  400 m/lap). That distance probably has either 3 or 4 significant figures, since the track distance is probably known to at least the nearest meter for competition purposes. The displacement is 0, because the ending point is the same as the starting point.

(a) Average speed =  $\frac{d}{\Delta t} = \frac{3200 \text{ m}}{14.5 \text{ min}} \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = \boxed{3.68 \text{ m/s}}$

(b) Average velocity =  $\bar{v} = \Delta x / \Delta t = \boxed{0 \text{ m/s}}$

10. The average speed is the distance divided by the time.

$$\bar{v} = \frac{d}{t} = \left( \frac{1 \times 10^9 \text{ km}}{1 \text{ yr}} \right) \left( \frac{1 \text{ yr}}{365.25 \text{ d}} \right) \left( \frac{1 \text{ d}}{24 \text{ h}} \right) = 1.141 \times 10^5 \text{ km/h} \approx \boxed{1 \times 10^5 \text{ km/h}}$$

- 11.** Both objects will have the same time of travel. If the truck travels a distance  $d_{\text{truck}}$ , then the distance the car travels will be  $d_{\text{car}} = d_{\text{truck}} + 210 \text{ m}$ . Using the equation for average speed,  $\bar{v} = d/\Delta t$ , solve for time, and equate the two times.

$$\Delta t = \frac{d_{\text{truck}}}{\bar{v}_{\text{truck}}} = \frac{d_{\text{car}}}{\bar{v}_{\text{car}}} \quad \frac{d_{\text{truck}}}{75 \text{ km/h}} = \frac{d_{\text{truck}} + 210 \text{ m}}{95 \text{ km/h}}$$

Solving for  $d_{\text{truck}}$  gives  $d_{\text{truck}} = (210 \text{ m}) \frac{(75 \text{ km/h})}{(95 \text{ km/h} - 75 \text{ km/h})} = 787.5 \text{ m}$ .

The time of travel is

$$\Delta t = \frac{d_{\text{truck}}}{\bar{v}_{\text{truck}}} = \left( \frac{787.5 \text{ m}}{75,000 \text{ m/h}} \right) \left( \frac{60 \text{ min}}{1 \text{ h}} \right) = 0.63 \text{ min} = 37.8 \text{ s} \approx \boxed{38 \text{ s}}$$

Also note that  $\Delta t = \frac{d_{\text{car}}}{\bar{v}_{\text{car}}} = \left( \frac{787.5 \text{ m} + 210 \text{ m}}{95,000 \text{ m/h}} \right) \left( \frac{60 \text{ min}}{1 \text{ h}} \right) = 0.63 \text{ min} = 37.8 \text{ s}.$

ALTERNATE SOLUTION:

The speed of the car relative to the truck is  $95 \text{ km/h} - 75 \text{ km/h} = 20 \text{ km/h}$ . In the reference frame of the truck, the car must travel 210 m to catch it.

$$\Delta t = \frac{0.21 \text{ km}}{20 \text{ km/h}} \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = 37.8 \text{ s}$$

12. The distance traveled is 500 km (250 km outgoing, 250 km return, keep 2 significant figures). The displacement ( $\Delta x$ ) is 0 because the ending point is the same as the starting point.

To find the average speed, we need the distance traveled (500 km) and the total time elapsed.

During the outgoing portion,  $\bar{v}_1 = \frac{\Delta x_1}{\Delta t_1}$ , so  $\Delta t_1 = \frac{\Delta x_1}{\bar{v}_1} = \frac{250 \text{ km}}{95 \text{ km/h}} = 2.632 \text{ h}$ . During the return portion,

$\bar{v}_2 = \frac{\Delta x_2}{\Delta t_2}$ , so  $\Delta t_2 = \frac{\Delta x_2}{\bar{v}_2} = \frac{250 \text{ km}}{55 \text{ km/h}} = 4.545 \text{ h}$ . Thus the total time, including lunch, is

$$\Delta t_{\text{total}} = \Delta t_1 + \Delta t_{\text{lunch}} + \Delta t_2 = 8.177 \text{ h}.$$

$$\bar{v} = \frac{\Delta x_{\text{total}}}{\Delta t_{\text{total}}} = \frac{500 \text{ km}}{8.177 \text{ h}} = \boxed{61 \text{ km/h}}$$

To find the average velocity, use the displacement and the elapsed time.

$$\boxed{\bar{v} = \Delta x / \Delta t = 0}$$

13. Since the locomotives have the same speed, they each travel half the distance, 4.25 km. Find the time of travel from the average speed.

$$\bar{v} = \frac{d}{\Delta t} \rightarrow \Delta t = \frac{d}{\bar{v}} = \frac{4.25 \text{ km}}{155 \text{ km/h}} = 0.0274 \text{ h} \left( \frac{60 \text{ min}}{1 \text{ h}} \right) = 1.645 \text{ min} \approx \boxed{1.6 \text{ min}} \approx 99 \text{ s}$$

14. (a) The area between the concentric circles is equal to the length times the width of the spiral path.

$$\pi R_2^2 - \pi R_1^2 = w\ell \rightarrow \ell \frac{\pi(R_2^2 - R_1^2)}{w} = \frac{\pi[(0.058 \text{ m})^2 - (0.025 \text{ m})^2]}{1.6 \times 10^{-6} \text{ m}} = 5.378 \times 10^3 \text{ m} \approx \boxed{5400 \text{ m}}$$

$$(b) \quad 5.378 \times 10^3 \text{ m} \left( \frac{1 \text{ s}}{1.2 \text{ m}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 74.69 \text{ min} \approx \boxed{75 \text{ min}}$$

15. The average speed of sound is given by  $v_{\text{sound}} = \Delta x / \Delta t$ , so the time for the sound to travel from the end of the lane back to the bowler is  $\Delta t_{\text{sound}} = \frac{\Delta x}{v_{\text{sound}}} = \frac{16.5 \text{ m}}{340 \text{ m/s}} = 4.85 \times 10^{-2} \text{ s}$ . Thus the time for the ball to travel from the bowler to the end of the lane is given by  $\Delta t_{\text{ball}} = \Delta t_{\text{total}} - \Delta t_{\text{sound}} = 2.80 \text{ s} - 4.85 \times 10^{-2} \text{ s} = 2.7515 \text{ s}$ . The speed of the ball is as follows:

$$v_{\text{ball}} = \frac{\Delta x}{\Delta t_{\text{ball}}} = \frac{16.5 \text{ m}}{2.7515 \text{ s}} = 5.9967 \text{ m/s} \approx \boxed{6.00 \text{ m/s}}$$

16. For the car to pass the train, the car must travel the length of the train AND the distance the train travels. The distance the car travels can thus be written as either  $d_{\text{car}} = v_{\text{car}}t = (95 \text{ km/h})t$  or  $d_{\text{car}} = \ell_{\text{train}} + v_{\text{train}}t = 1.30 \text{ km} + (75 \text{ km/h})t$ . To solve for the time, equate these two expressions for the distance the car travels.

$$(95 \text{ km/h})t = 1.30 \text{ km} + (75 \text{ km/h})t \rightarrow t = \frac{1.30 \text{ km}}{20 \text{ km/h}} = 0.065 \text{ h} = \boxed{3.9 \text{ min}}$$

Note that this is the same as calculating from the reference frame of the train, in which the car is moving at 20 km/h and must travel the length of the train.

The distance the car travels during this time is  $d = (95 \text{ km/h})(0.065 \text{ h}) = 6.175 \text{ km} \approx \boxed{6.2 \text{ km}}$

If the train is traveling in the opposite direction from the car, then the car must travel the length of the train MINUS the distance the train travels. Thus the distance the car travels can be written as either  $d_{\text{car}} = (95 \text{ km/h})t$  or  $d_{\text{car}} = 1.30 \text{ km} - (75 \text{ km/h})t$ . To solve for the time, equate these two expressions for the distance the car travels.

$$(95 \text{ km/h})t = 1.30 \text{ km} - (75 \text{ km/h})t \rightarrow t = \frac{1.30 \text{ km}}{170 \text{ km/h}} = 7.65 \times 10^{-3} \text{ h} \approx \boxed{28 \text{ s}}$$

The distance the car travels during this time is  $d = (95 \text{ km/h})(7.65 \times 10^{-3} \text{ h}) = \boxed{0.73 \text{ km}}$

17. The average acceleration is found from Eq. 2-4.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{95 \text{ km/h} - 0 \text{ km/h}}{4.3 \text{ s}} = \frac{(95 \text{ km/h})\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)}{4.3 \text{ s}} = \boxed{6.1 \text{ m/s}^2}$$

18. (a) The average acceleration of the sprinter is  $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{9.00 \text{ m/s} - 0.00 \text{ m/s}}{1.38 \text{ s}} = \boxed{6.52 \text{ m/s}^2}$ .

(b) We change the units for the acceleration.

$$\bar{a} = (6.52 \text{ m/s}^2)\left(\frac{1 \text{ km}}{1000 \text{ m}}\right)\left(\frac{3600 \text{ s}}{1 \text{ h}}\right)^2 = \boxed{8.45 \times 10^4 \text{ km/h}^2}$$

19. The initial velocity of the car is the average velocity of the car before it accelerates.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{120 \text{ m}}{5.0 \text{ s}} = 24 \text{ m/s} = v_0$$

The final velocity is  $v = 0$ , and the time to stop is 4.0 s. Use Eq. 2-11a to find the acceleration.

$$v = v_0 + at \rightarrow a = \frac{v - v_0}{t} = \frac{0 - 24 \text{ m/s}}{4.0 \text{ s}} = -6.0 \text{ m/s}^2$$

Thus the magnitude of the acceleration is  $\boxed{6.0 \text{ m/s}^2}$ , or  $(6.0 \text{ m/s}^2)\left(\frac{1 \text{ g}}{9.80 \text{ m/s}^2}\right) = \boxed{0.61 \text{ g's}}$

20. We assume that the speedometer can read to the nearest km/h, so the value of 120 km/h has three significant digits. The time can be found from the average acceleration,  $\bar{a} = \Delta v / \Delta t$ .

$$\Delta t = \frac{\Delta v}{\bar{a}} = \frac{120 \text{ km/h} - 65 \text{ km/h}}{1.8 \text{ m/s}^2} = \frac{(55 \text{ km/h}) \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right)}{1.8 \text{ m/s}^2} = 8.488 \text{ s} \approx \boxed{8.5 \text{ s}}$$

$$21. \quad (a) \quad \bar{v} = \frac{\Delta x}{\Delta t} = \frac{385 \text{ m} - 25 \text{ m}}{20.0 \text{ s} - 3.0 \text{ s}} = \boxed{21.2 \text{ m/s}}$$

$$(b) \quad \bar{a} = \frac{\Delta v}{\Delta t} = \frac{45.0 \text{ m/s} - 11.0 \text{ m/s}}{20.0 \text{ s} - 3.0 \text{ s}} = \boxed{2.00 \text{ m/s}^2}$$

22. The acceleration can be found from Eq. 2-11c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (28 \text{ m/s})^2}{2(88 \text{ m})} = \boxed{-4.5 \text{ m/s}^2}$$

$$23. \quad \text{By definition, the acceleration is } a = \frac{v - v_0}{t} = \frac{21 \text{ m/s} - 14 \text{ m/s}}{6.0 \text{ s}} = 1.167 \text{ m/s}^2 \approx \boxed{1 \text{ m/s}^2}.$$

The distance of travel can be found from Eq. 2-11b.

$$x - x_0 = v_0 t + \frac{1}{2} a t^2 = (14 \text{ m/s})(6.0 \text{ s}) + \frac{1}{2} (1.167 \text{ m/s}^2)(6.0 \text{ s})^2 = 105 \text{ m} \approx \boxed{110 \text{ m}}$$

It can also be found from Eq. 2-7 and Eq. 2-8.

$$x - x_0 = \bar{v} \Delta t = \frac{v_0 + v}{2} \Delta t = \frac{14 \text{ m/s} + 21 \text{ m/s}}{2} (6.0 \text{ s}) = 105 \text{ m} \approx \boxed{110 \text{ m}}$$

24. Assume that the plane starts from rest. The distance is found by solving Eq. 2-11c for  $x - x_0$ .

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow x - x_0 = \frac{v^2 - v_0^2}{2a} = \frac{(35 \text{ m/s})^2 - 0}{2(3.0 \text{ m/s}^2)} = 204.2 \text{ m} \approx \boxed{2.0 \times 10^2 \text{ m}}$$

25. For the baseball,  $v_0 = 0$ ,  $x - x_0 = 3.5 \text{ m}$ , and the final speed of the baseball (during the throwing motion) is  $v = 43 \text{ m/s}$ . The acceleration is found from Eq. 2-11c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(43 \text{ m/s})^2 - 0}{2(3.5 \text{ m})} = 264 \text{ m/s}^2 \approx \boxed{260 \text{ m/s}^2}$$

26. The sprinter starts from rest. The average acceleration is found from Eq. 2-11c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(11.5 \text{ m/s})^2 - 0}{2(18.0 \text{ m})} = 3.674 \text{ m/s}^2 \approx \boxed{3.67 \text{ m/s}^2}$$

Her elapsed time is found by solving Eq. 2-11a for time.

$$v = v_0 + at \rightarrow t = \frac{v - v_0}{a} = \frac{11.5 \text{ m/s} - 0}{3.674 \text{ m/s}^2} = \boxed{3.13 \text{ s}}$$

27. The words “slows down uniformly” imply that the car has a constant acceleration. The distance of travel is found from combining Eqs. 2-7 and 2-8.

$$x - x_0 = \frac{v_0 + v}{2} t = \left( \frac{28.0 \text{ m/s} + 0 \text{ m/s}}{2} \right) (8.00 \text{ s}) = \boxed{112 \text{ m}}$$



28. The final velocity of the car is zero. The initial velocity is found from Eq. 2-11c with  $v = 0$  and solving for  $v_0$ . Note that the acceleration is negative.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow v_0 = \sqrt{v^2 - 2a(x - x_0)} = \sqrt{0 - 2(-4.00 \text{ m/s}^2)(65 \text{ m})} = \boxed{23 \text{ m/s}}$$

29. (a) The final velocity of the car is 0. The distance is found from Eq. 2-11c with an acceleration of  $a = -0.50 \text{ m/s}^2$  and an initial velocity of 85 km/h.

$$x - x_0 = \frac{v^2 - v_0^2}{2a} = \frac{0 - \left[ (85 \text{ km/h}) \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \right]^2}{2(-0.50 \text{ m/s}^2)} = 434 \text{ m} \approx \boxed{430 \text{ m}}$$

- (b) The time to stop is found from Eq. 2-11a.

$$t = \frac{v - v_0}{a} = \frac{0 - \left[ (85 \text{ km/h}) \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \right]}{(-0.50 \text{ m/s}^2)} = 41.67 \text{ s} \approx \boxed{42 \text{ s}}$$

- (c) Take  $x_0 = x(t = 0) = 0$ . Use Eq. 2-11b, with  $a = -0.50 \text{ m/s}^2$  and an initial velocity of 75 km/h. The first second is from  $t = 0 \text{ s}$  to  $t = 1 \text{ s}$ , and the fifth second is from  $t = 4 \text{ s}$  to  $t = 5 \text{ s}$ .

$$x(0) = 0; \quad x(1) = 0 + (75 \text{ km/h}) \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) (1 \text{ s}) + \frac{1}{2}(-0.50 \text{ m/s}^2)(1 \text{ s})^2 = 20.58 \text{ m} \rightarrow$$

$$x(1) - x(0) = 20.58 \text{ m} \approx \boxed{21 \text{ m}}$$

$$x(4) = 0 + (75 \text{ km/h}) \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) (4 \text{ s}) + \frac{1}{2}(-0.50 \text{ m/s}^2)(4 \text{ s})^2 = 79.33 \text{ m}$$

$$x(5) = 0 + (75 \text{ km/h}) \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) (5 \text{ s}) + \frac{1}{2}(-0.50 \text{ m/s}^2)(5 \text{ s})^2 = 97.92 \text{ m}$$

$$x(5) - x(4) = 97.92 \text{ m} - 79.33 \text{ m} = 18.59 \text{ m} \approx \boxed{19 \text{ m}}$$

30. The origin is the location of the car at the beginning of the reaction time. The initial speed of the car is  $(95 \text{ km/h}) \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) = 26.39 \text{ m/s}$ . The location where the brakes are applied is found from the equation for motion at constant velocity.

$$x_0 = v_0 t_R = (26.39 \text{ m/s})(0.40 \text{ s}) = 10.56 \text{ m}$$

This is now the starting location for the application of the brakes. In each case, the final speed is 0.

- (a) Solve Eq. 2-11c for the final location.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow x = x_0 + \frac{v^2 - v_0^2}{2a} = 10.56 \text{ m} + \frac{0 - (26.39 \text{ m/s})^2}{2(-3.0 \text{ m/s}^2)} = 126.63 \text{ m} \approx \boxed{130 \text{ m}}$$

- (b) Solve Eq. 2-11c for the final location with the second acceleration.

$$x = x_0 + \frac{v^2 - v_0^2}{2a} = 10.56 \text{ m} + \frac{0 - (26.39 \text{ m/s})^2}{2(-6.0 \text{ m/s}^2)} = \boxed{69 \text{ m}}$$

31. Calculate the distance that the car travels during the reaction time and the deceleration.

$$\Delta x_1 = v_0 \Delta t = (18.0 \text{ m/s})(0.350 \text{ s}) = 6.3 \text{ m}$$

$$v^2 = v_0^2 + 2a\Delta x_2 \rightarrow \Delta x_2 = \frac{v^2 - v_0^2}{2a} = \frac{0 - (18.0 \text{ m/s})^2}{2(-3.65 \text{ m/s}^2)} = 44.4 \text{ m}$$

$$\Delta x = 6.3 \text{ m} + 44.4 \text{ m} = 50.7 \text{ m}$$

Since she is only 20.0 m from the intersection, she will NOT be able to stop in time. She will be 30.7 m past the intersection.

32. Use the information for the first 180 m to find the acceleration and the information for the full motion to find the final velocity. For the first segment, the train has  $v_0 = 0 \text{ m/s}$ ,  $v_1 = 18 \text{ m/s}$ , and a displacement of  $x_1 - x_0 = 180 \text{ m}$ . Find the acceleration from Eq. 2-11c.

$$v_1^2 = v_0^2 + 2a(x_1 - x_0) \rightarrow a = \frac{v_1^2 - v_0^2}{2(x_1 - x_0)} = \frac{(18 \text{ m/s})^2 - 0}{2(180 \text{ m})} = 0.90 \text{ m/s}^2$$

Find the speed of the train after it has traveled the total distance (total displacement of  $x_2 - x_0 = 255 \text{ m}$ ) using Eq. 2-11c.

$$v_2^2 = v_0^2 + 2a(x_2 - x_0) \rightarrow v_2 = \sqrt{v_0^2 + 2a(x_2 - x_0)} = \sqrt{2(0.90 \text{ m/s}^2)(255 \text{ m})} = \boxed{21 \text{ m/s}}$$

33. Calculate the acceleration from the velocity–time data using Eq. 2-11a, and then use Eq. 2-11b to calculate the displacement at  $t = 2.0 \text{ s}$  and  $t = 6.0 \text{ s}$ . The initial velocity is  $v_0 = 65 \text{ m/s}$ .

$$a = \frac{v - v_0}{t} = \frac{162 \text{ m/s} - 85 \text{ m/s}}{10.0 \text{ s}} = 7.7 \text{ m/s}^2 \quad x = x_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow$$

$$\begin{aligned} x(6.0 \text{ s}) - x(2.0 \text{ s}) &= [(x_0 + v_0(6.0 \text{ s}) + \frac{1}{2}a(6.0 \text{ s})^2) - (x_0 + v_0(2.0 \text{ s}) + \frac{1}{2}a(2.0 \text{ s})^2)] \\ &= v_0(6.0 \text{ s} - 2.0 \text{ s}) + \frac{1}{2}a[(6.0 \text{ s})^2 - (2.0 \text{ s})^2] = (85 \text{ m/s})(4.0 \text{ s}) + \frac{1}{2}(7.7 \text{ m/s}^2)(32 \text{ s}^2) \\ &= 463.2 \text{ m} \approx \boxed{460 \text{ m}} \end{aligned}$$

34. (a) The train's constant speed is  $v_{\text{train}} = 5.0 \text{ m/s}$ , and the location of the empty box car as a function of time is given by  $x_{\text{train}} = v_{\text{train}}t = (5.0 \text{ m/s})t$ . The fugitive has  $v_0 = 0 \text{ m/s}$  and  $a = 1.4 \text{ m/s}^2$  until his final speed is  $6.0 \text{ m/s}$ . The elapsed time during the acceleration is  $t_{\text{acc}} = \frac{v - v_0}{a} = \frac{6.0 \text{ m/s}}{1.4 \text{ m/s}^2} = 4.286 \text{ s}$ .

Let the origin be the location of the fugitive when he starts to run. The first possibility to consider is, "Can the fugitive catch the empty box car before he reaches his maximum speed?" During the fugitive's acceleration, his location as a function of time is given by Eq. 2-11b,

$$x_{\text{fugitive}} = x_0 + v_0 t + \frac{1}{2} a t^2 = 0 + 0 + \frac{1}{2} (1.4 \text{ m/s}^2) t^2. \text{ For him to catch the train, we must have}$$

$x_{\text{train}} = x_{\text{fugitive}} \rightarrow (5.0 \text{ m/s})t = \frac{1}{2} (1.4 \text{ m/s}^2) t^2$ . The solutions are  $t = 0 \text{ s}$ ,  $7.1 \text{ s}$ . Thus the fugitive cannot catch the car during his  $4.286 \text{ s}$  of acceleration.

Now the equation of motion of the fugitive changes. After the  $4.286 \text{ s}$  of acceleration, he runs with a constant speed of  $6.0 \text{ m/s}$ . Thus his location is now given (for times  $t > 5 \text{ s}$ ) by the following:

$$x_{\text{fugitive}} = \frac{1}{2} (1.4 \text{ m/s}^2) (4.286 \text{ s})^2 + (6.0 \text{ m/s})(t - 4.286 \text{ s}) = (6.0 \text{ m/s})t - 12.86 \text{ m}$$

So now, for the fugitive to catch the train, we again set the locations equal.

$$x_{\text{train}} = x_{\text{fugitive}} \rightarrow (5.0 \text{ m/s})t = (6.0 \text{ m/s})t - 12.86 \text{ m} \rightarrow t = 12.86 \text{ s} \approx \boxed{13 \text{ s}}$$

(b) The distance traveled to reach the box car is given by the following:

$$x_{\text{fugitive}}(t = 15.0 \text{ s}) = (6.0 \text{ m/s})(12.86 \text{ s}) - 12.86 \text{ m} = \boxed{64 \text{ m}}$$

35. For the runners to cross the finish line side-by-side, they must both reach the finish line in the same amount of time from their current positions. Take Mary's current location as the origin. Use Eq. 2-11b.

$$\text{For Sally: } 22 = 5.0 + 5.0t + \frac{1}{2}(-0.40)t^2 \rightarrow t^2 - 25t + 85 = 0 \rightarrow$$

$$t = \frac{25 \pm \sqrt{25^2 - 4(85)}}{2} = 4.059 \text{ s}, 20.94 \text{ s}$$

The first time is the time she first crosses the finish line, so that is the time to be used for the problem. Now find Mary's acceleration so that she crosses the finish line in that same amount of time.

$$\text{For Mary: } 22 = 0 + 4t + \frac{1}{2}at^2 \rightarrow a = \frac{22 - 4t}{\frac{1}{2}t^2} = \frac{22 - 4(4.059)}{\frac{1}{2}(4.059)^2} = \boxed{0.70 \text{ m/s}^2}$$

36. Define the origin to be the location where the speeder passes the police car. Start a timer at the instant that the speeder passes the police car and find another time that both cars have the same displacement from the origin.

For the speeder, traveling with a constant speed, the displacement is given by the following:

$$\Delta x_s = v_s t = (135 \text{ km/h}) \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) (t) = (37.5 t) \text{ m}$$

For the police car, the displacement is given by two components. The first part is the distance traveled at the initially constant speed during the 1 second of reaction time.

$$\Delta x_{p1} = v_{p1}(1.00 \text{ s}) = (95 \text{ km/h}) \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) (1.00 \text{ s}) = 26.39 \text{ m}$$

The second part of the police car displacement is that during the accelerated motion, which lasts for  $(t - 1.00) \text{ s}$ . So this second part of the police car displacement, using Eq. 2-11b, is given as follows:

$$\Delta x_{p2} = v_{p1}(t - 1.00) + \frac{1}{2}a_p(t - 1.00)^2 = [(26.39 \text{ m/s})(t - 1.00) + \frac{1}{2}(2.60 \text{ m/s}^2)(t - 1.00)^2] \text{ m}$$

So the total police car displacement is the following:

$$\Delta x_p = \Delta x_{p1} + \Delta x_{p2} = (26.39 + 26.39(t - 1.00) + 1.30(t - 1.00)^2) \text{ m}$$

Now set the two displacements equal and solve for the time.

$$26.39 + 26.39(t - 1.00) + 1.30(t - 1.00)^2 = 37.5 t \rightarrow t^2 - 10.55t + 1.00 = 0$$

$$t = \frac{10.55 \pm \sqrt{(10.55)^2 - 4.00}}{2} = 9.57 \times 10^{-2} \text{ s}, \boxed{10.5 \text{ s}}$$

The answer that is approximately 0 s corresponds to the fact that both vehicles had the same displacement of zero when the time was 0. The reason it is not exactly zero is rounding of previous values. The answer of 10.5 s is the time for the police car to overtake the speeder.

as a check on the answer, the speeder travels  $\Delta x_s = (37.5 \text{ m/s})(10.5 \text{ s}) = 394 \text{ m}$ , and the police car travels  $\Delta x_p = [26.39 + 26.39(9.5) + 1.30(9.5)^2] \text{ m} = 394 \text{ m}$ .

37. Choose downward to be the positive direction, and take  $y_0 = 0$  at the top of the cliff. The initial velocity is  $v_0 = 0$ , and the acceleration is  $a = 9.80 \text{ m/s}^2$ . The displacement is found from Eq. 2-11b, with  $x$  replaced by  $y$ .

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow y - 0 = 0 + \frac{1}{2} (9.80 \text{ m/s}^2) (3.55 \text{ s})^2 \rightarrow y = \boxed{61.8 \text{ m}}$$

38. Choose upward to be the positive direction, and take  $y_0 = 0$  to be the height from which the ball was thrown. The acceleration is  $a = -9.80 \text{ m/s}^2$ . The displacement upon catching the ball is 0, assuming it was caught at the same height from which it was thrown. The starting speed can be found from Eq. 2-11b, with  $x$  replaced by  $y$ .

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 = 0 \rightarrow$$

$$v_0 = \frac{y - y_0 - \frac{1}{2} a t^2}{t} = -\frac{1}{2} a t = -\frac{1}{2} (-9.80 \text{ m/s}^2) (3.4 \text{ s}) = 16.66 \text{ m/s} \approx \boxed{17 \text{ m/s}}$$

The height can be calculated from Eq. 2-11c, with a final velocity of  $v = 0$  at the top of the path.

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (16.66 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = \boxed{14 \text{ m}}$$

39. Choose upward to be the positive direction, and take  $y_0 = 0$  to be at the height where the ball was hit. For the upward path,  $v_0 = 25 \text{ m/s}$ ,  $v = 0$  at the top of the path, and  $a = -9.80 \text{ m/s}^2$ .

- (a) The displacement can be found from Eq. 2-11c, with  $x$  replaced by  $y$ .

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (25 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = \boxed{32 \text{ m}}$$

- (b) The time of flight can be found from Eq. 2-11b, with  $x$  replaced by  $y$ , using a displacement of 0 for the displacement of the ball returning to the height from which it was hit.

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 = 0 \rightarrow t(v_0 + \frac{1}{2} a t) = 0 \rightarrow$$

$$t = 0, t = \frac{2v_0}{-a} = \frac{2(25 \text{ m/s})}{-9.80 \text{ m/s}^2} = \boxed{5.1 \text{ s}}$$

The result of  $t = 0 \text{ s}$  is the time for the original displacement of zero (when the ball was hit), and the result of  $t = 5.1 \text{ s}$  is the time to return to the original displacement. Thus the answer is  $t = 5.1$  seconds.

- (c) This is an estimate primarily because the effects of the air have been ignored. There is a non-trivial amount of air effect on a baseball as it moves through the air—that's why pitches like the "curve ball" work, for example. So ignoring the effects of air makes this an estimate. Another effect is that the problem says "almost" straight up, but the problem was solved as if the initial velocity was perfectly upward. Finally, we assume that the ball was caught at the same height as which it was hit. That was not stated in the problem either, so that is an estimate.

- 40.** Choose upward to be the positive direction, and take  $y_0 = 0$  to be at the floor level, where the jump starts. For the upward path,  $y = 1.2 \text{ m}$ ,  $v = 0$  at the top of the path, and  $a = -9.80 \text{ m/s}^2$ .

- (a) The initial speed can be found from Eq. 2-11c, with  $x$  replaced by  $y$ .

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow$$

$$v_0 = \sqrt{v^2 - 2a(y - y_0)} = \sqrt{-2(-9.80 \text{ m/s}^2)(1.2 \text{ m})} = 4.8497 \text{ m/s} \approx \boxed{4.8 \text{ m/s}}$$

- (b) The time of flight can be found from Eq. 2-11b, with  $x$  replaced by  $y$ , using a displacement of 0 for the displacement of the jumper returning to the original height.

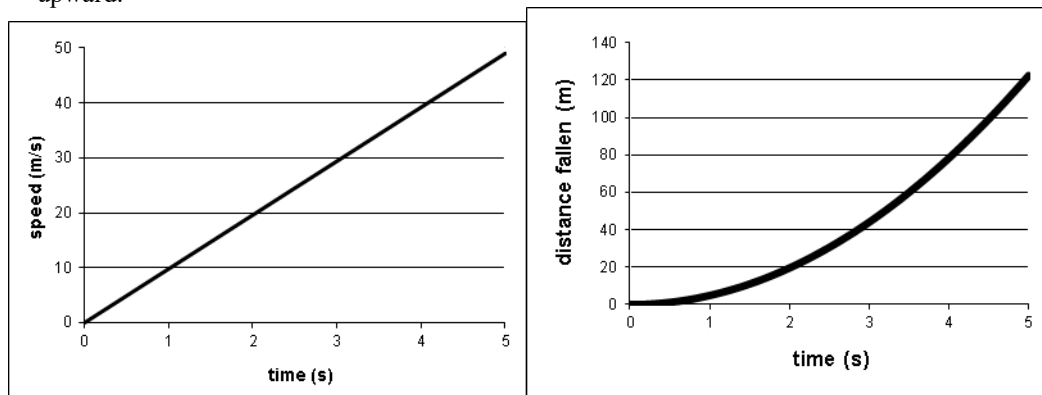
$$y = y_0 + v_0 t + \frac{1}{2} a t^2 = 0 \rightarrow t(v_0 + \frac{1}{2} a t) = 0 \rightarrow$$

$$t = 0, t = \frac{2v_0}{-a} = \frac{2(4.897 \text{ m/s})}{9.80 \text{ m/s}^2} = \boxed{0.99 \text{ s}}$$

The result of  $t = 0$  s is the time for the original displacement of zero (when the jumper started to jump), and the result of  $t = 0.99$  s is the time to return to the original displacement. Thus the answer is  $t = 0.99$  seconds.

41. Choose downward to be the positive direction, and take  $y_0 = 0$  to be at the height where the object was released. The initial velocity is  $v_0 = 0$ , and the acceleration is  $a = 9.80 \text{ m/s}^2$ .

- (a) The speed of the object will be given by Eq. 2-11a with  $v_0 = 0$ , so  $v = at = (9.80 \text{ m/s}^2)t$ . This is the equation of a straight line passing through the origin with a slope of  $9.80 \text{ m/s}^2$ .
- (b) The distance fallen will be given by Eq. 2-11b with  $v_0 = 0$ , so  $y = y_0 + v_0 t + \frac{1}{2} a t^2 = 0 + 0 + (4.90 \text{ m/s}^2)t^2$ . This is the equation of a parabola, with its vertex at the origin, opening upward.



42. Choose upward to be the positive direction, and  $y_0 = 0$  to be the height from which the stone is thrown. We have  $v_0 = 24.0 \text{ m/s}$ ,  $a = -9.80 \text{ m/s}^2$ , and  $y - y_0 = 13.0 \text{ m}$ .

- (a) The velocity can be found from Eq. 2-11c, with  $x$  replaced by  $y$ .

$$v^2 = v_0^2 + 2a(y - y_0) = 0 \rightarrow$$

$$v = \pm \sqrt{v_0^2 + 2ay} = \pm \sqrt{(24.0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(13.0 \text{ m})} = \pm 17.9 \text{ m/s}$$

Thus the speed is  $\boxed{|v| = 17.9 \text{ m/s}}$ .

- (b) The time to reach that height can be found from Eq. 2-11b.

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow t^2 + \frac{2(24.0 \text{ m/s})}{-9.80 \text{ m/s}^2} t + \frac{2(-13.0 \text{ m})}{-9.80 \text{ m/s}^2} = 0 \rightarrow$$

$$t^2 - 4.898t + 2.653 = 0 \rightarrow t = \frac{4.898 \pm \sqrt{(4.898)^2 - 4(2.653)}}{2} = \boxed{t = 4.28 \text{ s}, 0.620 \text{ s}}$$

- (c) There are two times at which the object reaches that height—once on the way up ( $t = 0.620 \text{ s}$ ) and once on the way down ( $t = 4.28 \text{ s}$ ).

43. Choose downward to be the positive direction, and take  $y_0 = 0$  to be the height from which the object is released. The initial velocity is  $v_0 = 0$ , and the acceleration is  $a = g$ . Then we can calculate the position as a function of time from Eq. 2-11b, with  $x$  replaced by  $y$ , as  $y(t) = \frac{1}{2} g t^2$ . At the end of each second, the position would be as follows:

$$y(0) = 0; \quad y(1) = \frac{1}{2} g; \quad y(2) = \frac{1}{2} g(2)^2 = 4y(1); \quad y(3) = \frac{1}{2} g(3)^2 = 9y(1)$$

The distance traveled during each second can be found by subtracting two adjacent position values from the above list.

$$d(1) = y(1) - y(0) = y(1); \quad d(2) = y(2) - y(1) = 3y(1); \quad d(3) = y(3) - y(2) = 5y(1)$$

We could do this in general. Let  $n$  be a positive integer, starting with 0.

$$y(n) = \frac{1}{2} g n^2 \quad y(n+1) = \frac{1}{2} g (n+1)^2$$

$$d(n+1) = y(n+1) - y(n) = \frac{1}{2} g (n+1)^2 - \frac{1}{2} g n^2 = \frac{1}{2} g ((n+1)^2 - n^2)$$

$$= \frac{1}{2} g (n^2 + 2n + 1 - n^2) = \frac{1}{2} g (2n + 1)$$

The value of  $(2n+1)$  is always odd, in the sequence 1, 3, 5, 7, ....

44. (a) Choose upward to be the positive direction, and  $y_0 = 0$  at the ground. The rocket has  $v_0 = 0$ ,  $a = 3.2 \text{ m/s}^2$ , and  $y = 775 \text{ m}$  when it runs out of fuel. Find the velocity of the rocket when it runs out of fuel from Eq. 2-11c, with  $x$  replaced by  $y$ .

$$v_{775 \text{ m}}^2 = v_0^2 + 2a(y - y_0) \rightarrow$$

$$v_{775 \text{ m}} = \pm \sqrt{v_0^2 + 2a(y - y_0)} = \pm \sqrt{0 + 2(3.2 \text{ m/s}^2)(775 \text{ m})} = 70.43 \text{ m/s} \approx \boxed{7.0 \times 10^1 \text{ m/s}}$$

The positive root is chosen since the rocket is moving upward when it runs out of fuel. Note that the value has 2 significant figures.

- (b) The time to reach the 775 m location can be found from Eq. 2-11a.

$$v_{775 \text{ m}} = v_0 + at_{775 \text{ m}} \rightarrow t_{775 \text{ m}} = \frac{v_{775 \text{ m}} - v_0}{a} = \frac{70.43 \text{ m/s} - 0}{3.2 \text{ m/s}^2} = 22.01 \text{ s} \approx \boxed{22 \text{ s}}$$

- (c) For this part of the problem, the rocket will have an initial velocity  $v_0 = 70.43 \text{ m/s}$ , an acceleration of  $a = -9.80 \text{ m/s}^2$ , and a final velocity of  $v = 0$  at its maximum altitude. The altitude reached from the out-of-fuel point can be found from Eq. 2-11c.

$$v^2 = v_{775 \text{ m}}^2 + 2a(y - 775 \text{ m}) \rightarrow$$

$$y_{\text{max}} = 775 \text{ m} + \frac{0 - v_{775 \text{ m}}^2}{2a} = 775 \text{ m} + \frac{-(70.43 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 775 \text{ m} + 253 \text{ m} = 1028 \text{ m} \approx \boxed{1030 \text{ m}}$$

- (d) The time for the “coasting” portion of the flight can be found from Eq. 2-11a.

$$v = v_{775 \text{ m}} + at_{\text{coast}} \rightarrow t_{\text{coast}} = \frac{v - v_0}{a} = \frac{0 - 70.43 \text{ m/s}}{2 \text{ } 9.80 \text{ m/s}^2} = 7.19 \text{ s}$$

Thus the total time to reach the maximum altitude is  $t = 22.01 \text{ s} + 7.19 \text{ s} = 29.20 \text{ s} \approx \boxed{29 \text{ s}}$ .

- (e) For this part of the problem, the rocket has  $v_0 = 0 \text{ m/s}$ ,  $a = -9.80 \text{ m/s}^2$ , and a displacement of  $-1028 \text{ m}$  (it falls from a height of  $1028 \text{ m}$  to the ground). Find the velocity upon reaching the Earth from Eq. 2-11c.

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow v = \pm \sqrt{v_0^2 + 2a(y - y_0)} = \pm \sqrt{0 + 2(-9.80 \text{ m/s}^2)(-1028 \text{ m})} = -141.95 \text{ m/s} \approx \boxed{-142 \text{ m/s}}$$

The negative root was chosen because the rocket is moving downward, which is the negative direction.

- (f) The time for the rocket to fall back to the Earth is found from Eq. 2-11a.

$$v = v_0 + at \rightarrow t_{\text{fall}} = \frac{v - v_0}{a} = \frac{-141.95 \text{ m/s} - 0}{-9.80 \text{ m/s}^2} = 14.48 \text{ s}$$

Thus the total time for the entire flight is  $t = 29.20 \text{ s} + 14.48 \text{ s} = 43.68 \text{ s} \approx \boxed{44 \text{ s}}$ .

45. Choose downward to be the positive direction, and take  $y_0 = 0$  to be the height where the object was released. The initial velocity is  $v_0 = -5.40 \text{ m/s}$ , the acceleration is  $a = 9.80 \text{ m/s}^2$ , and the displacement of the package will be  $y = 105 \text{ m}$ . The time to reach the ground can be found from Eq. 2-11b, with  $x$  replaced by  $y$ .

$$y = y_0 + v_0 t + \frac{1}{2}at^2 \rightarrow t^2 + \frac{2v_0}{a}t - \frac{2y}{a} = 0 \rightarrow t^2 + \frac{2(-5.40 \text{ m/s})}{9.80 \text{ m/s}^2}t - \frac{2(105 \text{ m})}{9.80 \text{ m/s}^2} = 0 \rightarrow t^2 - 1.102t - 21.43 = 0 \rightarrow t = \frac{1.102 \pm \sqrt{(1.102)^2 - 4(-21.43)}}{2} = 5.21 \text{ s}, -4.11 \text{ s}$$

The correct time is the positive answer,  $\boxed{t = 5.21 \text{ s}}$ .

46. (a) Choose  $y = 0$  to be the ground level and positive to be upward. Then  $y_0 = 15 \text{ m}$ ,  $a = -g$ , and  $t = 0.83 \text{ s}$  describe the motion of the balloon. Use Eq. 2-11b.

$$y = y_0 + v_0 t + \frac{1}{2}at^2 \rightarrow v_0 = \frac{y - y_0 - \frac{1}{2}at^2}{t} = \frac{0 - 15 \text{ m} - \frac{1}{2}(-9.80 \text{ m/s}^2)(0.83 \text{ s})^2}{(0.83 \text{ s})} = -14.01 \text{ m/s} \approx \boxed{-14 \text{ m/s}}$$

So the speed is  $\boxed{14 \text{ m/s}}$ .

- (b) Consider the change in velocity from being released to being at Roger’s room, using Eq. 2-11c.

$$v^2 = v_0^2 + 2a\Delta y \rightarrow \Delta y = \frac{v^2 - v_0^2}{2a} = \frac{-(-14.01 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = 10.01 \text{ m}$$

Thus the balloons are coming from two floors above Roger, or the fifth floor.

47. Choose upward to be the positive direction and  $y_0 = 0$  to be the location of the nozzle. The initial velocity is  $v_0$ , the acceleration is  $a = -9.80 \text{ m/s}^2$ , the final location is  $y = -1.8 \text{ m}$ , and the time of flight is  $t = 2.5 \text{ s}$ . Using Eq. 2-11b and substituting  $y$  for  $x$  gives the following:

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow$$

$$v_0 = \frac{y - \frac{1}{2} a t^2}{t} = \frac{-1.8 \text{ m} - \frac{1}{2} (-9.80 \text{ m/s}^2) (2.5 \text{ s})^2}{2.5 \text{ s}} = 11.53 \text{ m/s} \approx \boxed{12 \text{ m/s}}$$

48. Choose downward to be the positive direction and  $y_0 = 0$  to be the height from which the stone is dropped. Call the location of the top of the window  $y_w$ , and the time for the stone to fall from release to the top of the window is  $t_w$ . Since the stone is dropped from rest, using Eq. 2-11b with  $y$  substituting for  $x$ , we have  $y_w = y_0 + v_0 t + \frac{1}{2} a t^2 = 0 + 0 + \frac{1}{2} g t_w^2$ . The location of the bottom of the window is  $y_w + 2.2 \text{ m}$ , and the time for the stone to fall from release to the bottom of the window is  $t_w + 0.31 \text{ s}$ . Since the stone is dropped from rest, using Eq. 2-11b, we have the following:

$$y_w + 2.2 \text{ m} = y_0 + v_0 t + \frac{1}{2} a t^2 = 0 + 0 + \frac{1}{2} g (t_w + 0.31 \text{ s})^2$$

Substitute the first expression for  $y_w$  into the second one and solve for the time.

$$\frac{1}{2} g t_w^2 + 2.2 \text{ m} = \frac{1}{2} g (t_w + 0.31 \text{ s})^2 \rightarrow \frac{1}{2} g t_w^2 + 2.2 = \frac{1}{2} g (t_w^2 + 2 t_w (0.31) + (0.31)^2) \rightarrow$$

$$2.2 = \frac{1}{2} g (2 t_w (0.31) + (0.31)^2) \rightarrow 2.2 = t_w (0.31) g + \frac{1}{2} g (0.31)^2 \rightarrow$$

$$t_w = \frac{2.2 - \frac{1}{2} g (0.31)^2}{(0.31) g} = 0.569 \text{ s}$$

Use this time in the first equation to find the desired distance.

$$y_w = \frac{1}{2} g t_w^2 = \frac{1}{2} (9.80 \text{ m/s}^2) (0.569 \text{ s})^2 = 1.587 \text{ m} \approx \boxed{1.6 \text{ m}}$$

49. For the falling rock, choose downward to be the positive direction and  $y_0 = 0$  to be the height from which the stone is dropped. The initial velocity is  $v_0 = 0 \text{ m/s}$ , the acceleration is  $a = g$ , the final position is  $y = H$ , and the time of fall is  $t_1$ . Using Eq. 2-11b with  $y$  substituting for  $x$ , we have  $H = y_0 + v_0 t + \frac{1}{2} a t^2 = 0 + 0 + \frac{1}{2} g t_1^2$ . For the sound wave, use the constant speed equation that  $v_s = \frac{\Delta x}{\Delta t} = \frac{H}{T - t_1}$ , which can be rearranged to give  $t_1 = T - \frac{H}{v_s}$ , where  $T = 3.4 \text{ s}$  is the total time elapsed from dropping the rock to hearing the sound. Insert this expression for  $t_1$  into the equation for  $H$  from the stone, and solve for  $H$ .

$$H = \frac{1}{2} g \left( T - \frac{H}{v_s} \right)^2 \rightarrow \frac{g}{2 v_s^2} H^2 - \left( \frac{g T}{v_s} + 1 \right) H + \frac{1}{2} g T^2 = 0 \rightarrow$$

$$4.239 \times 10^{-5} H^2 - 1.098 H + 56.64 = 0 \rightarrow H = 51.7 \text{ m}, 2.59 \times 10^4 \text{ m}$$

If the larger answer is used in  $t_1 = T - \frac{H}{v_s}$ , a negative time of fall results, so the physically correct answer is  $\boxed{H = 52 \text{ m}}$ .

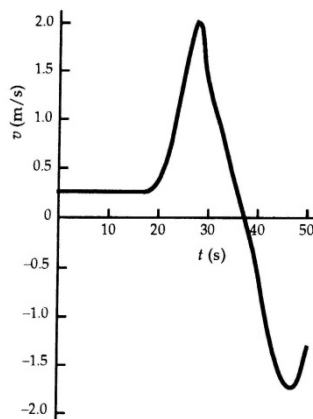
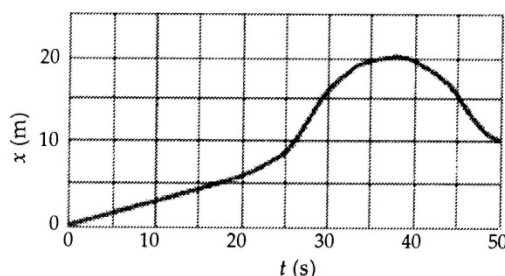


50. Slightly different answers may be obtained since the data come from reading the graph.
- (a) The greatest velocity is found at the highest point on the graph, which is at  $t \approx 48 \text{ s}$ .
  - (b) The indication of a constant velocity on a velocity vs. time graph is a slope of 0, which occurs from  $t = 90 \text{ s}$  to  $t \approx 108 \text{ s}$ .
  - (c) The indication of a constant acceleration on a velocity vs. time graph is a constant slope, which occurs from  $t = 0 \text{ s}$  to  $t \approx 42 \text{ s}$ , again from  $t \approx 65 \text{ s}$  to  $t \approx 83 \text{ s}$ , and again from  $t = 90 \text{ s}$  to  $t \approx 108 \text{ s}$ .
  - (d) The magnitude of the acceleration is greatest when the magnitude of the slope is greatest, which occurs from  $t \approx 65 \text{ s}$  to  $t \approx 83 \text{ s}$ .
51. Slightly different answers may be obtained since the data come from reading the graph. We assume that the short, nearly horizontal portions of the graph are the times that shifting is occurring, and those times are not counted as being “in” a certain gear.
- (a) The average acceleration in 2nd gear is given by  $\bar{a}_2 = \frac{\Delta v_2}{\Delta t_2} = \frac{24 \text{ m/s} - 14 \text{ m/s}}{8 \text{ s} - 4 \text{ s}} = 2.5 \text{ m/s}^2$ .
  - (b) The average acceleration in 4th gear is given by  $\bar{a}_4 = \frac{\Delta v_4}{\Delta t_4} = \frac{44 \text{ m/s} - 37 \text{ m/s}}{27 \text{ s} - 16 \text{ s}} = 0.64 \text{ m/s}^2$ .
52. Slightly different answers may be obtained since the data come from reading the graph.
- (a) The instantaneous velocity is given by the slope of the tangent line to the curve. At  $t = 10.0 \text{ s}$ , the slope is approximately  $v(10) \approx \frac{3 \text{ m} - 0}{10.0 \text{ s} - 0} = 0.3 \text{ m/s}$ .
  - (b) At  $t = 30.0 \text{ s}$ , the slope of the tangent line to the curve, and thus the instantaneous velocity, is approximately  $v(30) \approx \frac{20 \text{ m} - 8 \text{ m}}{35 \text{ s} - 25 \text{ s}} = 1.2 \text{ m/s}$ .
  - (c) The average velocity is given by  $\bar{v} = \frac{x(5) - x(0)}{5.0 \text{ s} - 0 \text{ s}} = \frac{1.5 \text{ m} - 0}{5.0 \text{ s}} = 0.30 \text{ m/s}$ .
  - (d) The average velocity is given by  $\bar{v} = \frac{x(30) - x(25)}{30.0 \text{ s} - 25.0 \text{ s}} = \frac{16 \text{ m} - 9 \text{ m}}{5.0 \text{ s}} = 1.4 \text{ m/s}$ .
  - (e) The average velocity is given by  $\bar{v} = \frac{x(50) - x(40)}{50.0 \text{ s} - 40.0 \text{ s}} = \frac{10 \text{ m} - 19.5 \text{ m}}{10.0 \text{ s}} = -0.95 \text{ m/s}$ .
53. Slightly different answers may be obtained since the data come from reading the graph.
- (a) The indication of a constant velocity on a position versus time graph is a constant slope, which occurs from  $t = 0 \text{ s}$  to  $t \approx 18 \text{ s}$ .
  - (b) The greatest velocity will occur when the slope is the highest positive value, which occurs at about  $t = 27 \text{ s}$ .
  - (c) The indication of a 0 velocity on a position versus time graph is a slope of 0, which occurs at about  $t = 38 \text{ s}$ .

- (d) The object moves in both directions. When the slope is positive, from  $t = 0$  s to  $t = 38$  s, the object is moving in the positive direction. When the slope is negative, from  $t = 38$  s to  $t = 50$  s, the object is moving in the negative direction.

54. The  $v$  vs.  $t$  graph is found by taking the slope of the  $x$  vs.  $t$  graph.

Both graphs are shown here.



55. Choose the upward direction to be positive and  $y_0 = 0$  to be the level from which the object was thrown. The initial velocity is  $v_0$  and the velocity at the top of the path is  $v = 0$ . The height at the top of the path can be found from Eq. 2-11c with  $x$  replaced by  $y$ .

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow y - y_0 = \frac{-v_0^2}{2a}$$

From this we see that the displacement is inversely proportional to the acceleration, so if the acceleration is reduced by a factor of 6 by going to the Moon, and the initial velocity is unchanged, then the displacement increases by a factor of 6

56. We are treating the value of 30 g's as if it had 2 significant figures. The initial velocity of the car is

$$v_0 = (95 \text{ km/h}) \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 26.39 \text{ m/s. Choose } x_0 = 0 \text{ to be location at which the deceleration}$$

begins. We have  $v = 0$  and  $a = -30g = -294 \text{ m/s}^2$ . Find the displacement from Eq. 2-11c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow x = x_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (26.39 \text{ m/s})^2}{2(-294 \text{ m/s}^2)} = 1.18 \text{ m} \approx \boxed{1.2 \text{ m}}$$

57. (a) For the free-falling part of the motion, choose downward to be the positive direction and  $y_0 = 0$  to be the height from which the person jumped. The initial velocity is  $v_0 = 0$ , acceleration is  $a = 9.80 \text{ m/s}^2$ , and the location of the net is  $y = 18.0 \text{ m}$ . Find the speed upon reaching the net from Eq. 2-11c with  $x$  replaced by  $y$ .

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow$$

$$v = \pm \sqrt{0 + 2a(y - 0)} = \pm \sqrt{2(9.80 \text{ m/s}^2)(18.0 \text{ m})} = 18.78 \text{ m/s}$$

The positive root is selected since the person is moving downward.

For the net-stretching part of the motion, choose downward to be the positive direction, and  $y_0 = 18.0 \text{ m}$  to be the height at which the person first contacts the net. The initial velocity is

$v_0 = 18.78 \text{ m/s}$ , the final velocity is  $v = 0$ , and the location at the stretched position is  $y = 19.0 \text{ m}$ . Find the acceleration from Eq. 2-11c with  $x$  replaced by  $y$ .

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow a = \frac{v^2 - v_0^2}{2(y - y_0)} = \frac{0^2 - (18.78 \text{ m/s})^2}{2(1.0 \text{ m})} = \boxed{-176 \text{ m/s}^2}$$

This is about  $18 g$ 's.

- (b) For the acceleration to be smaller, in the above equation we see that the displacement would have to be larger. This means that the net should be loosened.

58. The initial velocity is  $v_0 = 15 \text{ km/h}$ , the final velocity is  $v = 65 \text{ km/h}$ , and the displacement is  $x - x_0 = 4.0 \text{ km} = 4000 \text{ m}$ . Find the average acceleration from Eq. 2-11c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow$$

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{[(65 \text{ km/h})^2 - (15 \text{ km/h})^2] \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right)^2}{2(4000 \text{ m})} = \boxed{3.9 \times 10^{-2} \text{ m/s}^2}$$

59. The speed limit is  $40 \text{ km/h} \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 11.11 \text{ m/s}$ .

- (a) For your motion, you would need to travel  $(10 + 15 + 50 + 15 + 70) \text{ m} = 160 \text{ m}$  to get the front of the car to the third stoplight. The time to travel the  $160 \text{ m}$  is found using the distance and the speed limit.

$$\Delta x = \bar{v} \Delta t \rightarrow \Delta t = \frac{\Delta x}{\bar{v}} = \frac{160 \text{ m}}{11.11 \text{ m/s}} = 14.40 \text{ s}$$

No, you cannot make it to the third light without stopping, since it takes you longer than  $13.0$  seconds to reach the third light.

- (b) The second car needs to travel  $165 \text{ m}$  before the third light turns red. This car accelerates from  $v_0 = 0$  to a maximum of  $v = 11.11 \text{ m/s}$  with  $a = 2.00 \text{ m/s}^2$ . Use Eq. 2-11a to determine the duration of that acceleration.

$$v = v_0 + at \rightarrow t_{\text{acc}} = \frac{v - v_0}{a} = \frac{11.11 \text{ m/s} - 0 \text{ m/s}}{2.00 \text{ m/s}^2} = 5.556 \text{ s}$$

The distance traveled during that time is found from Eq. 2-11b.

$$(x - x_0)_{\text{acc}} = v_0 t_{\text{acc}} + \frac{1}{2} a t_{\text{acc}}^2 = 0 + \frac{1}{2} (2.00 \text{ m/s}^2) (5.556 \text{ s})^2 = 30.87 \text{ m}$$

Since  $5.556 \text{ s}$  have elapsed, there are  $13.0 - 5.556 = 7.444 \text{ s}$  remaining to clear the intersection.

The car travels another  $7.444 \text{ s}$  at a speed of  $11.11 \text{ m/s}$ , covering a distance of

$$\Delta x_{\text{constant speed}} = v_{\text{avg}} t = (11.11 \text{ m/s})(7.444 \text{ s}) = 82.70 \text{ m}. \text{ Thus the total distance is}$$

$30.87 \text{ m} + 82.70 \text{ m} = 113.57 \text{ m}$ . No, the car cannot make it through all three lights without stopping. The car has to travel another  $51.43 \text{ m}$  to clear the third intersection and is traveling at a speed of  $11.11 \text{ m/s}$ . Thus the front of the car would clear the intersection a time

$$t = \frac{\Delta x}{v} = \frac{51.43 \text{ m}}{11.11 \text{ m/s}} = \boxed{4.6 \text{ s}} \text{ after the light turns red.}$$

60. The average speed for each segment of the trip is given by  $\bar{v} = \frac{d}{\Delta t}$ , so  $\Delta t = \frac{d}{\bar{v}}$  for each segment.

For the first segment,  $\Delta t_1 = \frac{d_1}{\bar{v}_1} = \frac{2100 \text{ km}}{720 \text{ km/h}} = 2.917 \text{ h}$ .

For the second segment,  $\Delta t_2 = \frac{d_2}{\bar{v}_2} = \frac{2800 \text{ km}}{990 \text{ km/h}} = 2.828 \text{ h}$ .

Thus the total time is  $\Delta t_{\text{tot}} = \Delta t_1 + \Delta t_2 = 2.917 \text{ h} + 2.828 \text{ h} = 5.745 \text{ h} \approx \boxed{5.7 \text{ h}}$ .

The average speed of the plane for the entire trip is

$$\bar{v} = \frac{d_{\text{tot}}}{\Delta t_{\text{tot}}} = \frac{2100 \text{ km} + 2800 \text{ km}}{5.745 \text{ h}} = 852.9 \text{ km/h} \approx \boxed{850 \text{ km/h}}$$

Note that Eq. 2-11d does NOT apply in this situation.

61. Choose downward to be the positive direction and  $y_0 = 0$  to be at the roof from which the stones are dropped. The first stone has an initial velocity of  $v_0 = 0$  and an acceleration of  $a = g$ . Eqs. 2-11a and 2-11b (with  $x$  replaced by  $y$ ) give the velocity and location, respectively, of the first stone as a function of time.

$$v = v_0 + at \rightarrow v_1 = gt_1 \quad y = y_0 + v_0 t + \frac{1}{2}at^2 \rightarrow y_1 = \frac{1}{2}gt_1^2$$

The second stone has the same initial conditions, but its elapsed time is  $t_2 = t_1 - 1.30 \text{ s}$ , so it has velocity and location equations as follows:

$$v_2 = g(t_1 - 1.30 \text{ s}) \quad y_2 = \frac{1}{2}g(t_1 - 1.30 \text{ s})^2$$

The second stone reaches a speed of  $v_2 = 12.0 \text{ m/s}$  at a time given by

$$t_2 = \frac{v_2}{g} = t_1 - 1.30 \text{ s} \rightarrow t_1 = 1.30 \text{ s} + \frac{v_2}{g} = 1.30 \text{ s} + \frac{12.0 \text{ m/s}}{9.80 \text{ m/s}^2} = 2.524 \text{ s}$$

The location of the first stone at that time is

$$y_1 = \frac{1}{2}gt_1^2 = \frac{1}{2}(9.80 \text{ m/s}^2)(2.524 \text{ s})^2 = 31.22 \text{ m}$$

The location of the second stone at that time is

$$y_2 = \frac{1}{2}g(t_1 - 1.30 \text{ s})^2 = \frac{1}{2}(9.80 \text{ m/s}^2)(2.524 \text{ s} - 1.30 \text{ s})^2 = 7.34 \text{ m}$$

Thus the distance between the two stones is  $y_1 - y_2 = 31.22 \text{ m} - 7.34 \text{ m} = 23.88 \text{ m} \approx \boxed{23.9 \text{ m}}$

62. For the motion in the air, choose downward to be the positive direction and  $y_0 = 0$  to be at the height of the diving board. Then diver has  $v_0 = 0$  (assuming the diver does not jump upward or downward),  $a = g = 9.80 \text{ m/s}^2$ , and  $y = 4.0 \text{ m}$  when reaching the surface of the water. Find the diver's speed at the water's surface from Eq. 2-11c, with  $x$  replaced by  $y$ .

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow v = \pm \sqrt{v_0^2 + 2a(y - y_0)} = \sqrt{0 + 2(9.80 \text{ m/s}^2)(4.0 \text{ m})} = 8.85 \text{ m/s}$$

For the motion in the water, again choose down to be positive, but redefine  $y_0 = 0$  to be at the surface of the water. For this motion,  $v_0 = 8.85 \text{ m/s}$ ,  $v = 0$ , and  $y - y_0 = 2.0 \text{ m}$ . Find the acceleration from Eq. 2-11c, with  $x$  replaced by  $y$ .

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow a = \frac{v^2 - v_0^2}{2(y - y_0)} = \frac{0 - (8.85 \text{ m/s})^2}{2(2.0 \text{ m})} = -19.6 \text{ m/s}^2 \approx \boxed{-20 \text{ m/s}^2}$$

The negative sign indicates that the acceleration is directed upward.

63. First consider the “uphill lie,” in which the ball is being putted down the hill. Choose  $x_0 = 0$  to be the ball’s original location and the direction of the ball’s travel as the positive direction. The final velocity of the ball is  $v = 0$ , the acceleration of the ball is  $a = -1.8 \text{ m/s}^2$ , and the displacement of the ball will be  $x - x_0 = 6.0 \text{ m}$  for the first case and  $x - x_0 = 8.0 \text{ m}$  for the second case. Find the initial velocity of the ball from Eq. 2-11c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow v_0 = \sqrt{v^2 - 2a(x - x_0)} = \begin{cases} \sqrt{0 - 2(-1.8 \text{ m/s}^2)(6.0 \text{ m})} = 4.65 \text{ m/s} \\ \sqrt{0 - 2(-1.8 \text{ m/s}^2)(8.0 \text{ m})} = 5.37 \text{ m/s} \end{cases}$$

The range of acceptable velocities for the uphill lie is  $\boxed{4.65 \text{ m/s to } 5.37 \text{ m/s}}$ , a spread of  $0.72 \text{ m/s}$ .

Now consider the “downhill lie,” in which the ball is being putted up the hill. Use a very similar setup for the problem, with the basic difference being that the acceleration of the ball is now  $a = -2.6 \text{ m/s}^2$ . Find the initial velocity of the ball from Eq. 2-11c.

$$v^2 = v_0^2 + 2a(x - x_0) \rightarrow v_0 = \sqrt{v^2 - 2a(x - x_0)} = \begin{cases} \sqrt{0 - 2(-2.6 \text{ m/s}^2)(6.0 \text{ m})} = 5.59 \text{ m/s} \\ \sqrt{0 - 2(-2.6 \text{ m/s}^2)(8.0 \text{ m})} = 6.45 \text{ m/s} \end{cases}$$

The range of acceptable velocities for the downhill lie is  $\boxed{5.59 \text{ m/s to } 6.45 \text{ m/s}}$ , a spread of  $0.86 \text{ m/s}$ .

Because the range of acceptable velocities is smaller for putting down the hill, more control in putting is necessary, so putting the ball downhill (the “uphill lie”) is more difficult.

- 64.** Choose upward to be the positive direction and  $y_0 = 0$  to be at the throwing location of the stone. The initial velocity is  $v_0 = 15.5 \text{ m/s}$ , the acceleration is  $a = -9.80 \text{ m/s}^2$ , and the final location is  $y = -75 \text{ m}$ .

- (a) Using Eq. 2-11b and substituting  $y$  for  $x$ , we have the following:

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow (4.9 \text{ m/s}^2) t^2 - (15.5 \text{ m/s}) t - 75 \text{ m} = 0 \rightarrow$$

$$t = \frac{15.5 \pm \sqrt{(15.5)^2 - 4(4.9)(-75)}}{2(4.9)} = 5.802 \text{ s}, -2.638 \text{ s}$$

The positive answer is the physical answer:  $\boxed{t = 5.80 \text{ s}}$ .

- (b) Use Eq. 2-11a to find the velocity just before hitting.

$$v = v_0 + at = 15.5 \text{ m/s} + (-9.80 \text{ m/s}^2)(5.802 \text{ s}) = -41.4 \text{ m/s} \rightarrow |v| = \boxed{41.4 \text{ m/s}}$$

- (c) The total distance traveled will be the distance up plus the distance down. The distance down will be  $75 \text{ m}$  more than the distance up. To find the distance up, use the fact that the speed at the top of the path will be  $0$ . Then using Eq. 2-11c we have the following:

$$v^2 = v_0^2 + 2a(y - y_0) \rightarrow y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (15.5 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 12.26 \text{ m}$$

Thus the distance up is 12.26 m, the distance down is 87.26 m, and the total distance traveled is 99.5 m.

65. This problem can be analyzed as a series of three one-dimensional motions: the acceleration phase, the constant-speed phase, and the deceleration phase. The maximum speed of the train is as follows:

$$(95 \text{ km/h}) \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 26.39 \text{ m/s}$$

In the acceleration phase, the initial velocity is  $v_0 = 0$ , the acceleration is  $a = 1.1 \text{ m/s}^2$ , and the final velocity is  $v = 26.39 \text{ m/s}$ . Find the elapsed time for the acceleration phase from Eq. 2-11a.

$$v = v_0 + at \rightarrow t_{\text{acc}} = \frac{v - v_0}{a} = \frac{26.39 \text{ m/s} - 0}{1.1 \text{ m/s}^2} = 23.99 \text{ s}$$

Find the displacement during the acceleration phase from Eq. 2-11b.

$$(x - x_0)_{\text{acc}} = v_0 t + \frac{1}{2} at^2 = 0 + \frac{1}{2} (1.1 \text{ m/s}^2) (23.99 \text{ s})^2 = 316.5 \text{ m}$$

In the deceleration phase, the initial velocity is  $v_0 = 26.39 \text{ m/s}$ , the acceleration is  $a = -2.0 \text{ m/s}^2$ , and the final velocity is  $v = 0$ . Find the elapsed time for the deceleration phase from Eq. 2-11a.

$$v = v_0 + at \rightarrow t_{\text{dec}} = \frac{v - v_0}{a} = \frac{0 - 26.39 \text{ m/s}}{-2.0 \text{ m/s}^2} = 13.20 \text{ s}$$

Find the distance traveled during the deceleration phase from Eq. 2-11b.

$$(x - x_0)_{\text{dec}} = v_0 t + \frac{1}{2} at^2 = (26.39 \text{ m/s})(13.20 \text{ s}) + \frac{1}{2} (-2.0 \text{ m/s}^2) (13.20 \text{ s})^2 = 174.1 \text{ m}$$

The total elapsed time and distance traveled for the acceleration/deceleration phases are:

$$t_{\text{acc}} + t_{\text{dec}} = 23.99 \text{ s} + 13.20 \text{ s} = 37.19 \text{ s}$$

$$(x - x_0)_{\text{acc}} + (x - x_0)_{\text{dec}} = 316.5 \text{ m} + 174.1 \text{ m} = 491 \text{ m}$$

- (a) If the stations are spaced  $3.0 \text{ km} = 3000 \text{ m}$  apart, then there is a total of  $\frac{15,000 \text{ m}}{3000 \text{ m}} = 5$

interstation segments. A train making the entire trip would thus have a total of 5 interstation segments and 4 stops of 22 s each at the intermediate stations. Since 491 m is traveled during acceleration and deceleration,  $3000 \text{ m} - 491 \text{ m} = 2509 \text{ m}$  of each segment is traveled at an average speed of  $\bar{v} = 26.39 \text{ m/s}$ . The time for that 2509 m is given by  $\Delta x = \bar{v} \Delta t \rightarrow$

$$\Delta t_{\text{constant speed}} = \frac{\Delta x}{\bar{v}} = \frac{2509 \text{ m}}{26.39 \text{ m/s}} = 95.07 \text{ s. Thus a total interstation segment will take}$$

$37.19 \text{ s} + 95.07 \text{ s} = 132.26 \text{ s}$ . With 5 interstation segments of 132.26 s each, and 4 stops of 22 s each, the total time is given by  $t_{3.0 \text{ km}} = 5(132.26 \text{ s}) + 4(22 \text{ s}) = 749 \text{ s} = \boxed{12.5 \text{ min}}$

- (b) If the stations are spaced  $5.0 \text{ km} = 5000 \text{ m}$  apart, then there is a total of  $\frac{15,000 \text{ m}}{5000 \text{ m}} = 3$

interstation segments. A train making the entire trip would thus have a total of 3 interstation segments and 2 stops of 22 s each at the intermediate stations. Since 491 m is traveled during

acceleration and deceleration,  $5000 \text{ m} - 491 \text{ m} = 4509 \text{ m}$  of each segment is traveled at an average speed of  $\bar{v} = 26.39 \text{ m/s}$ . The time for that 4509 m is given by  $d = \bar{v}t \rightarrow$

$$t = \frac{d}{\bar{v}} = \frac{4509 \text{ m}}{26.39 \text{ m/s}} = 170.86 \text{ s. Thus a total interstation segment will take}$$

$37.19 \text{ s} + 170.86 \text{ s} = 208.05 \text{ s}$ . With 3 interstation segments of 208.05 s each, and 2 stops of 22 s each, the total time is given by  $t_{5.0 \text{ km}} = 3(208.05 \text{ s}) + 2(22 \text{ s}) = 668 \text{ s} = \boxed{11.1 \text{ min}}$

66. The car's initial speed is  $v_0 = (35 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 9.722 \text{ m/s}$ .

Case I: trying to stop. The constraint is, with the braking deceleration of the car ( $a = -5.8 \text{ m/s}^2$ ), can the car stop in a 28-m displacement? The 2.0 seconds has no relation to this part of the problem. Using Eq. 2-11c, the distance traveled during braking is as follows:

$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{0 - (9.722 \text{ m/s})^2}{2(-5.8 \text{ m/s}^2)} = 8.14 \text{ m} \quad \boxed{\text{She can stop the car in time.}}$$

Case II: crossing the intersection. The constraint is, with the acceleration of the car

$$\left[ a = \left( \frac{65 \text{ km/h} - 45 \text{ km/h}}{6.0 \text{ s}} \right) \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 0.9259 \text{ m/s}^2 \right], \text{ can she get through the intersection}$$

(travel 43 m) in the 2.0 seconds before the light turns red? Using Eq. 2-11b, the distance traveled during the 2.0 s is

$$(x - x_0) = v_0 t + \frac{1}{2} a t^2 = (9.722 \text{ m/s})(2.0 \text{ s}) + \frac{1}{2}(0.9259 \text{ m/s}^2)(2.0 \text{ s})^2 = 21.3 \text{ m}$$

$\boxed{\text{She should stop.}}$

67. The critical condition is that the total distance covered by the passing car and the approaching car must be less than 500 m so that they do not collide. The passing car has a total displacement composed of several individual parts. These are (i) the 10 m of clear room at the rear of the truck, (ii) the 20-m length of the truck, (iii) the 10 m of clear room at the front of the truck, and (iv) the distance the truck travels. Since the truck travels at a speed of  $\bar{v} = 18 \text{ m/s}$ , the truck will have a displacement of

$$\Delta x_{\text{truck}} = (18 \text{ m/s})t. \text{ Thus the total displacement of the car during passing is}$$

$$\Delta x_{\text{passing car}} = 40 \text{ m} + (18 \text{ m/s})t.$$

To express the motion of the car, we choose the origin to be at the location of the passing car when the decision to pass is made. For the passing car, we have an initial velocity of  $v_0 = 18 \text{ m/s}$  and an acceleration of  $a = 1.0 \text{ m/s}^2$ . Find  $\Delta x_{\text{passing car}}$  from Eq. 2-11b.

$$\Delta x_{\text{passing car}} = x_c - x_0 = v_0 t + \frac{1}{2} a t^2 = (18 \text{ m/s})t + \frac{1}{2}(1.0 \text{ m/s}^2)t^2$$

Set the two expressions for  $\Delta x_{\text{passing car}}$  equal to each other in order to find the time required to pass.

$$40 \text{ m} + (18 \text{ m/s})t_{\text{pass}} = (18 \text{ m/s})t_{\text{pass}} + \frac{1}{2}(1.0 \text{ m/s}^2)t_{\text{pass}}^2 \rightarrow 40 \text{ m} = \frac{1}{2}(1.0 \text{ m/s}^2)t_{\text{pass}}^2 \rightarrow$$

$$t_{\text{pass}} = \sqrt{\frac{80}{1.0}} \text{ s} = 8.94 \text{ s}$$

Calculate the displacements of the two cars during this time.

$$\Delta x_{\text{passing car}} = 40 \text{ m} + (18 \text{ m/s})(11.55 \text{ s}) = 247.9 \text{ m}$$

$$\Delta x_{\text{approaching car}} = v_{\text{approaching car}} t = (25 \text{ m/s})(11.55 \text{ s}) = 288.75 \text{ m}$$

Thus the two cars together have covered a total distance of  $247.9 \text{ m} + 288.75 \text{ m} = 536.65 \text{ m}$ , which is more than allowed. The car should not pass.

68. Choose downward to be the positive direction and  $y_0 = 0$  to be at the height of the bridge. Agent Bond has an initial velocity of  $v_0 = 0$ , an acceleration of  $a = g$ , and will have a displacement of  $y = 15 \text{ m} - 3.5 \text{ m} = 11.5 \text{ m}$ . Find the time of fall from Eq. 2-11b with  $x$  replaced by  $y$ .

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 \rightarrow t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(11.5 \text{ m})}{9.80 \text{ m/s}^2}} = 1.532 \text{ s}$$

If the truck is approaching with  $v = 25 \text{ m/s}$ , then he needs to jump when the truck is a distance away given by  $d = vt = (25 \text{ m/s})(1.532 \text{ s}) = 38.3 \text{ m}$ . Convert this distance into “poles.”

$$d = (38.3 \text{ m})(1 \text{ pole}/25 \text{ m}) = 1.53 \text{ poles}$$

So he should jump when the truck is about 1.5 poles away from the bridge.

69. The speed of the conveyor belt is found from Eq. 2-2 for average velocity.

$$\Delta x = \bar{v} \Delta t \rightarrow \bar{v} = \frac{\Delta x}{\Delta t} = \frac{1.2 \text{ m}}{2.8 \text{ min}} = 0.4286 \text{ m/min} \approx \text{0.43 m/min}$$

The rate of burger production, assuming the spacing given is center to center, can be found as follows:

$$\left( \frac{1 \text{ burger}}{0.25 \text{ m}} \right) \left( \frac{0.4286 \text{ m}}{1 \text{ min}} \right) = \text{1.7 } \frac{\text{burgers}}{\text{min}}$$

70. Choose downward to be the positive direction and the origin to be at the top of the building. The barometer has  $y_0 = 0$ ,  $v_0 = 0$ , and  $a = g = 9.8 \text{ m/s}^2$ . Use Eq. 2-11b to find the height of the building, with  $x$  replaced by  $y$ .

$$y = y_0 + v_0 t + \frac{1}{2} a t^2 = 0 + 0 + \frac{1}{2} (9.8 \text{ m/s}^2) t^2$$

$$y_{t=2.0} = \frac{1}{2} (9.8 \text{ m/s}^2) (2.0 \text{ s})^2 = 19.6 \text{ m} \quad y_{t=2.3} = \frac{1}{2} (9.8 \text{ m/s}^2) (2.3 \text{ s})^2 = 25.9 \text{ m}$$

The difference in the estimates is  $6.3 \text{ m}$ . If we assume the height of the building is the average of the two measurements, then the % difference in the two values is  $\frac{6.3 \text{ m}}{22.75 \text{ m}} \times 100 = 27.7\% \approx \text{30\%}$ .

The intent of the method was probably to use the change in air pressure between the ground level and the top of the building to find the height of the building. The very small difference in time measurements, which could be due to human reaction time, makes a  $6.3\text{-m}$  difference in the height. This could be as much as 2 floors in error.



71. Assume that  $y_0 = 0$  for each child is the level at which the child loses contact with the trampoline surface. Choose upward to be the positive direction.

(a) The second child has  $v_{02} = 4.0$  m/s,  $a = -g = -9.80$  m/s<sup>2</sup>, and  $v = 0$  m/s at the maximum height position. Find the child's maximum height from Eq. 2-11c, with  $x$  replaced by  $y$ .

$$v^2 = v_{02}^2 + 2a(y_2 - y_0) \rightarrow$$

$$y_2 = y_0 + \frac{v^2 - v_{02}^2}{2a} = 0 + \frac{0 - (4.0 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 0.8163 \text{ m} \approx \boxed{0.82 \text{ m}}$$

(b) Since the first child can bounce up to one-and-a-half times higher than the second child, the first child can bounce up to a height of  $1.5(0.8163 \text{ m}) = 1.224 \text{ m} = y_1 - y_0$ . Eq. 2-11c is again used to find the initial speed of the first child.

$$v^2 = v_{01}^2 + 2a(y_1 - y_0) \rightarrow$$

$$v_{01} = \pm \sqrt{v^2 - 2a(y_1 - y_0)} = \sqrt{0 - 2(-9.80 \text{ m/s}^2)(1.224 \text{ m})} = 4.898 \text{ m/s} \approx \boxed{4.9 \text{ m/s}}$$

The positive root was chosen since the child was initially moving upward.

(c) To find the time that the first child was in the air, use Eq. 2-11b with a total displacement of 0, since the child returns to the original position.

$$y = y_0 + v_{01}t_1 + \frac{1}{2}at_1^2 \rightarrow 0 = (4.898 \text{ m/s})t_1 + \frac{1}{2}(-9.80 \text{ m/s}^2)t_1^2 \rightarrow t_1 = 0 \text{ s}, 0.9996 \text{ s}$$

The time of 0 s corresponds to the time the child started the jump, so the correct answer is  $\boxed{1.0 \text{ s}}$ .

72. Choose downward to be the positive direction and the origin to be at the location of the plane. The parachutist has  $v_0 = 0$ ,  $a = g = 9.80$  m/s<sup>2</sup>, and will have  $y - y_0 = 3200 \text{ m} - 450 \text{ m} = 2750 \text{ m}$  when she pulls the ripcord. Eq. 2-11b, with  $x$  replaced by  $y$ , is used to find the time when she pulls the ripcord.

$$y = y_0 + v_0t + \frac{1}{2}at^2 \rightarrow t = \sqrt{2(y - y_0)/a} = \sqrt{2(2750 \text{ m})/(9.80 \text{ m/s}^2)} = 23.69 \text{ s} \approx \boxed{23.7 \text{ s}}$$

The speed is found from Eq. 2-11a.

$$v = v_0 + at = 0 + (9.80 \text{ m/s}^2)(23.69 \text{ s}) = 232.16 \text{ m/s} \approx \boxed{230 \text{ m/s}} \left( \frac{3.6 \text{ km/h}}{1 \text{ m/s}} \right) \approx \boxed{840 \text{ km/h}}$$

This is well over 500 miles per hour!

73. As shown in Example 2-15, the speed with which the ball was thrown upward is the same as its speed on returning to the ground. From the symmetry of the two motions (both motions have speed = 0 at top, have same distance traveled, and have same acceleration), the time for the ball to rise is the same as the time for the ball to fall, 1.4 s. Choose upward to be the positive direction and the origin to be at the level where the ball was thrown. For the ball,  $v = 0$  at the top of the motion, and  $a = -g$ . Find the initial velocity from Eq. 2-11a.

$$v = v_0 + at \rightarrow v_0 = v - at = 0 - (-9.80 \text{ m/s}^2)(1.4 \text{ s}) = 13.72 \text{ m/s} \approx \boxed{14 \text{ m/s}}$$

74. (a) Multiply the reading rate times the bit density to find the bit reading rate.

$$N = \frac{1.2 \text{ m}}{1 \text{ s}} \times \frac{1 \text{ bit}}{0.28 \times 10^{-6} \text{ m}} = \boxed{4.3 \times 10^6 \text{ bits/s}}$$

(b) The number of excess bits is  $N - N_0$ .

$$N - N_0 = 4.3 \times 10^6 \text{ bits/s} - 1.4 \times 10^6 \text{ bits/s} = 2.9 \times 10^6 \text{ bits/s}$$

$$\frac{N - N_0}{N} = \frac{2.9 \times 10^6 \text{ bits/s}}{4.3 \times 10^6 \text{ bits/s}} = 0.67 = \boxed{67\%}$$

## Solutions to Search and Learn Problems

- The two conditions are that the motion needs to be near the surface of the Earth and that there is no air resistance. An example where the second condition is not even a reasonable approximation is that of parachuting. The air resistance caused by the parachute results in the acceleration not being constant, with values much different than  $9.8 \text{ m/s}^2$ .
- The sounds will not occur at equal time intervals because the longer any particular bolt falls, the higher its speed. With equal distances between bolts, each successive bolt, having fallen a longer time when its predecessor reaches the plate, will have a higher average velocity and thus travel the interbolt distance in shorter periods of time. Thus the sounds will occur with smaller and smaller intervals between sounds.

To hear the sounds at equal intervals, the bolts would have to be tied at distances corresponding to equal time intervals. The first bolt (call it bolt #0) is touching the plate. Since each bolt has an initial speed of 0, the distance of fall and time of fall for each bolt are related to each other by  $d_i = \frac{1}{2}gt_i^2$ .

Thus for bolt #1,  $d_1 = \frac{1}{2}gt_1^2$ . For bolt #2, we want

$$t_2 = 2t_1, \text{ so } d_2 = \frac{1}{2}gt_2^2 = \frac{1}{2}g(2t_1)^2 = 4(\frac{1}{2}gt_1^2) = 4d_1.$$

Likewise,  $t_3 = 3t_1$ , which leads to  $d_3 = 9d_1$ ;  $t_4 = 4t_1$ , which leads to  $d_4 = 16d_1$ , and so on. If the distance from the bolt initially on the pan to the next bolt is  $d_1$ , then the distance from that bolt to the next one is  $3d_1$ , the distance to the next bolt is  $5d_1$ , and so on. The accompanying table shows these relationships in a simpler format.

Bolt #	Height above floor	Time to fall	Distance between bolts
0	0	0	
1	$d_1$	$t_1$	$d_1$
2	$d_2 = 4d_1$	$t_2 = 2t_1$	$3d_1$
3	$d_3 = 9d_1$	$t_3 = 3t_1$	$5d_1$
4	$d_4 = 16d_1$	$t_4 = 4t_1$	$7d_1$
5	$d_5 = 25d_1$	$t_5 = 5t_1$	$9d_1$

- We are given that  $x(t) = 2.0 \text{ m} - (3.6 \text{ m/s})t + (1.7 \text{ m/s}^2)t^2$ , where  $t$  is in seconds.
  - The value of 2.0 m is the initial position of the ball. The value of 3.6 m/s is the initial speed of the ball—the speed at  $t = 0$ . Note that the ball is initially moving in the negative direction, since  $-3.6 \text{ m/s}$  is used. The value of  $1.7 \text{ m/s}^2$  is the acceleration of the ball.
  - The units of 2.0 are meters. The units of 3.6 are m/s. The units of 1.7 are  $\text{m/s}^2$ .
  - $$x(1.0 \text{ s}) = 2.0 \text{ m} - (3.6 \text{ m/s})(1.0 \text{ s}) + (1.7 \text{ m/s}^2)(1.0 \text{ s})^2 = \boxed{0.1 \text{ m}}$$

$$x(2.0 \text{ s}) = 2.0 \text{ m} - (3.6 \text{ m/s})(2.0 \text{ s}) + (1.7 \text{ m/s}^2)(2.0 \text{ s})^2 = \boxed{1.6 \text{ m}}$$

$$x(3.0 \text{ s}) = 2.0 \text{ m} - (3.6 \text{ m/s})(3.0 \text{ s}) + (1.7 \text{ m/s}^2)(3.0 \text{ s})^2 = \boxed{6.5 \text{ m}}$$
  - $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{6.5 \text{ m} - 0.1 \text{ m}}{2.0 \text{ s}} = \boxed{3.2 \text{ m/s}}$$