

①

$$\omega_m = 0 \Rightarrow T_L = 0$$

$$\omega_m = 100 \text{ rad/s} \Rightarrow T_L = 12 \text{ Nm}$$

$$T_L(\omega_m) = \frac{12}{100} \cdot \omega_m = 0,12 \omega_m ; T_{em} = T_{L \text{ rated}} = 12 \text{ Nm}$$

$$J_{eq} = J_m + J_L = 1,5 + 2,5 = 4 \text{ kg m}^2$$

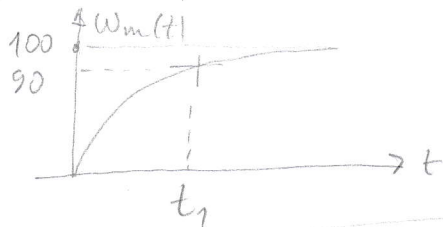
$$\frac{d\omega_m}{dt} = \frac{T_{em} - T_L(\omega_m)}{J_{eq}} \Rightarrow \frac{d\omega_m}{dt} = \frac{12 - 0,12 \omega_m}{4}$$

$$\frac{d\omega_m}{dt} + 0,03 \omega_m = 3$$

$$\omega_m(t) = \frac{3}{0,03} (1 - e^{-0,03t})$$

$$\omega_m(t_1) = 0,9 \cdot 100 = 100 (1 - e^{-0,03t_1})$$

$$e^{-0,03t_1} = 0,1 \Rightarrow 0,03t_1 = \ln 10 \Rightarrow t_1 = \frac{1}{0,03} \cdot \ln 10 \Rightarrow t_1 = 76,7 \text{ s}$$



②

$$V_a = k_e \omega_m + R_a I_a$$

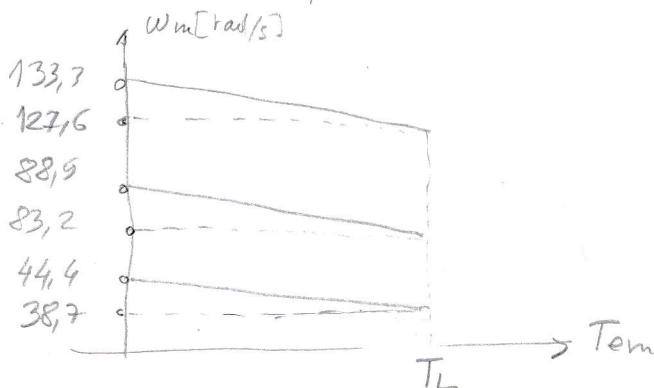
$$T_{em} = T_L = k_T I_a \Rightarrow I_a = \frac{T_L}{k_T} = \frac{9}{0,35} = 25,7 \text{ A}$$

$$V_{a1} = 0,9 \omega_{m1} + 0,2 \cdot 25,7 \Rightarrow \omega_{m1} = \frac{V_{a1} - 0,2 \cdot 25,7}{0,9}$$

$$\omega_{m1} = \frac{40 - 0,2 \cdot 25,7}{0,9} = 38,7 \text{ rad/s} \quad \omega_{m10} = \frac{40}{0,9} = 44,4 \text{ rad/s}$$

$$\omega_{m2} = \frac{80 - 0,2 \cdot 25,7}{0,9} = 83,2 \text{ rad/s} \quad \omega_{m20} = \frac{80}{0,9} = 88,9 \text{ rad/s}$$

$$\omega_{m3} = \frac{120 - 0,2 \cdot 25,7}{0,9} = 127,6 \text{ rad/s} \quad \omega_{m30} = \frac{120}{0,9} = 133,3 \text{ rad/s}$$



③ $P_{\text{rated}} = T_L \cdot \omega_{m \text{ rated}}$

$$\omega_{m \text{ rated}} = \frac{n}{60} \cdot 2\pi = 298 \text{ Nm}$$

$$T_{L \text{ rated}} = \frac{P_{\text{rated}}}{\omega_{m \text{ rated}}} = \frac{5000}{298} \approx 20 \text{ Nm}$$

$$T_L = C_T n_m^2$$

$$T_{L \text{ rated}} = C_T n_{m \text{ rated}}^2 \Rightarrow C_T = \frac{T_{L \text{ rated}}}{n_{m \text{ rated}}^2} = \frac{20}{2850^2} = 2,46 \cdot 10^{-6} \frac{\text{Nm}}{1/\text{min}^2}$$

$$k_T (n_{\text{syn}} - n_m) = C_T n_m^2$$

$$C_T n_m^2 + k_T n_m - k_T n_{\text{syn}} = 0$$

$$n_m = \frac{-k_T \pm \sqrt{k_T^2 + 4C_T k_T n_{\text{syn}}}}{2C_T}$$

$$k_T = \frac{T_L}{n_{\text{syn}} - n_{m \text{ rated}}} = \frac{20}{3000 - 2850} = \frac{20}{150} = 0,133 \frac{\text{Nm}}{1/\text{min}}$$

$$n_{m1} = \frac{-0,133 + \sqrt{0,133^2 + 4 \cdot 2,46 \cdot 10^{-6} \cdot 0,133 \cdot 3000}}{2 \cdot 2,46 \cdot 10^{-6}} = 2850 \frac{1}{\text{min}}$$

$$n_{m2} (f_{\text{syn}} = 30 \text{ Hz}) = \frac{-0,133 + \sqrt{0,133^2 + 4 \cdot 2,46 \cdot 10^{-6} \cdot 0,133 \cdot 1800}}{2 \cdot 2,46 \cdot 10^{-6}} = 1744 \frac{1}{\text{min}}$$

$$n_m (f_{\text{syn}} = 10 \text{ Hz}) = \frac{-0,133 + \sqrt{0,133^2 + 4 \cdot 2,46 \cdot 10^{-6} \cdot 0,133 \cdot 600}}{2 \cdot 2,46 \cdot 10^{-6}} = 593 \frac{1}{\text{min}}$$

